Electric charges & field

Presented by
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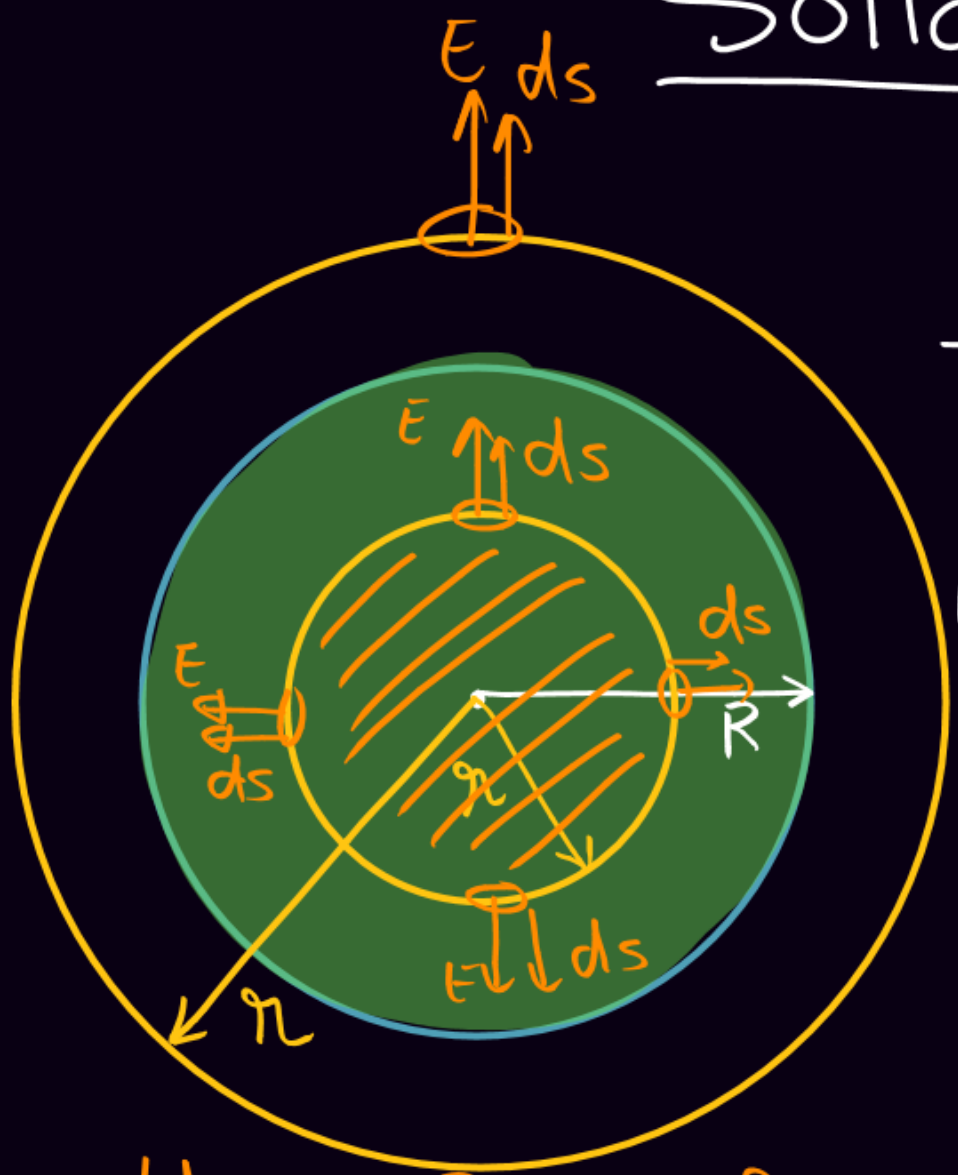
Todays Goal

Applications of Gauss law

Solid sphere



Q (Uniform distribution)

$$\boxed{\rho = \frac{Q}{\frac{4}{3}\pi R^3}}$$

Volume charge density

Inside $\boxed{r < R}$

$$\frac{4}{3}\pi R^3 \rightarrow Q$$

$$1 \rightarrow \frac{Q}{\frac{4}{3}\pi R^3}$$

$$\frac{4}{3}\pi r^3 \rightarrow \frac{Q}{\cancel{\frac{4}{3}}\pi R^3} \times \cancel{\frac{4}{3}}\pi r^3 = Q\, r^3/R^3$$

$$\oint \vec{E} \cdot d\vec{s} = \frac{q_{in}}{\varepsilon_0}$$

$$\oint E\, ds \cos 0 = \frac{q_{in}}{\varepsilon_0}$$

$$E \oint ds = \frac{Q\, r^3/R^3}{\varepsilon_0}$$

$$E\, 4\pi r^2 = \frac{Q\, r^3}{R^3 \varepsilon_0}$$

$$E = \frac{Q}{4\pi\varepsilon_0} \frac{r}{R^3}$$

$$\boxed{E = \frac{KQr}{R^3}}$$

$$\boxed{E_{in} \propto r}$$

$$\boxed{E = \frac{\rho r}{3\varepsilon_0}}$$

Outside point $(r > R)$

$$\oint \vec{E} \cdot d\vec{s} = \frac{q_{in}}{\varepsilon_0}$$

$$E \, 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$\boxed{E = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{KQ}{r^2}}$$

$$\boxed{E_{out} \propto \frac{1}{r^2} \quad \Big| \quad E_{in} \propto r}$$

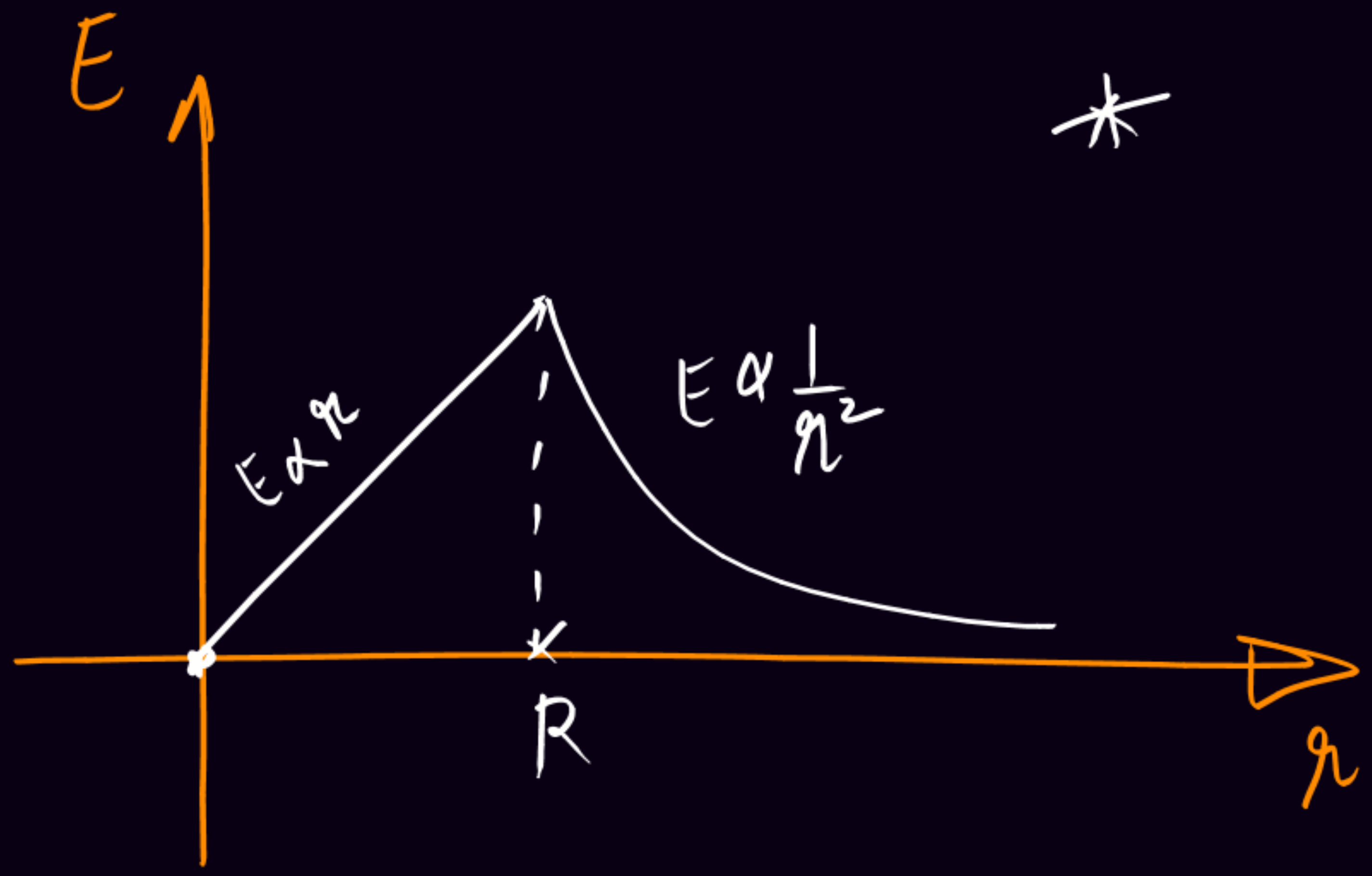
In terms of ρ

$$E = \frac{\rho \frac{4}{3}\pi R^3}{4\pi \varepsilon_0 r^2} = \boxed{\frac{\rho}{3\varepsilon_0} \frac{R^3}{r^2}}$$

(A solid sphere can also be treated as a point charge placed at the center)

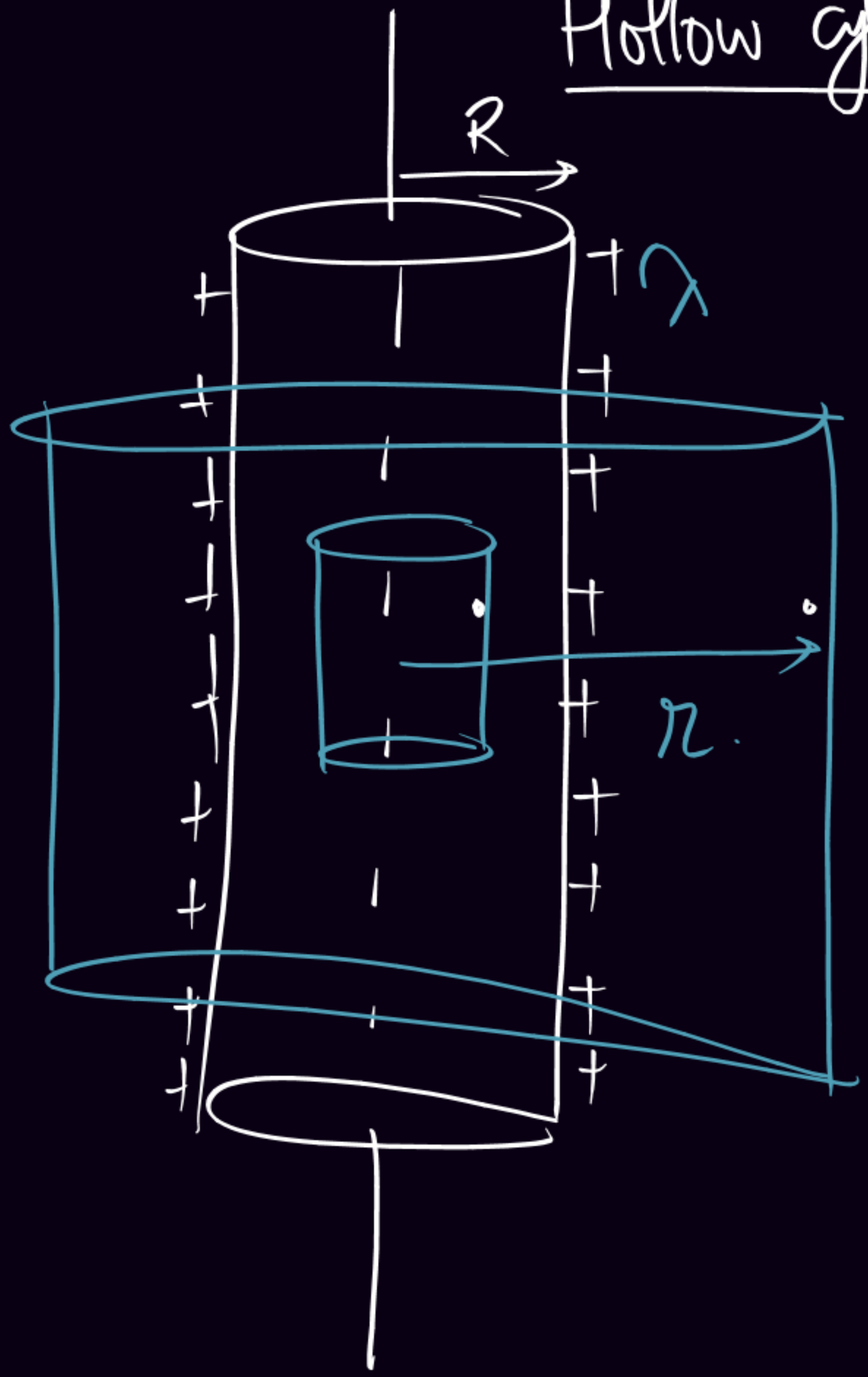
Sphere

- shell
  - Inside
    - $E_{in} = 0$
  - Outside
    - $E_{out} = \frac{KQ}{r^2}$
- solid
  - Inside
    - $E_{in} = \frac{KQr}{R^3}$
    - $E_{in} = \frac{\rho r}{3\varepsilon_0}$
  - Outside
    - $E_{out} = \frac{KQ}{r^2}$
    - $E_{out} = \left(\frac{\rho}{3\varepsilon_0} \frac{R^3}{r^2}\right)$



$$\boxed{\begin{array}{l} K \longrightarrow G \\ q \longrightarrow m \end{array}}$$

Hollow cylinder



$E_{in} = 0$

$E_{out} = \frac{2k\lambda}{r}$ (just like an infinite wire placed on the axis).

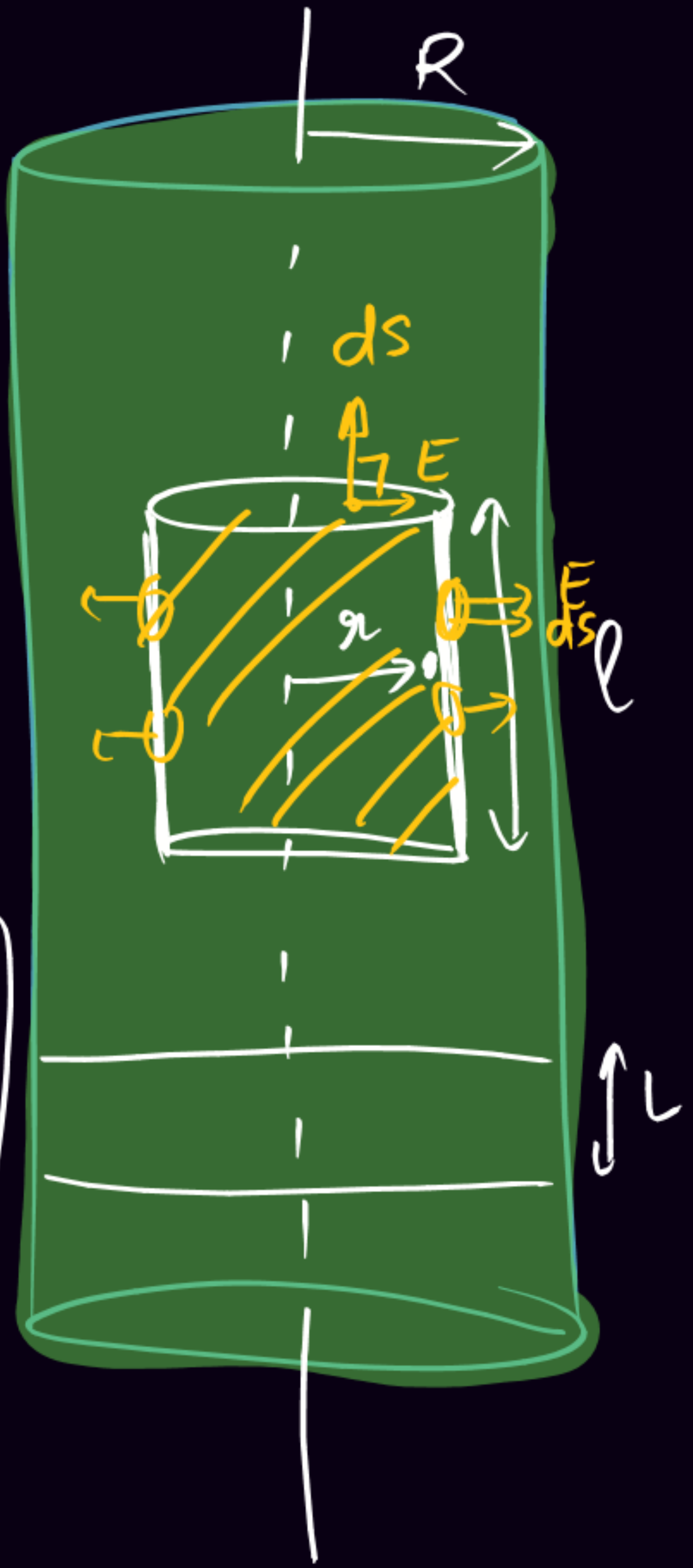
Solid cylinder

$\rho = \frac{Q}{Vol.}$

$\lambda = \frac{Q}{L}$

$\rho = \frac{Q}{\pi R^2 l}$

$\boxed{\rho = \frac{\lambda}{\pi R^2}}$



Inside point

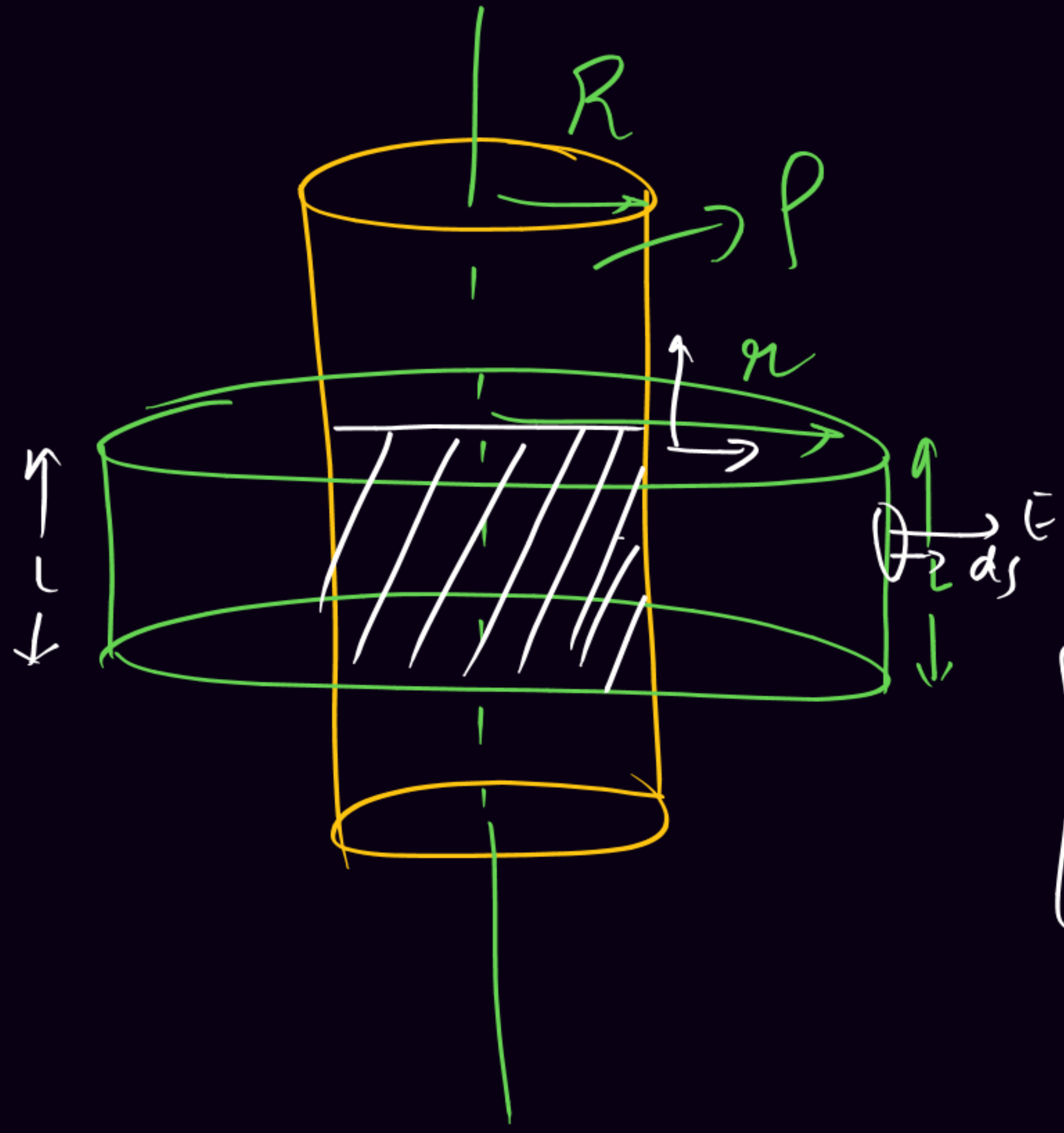
$\oint \vec{E} \cdot \vec{ds} = \frac{q_{in}}{\varepsilon_0}$

$\cancel{\int_{flat} E\, ds \cos 90^\circ} + \int_{curved} E\, ds \cos 0 = \frac{\rho \times \pi r^2 l}{\varepsilon_0}$

$E \int_{curved} ds = \frac{\rho \pi r^2 l}{\varepsilon_0}$

$E\, 2\cancel{\pi}\cancel{r}\cancel{l} = \frac{\rho \cancel{\pi} r^{\cancel{2}} \cancel{l}}{\varepsilon_0}$

$E = \frac{\rho r}{2\varepsilon_0}$



$$\underbrace{\int E\,ds\,\cancel{\cos 90}}_{\text{flat}} + \underbrace{\int E\,ds\cos 0}_{\text{curved}} = \frac{\rho \times (\pi R^2 L)}{\varepsilon_0}$$

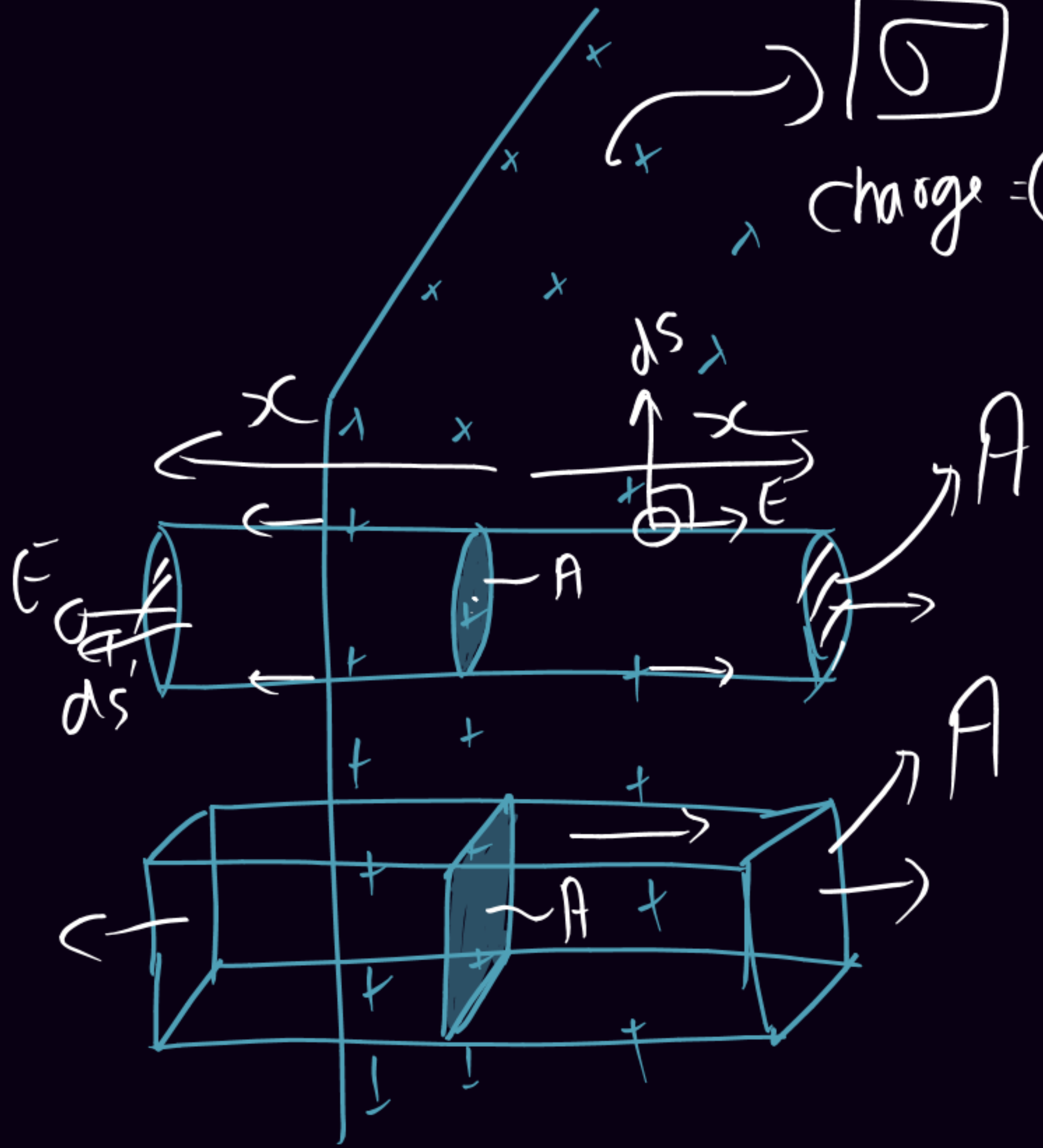
$$E\,2\cancel{\pi} r \cancel{L} = \frac{\rho \cancel{\pi} R^2 \cancel{L}}{\varepsilon_0}$$

$$\boxed{E = \frac{\rho}{2\varepsilon_0}\frac{R^2}{r}}$$

Nonconducting infinite charged sheet

$\boxed{\sigma} \Rightarrow$ surface charge density

charge $= \sigma \times Area$



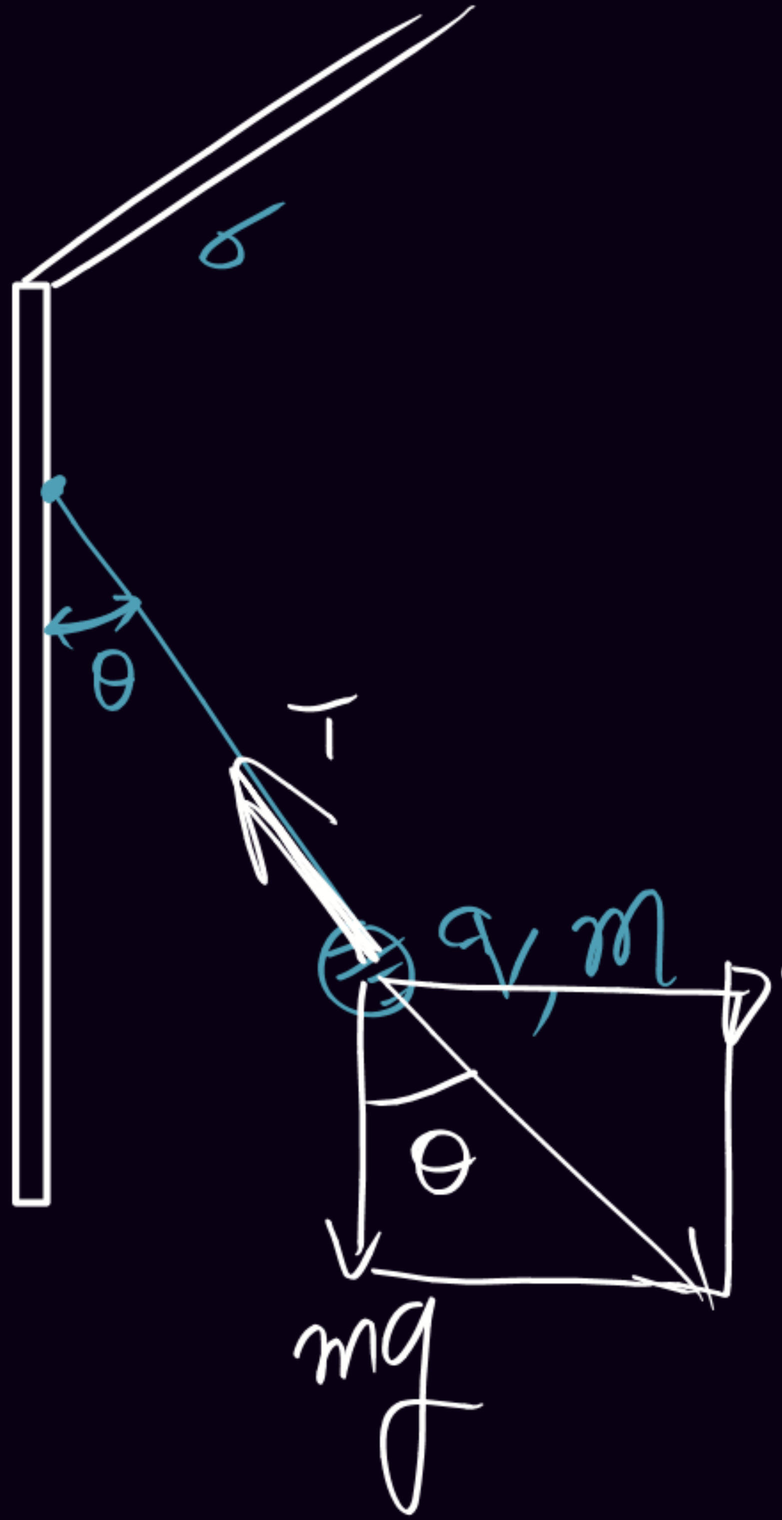
$$\int_{curved} E\,ds\,\cancel{\cos(90^\circ)} + \int_{flat} E\,ds\cos(0) = \frac{\sigma A}{\varepsilon_0}$$

$$E\int_{flat} ds = \frac{\sigma A}{\varepsilon_0}$$

$$E(2\cancel{A}) = \frac{\sigma \cancel{A}}{\varepsilon_0}$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

* (independent of x).

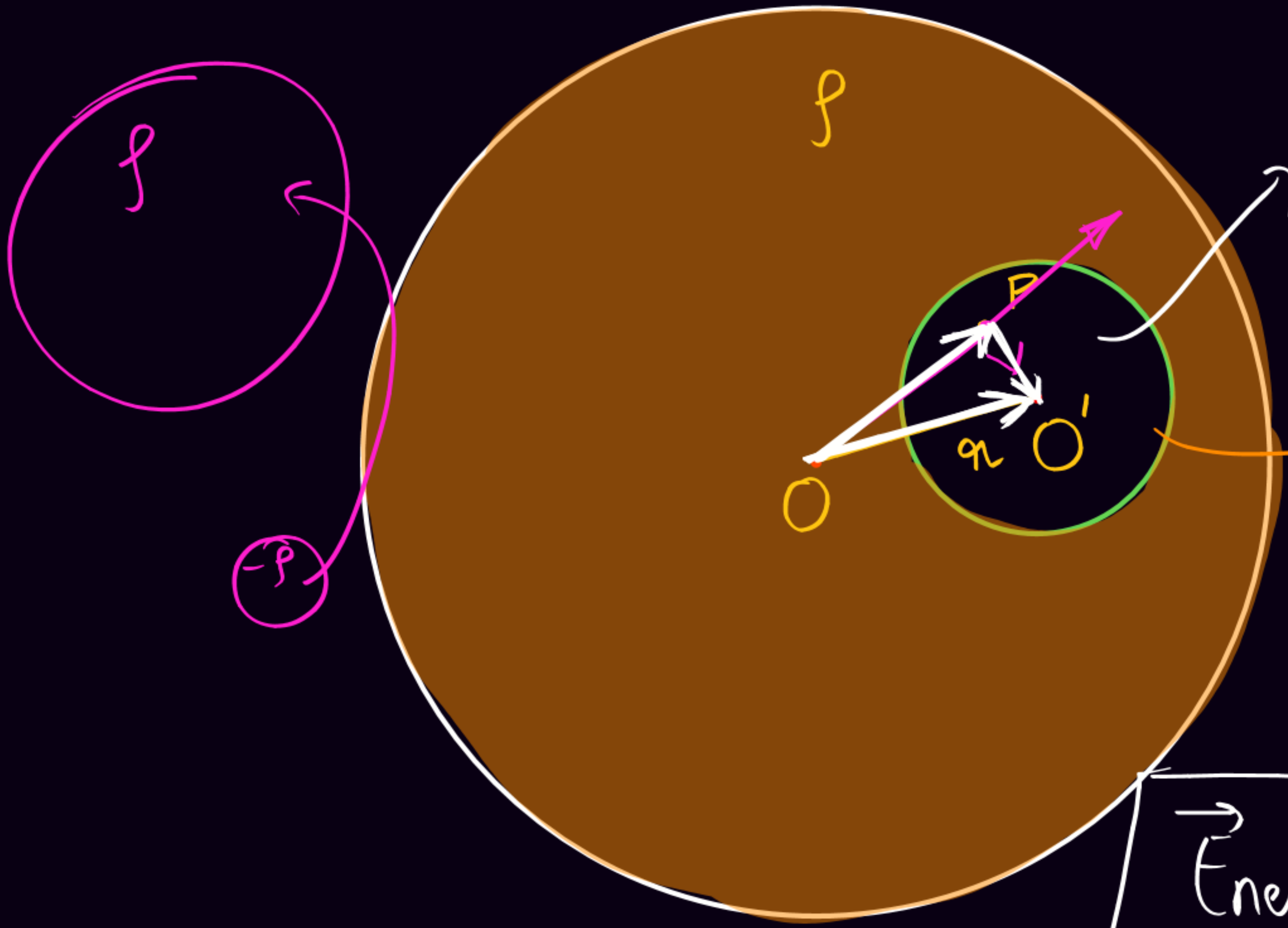


$$qE = \frac{\sigma}{2\varepsilon_0} q = F$$

$$\boxed{\text{Find } \theta = ?}$$

$$\tan\theta = \frac{\sigma q}{2\varepsilon_0 mg}$$

$$\theta = \tan^{-1}\left(\frac{\sigma q}{2\varepsilon_0 mg}\right)$$



$E_P = ?$

$E = \frac{\rho r}{3\varepsilon_0}$

Uniform d.elf

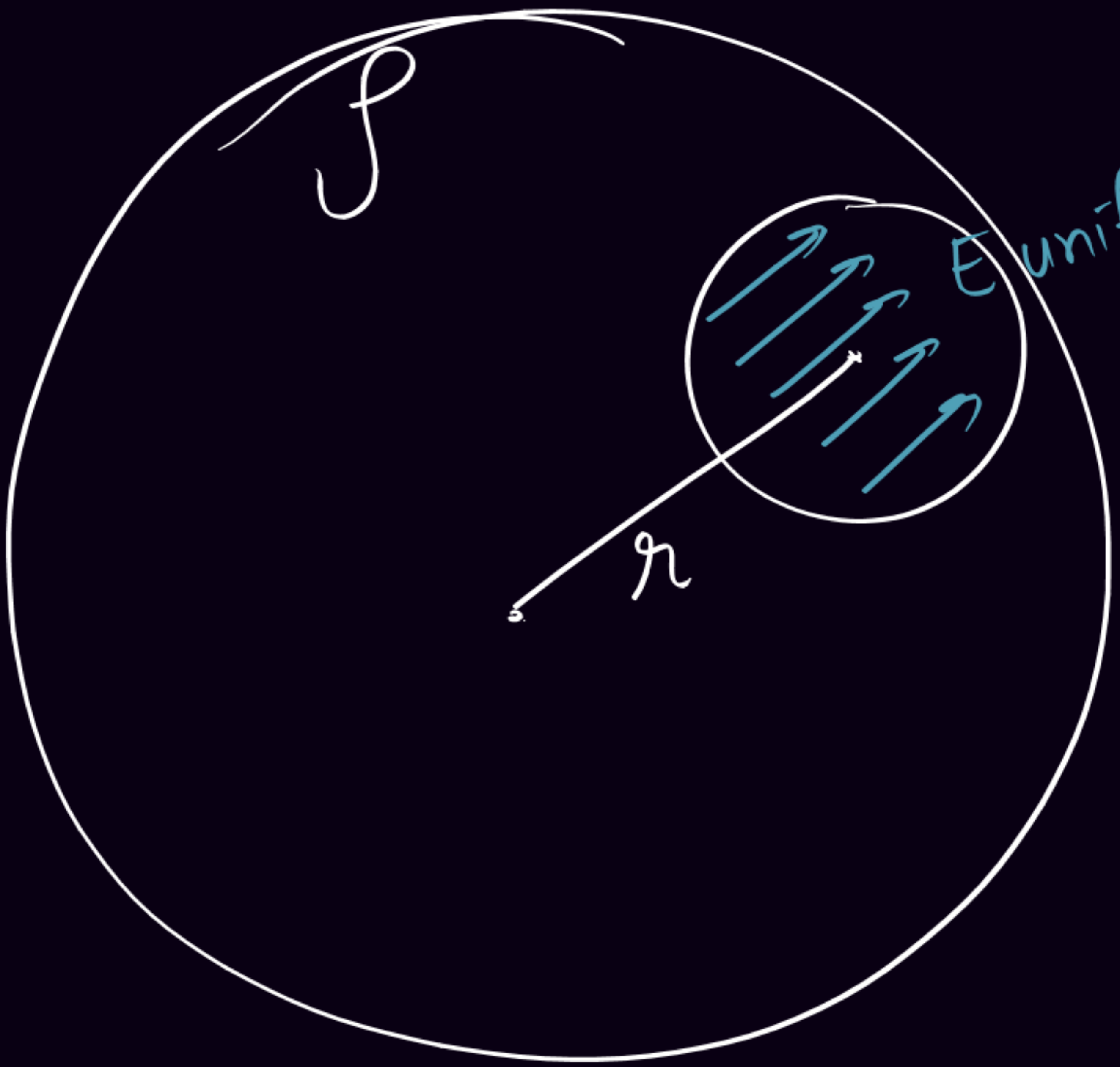
Cavity

$\vec{E}_{net} = \frac{\rho \vec{OP}}{3\varepsilon_0} + \frac{\rho \vec{PO'}}{3\varepsilon_0}$

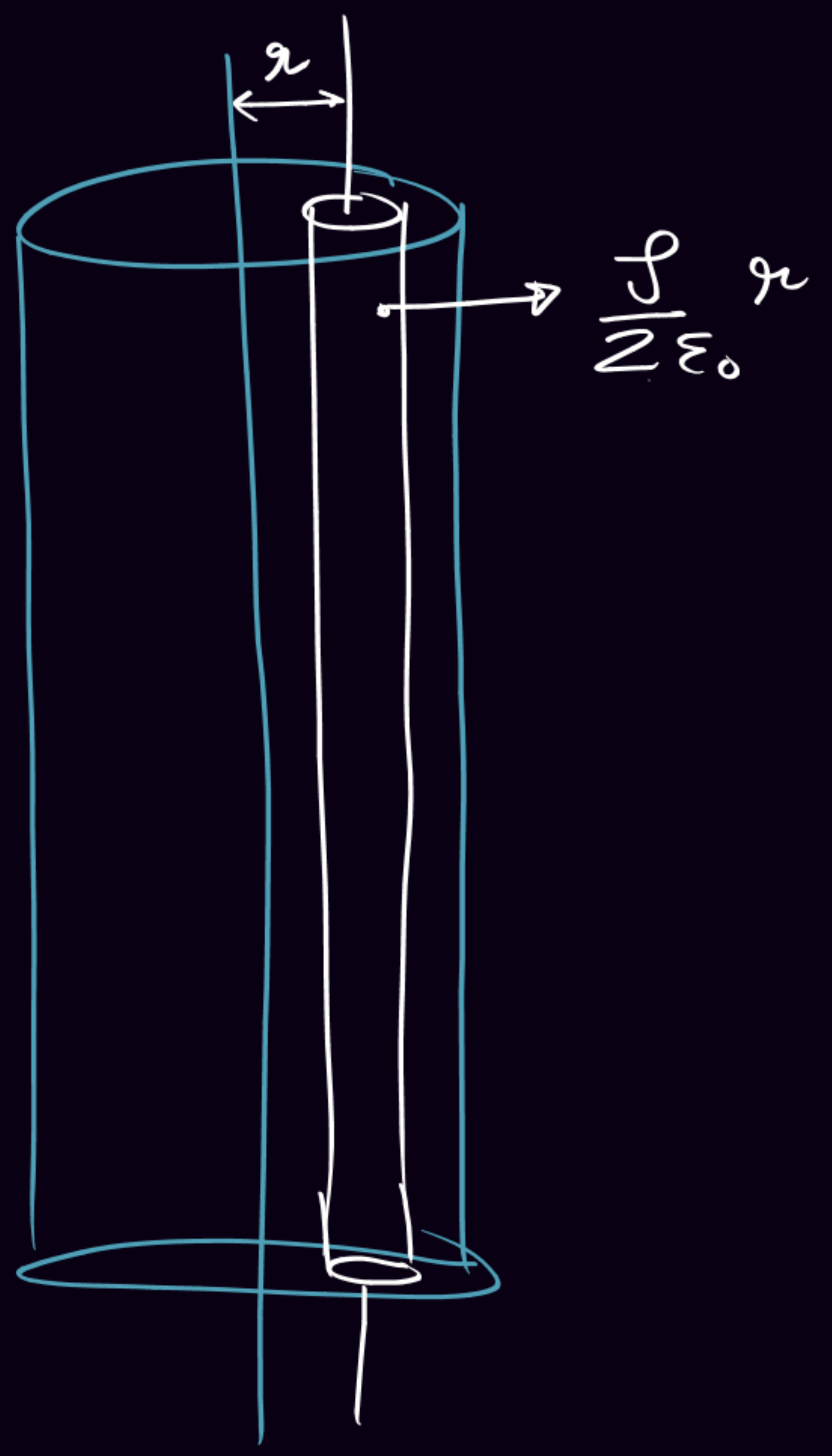
$= \frac{1}{3\varepsilon_0}\rho\left(\vec{OP} + \vec{PO'}\right)$

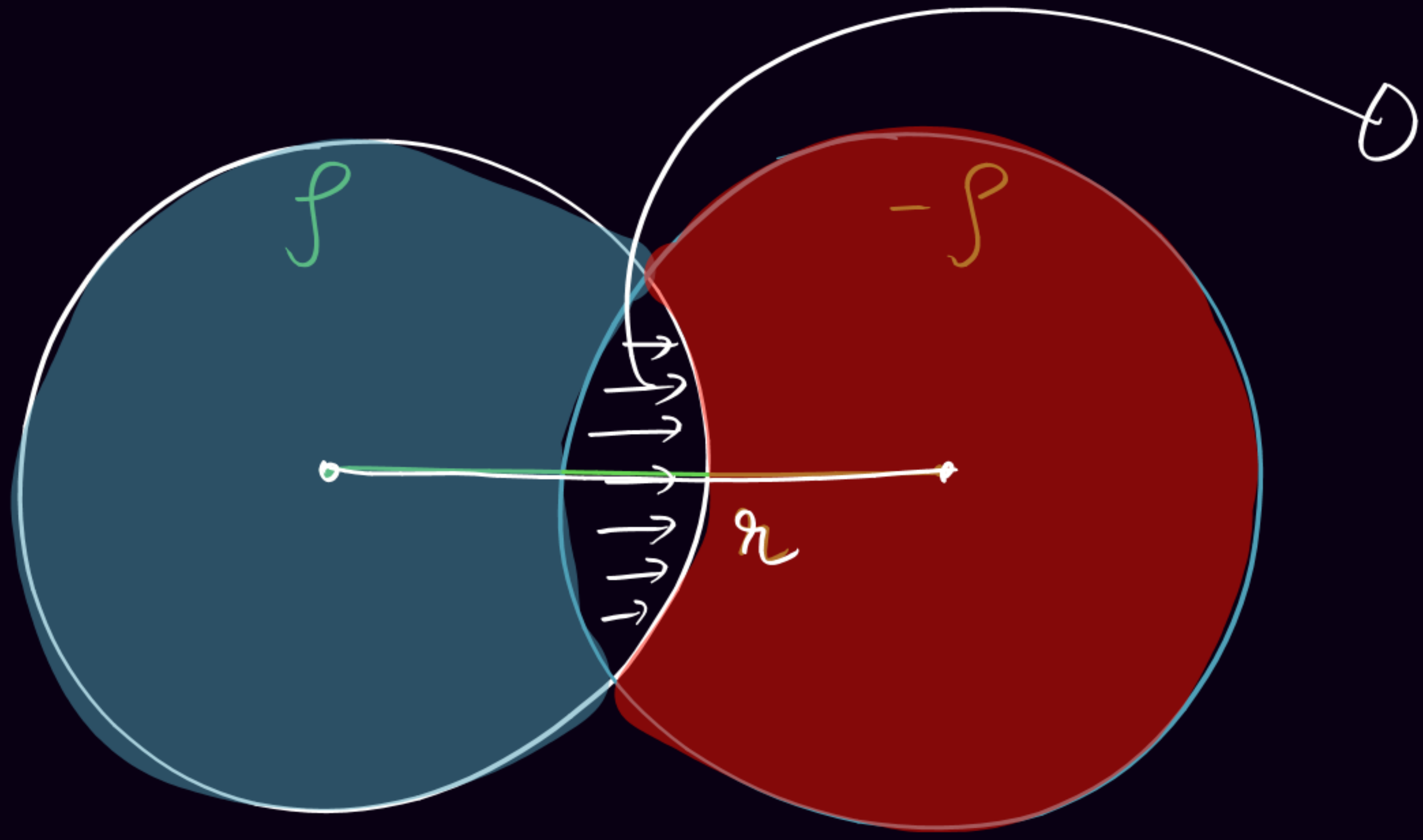
$\boxed{\vec{E}_{net} = \frac{\rho}{3\varepsilon_0}\vec{OO'}}$

$\boxed{E_{net} = \frac{\rho r}{3\varepsilon_0}}$

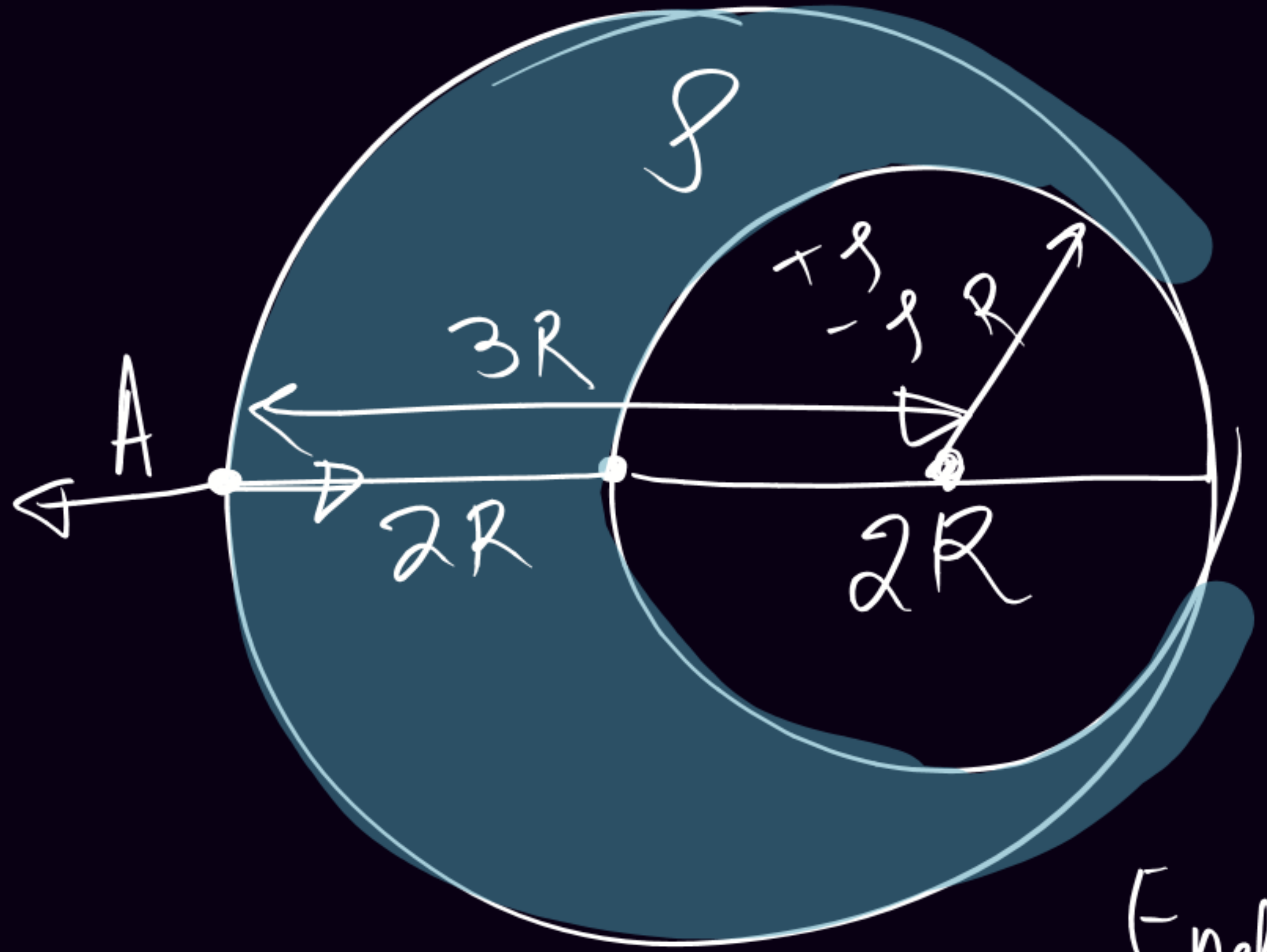


$$E_{uniform} = \frac{\rho r}{3\varepsilon_0}$$ → distance between centers of sphere & cavity.


r
$\frac{\rho}{2\varepsilon_0} r$



$$\boxed{\frac{\rho}{3\varepsilon_0}\, r}$$



$$\boxed{\text{Find } E_A = ?}$$

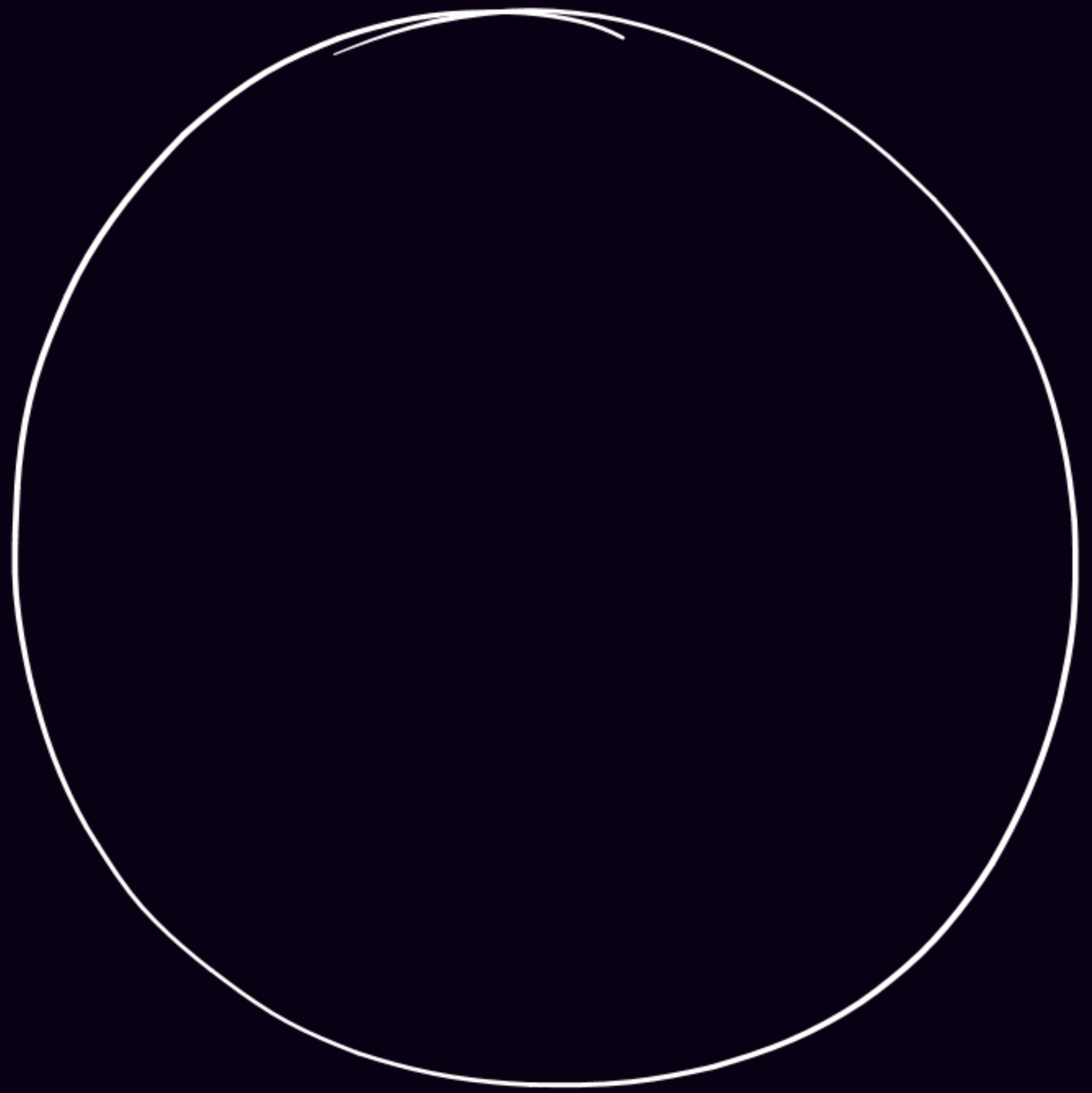
$$E_A \text{ complete sphere} = \frac{\rho}{3\varepsilon_0}(2R)\ (\text{left})$$

$$E_A(-\rho \text{ at cavity}) = \frac{\rho}{3\varepsilon_0}\frac{(R^3)}{(3R)^2}\ (\text{Right})$$

$$E_{net} = \frac{\rho}{3\varepsilon_0}2R - \frac{\rho}{3\varepsilon_0}\frac{R^{\cancel{3}}}{9\cancel{R^2}}$$

$$= \frac{\rho R}{3\varepsilon_0}\left(2 - \frac{1}{9}\right) = \frac{17}{9}\frac{\rho R}{3\varepsilon_0}.$$

H.W.



$$\rho = \rho_0 r \quad (0 < r \leq R)$$

$$\rho = 0 \quad (r > R)$$

Find $E_{R/2}$, E_{2R} ?

Don't stop when you're tired.
Stop when you're done!

THANK YOU!!