

Άσκηση 4

$$i) |x-4| \leq 2 \Leftrightarrow \dots \Leftrightarrow 2 \leq x \leq 6 \text{ . Άρα } A = \{2,3,4,5,6\} \text{ και } P(A) = \frac{N(A)}{N(\Omega)} = \frac{5}{10}$$

$$x^2 - 4x + 3 \leq 0 \Leftrightarrow 1 \leq x \leq 3 \text{ . Άρα } B = \{1,2,3\} \text{ και } P(B) = \frac{N(B)}{N(\Omega)} = \frac{3}{10}$$

$$(x-4)^2 \geq 3x-2 \Leftrightarrow \dots \Leftrightarrow x^2 - 11x + 18 \geq 0 \Leftrightarrow x \in (-\infty, 2] \cup [9, +\infty) \text{ . Άρα } \Gamma = \{1,2,9,10\}$$

$$\text{και } P(\Gamma) = \frac{N(\Gamma)}{N(\Omega)} = \frac{4}{10}$$

$$ii) A \cup B = \{1,2,3,4,5,6\} \text{ , } A \cap \Gamma = \{2\} \text{ , } P(A \cup B) = \frac{6}{10} \text{ , } P(A \cap \Gamma) = \frac{1}{10}$$

Άσκηση 5

$$i) A = (A-B) \cup (A \cap B) = \{1,2,3,4,6\} \text{ , } P(A) = \frac{N(A)}{N(\Omega)} = \frac{5}{10}$$

$$B-A = \{5\} \text{ , } B = \{1,3,4,5\} \text{ , } P(B) = \frac{N(B)}{N(\Omega)} = \frac{4}{10}$$

$$|2x-5| < 3 \Leftrightarrow \dots \Leftrightarrow 1 < x < 4 \text{ . Άρα } P(\Gamma) = \frac{N(\Gamma)}{N(\Omega)} = \frac{2}{10} \text{ .}$$

$$ii) B-\Gamma = \{1,4,5\} \text{ , } \Gamma-B = \{2\} \text{ . Άρα } P(B-\Gamma) = \frac{3}{10}$$

$$iii) P[(B-\Gamma) \cup (\Gamma-B)] = \frac{4}{10}$$

Άσκηση 9

$$i) P(A) = 1 - P(A') = 0,6$$

$$ii) P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0,4$$

$$iii) P[(A-B) \cup (B-A)] = 0,1$$

$$iv) P(\Gamma) + \frac{1}{P(\Gamma)} \geq -2 \Leftrightarrow P^2(\Gamma) + 2P(\Gamma) + 1 \geq 0 \Leftrightarrow (P(\Gamma) + 1)^2 \geq 0 \text{ ισχύει.}$$

Άσκηση 11

$$p(A) = \frac{1800}{2400} = \frac{3}{4} \text{ , } p(B) = \frac{800}{2400} = \frac{1}{3} \text{ , } p(A \cap B) = \frac{600}{2400} = \frac{1}{4}$$

$$i) p(A \cup B) = p(A) + p(B) - p(A \cap B) = \frac{3}{4} + \frac{1}{3} - \frac{1}{4} = \frac{5}{6}$$

$$\text{ii) } p[(A \cup B)'] = 1 - p(A \cup B) = 1 - \frac{5}{6} = \frac{1}{6}$$

$$\text{iii) } p(B \cap A') = p(B - A) = p(B) - p(A \cap B) = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$$

$$\text{iv) } p(A \cup B') = p(A) + p(B') - p(A \cap B') = p(A) + 1 - p(B) - p(A - B) =$$

$$\cancel{p(A)} + 1 - p(B) - \cancel{p(A)} + p(A \cap B) = 1 - p(B) + p(A \cap B) = 1 - \frac{1}{3} + \frac{1}{4} = \frac{11}{12}$$

Άσκηση 13

$$\text{Από τη σχέση που δίνει έχουμε: } P(A) = \frac{1}{3}, P(B') = \frac{1}{2} \Rightarrow P(B) = \frac{1}{2}, P(A \cap B) = \frac{1}{6}$$

$$\text{i) } P(A - B) = \frac{1}{6}$$

$$\text{ii) } P(B - A) = \frac{1}{3}$$

$$\text{iii) } P[(A - B) \cup (B - A)] = \frac{1}{2}$$

$$\text{iv) } P[(A \cup B)'] = 1 - P(A \cup B) = \frac{1}{3}$$

Άσκηση 25

$$\text{i) } (a - 2)^2 + (\beta + 3)^2 = 0 \text{ .Άρα } \alpha=2, \beta=-3$$

$$\text{ii) } \alpha) A = \frac{15}{\sqrt{5} \cdot (\sqrt{2} - 3)} = \frac{15\sqrt{5}(\sqrt{2} + 3)}{\sqrt{5}^2 \cdot (\sqrt{2} - 3) \cdot (\sqrt{2} + 3)} = \frac{15\sqrt{5}(\sqrt{2} + 3)}{-5} \dots\dots$$

$$\beta) B = \sqrt{3 + 2\sqrt{2}} - \sqrt{3 - 2\sqrt{2}} = \sqrt{(\sqrt{2} + 1)^2} - \sqrt{(\sqrt{2} - 1)^2} = \sqrt{2} + 1 - \sqrt{2} + 1 = 2$$

$$\gamma) \Gamma = \sqrt[3]{2^3 \cdot \sqrt{2^5 \sqrt{2}}} = \sqrt[30]{2^{30} \cdot 2^5 \cdot 2} = \sqrt[30]{2^{36}} = \sqrt[5]{2^6} = \sqrt[5]{64}$$

Άσκηση 26

$$\text{i) } a = \frac{2^{13} \cdot 2^{16}}{2^{27}} = 2^2 = 4$$

$$\beta = \sqrt[3]{9(3 - \sqrt{6})(3 + \sqrt{6})} = \sqrt[3]{9(3^2 - \sqrt{6}^2)} = \sqrt[3]{27} = 3$$

ii)

$$K = \sqrt[4]{\frac{1}{8} \cdot \frac{1}{27}} \cdot \sqrt[4]{\frac{1}{8} \cdot \sqrt{\left(\frac{1}{27}\right)^2}} = \sqrt[4]{\left(\frac{1}{6}\right)^3} \cdot \sqrt[4]{\frac{1}{8} \sqrt{\left(\frac{2}{9}\right)^3}} = \sqrt[4]{\left(\frac{1}{6}\right)^3} \cdot \sqrt[4]{\frac{1 \cdot 2}{8 \cdot 9}} = \sqrt[4]{\left(\frac{1}{6}\right)^3} \cdot \frac{1}{6} = \sqrt[4]{\left(\frac{1}{6}\right)^4}$$

Άσκηση 27

i)

$$a = \sqrt{\sqrt{3}^2 + 1 - 2\sqrt{3}} + \sqrt{5^2 + \sqrt{3}^2 - 10\sqrt{3}} = \sqrt{(\sqrt{3} - 1)^2} + \sqrt{(5 - \sqrt{3})^2} = \sqrt{3} - 1 + 5 - \sqrt{3} = 4$$

$$\beta = \sqrt[30]{2^{10} \cdot 3^2 \cdot 2} : \sqrt[30]{3^6 \cdot 2^3 \cdot 2} \cdot \sqrt[10]{2} \cdot \sqrt[15]{9} = \sqrt[30]{2^6 \cdot 3^{-4}} \cdot \sqrt[30]{2^3} \cdot \sqrt[30]{9^2} = \sqrt[30]{2^9} = \sqrt[10]{2^3} = \sqrt[10]{8}$$

$$\text{ii) } A = \sqrt[6]{4} \cdot \sqrt[5]{8} = \sqrt[3]{2} \cdot \sqrt[5]{2^3} = \sqrt[15]{2^5 \cdot 2^9} = \sqrt[15]{2^{14}}$$

$$\text{iii) } \sqrt[10]{8} \cdot x = 4 \Leftrightarrow x = \frac{4}{\sqrt[10]{8}} = \frac{4}{\sqrt[10]{2^3}} \cdot \frac{\sqrt[10]{2^7}}{\sqrt[10]{2^7}} = \frac{4 \sqrt[10]{2^7}}{2} = 2 \sqrt[10]{2^7}$$