22

Unsymmetrical Faults on an Unloaded Generator

Sequence Impedances — Sequence Network of Alternator — L-G Faults on Alternator — L-L Fault on Alternator — 2 L-G Fault on Alternator — Solved Examples.

22.1. SEQUENCE IMPEDANCES

The impedances offered by a rotating machine to positive sequence component of current, differ from those offered to negative sequence components of currents. Consider a circuit component, the voltage drop across it, for given sequence component of current will be equal to magnitude of that sequence current into impedance offered to it. Thus we come across positive sequence impedance or reactance, negative sequence impedance or reactance and zero sequence impedance or reactance.

The impedance offered by a circuit to positive sequence component current is called Positive Sequence Impedance of that circuit. Likewise the negative sequence impedance and zero sequence are defined.

22.2. SEQUENCE NETWORKS OF ALTERNATOR

(*I*) The positive sequence network of 3 phase alternator (shown in Fig. 22.1) consists of an e.m.f. source E_a in series with positive sequence impedance Z_1. E_a is the induced e.m.f. of one phase. Z_1 is replaced by jX_1 if the resistance is neglected. X_1 is positive sequence reactance of generator.

E_a = Voltage behind the reactance (induced e.m.f.) per phase

X_1 = Positive sequence reactance Direct axis reactance

$V_1 = E_a - I_{a1} X_1$

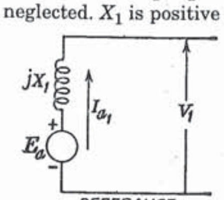


Fig. 22.1. Positive sequence network of an alternator.

It is same as direct axis reactance. It may be sub-transient or transient or steady state reactance depending upon the problem to be solved. Typical values of reactance are given in Table 19.2.

The generator e.m.f's are balanced voltages and, therefore, considered to be positive sequence e.m.f's. Generator does not induce negative or zero sequence e.m.f. The phase sequence of positive sequence voltages is the same as the phase sequence of induced e.m.f.

X_1 = positive sequence reactance of generator

E_a = e.m.f. induced in phase *a*.

(*II*) *Negative Sequence Network of Generator.* The negative sequence network of a generator consists simply of negative sequence reactance, jX_2 (Fig. 22.2) as there are no negative sequence e.m.f's induced by the alternator. Only

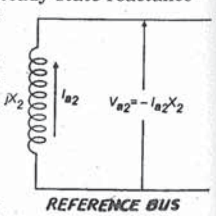


Fig. 22.2. Negative sequence network of a generator.

negative sequence current flows through negative sequence impedance (or reactance) and the voltages drop in the negative sequence network is given by

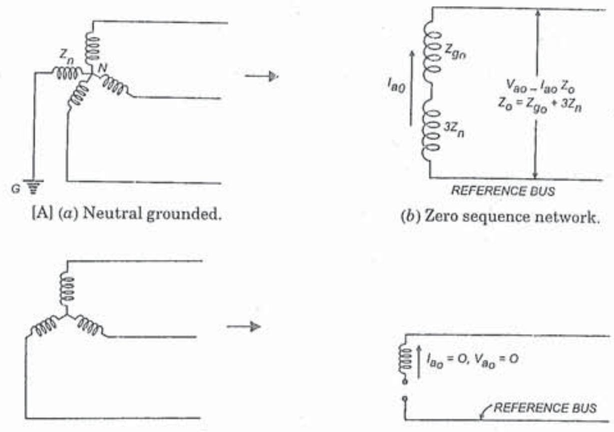
$V_{a2} = -I_{a2} Z_2$ or $-jI_{a2} X_2$

jX_2 = Negative sequence of reactance generator.

(*III*) *Zero Sequence Network.* Zero sequence network of an alternator consists of the zero sequence impedance of alternator per phase, plus three times the impedance in neutral to ground circuit, *i.e.*,

$$Z_0 = Z_{g0} + 3Z_n \text{ ; voltage drop} = I_n Z_0$$

It may be recalled* that the current in the neutral circuit is $I_n = 3I_0$. Hence the voltage drop is equal to $3I_{a0}Z_n$, where Z_n, is the reactance in neutral to ground circuit. We consider that only current I_{a0} flows through the neutral circuit. Hence Z_n is multiplied by 3 to get the voltage drop $3I_{a0} Z_n$. The zero sequence network of an alternator is shown in Fig. 22.3. If neutral is not grounded there is a gap in the zero sequence network and zero sequence component of current I_a is zero. Hence I_n is also zero.



[A] (*a*) Neutral grounded. (*b*) Zero sequence network.

[B] (*a*) Neutral ungrounded. (*b*) Zero sequence network.

Fig. 22.3. Zero sequence network of alternator.
[A] With neutral grounded. [B] Without neutral grounding.

22.3. VOLTAGE EQUATIONS

We observe that the currents of a particular sequence produce voltage drop of like sequence. Referring to Figs. 22.1, 22.2, 22.3 we write the following equations :

$$V_{a1} = E_a - I_{a1}Z_1 \quad \text{or} \quad E_a - jI_{a1} X_1$$

$$V_{a2} = -I_{a2}Z_2 \quad \text{or} \quad -jI_{a2}X_2$$

$$V_{a0} = -I_{a0}Z_0 \quad \text{or} \quad -jI_{a0} X_0$$

$$Z_0 = Z_{g0} + 3Z_n \quad \text{or} \quad X_0 = X_{g0} + 3X_n$$

* Refer Sec. 21.5 $I_{a0} = I_{b0} = I_{c0} = I_{n3}$

The word line refers to one conductor of 3 phase system.

22.4. SINGLE LINE* TO GROUND FAULT ON AN UNLOADED THREE-PHASE ALTERNATOR AT RATED TERMINAL VOLTAGE

Solution. Let a, b, c be the terminals of the unloaded generator whose star point N is grounded through impedance Z_n. A single line to ground fault occurs on terminal a. We have to determine fault current and voltage of the lines.

V_a = Voltage of terminal a with respect to N.

$I_b = 0$ and $I_c = 0$. Since generator is on no load.

$V_a = 0$. Neglecting drop in Z_n.

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix}$$

$$= \frac{1}{3} I_a.$$

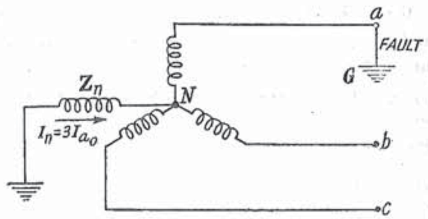


Fig. 22.4. Circuit condition of L-G fault.

Hence for single line to ground fault on terminal a, we get

$$I_{a0} = I_{a1} = I_{a2} = \frac{1}{3} I_a \qquad ...(a)$$

Coming to voltage equations,

$$V_{a1} = E_a - I_{a1} Z_1$$
$$V_{a2} = - I_{a2} Z_2$$
$$V_{a0} = - I_{a0} Z_0$$

Since $\quad I_{a1} = I_{a2} = I_{a0}$

$$V_a = V_{a1} + V_{a2} + V_{a0} = E_a - I_{a1}(Z_1 + Z_2 + Z_0) = 0$$

As $\quad V_a = 0,$

Hence $\quad I_{a1} = \dfrac{E_a}{Z_0 + Z_1 + Z_2} \qquad ...(b)$

Taking a look at equations (a) and (b), we feel that there should be some easy way to remember these expressions. And there is ! This is a wonderful part of the method of symmetrical components. The apparently dull and complicated complexities can be brought to a simple systematic form which makes the analysis interesting and easy.

Connect the three sequence networks of the generator in series. Equal current flows through the three networks and the above equations are satisfied. Fig. 22.5 shows the connection of sequence network to represent Single Line to Ground fault.

The sequence currents can be easily calculated from this simple series circuit.

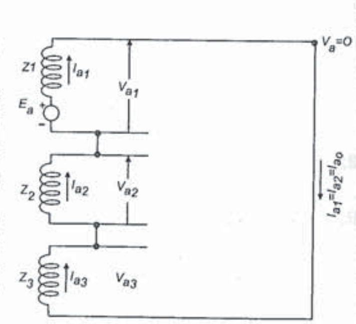


Fig. 22.5. Connection of sequence networks to represent single line to ground fault on phase a.

Example 22.1. *A 25,000 kV, 11 kV, 3 phase alternator has direct axis sub-transient reactance of 0.25 per unit, negative sequence reactance and zero sequences are respectively 0.35 and 0.1 p.u.*

Neutral of the generator is solidly grounded. Determine subtransient line currents and line-to-line voltages, for

(*a*) *Single line to ground fault.*

(*b*) *Double line to ground fault.*

(*c*) *Line to line fault.*

Generator is on no load and rated terminal voltage. Resistance is negligible.

Solution. (a) Let a, b, c be three terminals of the generator. Let fault occur between terminal a and ground. Let the induced voltage of phase a line to neutral E_a be 1 p.u.

$$E_a = 1 + j0 \text{ p.u.} = \frac{11}{\sqrt{3}} \text{ kV, actual}$$

$$X_n - 0, X_0 = X_{g0} + 0$$

Fig. 22.5 represents the fault condition. For L-G fault :

$$I_{a0} = I_{a1} = I_{a2} = \frac{E_2}{X_{g0} + X_1 + X_2}$$

$$= \frac{1 + j0}{j0.25 + j0.35 + j0.1} = \frac{1 + j0}{j0.70} = -j\,1.43 \text{ p.u.}$$

$$I_a = I_{a0} + I_{a1} + I_{a2} = 3I_{a0} = -j\,4.29 \text{ p.u.}$$

Base current $\quad = \dfrac{\text{Base kVA}}{\sqrt{3}\text{ Base kV}} = \dfrac{25{,}000}{\sqrt{3} \times 11} = 1310$ Amp.

Fault current $\quad I_a = -j4.29 \times 1310 = 5630 \text{ A } \underline{/-90^\circ}$

$$V_{a1} = E_a - I_{a1} Z_1$$
$$= 1 - (-j1.43)\,(j0.25) = 1 - 0.357 = 0.643 \text{ p.u.}$$
$$V_{a2} = - I_{a2} Z_2 = (-j1.43)\,(j0.35) = -0.50 \text{ p.u.}$$
$$V_{a0} = - I_{a0} Z_0 = -(-j\,1.43)\,(j0.1) = -0.143 \text{ p.u.}$$

Line to ground voltages

$$V_a = V_{a1} + V_{a0} = 0.643 - 0.50 - 0.143 = 0 \text{ [check]}$$
$$V_b = a^2 V_{a1} + aV_{a2} + V_{a0}$$
$$= 0.643\,(-0.5 - j0.866) + (-0.5 + j0.866)\,(-0.5) - 0.143 = 0.215 - j0.989 \text{ p.u.}$$
$$V_c = aV_{a1} + a^{2V_{a2}} + V_{a0}$$
$$= 0.643\,(-0.5 + j0.866) - 0.5\,(-0.5 - j0.866) - 0.143 = 0.215 + j0.989$$

Line to line voltages

$$V_{ab} = V_a - V_b = 0.215 + j0.989 = 1.01\angle 77^\circ \text{p.u.}$$
$$V_{bc} = V_b - V_c = 0 - j1.978 = 1.978\angle 270^\circ \text{ p.u.}$$
$$V_{ac} = V_c - V_a = +\,0.215 + j989 = 1.01\angle 102.3^\circ \text{ p.u.}$$

Line to line voltages in kV

Since 1 p.u. voltage $= \dfrac{11}{\sqrt{3}}$ kV

$$V_{ab} = 1.01 \times \frac{11}{\sqrt{3}} = 6.42\angle 77^\circ \text{ kV}$$

$$V_{bc} = 1.978 \times \frac{11}{\sqrt{3}} = 12.55\ \angle 270^\circ \text{ kV}$$

$$V_{ca} = 1.01 \times \frac{11}{\sqrt{3}} = 6.42\ \underline{/102.3} \text{ kV}$$

Fault current I_a = 3630 /90 A

Parts (B) and (C) will be solved later.

Example 22.2. A 3-phase 11 kV. 10,000 kVA alternator has $X_0 = 0.05$ p.u., $X'' = 0.15$ p.u., $X_2 = 0.15$ p.u. It is on no load and rated terminal voltage. Find the ratio of the line currents for a Single Line to Ground Fault, to 3 phase fault. Neutral is solidly grounded.

Solution. Let $E_a = 1$ p.u.

(1) **L-G Fault**

$$I_{a1} = \frac{E_a}{X_1 + X_2 + X_0} = \frac{1}{j0.05 + j0.15 + j0.15} = \frac{1}{j0.35}$$

$$I_a = 3I_{a1} = \frac{3}{j0.35} = -j8.57 \text{ p.u.}$$

(2) **Three-Phase Fault**

$$I_c = \frac{E_a}{X_1} = \frac{E_a}{X'} \qquad \{X_1 = X' \text{ or } X'' \text{ or } X_s\}$$

$$= \frac{1}{j0.15} = -j6.66 \text{ p.u.}$$

Ratio of line currents $= \dfrac{8.57}{6.66} = 1.285$

Single line to ground fault current is 1.285 times three phase fault current. **Ans.**

Example 22.3. *A 3-phase, 11 kV, 25,000 kVA alternator with $X_{g0} = 0.05$ p.u., $X_1 = 0.15$ p.u. and $X_2 = 0.15$ p.u. is grounded through a reactance of 0.3 ohms. Calculate the line current for a single line to ground fault.*

Solution. $\text{Base Z} = \dfrac{\text{Base kV}^2 \times 1000}{\text{Base kVA}}$

Let Base kV = 11

and Base kVA = 25,000

Base Z $= \dfrac{121 \times 1000}{25{,}000} = 4.84$ ohms

P.u. X_d of neutral connection $= \dfrac{0.3}{4.84} = 0.062$ p.u.

$$X_0 = X_{g0} + 3X_n$$

$$= j0.05 + 3\,[0.062] = j0.05 + j0.15 + j0.186 = j0.236 \text{ p.u.}$$

$$X_1 + X_2 + X_0 = j0.15 + j0.236 = j0.536 \text{ p.u.}$$

For single to ground fault, refer Fig. 22.5.

$$I_{a0} = \frac{E_a}{X_1 + X_2 + X_0} = \frac{1 + j0}{j0.536} = -j1.86 \text{ p.u.}$$

$$I_a = 3 \times I_{a0} = 3 \times 1.86 = 5.58 \text{ p.u.}$$

I_a in amperes $= 5.58 \times \dfrac{25{,}000}{\sqrt{3} \times 11} = 5.58 \times 1310 =$ **7312 amperes.**

22.5. DOUBLE LINE TO GROUND FAULT ON AN UNLOADED GENERATOR

Let fault involve terminals b, c and ground (Fig. 22.6)

Observing the fault condition, $V_b = 0$, $V_c = 0$ and $I_a = 0$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ 0 \\ 0 \end{bmatrix}$$

Hence $V_{a1} - V_{a2} = V_{a0} = \dfrac{V_a}{3}$

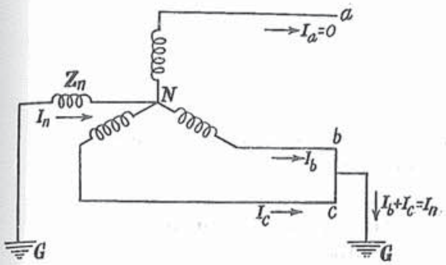


Fig. 22.6. 2 *L-G* fault.

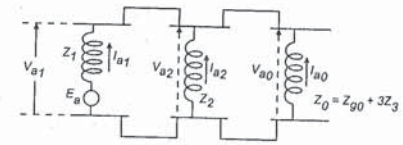


Fig. 22.7. Connection of sequence networks of 2 *L-G* fault.

Further $I_a = I_{a1} + I_{a2} + I_{a0} = 0$

This suggests that the three sequence networks of the generator may be connected in parallel as shown in Fig. 22.7.

From the figure, for double line to ground fault, we get

$$I_{a1} = \frac{E_a}{Z_1 + 1/\left(\dfrac{1}{Z_2} + \dfrac{1}{Z_0}\right)} = \frac{E_a}{Z_1 + \dfrac{Z_2 Z_0}{Z_2 + Z_0}}$$

$$V_{a1} = E_a - I_{a1} Z_1$$

$$V_{a1} = V_{a/3}$$

$$V_{a0} = V_{a1} = V_{a2}$$

$$I_{c2} = \frac{-V_{a1}}{Z_2} \qquad \text{...[Note the – ve sign]}$$

$$I_{a0} = \frac{-V_{a1}}{Z_0}$$

From these symmetrical components the currents and voltages can be determined.

Example 22.4. *(A) Part (b) of example 22.1.*

Given : Generator 11 kV, 25,000 kVA

$$X_1 = j0.25,\ X_2 = j0.35 \text{ p.u.},\ X_{g0} = j0.1,\ X_n = 0$$

Fault : Double line to ground, between terminals b, c and ground.

Solution. Let $E_a = 1 + j0$ p.u. $= \dfrac{11}{\sqrt{3}}$ kV

Refer Fig. 22.7,

$$I_{a1} = \frac{E_a}{X_1 + \dfrac{Z_0 Z_2}{Z_0 + Z_2}} = \frac{1 + j0}{j0.25 - \dfrac{j0.35 \times j0.10}{j0.35 + j0.10}}$$

$$= \frac{1}{j0.25 + j0.0778} = \frac{1}{j0.3278} = -j3.05 \text{ p.u.}$$

$$V_{a1} = V_{a2} = V_{c0} \qquad \text{...for LL-G Fault.}$$

$$= E_a - I_{a1} X_1$$

$$= 1 + j0 - (-j3.05)(j0.25) = 1 - 0.763 = 0.237 \text{ p.u.}$$

$$I_{a2} = \frac{-V_{a2}}{jX_2} = -\frac{0.237}{j0.35} = j0.68 \text{ p.u.}$$

$$I_{a0} = \frac{-V_{a0}}{X_{g0}} = -\frac{-0.237}{j0.10} = j2.37$$

$$I_a = I_{a1} + I_{a2} + I_{a0} = -j3.05 + j0.68 + j2.37 = 0$$

$$I_b = I_{a0} + a^2 I_{a1} + a I_{a2}$$
$$= j2.37 + (-0.5 - j0.866)(-j3.05) + (-0.5 + j0.866)(+j0.68)$$
$$= -3.229 + j3.55 = 4.81 \angle 132.5 \text{ p.u.}$$

$$I_c = I_{a0} + a I_{a1} + a^2 I_{a2}$$
$$= j2.37 + (-0.5 + j0.866)(-j3.05) + (-0.5 - j0.866)(j0.68)$$
$$= j2.37 + j1.525 + 2.64 - j0.34 + 0.589$$
$$= 3.229 + j3.555 = 4.81 \underline{/47.5^\circ} \text{ p.u. (Amp.)}$$

$$I_n = 3I_{a0} = 3 \times j2.37 = j7.11 \text{ p.u.}$$

or $$I_n = I_b + I_c$$
$$= -3.229 + j3.555 + 3.229 + j3.555 = j7.11 \text{ p.u. (Check)}$$

Phase voltages $V_a = V_{a1} + V_{a2} + V_{a0} = 3V_{a1}$
$$= 3 \times 0.237 = 0.711 \text{ p.u.}$$
$$V_b = V_c = 0$$

Line to line voltages

$$V_{ab} = V_a - V_b = 0.711 \text{ p.u.}$$
$$V_{bc} = V_b - V_c = 0$$
$$V_{ca} = V_c - V_a = -0.711 \text{ p.u.}$$

Base voltage $1 \text{ p.u.} = \frac{11}{\sqrt{3}} \text{ kV since } E_a = 1 + j0 = 11/1.73 = 6.35 \text{ kV}$

$$V_{ab} = 0.711 \times \frac{11}{\sqrt{3}} = 4.52 \angle 0^\circ \text{ kV} \quad \textbf{Ans.}$$

$$V_{ca} = -0.711 \times \frac{11}{\sqrt{3}} = 4.52 \angle 180^\circ \text{ kV}$$

Base Current $= \frac{\text{Base kVA}}{\sqrt{3} \text{ Base kV}} = \frac{25{,}000}{\sqrt{3} \times 11} = 1310 \text{ Amp.}$

$$|I_b| = 4.81 \times 1310 = 6300 \text{ A} \underline{/132.5^\circ}$$
$$|I_c| = 4.81 \times 1310 = 6300 \text{ A} \underline{/47.5^\circ}$$
$$I_a = 0 \quad \textbf{Ans.}$$
$$I_n = I_b + I_c = 7.11 \times 1310 \equiv 9320 \text{ A}$$

Example 22.4 (B) Neutral Reactor

In Ex. 22.4 (A), the Neutal to Ground Circuit has reactance of 0.1 p.u. instead of zero. [Add $X_n = 0.1$ p.u. in Fig. 22.6].

Calculate Fault Current following through the Neutral Reactor

Solution. Ref. Fig. 22.7.

Now $X_0 = X_{g0} + 3X_n$

where X_0 = Total Eq. Zero Seq. Reactance

X_{g0} = Zero Seq. Reactance of Generator

X_n = Reactance in Neutral to Ground Circuit.

X_1 and X_2 remains unchanged.

In this example, $X_{g0} = j0.1$ p.u. ; $X_n = j0.1$ p.u.

$$X_0 = j0.1 + 3 \times j0.1 = j0.4 \text{ p.u.}$$

$$I_{a1} = \frac{E_a}{X_1 + \frac{X_0 X_2}{X_0 + X_2}} = \frac{1 + j0}{j0.25 + \frac{j0.4 \times j0.35}{j0.4 + j0.35}}$$
$$= \frac{1 + j0}{j0.25 + j0.187} = \frac{1 + j0}{j0.437} = -j2.29 \text{ p.u.}$$

$$V_{a1} = V_{a2} = V_{a0} = E_a - I_{a1} X_1$$
$$= (1 - j0) - (-j2.29)(j0.25) = 1 - 0.572 = 0.428 \text{ p.u.}$$

$$I_{a2} = \frac{-V_{a2}}{X_2} = \frac{-V_{a1}}{X_2} = \frac{-0.428}{j0.35} = j1.225 \text{ p.u.}$$

$$I_{a0} = \frac{-V_{a0}}{X_0} = \frac{-0.428}{j0.4} = j1.072$$

$$I_a = I_{a1} + I_{a2} + I_{a0}$$
$$= -j2.29 + j1.225 + j1.072 \approx 0 \text{ (check)}$$
$$I_b = I_{a0} + a^2 I_a + a I_{a2} = -3.04 + j0.6045$$
$$I_c = I_{a0} + a I_a + a^2 I_{a2} = +3.04 + j0.6045$$
$$I_n = I_b + I_c + I_a = -3.04 + j0.6045 + 3.04 + j0.6045 = j1.2090 \text{ p.u.}$$

Base current was 1310 A.

Hence current in Neutral-to-ground Reactor = 1.209 × 1310 = **1570 Amp. Ans.**

Note. Reason for Reactance Grounding. Ex. 22.4 (A) Neutral-to-Ground Current without neutral reactor was 9320 A. Ex. 22.4 (B) Neutral-to-Ground Current with neutral reactor was 1570 A. Reactance in Neutral to-Ground circuit reduces fault current. Hence modern practice is in favour of reactance grounding. [Ref. Sec. 33.6]

Example 22.5. *A 3-phase, 10 MVA star connected alternator having solid neutral earthing supplies a feeder. The per unit reactances are as follows :*

Generator : $X_1 = j0.16,\ X_2 = j0.08,\ X_0 = j0.06$

Feeder : $X_1 = j0.1,\ X_2 = j0.1,\ X_0 = j0.3.$

Determine fault current and line to neutral voltages at the generator terminals for a double line to ground fault at the other end of the feeder. Generator rated voltage is 11 kV.

Solution.

Total $X_1 = j0.26,\ X_2 = j0.18,\ X_0 = j0.36.$

Let $E_a = 1 + j0$ p.u. be reference. Fault occurs between b, c and ground, Figs. 22.6 and 22.7.

$$I_{a1} = \frac{1 + j0}{j0.26 + \frac{j0.18 \times j0.36}{j0.18 + j0.36}} = -j2.63 \text{ p.u.}$$

or $$V_{a1} = E_a - I_{a1} Z_1 = 1 - (-j2.63)(j0.26) = 1 - 0.684 = 0.316$$

$$I_{a2} = \frac{-V_{a2}}{Z_2} = \frac{-0.316}{j0.18} = j1.75 \text{ p.u.}$$

$$I_{a0} = \frac{-0.316}{j0.36} = j0.875 \text{ p.u.}$$

Check : $I_{a2} + I_{a0} + I_{a1} = j1.75 + j0.875 - j2.63 = j2.625 - j2.63$

is approximate $= 0$

∴ $I_a = 0$ (Check) since fault is on b, c.

$$I_b = a^2 I_{a1} + a I_{a2} + I_{a0} = -3.8 + j1.32 = 4.02 \angle 160^\circ \text{ p.u.}$$
$$I_c = a I_{a2} + a^2 I_{a2} + I_{a0} = 3.8 + j0.32 = 4.02 \angle 20^\circ \text{ p.u.}$$

Base current $= \dfrac{10 \times 10^6}{\sqrt{3} \times 11 \times 10^3} = 5.25 \times 10^2 \text{ A} = 225 \text{ A}$

$$I_b = 4.02 \times 524 = 2110\angle 160^\circ$$

$$I_c = 4.02 \times 524 = 2110\angle 20^\circ \text{ A}$$

Fault current $= I_b + I_c$

$$= -3.8 + j1.32 - 3.8 + j1.32$$

$$= j2.64 \text{ p.u.} = j2.64 \times 524 = 1385 \text{ A}\angle 90^\circ$$

or $\quad I_a = I_b + I_c = 3I_a = j0.875 \times 3 = j2.625$ p.u.

This current flows through ground.

Voltage at the terminal of generator

$$V_{a1} = E_1 - I_{a1} Z_1 = (1 + j0) - (-j2.63)(j0.16) = 0.58\angle 0^\circ \text{ p.u.}$$

$$V_{a2} = -I_{a2}Z_2 = -j1.75 \times j0.08 = 0.14\angle 0^\circ \text{ p.u.}$$

$$V_{a0} = 0 - I_{a0}Z_0 = -j0.875 \times j0.06 = 0.0525\angle 0^\circ \text{ p.u.}$$

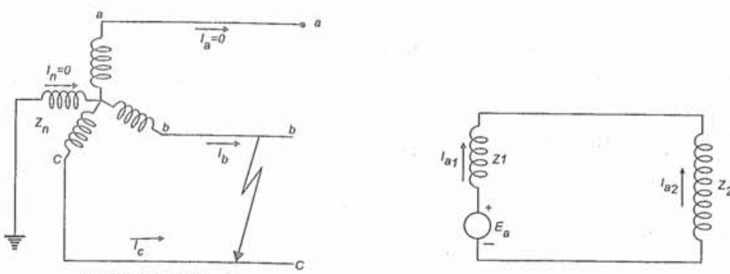
$$V_a = V_{a0} + V_{a1} + V_2 = 0.77 \text{ p.u. } \angle 0^\circ = 4.9\angle 0^\circ \text{ kV}$$

$$V_b = a^2V_{a1} + aV_{a2} + V_{a0} = -0.306 - j0.383 \text{ p.u.} = 3.12\angle 232^\circ \text{ kV}$$

$$V_c = aV_{a1} + a^2V_{a2} + V_{a0} = 0.306 + j0.382 \text{ p.u.} = 3.12\angle 128^\circ \text{ kV.}$$

22.6. LINE TO LINE FAULT ON UNLOADED ALTERNATOR (GENERATOR)

Let a, b, c be the three terminals of a generator whose neutral is grounded through an impedance Z_n. A fault occurs between lines b and c (Fig. 22.8). We have to determine the current and voltages for the fault condition, neglecting load current.



(*a*) Condition of fault (*b*) Connection of sequence network.

Fig. 22.8. *L/L* Fault.

Taking a look at the circuit conditions (Fig. 22.8) we can directly say the following :

$$I_a = 0$$

$I_n = 0$. Since the fault does not involve earth.

$$V_b = V_c$$

$$I_b = -I_c \text{ means } I_c = -I_b$$

Fault does not involve ground. Putting these conclusions in the equations we proceed as follows :

$$I_{a0} = \frac{I_n}{3} = 0$$

$$I_{a0} = -I_{a0}Z_0 = 0.$$

No zero sequence network for *L-L* fault need be considered.

$$I_{a1} = \frac{1}{3}[I_a + aI_b + a^2I_c]$$

$$I_{a2} = \frac{1}{3}[I_a + a^2I_b + aI_b]$$

Since $\quad I_a = 0$ and $I_b = -I_c$, we get

$$I_{a1} = \frac{1}{3}(a - a^2)\, I_b$$

$$I_{a2} = \frac{1}{3}(a^2 - a)\, I_b$$

$\therefore \quad I_{a1} = -I_{a2}.$

To get these conditions the sequence networks of the generator are connected in parallel as shown in Fig. 22.2 (*b*).

Given the sequence impedances, we proceed as follows :

$$I_{a1} = \frac{E_a}{Z_1 + Z_2}$$

$$I_{a1} = -I_{a2}$$

$$I_{a0} = 0.$$

Thus $I_{a0}\, I_{a1}, I_{a3}$ are known from which I_a, I_b, I_c can be determined.

$$V_{a1} = E_a - I_{a1}Z_1 = V_{a2}$$

$$V_{a0} = 0,$$

From V_{a2}, V_{a1}, V_{a2} the voltages can be determined.

Example 22.6. *Part* (*c*) of example 20.1.

Given : *11 kV ; 25,000 kVA generator*

$X_1 = j0.25$, $X_2 = j0.35$, $X_0 = j0.1$ p.u.

Line to the fault on terminal b, c.

Solution.

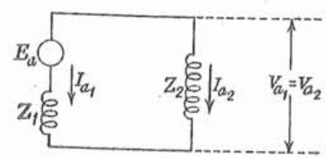


Fig. 22.9 of Ex. 22.6.

Let $E_a = 1$ p.u. $\quad = 1 + j0 = \dfrac{11}{\sqrt{3}}$ kV

$$I_{a1} = -I_{a2} = \frac{E_0}{X_1 + X_2} = \frac{1 + j0}{j0.25 + j0.35} = -j1.667 \text{ p.u.}$$

$I_{a0} = 0$. Line to line fault does not involved ground. Hence $I_n = 0$. Hence $I_a = 0$

$$I_{a0} = 0$$

$$I_a = I_{a0} + I_{a2} + I_{a2} = 0 - j1.667 + j1.667 = 0$$

$$I_b = I_{a0} + a^2 i_{a1} + aI_{a2},$$

$$= 0 + (-0.5 - j0.866)(-j1.667) + (-0.5 + j0.866)(j1.667) = -2.892 \text{ p.u.}$$

$$I_c = -I_b = 2.892 \text{ p.u.}$$

Base current $= \dfrac{\text{Base kVA}}{\sqrt{3}\text{ Base kV}} = \dfrac{25{,}000}{\sqrt{3} \times 11} = 1310$ A

$$I_B = 2.892 \times 1310 = -3786 \text{ A}$$

$$I_C = +3786 \text{ A}$$

Voltages $\quad V_{a1} = V_{a2} = -I_{a2}X_2 = -(j0.35)(+j1.667) = 0.584$ p.u.

Also $\quad V_{a1} = E_a - Ia_1 X_1 = (1 + j0) - (-j1.667)(j0.25) = 1 - 0.416 = 0.584$ p.u.

$$V_{a0} = 0$$

$$V_a = V_{a0} + V_{a1} + V_{a2} = 0.584 + 0.584 = 1.168\angle 0^\circ \text{ p.u.}$$

$$V_b = V_{a0} + a^2 V_{a1} + aV_{a2}$$
$$= 0 + (-0.5 - j0.866)(0.584) + (-0.5 + j0.866)(0.584) = -0.584 \text{ p.u.}$$
$$V_b = V_c = -0.584 \text{ p.u.}$$
$$V_{ab} = V_a - V_b = 1.168 + 0.584 = 1.752 \angle 0^\circ \text{ p.u.}$$
$$V_{bc} = V_b - V_c = -0.584 + 0.584 = 0$$
$$V_{ca} = V_c - V_a = -0.584 - 1.168 = 1.752 \angle 180^\circ \text{ p.u.}$$

Voltage in kV

$$\left.\begin{aligned} V_{ab} &= 1.752 \times \frac{11}{\sqrt{3}} = 11.15 \angle 0^\circ \text{ kV} \\ V_{bc} &= 0 \\ V_{ca} &= 11.15 \angle 180^\circ \text{ kV} \end{aligned}\right\} \textbf{ Ans.}$$

Fault current $I_b = 3786 \text{ Amp.} = I_c$

Example 22.7. *A 3-phase generator has $X_1 = 0.15$ p.u., $X_2 = 0.15$ p.u. is with solidly grounded neutral. Calculate the ratio of the line currents for line-to-line fault to three phase fault.*

Solution.

Let e.m.f. $= E_a = 1 + j0$ p.u.

(I) Line-to-line fault on phases b, c.

$$I_{a1} = \frac{E_a}{X_1 + X_2} = \frac{1 + j0}{j0.15 + j0.15} = \frac{1}{j0.3}$$

Fault current $= I_a = a^2 Ia_1 + aI_{a2} = (a^2 - a) I_{a1}$

$$= -1.732\, I_{a1} = 1.732 \left(\frac{1}{j0.3}\right) = j5.78 \text{ p.u.}$$

3-phase fault current

$$I_a = \frac{E_a}{X_1} = \frac{1}{j0.15} = -6.66 \text{ p.u.}$$

Ratio $\dfrac{5.78}{6.66} = \dfrac{\text{Line to line fault}}{\text{3 phase fault}} = \mathbf{0.868.}$ **Ans.**

Example 22.8. *A generator has the following sequence reactances : $X_1 = 60\%$, $X_2 = 25\%$ and $X_0 = 15\%$.*

(a) Calculate percentage reactance that should be added in the generator neutral such that the current for single line to ground fault does not exceed the rated current.

(b) Calculate value of resistance to be connected to neutral to achieve the same purpose.

Solution.

(*a*) $E_a = 1$ p.u. Let X_a be p.u. reactance added in the neutral connection.

Fault current for single line to ground fault

$$= 3\left(\frac{1 + j0}{X_1 + X_2 + X_0}\right) = \frac{3}{0.6 + 0.25 + 0.15 + 3X_n} = \frac{3}{1.00 + 3X_n}$$

To limit fault current to rated current, *i.e.* 1 p.u., we must get

$$1 = \frac{3}{1 + 3X_n}. \qquad \text{Hence } 3X_n = 2$$

$$X_n = \frac{2}{3} = 0.66 \text{ p.u.} = 66.6\%.$$

(*b*) Let resistance r p.u. be added to neutral connection

$$I_f = \frac{3E_a}{X_1 + X_2 + X_0} = \frac{3E_a}{X_1 + X_2 + X_{g0} + 3r} = \frac{3}{[3r + j1.0]}$$

$I_f = 1$ since current is to be limited to 1

$$1 = \frac{3}{[3r + j1]}$$
$$|3r + j1| = 3$$
$$\sqrt{1^2 + 9r^2} = 3$$
$$9r^2 = 9 - 1 = 8$$
$$r^2 = \frac{8}{9}$$
$$r = \frac{\sqrt{8}}{3} = \frac{2.85}{3} = 0.95 \text{ p.u.}$$

Hence resistance to be added in neutral to ground circuit to achieve the same purpose is 95 per cent.

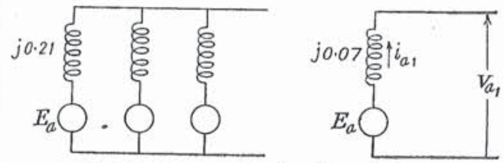
Example 22.9. *Three alternators have identical constants given by $X_d' = 21\%$, $X_2 = 12\%$, $X_0 = 10\%$ are operating in parallel.*

Neutral of only one is grounded solidly. Other machines have ungrounded neutral.

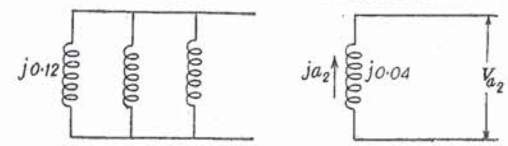
(1) Find short circuit current for line to ground fault.

(2) Determine the ratio in which the alternators contribute to the fault mentioned above

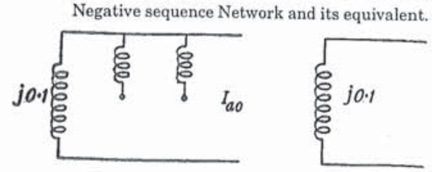
(3) How does a 3-phase short circuit current compare with line to ground fault current ?



Positive sequence network and its equivalent.



Negative sequence Network and its equivalent.



Zero sequence Network and its equivalent.
Fig. of Ex. 22.9.

Solution. Draw Thevenin's equivalents of three networks. Note that the zero sequence networks of ungrounded generators are open. Hence zero sequence component is contributed only by generator 1.

(*a*) For single line to ground fault, connect the equivalent networks in series

$$I_a = \frac{3E_a}{X_1 + X_2 + X_0} = \frac{3 \times 1}{j0.07 + j0.04 + j0.1} = \frac{3}{j0.21} = \mathbf{14.3 \text{ p.u.}} \quad \textbf{Ans.}$$

(*b*) **Contributions of Generators.** Only generator 1 whose neutral is grounded contributes to zero sequence component.

Total $I_{a0} = \dfrac{14.3}{3} = 4.77$ p.u.

Total $I_{a1} = \dfrac{14.3}{3} = 4.77$ p.u.

Total $I_{a2} = \dfrac{14.4}{3} = 4.77$ p.u.

Contributions

I_{a0} completely by generator $1 = 4.77$ p.u.

I_{a1} equally by 3 generators each $= 1.59$ p.u.

I_{a2} equally by 3 generators each $= 1.59$ p.u.

Contribution of generator I

$$= I_{a0} + I_{a1} + I_{a2} = 4.77 + 1.59 + 1.59 = 7.86 \text{ p.u.}$$

Contribution of generators II and III

$$= I_{a1} + I_{a2} = 1.59 + 1.59 = 3.18$$

Ratio of share = *7.86 : 3.18 : 3.18*

10 : 4.05 : 4.05

Generators 1, 2, 3 will share the short circuit currents in proportion 10 : 4.05 : 4.05.

(c) 3-phase fault current

$$= \frac{E_a}{X_{1eq}} = \frac{1}{j0.07} = -j14.3 \text{ p.u.}$$

Line to ground fault current

$$= \frac{3E_a}{X_1 + X_2 + X_0} = \frac{3 \times 1}{-j0.21} = -j14.3 \text{ p.u.}$$

Hence the two currents are equal.

Example 22.10. *Compare the fault currents of a generator with three-phase fault current.*

Solution. Let a, b, c be the terminals.

$$E_a = 1 \text{ p.u.}$$

For fault on terminal a to ground.

Single line to ground fault.

$$I_f = \frac{3E_a}{X_1 + X_2 + X_0} = \frac{3}{X_1 + X_2 + X_0} \quad \text{...(I)}$$

3-phase fault

$$I_f = \frac{E_a}{X_1} = \frac{1}{X_1} \quad \text{...(II)}$$

Line to line fault between B, C.

$$I_{a1} = \frac{E_a}{X_1 + X_2}$$

$$I_{a2} = \frac{E_a}{X_1 + X_2}$$

$$I_a = (a^2 - a)\frac{E_a}{X_1 + X_2} = 1.732\frac{E_a}{X_1 + X_2} \quad \text{...(III)}$$

$$= \frac{1.732}{X_1 + X_2}$$

Ratio of fault currents

$$\frac{L\text{–}G \text{ fault}}{3 \text{ phase fault}} = \frac{(\text{I})}{(\text{II})} = \frac{3X_1}{X_1 + X_2 + X_0}$$

$$\frac{L\text{–}L \text{ fault}}{3 \text{ phase fault}} = \frac{(\text{III})}{(\text{II})} = \frac{1.732\, X_1}{X_1 + X_2}$$

Summary

1. The generator positive sequence network consists of an e.m.f. source in series with reactance, which is transient/sub-transient or steady state reactance.

2. The negative sequence network has only negative sequence reactance.

3. The zero sequence network consists of $X_{g0} + 3X_n$, where X_n is the reactance in neutral connection.

These three networks connected as follows :

(*i*) *Single line to ground fault :* $I_{a0} = I_{a1} = I_{a2}$. Connect networks in series.

(*ii*) *Line to line fault :* $I_{a0} = 0$, $V_{a0} = 0$, $I_{a1} = -I_{a2}$.

Positive sequence network in parallel to negative sequence network.

(*iii*) *2 LG fault :* Connect the three networks in parallel

$$V_{a1} = V_{a2} = V_{a0}.$$

QUESTIONS

1. Define positive sequence impedance, negative sequence impedance, zero sequence impedance. Derive expressions for fault currents on an unloaded generator for single line to ground fault, line to line fault and 3 phase fault.
2. A single line to ground fault occurs on a cable connected to a 10,000 kVA, 3 phase, alternator with solidly earthed neutral. The positive negative and zero impedances of the generator are
$$0.5 + j4.7 \text{ ohms}, 0.2 + j0.6, j0.43 \text{ ohms}$$
respectively. The corresponding line values of cable are
$$0.36 + j0.25, 0.36 + j0.25, 2.9 + j0.95 \text{ ohms}$$
respectively. Line voltage 6.6 kV, calculate
(*i*) fault current,
(*ii*) voltages of sound lines to earth point.
3. Three 6600 V, 10,000 kVA, 3 phase alternators are connected in a parallel each has $X_1 = 0.15$ p.u., $X_2 = 0.75$ p.u., $X_0 = 0.30$ p.u. an earth fault occurs on one bus bar. Calculate fault current if
(*a*) all the alternators have solid neutral earthing :
(*b*) if one alternator neutral is solidly earthed ;
(*c*) if all the neutrals are isolated.
4. Derive the expressions for the ratios given below for a generator on no load.
(*a*) $\dfrac{\text{Line to ground fault current}}{\text{Line to line fault current}}$ (*b*) $\dfrac{\text{Line to line fault current}}{\text{3-phase fault current}}$
[**Hint.** Refer Ex. 22.10]
5. A 20,000 kVA, 13.8 kV generator has the following reactances :
Direct axis sub-transient reactance = 0.25 p.u.
Negative sequence reactance = 0.35 p.u.
Zero sequence reactance = 0.1 p.u.
Neutral is solidly earthed.
Fault occurs when the generator is on no load and rated terminal voltage. Calculate the fault currents and line to line voltage for the following :
(*a*) Line to ground fault (*b*) Line to line fault (*c*) 2L-G fault.
[**Ans.** (*a*) 3585 A ; 8.05 kV ; 15.73 kV ; 8.05 kV
(*b*) 2420 A ; 13.95 kV, 0 kV, 13.95 kV
(*c*) 4025 A ; 4025 A, 5.66 kV, 0 kV, 5.66 kV]
6. A generator has positive sequence reactance of 0.25 p.u. Calculate the p.u. reactance to be connected in series to limit the fault current for 3 phase fault to rated current.
7. A 3-phase generator has the following reactance :
$$X_1 = 0.20 \text{ p.u.}, X_2 = 0.20 \text{ p.u.}, X_0 = 0.1 \text{ p.u.}$$

reactance connected to neutral 0.3 p.u. calculate p.u. fault current for a

(a) Single line to ground fault

(b) Three-phase fault.

8. Calculate the single line to ground fault current for the generator if the neutral is solidly grounded. Given $X_1 = 0.58$ p.u., $X_3 = 0.25$ p.u., $X_0 = 0.1$ p.u.

9. Two generators rated 11 kV, 100 kVA having $X_1 = 0.15$, $X_3 = 0.12$, $X_0 = 0.1$ p.u. are operating in parallel a single line to ground fault occurs on the bus bar. Calculate the fault current if

(a) Both generator neutrals are solidly earthed ;

(b) only one generator neutral is solidly earthed ;

(c) both neutrals are isolated. (**Hint.** Refer Ex. 22.9)

10. For a generator the ratio of fault current for line to line fault and three phase faults is 0.866. The positive sequence reactance is 0.15 p.u. Calculate negative sequence reactance. (**Hint.** Refer Ex. 22.10)

11. A fault occurs on an unloaded generator. The zero sequence component of fault current for a single line to ground fault has to magnitudes of 100 Amperes. Calculate the current in the neutral to ground connection.

12. A 3 phase 132 kV system can be represented by a solidly earthed source, feeding a 132/33 kV star delta transformer whose star point is solidly earthed. An earth fault occurs on one of the 132 kV terminals when 33 kV side is not connected to load, determine the fault current to earth and current in transformer delta winding. The sequence reactances based on 100 MVA base are as follows :

Source : P.S. Reactance 20%
N.S. Reactance 15%
Zero seq. Reactance 10%

Transformer : 15% reactance [**Ans.** 3200 A, 985 A]

23

Faults on Power Systems

Sequence Networks — Connections of Transformers — Connections of Sequence Networks — Single Line to Ground Fault — Line to Line Fault — Double Line to Ground Fault on Power Systems — Solved Examples.

23.1. Sequence Networks

The positive sequence network was considered in analysing symmetrical faults. In positive sequence network only positive sequence voltages, positive sequence impedance and positive sequence current are effective. Positive sequence network is same as impedance or reactance diagram. The negative sequence network is one in which the negative sequence voltages, negative sequence currents and negative sequence reactances exist. Negative sequence networks are very much like positive sequence networks but differ in the following aspects :

(1) Normally there are no negative sequence e.m.f. sources.

(2) Negative sequence impedances of rotating machine is generally different from their positive sequence impedances.

The phase displacement of transformer banks for negative sequence is of opposite sign to that of positive sequence.

The zero sequence network differs greatly from the positive sequence, negative sequence networks in the following aspects :

(1) Z.S. Reactance of transmission lines is higher than that for positive sequence.

(2) Equivalent circuits for transformers are different.

(3) The neutral grounding should be considered in zero sequence network.

Zero sequence networks. As the zero sequence currents in three phases (I_{a0}, I_{b0}, I_{c0}) are equal and of same phase, three systems operate like single phase as regards zero sequence currents. Zero sequence currents flow only if return path is available through which circuit is completed.

Case I. **Star Connections**

Star connection without neutral wire or neutral ground. Zero sequence currents have no return path and, therefore, the zero sequence network is open. Beyond the neutral point the zero sequence currents find infinite impedance hence no zero sequence current flows [Fig. 23.1 (*a*)].

Case II. **Star connection with solid grounding of neutral**

The zero sequence current flows through the ground connection. Hence in the zero sequence network the point N is connected directly to the reference bus [Fig. 23.1 (*b*)].

Case III. **Star connection with impedance grounding.** The neutral current $I_n = 3I_{a0}$ flows through the impedance Z_n connected between the neutral and ground. In the zero sequence network impedance $3Z_n$ is connected between N and reference, current flowing being, I_{a0}.

Case IV. Delta Connection

In delta connection the return neutral path is absent. Hence the zero sequence currents have no road to go ahead, they find the road to be suddenly stopped with infinite impedance ahead. How-