***124 MIDTERM TWO REVIEW OUTLINE ©***

# Outline

1. **Explicit Derivatives**
   - (a) Chain Rule, Product Rule, Quotient Rule
   - (b) Trig and ArcTrig
   - (c) Periodic Derivatives
2. **Implicit Derivatives**
   - (a) Logarithmic Differentiation
   - (b) Strange Curves
   - (c) Parametric
3. **Tangent Line Approximation**
   - (a) Single Variable
   - (b) Implicit
   - (c) Parametric
   - (d) Over or Underestimate?
4. **Related Rates**
   - (a) With Angles
     - i. Basic Trig
     - ii. Law of Sines
     - iii. Law of Cosines
   - (b) Pythagorean
   - (c) Similar Triangles
   - (d) Volumes or other Geometric Formulas
   - (e) Curves

# Problems

1. **Explicit Derivatives**
   (a) Chain Rule, Product Rule, Quotient Rule
      i. $e^{sin^2(cos(x^2))}$
      ii. $\left(\frac{x}{x^2+1}\right)^4$
   (b) Trig and ArcTrig
      i. $\tan(\sin(x)) + \cos(\cot(x))$
      ii. $\frac{d^2}{dx}\arcsin(x)$
   (c) Periodic Derivatives
      i. $f(x) = e^{-x} + (x^{70} + 1)(x^{28} + 17x^2) - \sin(2x)$ find $f^{(101)}(x)$.

2. **Implicit Derivatives**
   (a) Logarithmic Differentiation
      i. $x^{x^x}$
      ii. $\ln(x)^{\cos(x)}$
   (b) Strange Curves
      i. $y + xy + xy^2 = e^x$ find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$
      ii. $\arctan(e^w) = \sin(\theta)$. Find both $\frac{dw}{d\theta}$ and $\frac{d\theta}{dw}$
      iii. $xy^3 - y = 1$ contains the point $(2, 1)$. Compute $\frac{d^2y}{dx^2}$ at $(2, 1)$.
   (c) Parametrics
      i. $\frac{dy}{dx} = \frac{y'(t)}{x'(t)}$
      ii. $\frac{d^2y}{dx^2} = \frac{y''x'-x''y'}{(x')^3} = \frac{\frac{d}{dt}\frac{dy}{dx}}{\frac{dx}{dt}}$
      iii.
      $$(x, y) = (t^t + t, 1 + \ln t)$$
      Compute $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $t = e$ and find the tangent line at $t = e$.

3. **Tangent Line Approximation**
   (a) Single Variable
      i. Estimate $(16.01)^{1/4}$ using tangent line approximation. Over or underestimate?
      ii. Estimate $(1003)^{2/3}$.
   (b) Implicit
      i. $y + xy + y^2 = e^x$ at the point $(0, 1)$ use tangent line approximation to estimate $y$ if $x = .5$.
   (c) Parametric
      i. Ralph walks to the grocery store with coordinate functions
      $$(x(t), y(t)) = \left(e^{1-t} + 1, \sin^2(\frac{\pi}{2}t) + t^3\right)$$
      Using tangent lie approximation, estimate Ralph's $y$-coordinate at the point $(2.05, y)$.

ii. Ryu the sea cucumber has position on the ocean floor given for $t \geq 0$ by

$$x(t) = 2t^2 - 10t + 12$$

$$y(t) = t\sin((\pi/2)t)$$

Using linear approximation estimate the $x$-coordinate of the point $(x, -\pi)$. *(Hint: $y(3) = -3$ which is pretty close to $-\pi$.)*

4. **Related Rates**
   (a) With Angles
      i. Basic Trig
         A. The moon is flying at a constant elevation through the sky so that it will pass directly over you. It has a velocity of 8 moon units per second. Obviously the moon is $2\sqrt{3}$ moon units above you. When the moon's moonshadow is 2 moon units away from you (the moonshadow is always directly below the moon), at what rate is the angle of elevation between you and the moon changing?
         B. A lamprey is moving along the curve $y = xe^{x-1}$. Let $\theta$ be the angle of elevation relative to the lamprey and the origin. When $\theta = \pi/4$ the rate of change of $\theta$ is .2 rad/sec. How fast is the x-coordinate of the lamprey changing at this time?
      ii. Law of Sines
      iii. Law of Cosines
   (b) Pythagorean
      i. Bilbo is at first base and Frodo is at second in a baseball game. After the ball is hit they begin simultaneously running towards the proper base, Bilbo runs at 4 ft/sec and Frodo runs at 3 ft/sec. After 4 seconds, at what rate is the distance between them changing? Is it positive or negative?
      ii. Two people start from the same point. One walks east at 3 mi/hr and the other walks northeast at 2 mi/hr. How fast is the distance between the people changing after 15 minutes?
   (c) Similar Triangles
      i. A firefighter is scrambling up a 20 foot ladder at 2 ft/sec. The base of the ladder is sliding away at a constant rate of 3 ft/sec. When the firefighter has climbed up 6 feet of ladder the base of the ladder is 10 feet from the wall. Let $h$ be the height of the firefighter. Find $\frac{dh}{dt}$.
   (d) Volumes or other Geometric Formulas
      i. Water is being pumped into an inverted conical tank at a variable rate. The tank has height 8 meters and the diameter at the top is 6 meters. If water is being pumped into the tank at the rate of 2 $m^3$/min at the the instant when the height of the water is 3 meters. Find the rate at which the height is increasing at that instant. Give an exact answer.
      ii. Matt is making a pizza and throwing dough. It is a perfect disk that has radius increasing at 1 m/s. When the radius is 12 m, how fast is the area changing?
      iii. Matt is baking a cylindrical cake. Both the radius and the height of the cake are expanding. The height is increasing at a rate of 2 m/s and the surface area is increasing at a rate of $36\pi$ $m^2$/s. When the height is 2 m and the radius is 4 m at what rate is the volume of the cake changing?

iv. Matt is filling a spherical water balloon from a rigid cylindrical canister of radius 1 meter. Suppose that the moment the water balloon has radius $\sqrt{2}$ meters, the height of the water level in the canister is decreasing at a rate of $2\pi \frac{m}{min}$. At this instant, find the rate at which the radius of the water balloon is changing. ($V_{cylinder} = \pi r^2 h$, $V_{sphere} = \frac{4}{3}\pi r^3$)

(e) Curves

i. A bug crawls along the curve $x^4 + xy + y^4 = 3$. If the bugs $x$-coordinate is increasing at 2 ft/sec at the point (1,1) then how fast is his $y$-coordinate changing?