21

Symmetrical Components

Introduction — Symmetrical Components of 3 Phase System — Operator '*a*' — Some Trigonometrical Relations — Zero Sequence Currents-Unbalanced Supply Voltage. **Example 21.1 to 21.10.**

21.1. INTRODUCTION

For unsymmetrical faults such a single phase to ground fault, phase to phase fault, double phase to ground fault, simple single phase representation is not valid. The method of *'Symmetrical Components'* is generally used. The method of symmetrical components is a very powerful approach and has simplified the procedure of fault calculation in a miraculous way. Dr. C.L. Fortesque introduced the method of symmetrical components to the solution of polyphase networks in his paper presented in the year 1918. The principle of symmetrical components is as follows. Suppose we have to solve an unbalanced systems of n vectors. It is then resolved into n balanced systems, each of which consists of n vectors. These balanced vectors are called symmetrical components of the original components. Let us concentrate on unbalanced three-phase systems.

21.2. SYMMETRICAL COMPONENTS OF 3-PHASE SYSTEMS

In unbalanced systems of three vectors [I_a, I_b, I_c, or V_a, V_b, V_c] can be resolved into three balanced systems of vectors, the vectors of the balanced system are called symmetrical components of the original system, which are :

1. **Positive Sequence Components** V_{a1}, V_{b1}, V_{c1} or I_{a1}, I_{b1}, I_{c1} comprising three balanced systems, of vectors, displaced mutually by 120° and having the same phase sequence as that of the original system.

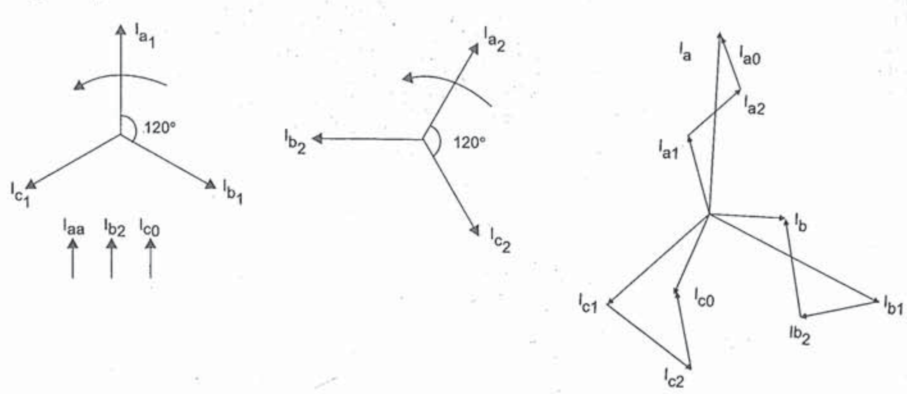


Fig. 21.1. Symmetrical components.

2. **Negative Sequence Components.** [V_{a2}, V_{b2}, V_{c2} or I_{a2}, I_{b2}, I_{c2}] comprising three balanced vectors of equal magnitude displaced mutually by 120° and having phase sequence opposite that of the original system of vectors. (If the original system of vectors have a phase sequence $a - b - c$; then positive sequence components too have phase sequence $a_1 - b_1 - c_1$ but negative phase sequence components have phase $(a_2 - c_2 - b_2)$.

3. **Zero Phase Sequence Components.** [V_{a0}, V_{b0}, V_{c0} or I_{a0}, I_{b0}, I_{c0}] comprising three equal vectors having zero phase displacement, *i.e.* having same phase.

Symbolically,

Subscript 1 for positive sequence entities.

Subscript 2 is for negative sequence entities.

and subscript 0 for zero sequence entities.

$$\left.\begin{matrix} V_a, V_b, V_c, \\ I_a, I_b, I_c \end{matrix}\right\}$$ Original System of Unbalanced Vectors [Meaning : They may not be equal in magnitude or/and do not have same phase displacement.]

$$\left.\begin{matrix} V_{a1}, V_{b1}, V_{c1} \\ I_{a1}, I_{b1}, I_{c1} \end{matrix}\right\}$$ Positive Sequence Components

$$\left.\begin{matrix} V_{a2}, V_{b2}, V_{c2} \\ I_{a2}, I_{b2}, I_{c2} \end{matrix}\right\}$$ Negative Sequence Components.

$$\left.\begin{matrix} V_{a0}, V_{b0}, V_{c0} \\ I_{a0}, I_{b0}, I_{c0} \end{matrix}\right\}$$ Zero Sequence Components.

The original unbalanced system of vectors can be resolved into their symmetrical components or the respective symmetrical components can be added to get the original system of vectors.

$$\Leftrightarrow$$

Thus
$$V_a = V_{a0} + V_{a1} + V_{a2} \qquad ...(I)$$
$$V_b = V_{b0} + V_{b1} + V_{b2}$$
$$V_c = V_{c0} + V_{c1} + V_{c2}$$

$$\Leftrightarrow$$

and
$$I_a = I_{a0} + I_{a1} + I_{a2} \qquad ...(II)$$
$$I_b = I_{b0} + I_{b1} + I_{b2}$$
$$I_c = I_{c0} + I_{c1} + I_{c2}$$

Fig. 21.1, illustrates the Eqn. (II).

21.3. OPERATOR '*a*'

Letter '*a*' is commonly used to designate the operator that causes a counter-clockwise rotation of 120°.

It has unit magnitude and angle 120°. The vector operator '*a*' is defined as :

$$a = 1e^{+j\frac{2\pi}{3}}$$
$$= \cos\frac{2\pi}{3} + j\sin\frac{2\pi}{3} = -0.5 + j0.866$$
$$a = 1\angle 120° = -0.5 + j0.866$$
$$a^2 = 1e^{+j\frac{4\pi}{3}}$$
$$= \cos\frac{4\pi}{3} + j\sin\frac{4\pi}{3}$$
$$= -0.5 - j0.866 = 1\angle 240°$$
$$a^3 = 1e^{+j2\pi} = 1\angle 360° = 1 + j0$$

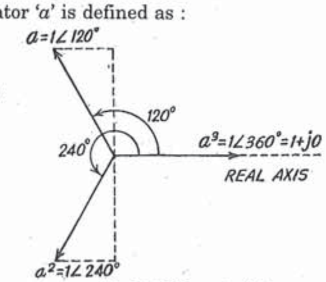


Fig. 21.2. Operator '*a*'.

21.4. SOME TRIGONOMETRIC RELATIONS

$$\sin\theta = \frac{\text{Opposite side}}{\text{Hypotenuse}}$$

$$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}}$$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta = \frac{\text{Opposite side}}{\text{Base}}$$

$$\sin 30° = \frac{1}{2} = 0.5$$

$$\sin 60° = \frac{\sqrt{3}}{2} = 0.866$$

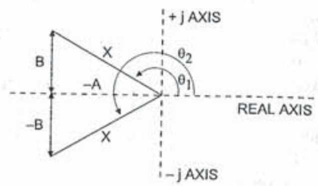


Fig. 21.3. Trigonometric relations.

$$\sin 120° = \sin(180° - 60°) = \sin 60° = 0.866$$
$$\sin 240° = \sin(180° + 60°) = -\sin 60° = -0.866$$
$$\cos 30° = \frac{\sqrt{3}}{2} = 0.866$$
$$\cos 60° = 0.5$$
$$\cos\theta_1 = -\frac{A}{X}$$
$$\cos\theta_2 = -\frac{A}{X}$$
$$\sin\theta_1 = +\frac{B}{X}$$
$$\sin\theta_2 = -\frac{B}{X}$$
$$\cos 120° = \cos(180° - 60°) = -0.5$$
$$\cos 240° = \cos(180° + 60°) = -0.5$$

Remember :

$$a = 1\angle 120° = \cos 120° + j\sin 120° = -0.5 + j0.866$$
$$a^2 = 1\angle 240° = \cos 240° + j\sin 240° = -0.5 - 0.866$$
$$a^3 = 1 - j0$$

Table of Operator 'a'

$$a = 1\angle 120° = -0.5 + j0.866$$
$$a^2 = 1\angle 240° = -0.5 - j0.866$$
$$a^3 = 1\angle 360° = 1 + j0$$
$$1 + a = 1\angle 60° = 0.5 + j0.866$$
$$1 - a = \sqrt{3}\angle -30° = 1.5 - j0.866$$
$$1 + a = 1\angle -60° = 0.5 + j0.866$$
$$a^3 - a^2 = \sqrt{3}\angle 30° = 1.5 + j0.866$$
$$a + a^2 = 1\angle 180° = -1 - j0$$
$$1 + a + a^2 = 0 \qquad = 0 + j0.$$

From Balanced Vector to Symmetrical Components

Positive Sequence	*Negative Sequence*	*Zero Sequence*
V_{a1}	V_{a2}	V_{a0}
$V_{b1} = a^2 Va_1$	$V_{b2} = aVa_2$	$V_{b0} = Va_0$
$V_{c1} = aVa_1$	$V_{c2} = a^2 V_{a2}$	$V_{c0} = V_{a0}$

From set equation I

$$V_a = V_{a0} + V_{a1} + V_{a2} \qquad \text{...(III)}$$
$$V_b = V_{a0} + a^2 V_{a1} + aVa_2$$
$$V_c = V_{a0} + aVa_1 + a^2 Va_2$$

Written in matrix form,

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} \qquad \text{...(IV)}$$

Let $\quad A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$

Then $\quad A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$

Multiplying both sides of Eq. (IV) by A^{-1}, we get

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \qquad \text{...(V)}$$

$$V_{a0} = \frac{1}{3}(V_a + V_b + V_c) \qquad \text{...(VI)}$$
$$V_{a1} = \frac{1}{3}(V_a + aV_b + a^2 V_c)$$
$$V_{a2} = \frac{1}{3}(V_a + a^2 V_b + aV_c)$$

From these equations, we can get symmetrical components of unbalanced system of vectors. Summarizing

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$V_a = V_{a0} + V_{a1} + V_{a2}$ $V_b = V_{a0} + a^2 V_{a1} + aV_{a2}$ $V_c = V_{a0} + aV_{a1} + a^2 V_{a2}$	$I_a = I_{a0} + I_{a1} + I_{a2}$ $I_b = I_{a0} + a^2 I_{a1} + aI_{a2}$ $I_c = I_{a0} + aI_{a1} + a^2 I_{a2}$
$V_{a0} = \frac{1}{3}[V_a + V_b + V_c]$ $V_{a1} = \frac{1}{3}[V_a + aV_b + a^2 V_c]$ $V_{a2} = \frac{1}{3}[V_a + a^2 V_b + aV_c]$	$I_{a0} = \frac{1}{3}[I_a + I_b + I_c]$ $I_{a1} = \frac{1}{3}[I_a + aI_b + a^2 I_c]$ $I_{a2} = \frac{1}{3}[I_a + a^2 I_b + aI_c]$

V_{a1}, $\quad V_{b1} = a^2 V_{a1}$, $\quad V_{c1} = aV_{a1}$

V_{a2}, $\quad V_{b2} = aVa_2$, $\quad V_{c2} = a^2 V_{a2}$

$V_{a0} = V_{b0} = V_{c0}$

$I_n = I_a + I_b + I_c = 3I_{a0}$

$V_{a0} = \frac{1}{3}(V_a + V_b + V_c)$

In the following text, the word phase currents implies currents in R, Y, B phases of a 3 phase supply line.

21.5. ZERO SEQUENCE CURRENTS

In three-phase systems, when there is a neutral return path for currents,

$$I_n = I_a + I_b + I_c$$

We get

$$I_a + I_b + I_c = 3\,[Ia_0]$$

$$\therefore \quad I_{a0} = I_n/3$$

In delta connected load, the line currents do not find return neutral path. Hence line currents do not have zero sequence components.

In star connected system without neutral path or neutral grounding, I_n is zero. Hence zero sequence currents are zero.

Example 21.1. *Calculate the symmetrical components of the following unbalanced line to neutral voltages.*

$$V_a = 100\angle 90° \,;\; V_b = 116\angle 0° : V_c = 71\angle 224.8°.$$

Solution.

$$V_{a0} = \frac{1}{3}\,[V_a + V_b + V_c]$$

$$V_{a1} = \frac{1}{3}\,[V_a + aV_b + a^2V_c]$$

$$V_{a2} = \frac{1}{3}\,[V_a + a^2V_b + aV_c]$$

$$V_{a0} = \frac{1}{3}\,[100\,\angle 90° + 116\,\angle 0° + 71\,\angle 224.8°]$$

$$= \frac{1}{3}\,[0 + j100 + 116 + j0 + (-50 - j50)]$$

$$= \frac{1}{3}\,[66 + j50] = \frac{83}{3}\,\angle 37°$$

$$V_{a0} = 22 + j16.66 = 27.77\angle 37°$$

$$V_{a1} = \frac{1}{3}\,[V_a + aV_b + a^2V_c]$$

$$V_{a1} = \frac{1}{3}\,[100\angle 90° + 116\angle 0° + 120° + 71\,\angle 224.8° + 240°]$$

$$= \frac{1}{3}\,[100\,\angle 90° + 116\,\angle 120° + 71\,\angle 104.8°]$$

$$= \frac{1}{3}[(0 + j100) + (-58 + j100) + (-18 + j68)]$$

$$= \frac{1}{3}\,(-76 + j268) = -25.33 + j89.33 = 98\angle 106°$$

$$V_{a2} = \frac{1}{3}\,(V_a + a^2V_b + aV_c)$$

$$= \frac{1}{3}\,[100\,\angle 90° + 116\,\angle 0 + 240 + 71\angle 224.8° + 120]$$

$$= \frac{1}{3}\,[100\angle 90° + 116\angle 240° + 71\angle 344.8°]$$

$$= \frac{1}{3}\,[0 + j100 + (-58 - j100 + 68 - j18)]$$

$$= \frac{1}{3}\,[10 - j18] = 3.33 - j6 = 6.85\angle 299.$$

Check :

$$V_a = V_{a0} + V_{a1} + V_{a2}$$

$$= (22 + j6.66) - (25.33 + j89.33) + (3.33 - j6.0)$$

$$= 0 + j99.99 = \text{app. } j100 = V_a = 100\,\angle 90°$$

Example 21.2. *The given symmetrical components are $V_{a0} = 22 + j16.66$, $V_{a1} = -25.33 + j89.34$ and $V_{a2} = 3.33 - j6.00$; calculate V_a, V_b, and V_c.*

Solution.

$$V_a = V_{a0} + V_{a1} + V_{a2}$$

$$V_b = V_{c0} + a^2V_{a1} + aV_{a2}$$

$$V_c = V_{ag} + aV_{c1} + a^2V_{c2}$$

$$V_a = V_{a0} + V_{a1} + V_{a2}$$

$$= (22 + j16.66) + (-25.33 + j89.34) + (3.33 - j6.00)$$

$$= 0 + j100$$

$$V_b = V_{a0} + a^2V_{a1} + aV_{a2}$$

$$= (22 + j16.16) + (-0.5 - j0.866)\,(-25.33 + j89.34) + (-0.5 + j0.866)\,(3.33 - j6.00)$$

$$= 115.6 + j0 = 115.6\angle 0°$$

$$V_c = V_{a0} + aV_{a1} + a^2V_{a2}$$

$$= (22 + j16.16) + (-0.5 + j0.866)\,(-25.33 + j89.34) + (-0.5 - j0.866)\,(3.33 - j6.0)$$

$$= -50 - j50.$$

Example 21.3. *Determine I_a, I_b, I_c, from the symmetrical components :*

$$I_{a1} = 50\angle 0°,\; I_{a2} = 10\angle 90°,\; I_{a3} = 10\angle 180°.$$

Solution.

$$I_a = I_{a0} + I_{a1} + I_{a2}$$

$$I_b = I_{a0} + a^2I_{a1} + aI_{a2}$$

$$I_c = I_{a0} + aI_{a1} + a^2I_{a2}$$

$$I_a = 10\angle 180° + 50\angle 0° + 10\angle 90°$$

$$= 10\,[-1 + j0) + 50[1 + j0] + 10[0 + j1]$$

$$= -10 + j\,0 + 50 + j0 + 0 + j10 = 40 + j10$$

$$I_b = 10\angle 180° + 50\angle 0 + 240° + 10\angle 90 + 120°$$

$$= -10 + 50\,(-0.5 - j0.866) + 10\,(-0.866 - j0.5)$$

$$= -10 - 25 - 8.66 - j43 - j5$$

$$= -43.66 - j48$$

$$I_c = I_{a0} + aI_{a1} + a^2I_{a2} = -30 + j34.84.$$

Example 21.4. *A delta connected load is connected to three–phase supply. One line of supply is open. The current in other two lines is 20 A. Find the symmetrical components of the line currents.*

Solution. Let a, b, c be the supply lines and c is open. Therefore, currents in the two liens are equal in magnitude.

$$I_a = 20\angle 0°; \qquad I_b = 20\angle 180° \,; \qquad I_c = 0$$

$$I_{ao} = \frac{1}{3}\,[I_a + I_b + I_c] = \frac{1}{3}\,[20\angle 0° + 20\angle(180°) + 0] = 0$$

$$I_{a1} = \frac{1}{3}\,[20\angle 0° + 20\angle(180° + 120) + 0]$$

$$= \frac{1}{3}\,[20 + j0 + 20\,(0.5 + j0.866) + 0] = \frac{1}{3}\,[30 - j17.32]$$

$$= 10 - j5.77 = 11.56\,\angle -30° \text{ Amp.}$$

$$I_{a2} = \frac{1}{3}\,[20\angle 0° + 20\angle(180° + 240°) + 0]$$

$$= 10 + j5.77 = 11.56\angle 30° \text{ Amp.}$$

$$I_{b1} = a^2 I_{a1} = 11.56\angle -30 + 240° = 11.56\angle -210°$$

$$I_{b2} = a I_{a2} = 11.56\angle 30° + 120° = 11.56\angle 150°$$

$$I_{b0} = I_{c0} = 0.$$

$$I_{c1} = a I_{a1} = 11.56\angle -30° + 120° = 11.56\angle 90°$$

$$I_{c2} = a^2 I_{a2} = 11.56\angle 30° + 240° = 11.56\angle -90°$$

$$I_{c0} = I_{a0} = 0. \quad \text{(Ref. Sec. 21.5)}$$

Check :

$$I_c = I_{c0} + I_{c1} + I_{c2}$$

$$= 0 + 11.56\angle -90° + 11.56\angle +90° = 0.$$

Example 21.5. *In a three-phase system, the voltage of phase of neutral had the following sequence components during phase to phase fault condition. Calculate the voltages of the phases with respect to neutral, voltages between phases. Given :*

$$V_{a1} = 0.584 + j0 \text{ p.u.}, V_{a2} = 0.584 + j.0 \text{ p.u.}, V_{a0} = 0$$

Solution. $V_a = V_{a1} + V_{a2} + V_{a0}$

$$= 0.584 + 0.584 + 0 = 1.68\angle 0° \text{ p.u.}$$

$$V_b = a^2 V_{a1} + a V_{a2} + V_{a0}$$

$$= (-0.5 - j0.866)\, V_{a1} + (-0.5j + 0.866)\, V_{a2} + 0 = -0.584 \text{ p.u.}$$

$$V_c = V_b = -0.584 \text{ p.u.}$$

$$V_{ab} = V_a - V_b = 1.168 + 0.584 = 1.752\angle 0° \text{ p.u.}$$

$$V_{bc} = V_b + V_c = -0.584 + 0.584 = 0$$

$$V_{ca} = -0.584 - 1.168 = -1.752\angle 180° \text{ p.u.}$$

Fault is between phases b and c. Hence V_{bc} is zero. $V_{ab} = -V_{ca}$.

Example 21.6. *In a single phase equivalent circuit the positive sequence components of current is given by*

$$I_{a0} = \frac{E_a}{X_1 + X_2 + X_0}$$

Further I_{a0} is equal to I_{a1} and I_{a2}. Calculate I_a. Given $E_a = 1 + j0$ p.u., $X_1 = j0.25$, $X_2 = j0.35$, $X_0 = j0.10$.

Solution. Given $I_{a0} = \dfrac{E}{X_1 + X_2 + X_0}$

Substituting the given values

$$I_{a0} = \frac{1 + j0}{j0.25 + j0.35 + j0.10} = \frac{1}{j0.70} = -j1.43 \text{ p.u.}$$

It is given, $I_{a0} = I_{a1} = I_{a2}$

$\therefore$ $I_a = I_{a0} + I_{a1} + I_{a2} = 3\,[I_{a0}]$

$$= 3 \times [-j1.43] = -j4.29 \text{ p.u.}$$

Example 21.7. *In a problem on fault calculations, following expressions were obtained.*

$$I_a = \frac{E_a}{Z_1 + \dfrac{1}{\left(\dfrac{1}{Z_2} + \dfrac{1}{Z_0}\right)}}$$

$$V_{a1} = V_{a2} = V_{a0} = E_a - I_{a1} Z_1$$

$$I_2 = \frac{-V_{a2}}{Z_2}, \; I_0 = \frac{-V_{a0}}{Z_0}$$

Determine I_a and V_a

Given : $Z_1 = j0.25$, $Z_2 = j0.35$, $Z_o = j0.1$ p.u., $E_a = 1 + j0$ p.u.

Solution.

$$I_{a1} = \frac{E_a}{Z_1 + \dfrac{Z_2 Z_0}{Z_2 + Z_0}} = \frac{1 + j\,0}{j\,0.25 + \dfrac{j0.35 \times j0.10}{j0.35 + j0.10}}$$

$$= \frac{1.0}{j0.25 + j0.0778} = \frac{1.0}{j0.3278} = -j3.05 \text{ p.u.}$$

Given : $V_{a1} = V_{a2} = V_{a0} = E_a - I_{a1} Z_1$

$$= 1 + j0 - (-j3.05)\,(j0.25) = 1 - 0.763 = 0.237 \text{ p.u.}$$

$$I_{a0} = \frac{-V_{a0}}{Z_0} = \frac{-0.237}{j0.1} = j2.37 \text{ p.u.}$$

$$I_{a2} = \frac{-V_{a2}}{Z_2} = \frac{-0.237}{j0.35} = j0.68 \text{ p.u.}$$

$$I_a = I_{a1} + I_{a2} + I_{a0} = -j3.05 + j0.68 + j2.37 = 0$$

$$V_a = V_{a1} + V_{a2} + V_{a0} = 3V_{a1} = 3 \times 0.237 = 0.711 \text{ p.u.}$$

Example 21.8. *The positive sequence network of a system is shown in the figure given below. Draw Thevenin's equivalent network and determine the positive sequence component of fault current, assuming zero fault impedance and voltage at fault point 1.0 p.u.*

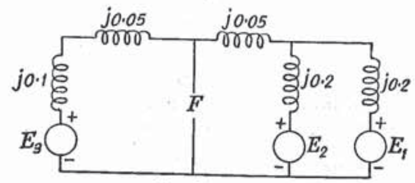
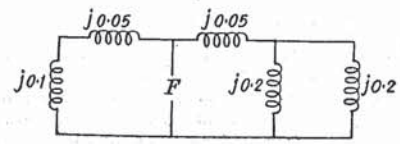


Fig. of Ex. 21.8.

Solution. Thevenin's equivalent impedance

V_f = Thevenin's O.C. voltage 1 p.u.



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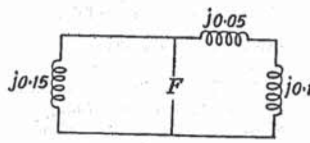

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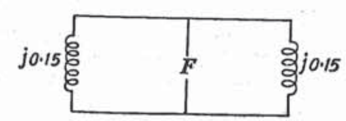

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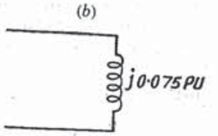

(d)

Positive sequence current

$$= \frac{V_t}{X_{h\,(eq)}} = \frac{1 + j0}{j0.075} = 13.35 \text{ p.u.}$$

Example 21.9. *In a three-phase 4-wire system the phase currents in R, Y, B phases are $I_R = 100\angle 30°$, $I_Y = 60\angle 300$ A, $I_B = 30\angle 180°$. Calculate positive, negative and zero sequence components of current in the phase R and current in the return neutral current.*

Solution. $I_R = 100\angle 30° = 100\,[\cos 30° + j \sin 30°]$

$$= 100\,[0.866 + j0.5] = 86.6 + j50$$

$$I_Y = 50\ \angle 300°$$
$$= 50[\cos 300° + j \sin 300] = 50\ [0.5 - j0.866] = 25 - j43.3$$
$$I_B = 30\angle 180° = 30[-1 + j0] = -30$$
I_n = Current in return neutral path
$$= I_R + I_Y + I_B = 86.6 + j50 + 25 - j43.3 - 30 = 81.6 + j6.7 = 82 \text{ A}$$
$$I_{R0} = \tfrac{1}{3}\ [I_R + I_Y + I_B] = \frac{1}{3}\ [81.6 - j6.7] = 27.2 + j2.25$$
$$I_{R1} = \frac{1}{3}\ [I_R + aI_Y + a^2 I_B]$$
$$= \frac{2}{3}[(86.5 + j50.0)] + (-0.5 + j0.866) \times (25 - j43.3) + (-0.5 - j0.866)\ (-30)]$$
$$= 42.2 + j39.8$$
$$I_{R2} = \frac{1}{3}[I_R + a^2 I_Y + aI_B] = 17.2 + j8$$

Check : $I_R = I_{R0} + I_{R1} + I_{R2}$
$$= 27.2 + 42.2 + 17.2 + j2.25 + j39.8 + j8.0 = 86.6 + j50.05$$

which agrees closely with I_R.

Ans. $I_n = 82$ A Check : $I_n = 3I_0$

$I_{R0} = 27.2 + j2.25$ A

$I_{R1} = 42.2 + j39.8$ A

$I_{R2} = 17.2 + j8$ A.

Example 21.10. *Given current in neutral to ground connection 1.9 Amperes. Calculate zero sequence component of current in phases.*

Solution. $I_{a0} = I_{bo} = I_{c0} = \dfrac{I_n}{3} = \dfrac{9}{3} = 3$ Amperes. **Ans.**

21.6. PHASE DISPLACEMENT IN STAR-DELTA TRANSFORMERS

The angular difference between vectors representing the voltages induced between h.v. and l.v. terminals having same marking letter and the corresponding neutral points (real or imaginary), expressed with reference to h.v. side is termed as phase displacement of transformer. Even under normal condition, the phase to phase voltages and phase to neutral voltages of h.v. side are displaced from corresponding voltages of l.v. side, in case of star-delta transformers. The phase displacement of + 30° comes in Group 4 and that of – 30° comes in Group 3 (IS : 2026, 1962 reprint 1972—Specifications for power transformers). Similarly the currents on the two sides are also displaced. While applying the method of symmetrical components, the inherent phase shirt should be considered. A phase shift can be expected in p.s., n.s., z.s. components on either sides.

Generally in short-circuits calculations the phase displacement is neglected. The procedure is as follows :

Consider a star-delta transformer. The analysis of the positive sequence currents and positive sequence voltages can be corrected if necessary for phase displacement by taking into account the inherent phase displacement. Similarly, the same reasoning applies to negative sequence currents and negative sequence voltages. Magnitude of phase shift is same for positive sequence components and negative sequence components. However, the direction of phase shift in case of negative phase sequence components is reverse of that applicable to the positive sequence components (Due to reverse phase sequence). The phase shifts pf p.s. components and n.s. components are equal in magnitude but opposite in direction. The magnitude and direction of phase displacement depends on transformer group and allocation of phase references. Phase displacement of zero-sequence quantities need not be considered in star-delta transformer. Since the zero sequence currents do not flow in lines on delta-connected side.

QUESTIONS

1. Given $V_a = 100 + j0$, $V_b = -2.7 + j32.3$, $V_c = -37.3 + j2.3$. Find the symmetrical components V_{a0}, V_{a1}, V_{a2}. **[Ans.** $20 - j18$, $50 + j15$, $30 - j5$]
2. Given $V_a = 100\angle 0$, $V_b = 50\angle 225$, $V_c = 111\angle 2.6$. Find the sequence components. **[Ans.** $66.66 + j16.66$, $60.1 - j8.17$, $-26.5 - j8.3$]
3. The current in three line conductors, a, b, c are $40 + j60$, -90, $-80 + j10$ Amp.
   If the reactance per phase for positive, negative and zero sequence current is respectively 20, 20, 50 ohms, find the voltage drop in conductor 'a'. **[Ans.** $-900 + j400$ V]
4. (*a*) Show that
   $$a + a^2 + 1 = 0$$
   $$a - a^2 = j\sqrt{3}$$
   $$a - 1 = 1.5 - j0.866,$$
   (*b*) Evaluate the following : $\dfrac{1+a^2}{1-a}, \dfrac{1-a}{1+a}, \dfrac{1+a}{1+a^2}$
5. Given $E_a, E_b, E_c = 60 + j0$, $45 - j75$, $-51 + j120$ respectively.
   Determine E_{a0}, E_{a1}, E_{a2}. **[Ans.** $28 + j15$, $72.2 + j11.5$, $-40.2 - j26.5$]
6. Given $I_a = 0 + j100$, $I_b = 20 + j0$, $I_c = 0$. Find I_{a1}, I_{a2}, I_{a0}.
7. Assume $I_a = 100$ Amp., $I_b = I_c = -50$ Amp. What are the sequence currents ?
8. The phase to phase voltage of a 3-phase system are 100, 150, 200 volts. Find the magnitudes of positive and negative sequence components.
9. A grounded neutral system has positive sequence voltage of E_{a1}, show that if neutral ground be removed and one phase wire grounded the sequence voltage remains unchanged.
10. Three resistors of 5, 10, 20 ohms are connected in delta across the bases A, B, C respectively to a balanced supply to 100 volts. What are the sequence components of current in the resistors and the supply line ?
11. Given $I_{a1} = j1.56$ p.u.
    $I_{a2} = j1.56$ p.u.
    $I_{a0} = 0$
    Calculate I_a, I_b, I_c. **[Ans.** $I_a = 0$, $I_b = -2.70 + j0$ p.u. $I_c = 2.70 + j0$ p.u.]
12. Given $I_{a1} = I_{a2} = I_{a0} = 0.255 - j0.32$ p.u.
    Calculate I_a, I_b, I_c **[Ans.** $I_a = 0.765 - j0.156$; $I_b = 0$; $I_c = 0$]
13. In a problem on fault calculations.
    $$I_{a1} = I_{a2} = I_{a0} = \frac{E}{Z_2 + Z_2 + Z_0} \text{ p.u.}$$
    $$E = 1 + j0$$
    Calculate I_a, I_b, I_c
    Given : $Z_1 = j0.5$, $Z_2 = j0.5$, $Z_0 = 0$.
14. Given $I_a = 0$ p.u., $I_b = -2.70 + j0$. p.u., $I_c = -0.70 + j0$. p.u.
    Calculate I_{a1}, I_{a2}, I_{a0}. **[Ans.** $-j1.560$ p.u., j 1.560 p.u.]
15. Explain the principle of symmetrical components. What is the difference between positive sequence and negative sequence components ? Derive the relation between V_a, V_b, V_c and $V_{a0}\ V_{a1}\ V_{a2}$.
16. Given V_{a0}, V_{a1}, V_{a2} derive an expression to obtain V_a, V_b, V_c.
17. Show that the current is neutral to ground connection is three times zero sequence component of current, *i.e.* $I_n = 3I_{a0}$.
18. A star connected three phase winding is with earthed neutral. During a fault, the current in neutral to ground path was 9 Amp. Calculate the zero sequence component of current in the winding.