LAKSHYA JEE

LAKSHYA KO HAR HAAL ME PAANA HAI

Relations & Functions

Lecture: 07

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Today's Goal: :

Standard Problems of Domain of the Functions:

Standard Problems of Range of the Functions:

Quick Recap

Absolute Value: $|x|$

greatest integer function $[x]$

least integer function: (x)

from theory:

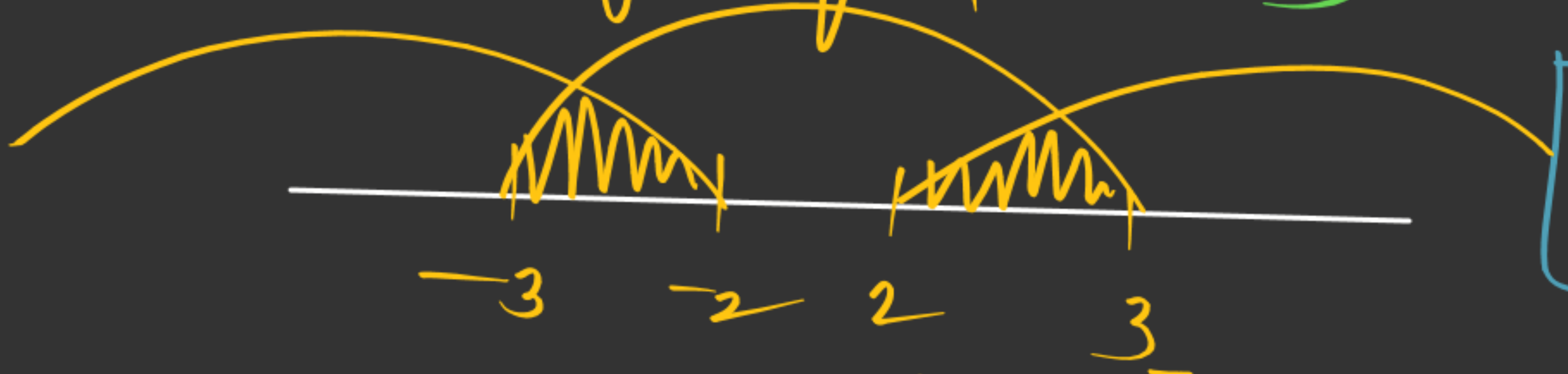
$$[|x|] = 2$$

$$2 \leq |x| < 3$$

$$\Downarrow$$

$$-3 < x < 3$$

$$x \geq 2 \text{ or } x \leq -2$$



$$x \in (-3,-2] \cup [2,3)$$

Fractional part function:

$$f(x) = \{x\} = \boxed{x - [x]} \Rightarrow \boxed{0 \leq \{x\} < 1}$$

$2 = 2 + 0$

$\{2.3 = 2 + 0.3$

$x = [x] +$ fractpa

e.g. $\{2.4\} = 0.4, \quad \{4\} = 0$

$\{0.3\} = 0.3$

$\{-2.4\} = 0.6 = (-2.4) - [-2.4]$

$= -2.4 + 3$

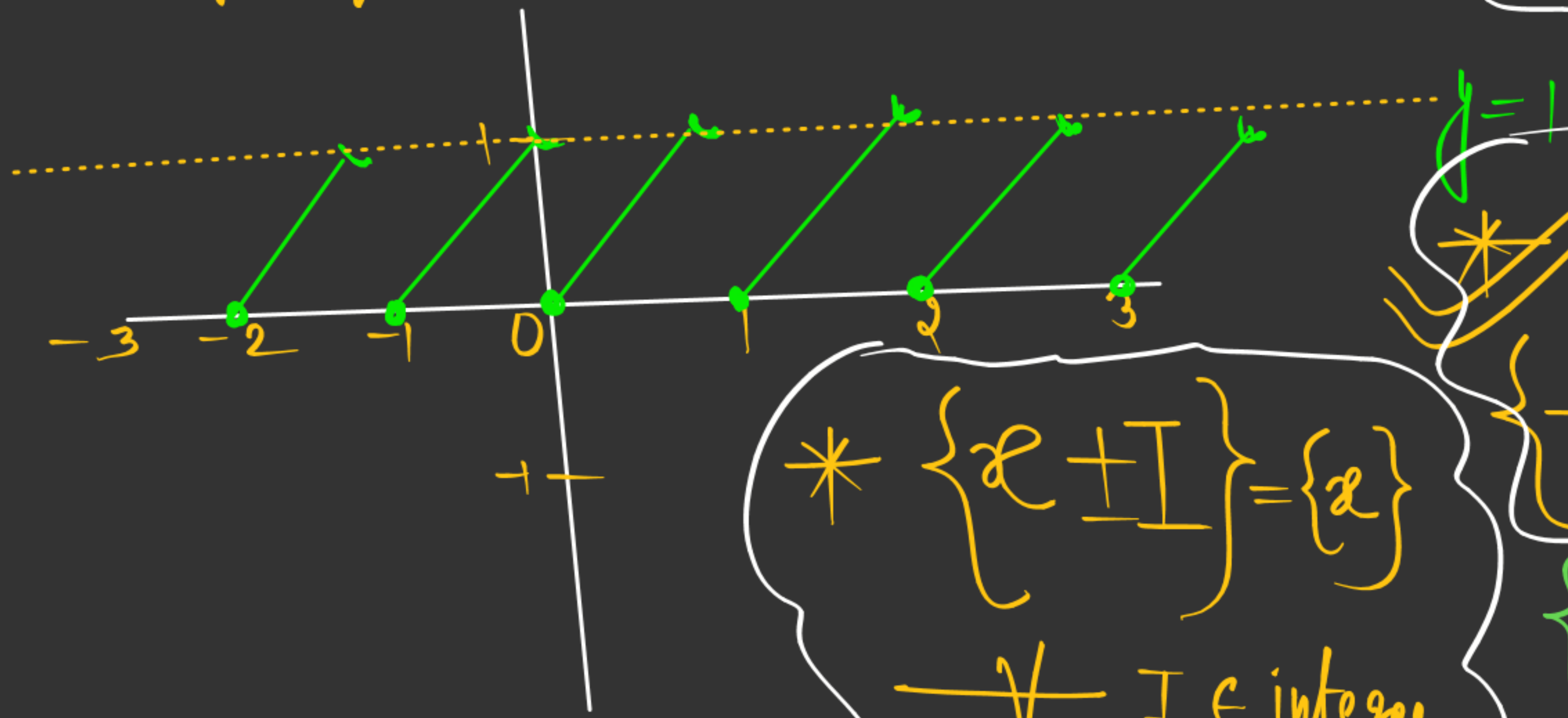
$= 0.6$

$$f(x) = \{x\} = x - [x]$$

y

$$= \begin{cases} x \; ; & 0 \leq x < 1 \\ x - 1 \; ; & 1 \leq x < 2 \\ x - 4 \; ; & 4 \leq x < 5 \\ x + 3 \; ; & -3 \leq x < -2 \end{cases}$$

Graph of $f(x) = \{x\}$



* $\{x \pm I\} = \{x\}$
$\forall$ I ∈ integer

* $x \in I$
$\{x\} = \{-x\} = 0$

* $x \notin I$
$\{-x\} = 1 - \{x\}$

$\{-2.4\} = 1 - \{2.4\}$
$= 1 - 0.4$
$= 0.6$

find $\sum_{r=1}^{2020} \frac{\{x+r\}}{2020} = ?$ $\quad x \quad [x] \quad \{x\} \quad$ none

$$= \frac{\{x+1\}+\{x+2\}+\{x+3\}+\cdots\cdots+\{x+2020\}}{2020}$$

$$= \frac{\{x\}+\{x\}+\{x\}+\cdots\cdots 2020 \text{ times}}{2020} = \frac{\cancel{2020}\{x\}}{\cancel{2020}}$$

$$= \{x\}$$

Find x satisfying $4\{x\} = x + [x]$?

x का त्याग

using $x = [x] + \{x\}$

Now, $4\{x\} = [x] + \{x\} + [x]$

$\Rightarrow 3\{x\} = 2[x]$

$\Rightarrow \{x\} = \frac{2[x]}{3}$ — (1)

As, $0 \leq \{x\} < 1$

$\Rightarrow 0 \leq \frac{2[x]}{3} < 1$

$\Rightarrow 0 \leq [x] < \frac{3}{2}$

$\Rightarrow [x] = 0, 1$

from (1) $\{x\} = 0, \frac{2}{3}$

$x = [x] + \{x\}$

$\Rightarrow \boxed{0, 5/3}$

Domain:

$y = f(x)$

The Values (interval) of x for which $f(x)$ is real, is known as Domain of $f(x)$

ILATE:

Algebraic f^n

$\sqrt{f(x)}$ is real iff $f(x) \geq 0$

$\frac{1}{f(x)}$ is real iff $f(x) \neq 0$

Logarithmic f^n: $\log_{g(x)} f(x)$ is real, iff $f(x) > 0$

$g(x) > 0$ & $g(x) \neq 1$

Domain of Definition of the Function:

Find the domain of the function

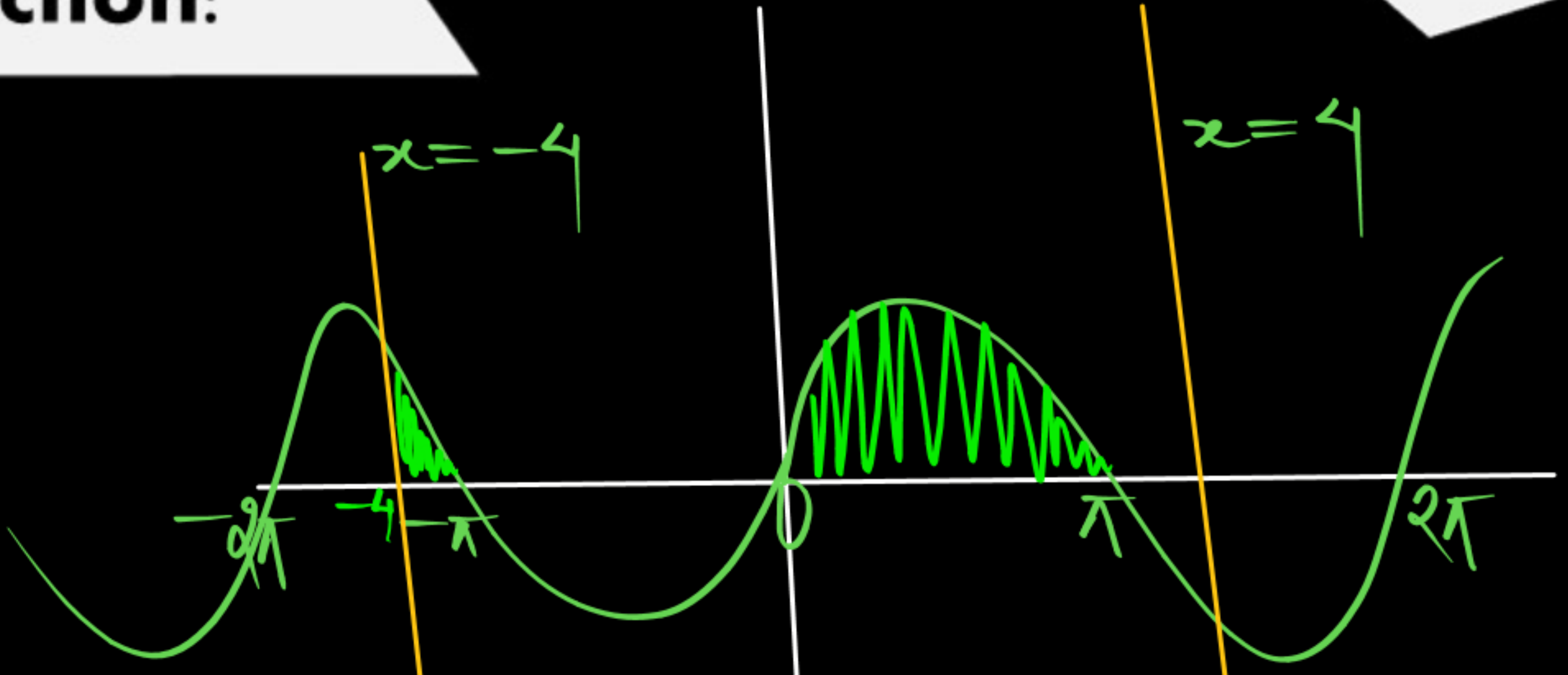
1. $f(x) = \sqrt{\sin x} + \sqrt{16 - x^2}$

For $f(x)$ is to be real

$\sin x \geq 0$ — (1)

& $\underbrace{16 - x^2 \geq 0}$

$\boxed{-4 \leq x \leq 4}$ — (2)



$D_f = [-4, -\pi] \cup [0, \pi]$

Domain of Definition of the Function:

Find the domain of the function

2. $f(x) = \log_{100x}\left(\dfrac{2\log_{10} x+1}{-x}\right)$

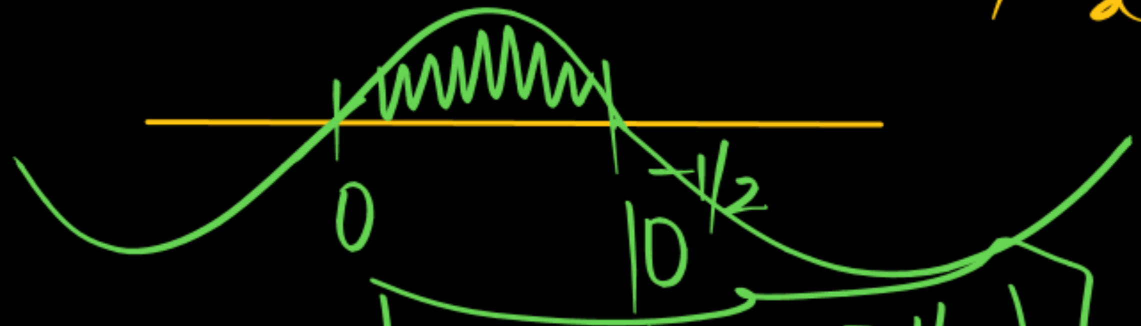
For $f(x)$ is to be real,

$\dfrac{2\log_{10} x+1}{-x} > 0$ — (1)

$2\log_{10} x+1=0 \Rightarrow 2\log_{10} x=-1$

$\Rightarrow \log_{10} x = -1/2$

$\Rightarrow x = 10^{-1/2}$



$\boxed{x \in (0, 10^{-1/2})}$

$100x > 0 \Rightarrow \boxed{x>0}$

& $100x \neq 1$ $\Rightarrow \boxed{x \neq \frac{1}{100}}$

finally $D_f \equiv (0, 10^{-1/2}) - \left\{\frac{1}{100}\right\}$

Domain of Definition of the Function:

The domain of definition of the function:

$$y=\frac{1}{\log_{10}(1-x)}+\sqrt{x+2}$$ {IIT 1983}

for $f(x)$ is to be real,

$x+2 \geq 0 \Rightarrow \boxed{x \geq -2}$ — (1)

& $\log_{10}(1-x) \neq 0$

$\Rightarrow 1-x \neq 10^0 \Rightarrow 1-x \neq 1$

$\Rightarrow \boxed{x \neq 0}$ — (2)

& $1-x > 0 \Rightarrow \boxed{x < 1}$ — (3)

from (1), (2), (3); we get

$D_f \equiv [-2, 1) - \{0\}$

Range of the Function:

Let $f: R \to R$ be defined by $f(x) = \dfrac{x}{1+x^2}$, $x \in R$. Then the Range of f is-

[JEE MAIN 2019]

$f(x) = \dfrac{x}{1+x^2}$

as $1+x^2 \neq 0 \ \forall \ x \in R$

$\Rightarrow \boxed{D_f \equiv R}$ $x \in R$

for Range:

let $f(x) = y$

$\Rightarrow \dfrac{x}{1+x^2} = y$

$\Rightarrow x = y + yx^2$

$\Rightarrow \boxed{yx^2 - x + y = 0}$

Quadratic in x,

as $x \in R$

$\Rightarrow D \geq 0$

$\Rightarrow (-1)^2 - 4y \cdot y \geq 0$

$\Rightarrow \boxed{1 - 4y^2 \geq 0}$

$y^2 \leq \frac{1}{4}$

$\Rightarrow \boxed{y \in \left[-\frac{1}{2}, \frac{1}{2}\right]}$

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Range of the Function:

Range of the Function:

Find the domain & Range of the function:

$f(x) = {}^{x+1}C_{2x-8}$

Home assignment

${}^{n}C_{r}$ is defined

iff $\left.\begin{array}{l} n > 0 \\ r \geq 0 \\ n \geq r \end{array}\right\} n, r \in I$

Thank You Lakshyians