125 MIDTERM TWO REVIEW OUTLINE ©

****Note this is just me thinking about what might be on the test. I could be missing stuff and I have no actual idea what will be on your exam. This just gives you an idea of the general type of question, but you should definitely do as many old midterm questions as possible to prepare.*

Outline

1. Work
   (a) Pumping water
   (b) Lifting Ropes
   (c) Spring Stuff
   (d) Grab Bag Weird Stuff
2. Average Value
3. Integration by Parts
4. Trigonometric Identity Integrals
5. Trig Sub
6. Making Stuff Look Like an Arctrig Function
7. Partial Fractions
8. Approximation Techniques
   (a) Trapezoid Rule
   (b) Simpsons' Rule
9. Improper Integrals
10. Arc Length

Matt's Power Notesheet would include...

Work

First take a dy slice (or dx, dr whatever is appropriate).

(i) If dy represents the width of something then ask two questions.

- (a) How heavy is the dy slice?
- (b) How far do I have to move it?
- (c) Then integrate over the appropriate distance the product $\int(\text{How Heavy})(\text{How Far})dy$.

(ii) If dy represents a small change in distance then ask

- (a) What is the force required at this instant?
- (b) Then integrate over the appropriate distance $\int(\text{Force})\ dy$.

Average Value

The average value of a function $f(x)$ on the interval $[a, b]$ is $\frac{1}{b-a}\int_a^b f(x)dx$.

Integration by Parts

- $\int x^n e^x dx$ requires n applications of integration by parts.
- $\int e^x \sin x dx$ requires 2 application of integration by parts then you end up with a $-\int e^x \sin x$ term which you add to both sides.
- $\int(\text{an arctrig function})dx$. Let $u =$(arctrig function) and $dv = dx$.

Trig Integrals

Be sure the following are on your notesheet

- Double angle identities.
- $\sin^2 x + \cos^2 x = 1$, $\tan^2 x + 1 = \sec^2 x$, $1 + \cot^2 x = \csc^2 x$.
- Derivative of $\sin, \cos, \tan, \csc, \sec, \cot$.
- Derivatives of $\arcsin, \arccos, \arctan$.

Some key trig integrals to know are

- $\int \sin^n x \cos^m x dx$.
- $\int \sec x dx$.
- $\int \sec^3 x dx$.
- $\int \tan^2 x \sec x$.

When solving a trig integral keep the following tips in mind.

(i) First consider reducing to just sin and cos in your integral or only tan and sec.

(ii) Try using either $\sin^2 x + \cos^2 x = 1$ or $\tan^2 x + 1 = \sec^2 x$.

(iii) Usually you will finish with a u-sub with $u = \sin x$, $u = \tan x$ or $u = \sec x$. Always be looking for this option.

Trig-Sub

Here are the steps.

1. Get into form $u^2 - a^2$, $a^2 - u^2$ or $a^2 + u^2$. Possibly need to complete the square.
   (a) To complete the square. First factor out a constant so it looks like $A[x^2 + Bx + C]$
   (b) then rewrite as $A[(x + \frac{B}{2})^2 - (B/2)^2 + C]$.
2. Make the appropriate trig sub. $u = a \sec\theta$, $u = a \sin\theta$ or $u = a \tan\theta$. (these correspond to the situations in (1).)
3. Don't forget to substitute in the du term!
4. Use a quadratic trig identity.
5. Cancel as much as possible.
6. Solve the resulting trig integral.
7. Make a reference triangle.
   (a) Draw a triangle.
   (b) Write the formula for your trig substitution next to it.
   (c) Intuit what the other sides should be.

Partial Fractions

- Try and factor the denominator.
- If it factors then use the appropriate technique. Remember

$$\text{Number of Constants } A, B, C, \ldots = \text{ Degree of Denominator.}$$

Approximation Techniques

My only tip is to write an exact answer (if you aren't asked for a decimal). Also, practice with your scientific calculator beforehand. Write your steps very clearly in the even of a calculator error.

Improper Integrals

(i) First solve the indefinite integral.

(ii) Then plug in the appropriate limits.

Arc Length

Use the formula. Most likely the resulting inside will be a perfect square or something you can solve using trig-sub.

Practice Problems

1. Work

   (a) Pumping water

   A tank of liquid has the shape of the top half of a sphere of radius 4 ft. It has a spigot located at its topmost point. Find the work done in pumping the liquid out of the tank. Assume that the liquid has density 1 pound per cubic foot. $(= \frac{320\pi}{3})$

   (b) Lifting Ropes

   Water is drawn from a well that is 25 meters deep using a leaky bucket that initially scoops up 20 kg of water from the bottom of the well. The mass of the bucket is 2 kg and the mass of the rope that is attached is .2 kg/m. The rope is being pulled at a constant rate of .5 m/s. The bucket has a hole in it and water leaks from the bucket at a rate of .1 kg/s. *How much work is done to get the bucket to the top of the well?* $(= 7546)$ Joules

   (c) Spring Stuff

   A spring has natural length 10 cm. The spring is now allowed to hang vertically with the top end attached to a rigid support and the other end attached to a mass of 1 kg. This causes the spring to stretch 3 cm, to a length of 13 cm. A small child pulls down the mass further, stretching it to 15 cm. How much work does the child do? (Hint: First find the spring constant).$(k = \frac{9.8}{.03},\ answer = .0002k \approx .0653)$

   It takes 18ft-lb to stretch a spring from 2 ft beyond its natural length to 4 ft beyond its natural length. How far beyond its natural length can the spring be stretched with a force not exceeding 24 pounds? $(= 8ft)$

   (d) Grab Bag Weird Stuff

   If something weird is asked it will probably be very straightforward to do, so don't do anything too complicated. Still, expect one of the first three types of work problems.

2. Average Value

   Find the average value of the function $f(x) = \sqrt{x}$ on $0 \le x \le 2$. Also, find the number c so that $f(x)$ has average value 1 on $[0, c]$.

3. Integration by Parts

$$\int_0^{\pi} x \sin^2 x \cos x \, dx \left(= -\frac{4}{9}\right)$$

4. Trigonometric Identity Integrals

$$\int_0^{\frac{\pi}{4}} \cos^6(3x) \sec^3(3x) \, dx \left(= \frac{5\sqrt{2}}{36}\right)$$

5. Trig Sub

$$\int_{\frac{1}{2}}^{1} \frac{\sqrt{1-x^2}}{x} \, dx \left(= \ln(2+\sqrt{3}) - \frac{\sqrt{3}}{2}\right)$$

$$\int \frac{\sqrt{3-2x-x^2}}{x+1} \left(= 2\ln\left|\frac{2-\sqrt{3-2x-x^2}}{x+1}\right| + \sqrt{3-2x-x^2} + C \right)$$

6. Making Stuff Look Like an Arctrig Function

$$\int \frac{1}{x^{\frac{2}{3}}(x^{\frac{2}{3}}+3)} \left(= \frac{\sqrt{3}}{3} \arctan\left(\frac{x^{\frac{1}{3}}}{\sqrt{3}}\right) + C \right)$$

7. Partial Fractions

$$\int \frac{2x+3}{x^2-2x+1}\,dx \left(= 2\ln(x-1) - \frac{5}{x-1} + C \right)$$

$$\int \frac{\sin(3t)\cos(3t)}{\cos^2(3t) - 3\cos(3t) + 2}\,dt \left(= \frac{1}{3}\ln|\cos(3t)-1| - \frac{2}{3}\ln|\cos(3t)-2| + C \right)$$

8. Approximation Techniques

   (a) Trapezoid Rule

   (b) Simpsons' Rule

Use Simpson's Rule and Trapezoid Rule with $n = 4$ to approximate $\int_3^5 \frac{e^x}{x}\,dx$
(= 30.25 (with Simpsons' and I didn't figure out Trapezoid. It should be close to 30...))

9. Improper Integrals

Decide whether the following integrals are convergent or divergent. If they converge evaluate the integral.

   (a) $\int_1^\infty \frac{2x+3}{x^2+3x+1}\,dx$ (Diverges)

   (b) $\int_0^\infty xe^{-x}\,dx (= 1)$

   (c) $\int_2^\infty \frac{\ln x}{x^4}\,dx \left(= \frac{\ln 2}{24} + \frac{1}{72}\right)$

10. Arc Length

I would expect an application of Simpson's rule with arc length (since arc length integrals are typically uncomputable). So try to approximate the length of $y = e^{x^2}$ from $0 \le x \le 1$ using Simpson's and $n = 4$. Answer is too messy to type.

Compute the length of the curve $y = -\ln(\cos\theta)$ on $0 \le \theta \le \frac{\pi}{4}$ $\left(= \ln(\sqrt{2}+1)\right)$