CHAPTER 4

BUOYANCY AND FLOATATION

4.1 INTRODUCTION

In this chapter, the equilibrium of the floating and sub-merged bodies will be considered. Thus the chapter will include : 1. Buoyancy, 2. Centre of buoyancy, 3. Metacentre, 4. Metacentric height, 5. Analytical method for determining metacentric height, 6. Conditions of equilibrium of a floating and sub-merged body, and 7. Experimental method for metacentric height.

4.2 BUOYANCY

When a body is immersed in a fluid, an upward force is exerted by the fluid on the body. This upward force is equal to the weight of the fluid displaced by the body and is called the force of buoyancy or simply buoyancy.

4.3 CENTRE OF BUOYANCY

It is defined as the point, through which the force of buoyancy is supposed to act. As the force of buoyancy is a vertical force and is equal to the weight of the fluid displaced by the body, the centre of buoyancy will be the centre of gravity of the fluid displaced.

Problem 4.1 *Find the volume of the water displaced and position of centre of buoyancy for a wooden block of width 2.5 m and of depth 1.5 m, when it floats horizontally in water. The density of wooden block is 650 kg/m³ and its length 6.0 m.*

Solution. Given :

Width $= 2.5$ m

Depth $= 1.5$ m

Length $= 6.0$ m

Volume of the block $= 2.5 \times 1.5 \times 6.0 = 22.50 \text{ m}^3$

Density of wood, $\rho = 650 \text{ kg/m}^3$

$\therefore$ Weight of block $= \rho \times g \times \text{Volume}$

$= 650 \times 9.81 \times 22.50 \text{ N} = 143471 \text{ N}$

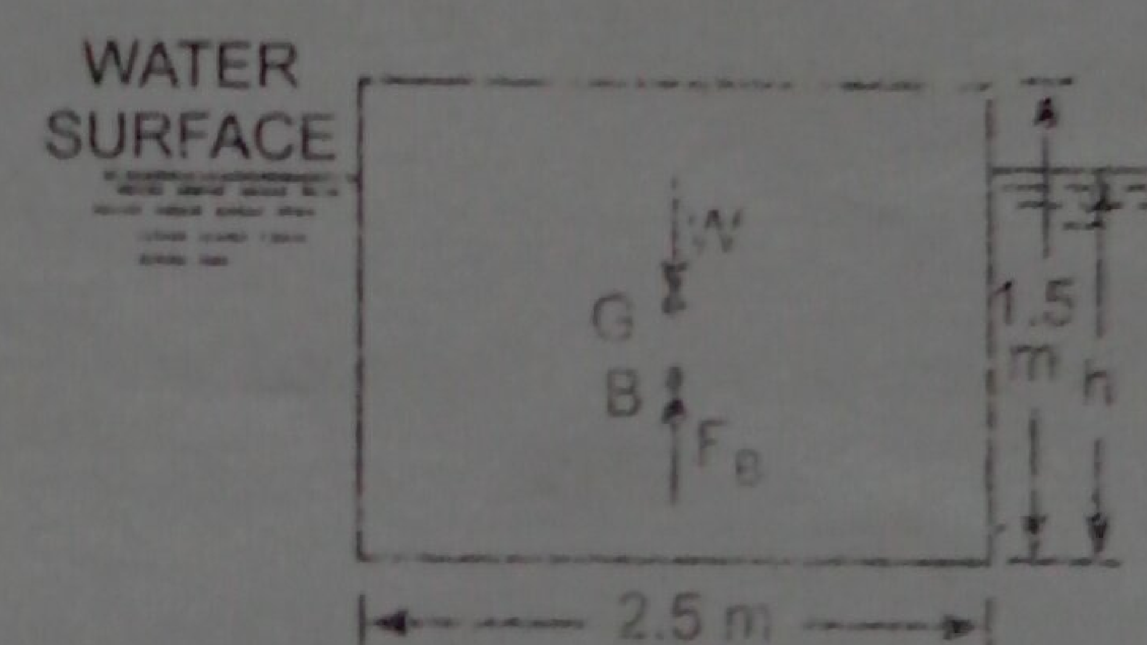


Fig. 4.1

For equilibrium the weight of water displaced = Weight of wooden block

$= 143471$ N

$\therefore$ Volume of water displaced

$$= \frac{\text{Weight of water displaced}}{\text{Weight density of water}} = \frac{143471}{1000 \times 9.81} = \mathbf{14.625\ m^3.\ Ans.}$$

($\because$ Weight density of water $= 1000 \times 9.81$ N/m^3)

Position of Centre of Buoyancy. Volume of wooden block in water

= Volume of water displaced

or $2.5 \times h \times 6.0 = 14.625$ m^3, where h is depth of wooden block in water

$$\therefore \quad h = \frac{14.625}{2.5 \times 6.0} = 0.975 \text{ m}$$

$$\therefore \quad \text{Centre of Buoyancy} = \frac{0.975}{2} = \mathbf{0.4875\ m\ from\ base.\ Ans.}$$

Problem 4.2 *A wooden log of 0.6 m diameter and 5 m length is floating in river water. Find the depth of the wooden log in water when the sp. gravity of the log is 0.7.*

Solution. Given :

Dia. of log $= 0.6$ m

Length, $L = 5$ m

Sp. gr., $S = 0.7$

$\therefore$ Density of log $= 0.7 \times 1000 = 700$ kg/m^3

$\therefore$ **Weight density of log,** $w = \rho \times g$

$= 700 \times 9.81$ N/m^3

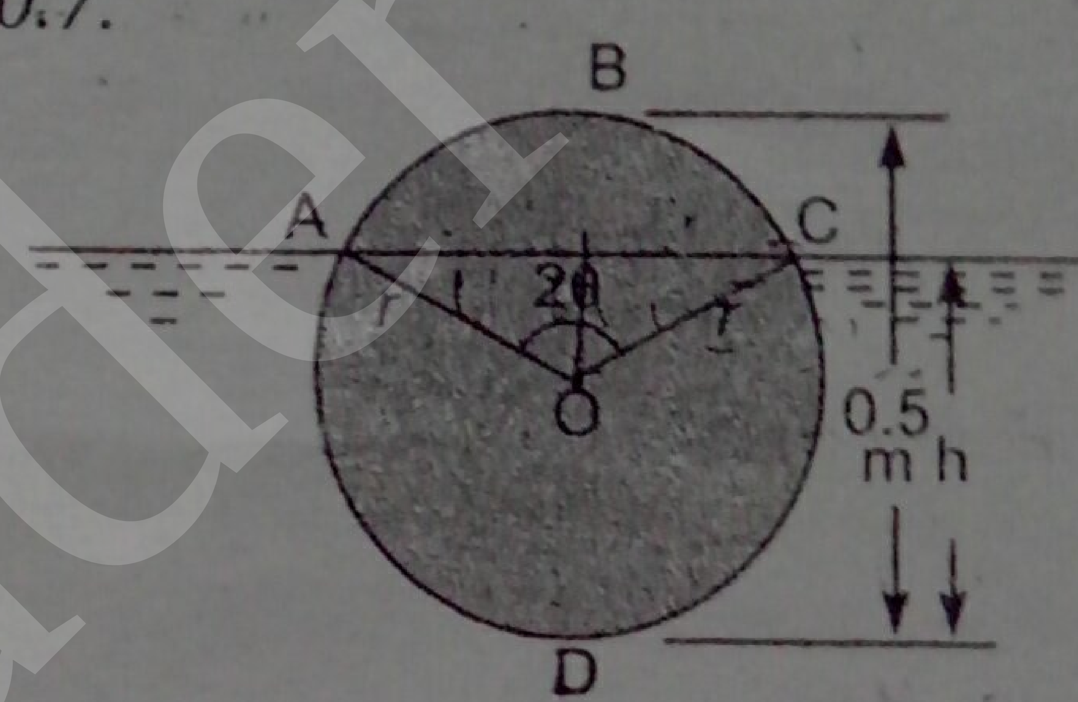


Fig. 4.2

Find depth of immersion or h

Weight of wooden log = Weight density $\times$ Volume of log

$$= 700 \times 9.81 \times \frac{\pi}{4}(D)^2 \times L$$

$$= 700 \times 9.81 \times \frac{\pi}{4}(.6)^2 \times 5 \text{ N} = 989.6 \times 9.81 \text{ N}$$

For equilibrium,

Weight of wooden log = Weight of water displaced

= Weight density of water $\times$ Volume of water displaced

$$\therefore \quad \text{Volume of water displaced} = \frac{989.6 \times 9.81}{1000 \times 9.81} = 0.9896 \text{ m}^3$$

($\because$ Weight density of water $= 1000 \times 9.81$ N/m^3)

Let h is the depth of immersion

$\therefore$ Volume of log inside water = Area of $ADCA \times$ Length

= Area of $ADCA \times 5.0$

But volume of log inside water = Volume of water displaced $= 0.9896$ m^3

$$\therefore \quad 0.9896 = \text{Area of } ADCA \times 5.0$$

$$\therefore \text{ Area of } ADCA = \frac{0.9896}{5.0} = 0.1979 \text{ m}^2$$

But area of $ADCA$ = Area of curved surface $ADCOA$ + Area of ΔAOC

$$= \pi r^2 \left[\frac{360° - 2\theta}{360°}\right] + \frac{1}{2} r \cos\theta \times 2r \sin\theta$$

$$= \pi r^2 \left[1 - \frac{\theta}{180°}\right] + r^2 \cos\theta \sin\theta$$

$$\therefore \quad 0.1979 = \pi (.3)^2 \left[1 - \frac{\theta}{180°}\right] + (.3)^2 \cos\theta \sin\theta$$

$$0.1979 = .2827 - .00157\,\theta + 0.9 \cos\theta \sin\theta$$

or $\quad .00157\,\theta - .09 \cos\theta \sin\theta = .2827 - .1979 = 0.0848$

$$\theta - \frac{.09}{.00157} \cos\theta \sin\theta = \frac{.0848}{.00157}$$

or $\quad \theta - 57.32 \cos\theta \sin\theta = 54.01.$

or $\quad \theta - 57.32 \cos\theta \sin\theta - 54.01 = 0$

For $\theta = 60°$, $\quad 60 - 57.32 \times 0.5 \times .866 - 54.01 = 60 - 24.81 - 54.01 = -18.82$

For $\theta = 70°$, $\quad 70 - 57.32 \times .342 \times 0.9396 - 54.01 = 70 - 18.4 - 54.01 = -2.41$

For $\theta = 72°$, $\quad 72 - 57.32 \times .309 \times .951 - 54.01 = 72 - 16.84 - 54.01 = +1.14$

For $\theta = 71°$, $\quad 71 - 57.32 \times .325 \times .9455 - 54.01 = 71 - 17.61 - 54.01 = -0.376$

$\therefore \quad \theta = 71.5°$, $\quad 71.5 - 57.32 \times .3173 \times .948 - 54.01 = 71.5 - 17.24 - 54.01 = +.248$

Then $\quad h = r + r\cos 71.5°$

$$= 0.3 + 0.3 \times 0.3173 = \mathbf{0.395\ m.} \text{ Ans.}$$

Problem 4.3 ***A stone weighs 392.4 N in air and 196.2 N in water. Compute the volume of stone and its specific gravity.***

Solution. Given :

Weight of stone in air = 392.4 N

Weight of stone in water = 196.2 N

For equilibrium,

Weight in air – Weight of stone in water = Weight of water displaced

or $\quad 392.4 - 196.2 = 196.2 = 1000 \times 9.81 \times$ Volume of water displaced

$\therefore$ Volume of water displaced

$$= \frac{196.2}{1000 \times 9.81} = \frac{1}{50} \text{ m}^3 = \frac{1}{50} \times 10^6 \text{ cm}^3 = \mathbf{2 \times 10^4\ cm^3}. \text{ Ans.}$$

$$= \text{Volume of stone}$$

$\therefore$ Volume of stone $= \mathbf{2 \times 10^4\ cm^3}$. Ans.

Specific Gravity of Stone

Mass of stone $= \dfrac{\text{Weight in air}}{g} = \dfrac{392.4}{9.81} = 40 \text{ kg}$

Density of stone $= \dfrac{\text{Mass in air}}{\text{Volume}} = \dfrac{40.0 \text{ kg}}{\dfrac{1}{50} \text{m}^3} = 40 \times 50 = 2000 \dfrac{\text{kg}}{\text{m}^3}$

$\therefore$ Sp. gr. of stone $= \dfrac{\text{Density of stone}}{\text{Density of water}} = \dfrac{2000}{1000} = 2.0.$ **Ans.**

Problem 4.4 *A body of dimensions 1.5 m × 1.0 m × 2 m, weighs 1962 N in water. Find its weight in air. What will be its specific gravity ?*

Solution. Given :

Volume of body $= 1.50 \times 1.0 \times 2.0 = 3.0 \text{ m}^3$

Weight of body in water $= 1962$ N

Volume of the water displaced = Volume of the body = 3.0 m^3

$\therefore$ Weight of water displaced $= 1000 \times 9.81 \times 3.0 = 29430$ N

For the equilibrium of the body

Weight of body in air – Weight of water displaced = Weight in water

$$\therefore \quad W_{air} - 29430 = 1962$$

$$W_{air} = 29430 + 1962 = 31392 \text{ N}$$

Mass of body $= \dfrac{\text{Weight in air}}{g} = \dfrac{31392}{9.81} = 3200 \text{ kg}$

Density of the body $= \dfrac{\textbf{Mass}}{\textbf{Volume}} = \dfrac{3200}{3.0} = 1066.67$

$\therefore$ **Sp. gravity of the body** $= \dfrac{1066.67}{1000} = 1.067.$ **Ans.**

Problem 4.5 *Find the density of a metallic body which floats at the interface of mercury of sp. gr. 13.6 and water such that 40% of its volume is sub-merged in mercury and 60% in water.*

Solution. Let the volume of the body $= V$ m^3

Then volume of body sub-merged in mercury

$$= \frac{40}{100} V = 0.4\, V \text{ m}^3$$

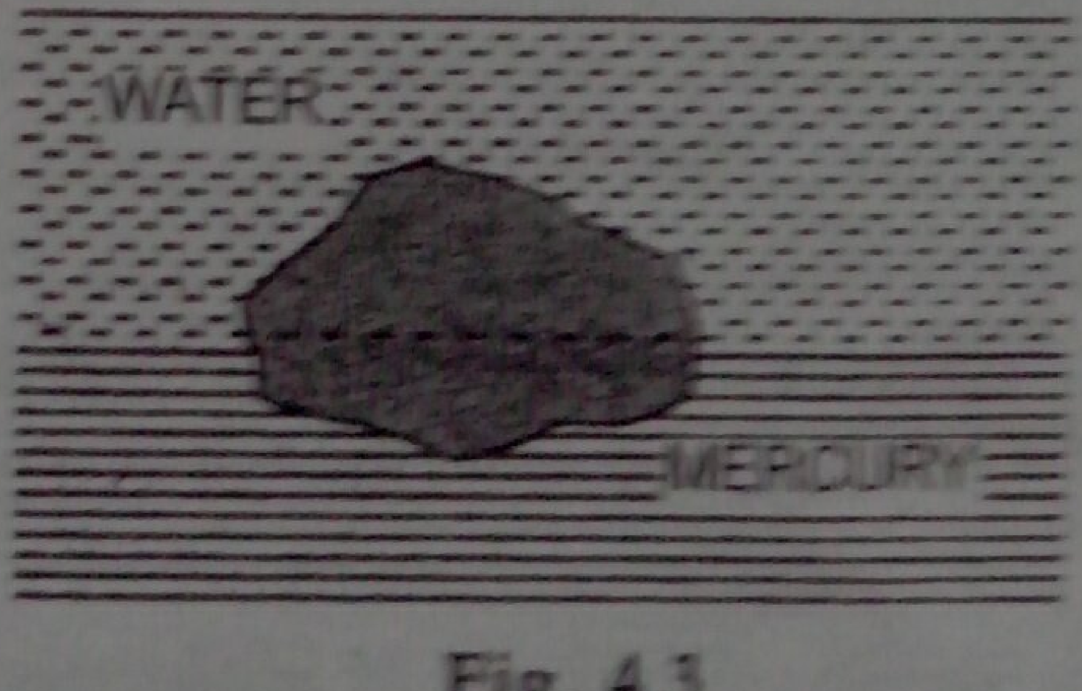


Fig. 4.3

Volume of body sub-merged in water

$$= \frac{60}{100} \times V = 0.6\, V \text{ m}^3$$

For the equilibrium of the body

Total buoyant force (upward force) = Weight of the body

But total buoyant force = Force of buoyancy due to water + Force of buoyancy due to mercury

Force of buoyancy due to water = Weight of water displaced by body

= Density of water × g × Volume of water displaced

= 1000 × g × Volume of body in water

$= 1000 \times g \times 0.6 \times V$ N

and Force of buoyancy due to mercury $=$ Weight of mercury displaced by body

$= g \times$ Density of mercury $\times$ Volume of mercury displaced

$= g \times 13.6 \times 1000 \times$ Volume of body in mercury

$= g \times 13.6 \times 1000 \times 0.4\, V$ N

Weight of the body $=$ Density $\times g \times$ Volume of body $= \rho \times g \times V$

where ρ is the density of the body

$\therefore$ For equilibrium, we have

Total buoyant force $=$ Weight of the body

$$1000 \times g \times 0.6 \times V + 13.6 \times 1000 \times g \times .4\, V = \rho \times g \times V$$

or $$\rho = 600 + 13600 \times .4 = 600 + 54400 = 6040.00 \text{ kg/m}^3$$

$\therefore$ Density of the body $=$ **6040.00 kg/m³. Ans.**

Problem* 4.6 *A float valve regulates the flow of oil of sp. gr. 0.8 into a cistern. The spherical float is 15 cm in diameter. AOB is a weightless link carrying the float at one end, and a valve at the other end which closes the pipe through which oil flows into the cistern. The link is mounted in a frictionless hinge at O and the angle AOB is 135°. The length of OA is 20 cm, and the distance between the centre of the float and the hinge is 50 cm. When the flow is stopped AO will be vertical. The valve is to be pressed on to the seat with a force of 9.81 N to completely stop the flow of oil into the cistern. It was observed that the flow of oil is stopped when the free surface of oil in the cistern is 35 cm below the hinge. Determine the weight of the float.*

Solution. Given :

Sp. gr. of oil $= 0.8$

$\therefore$ **Density of oil,** $\rho_0 = 0.8 \times 1000 = 800$ kg/m³

Dia. of float, $D = 15$ cm

$\angle AOB = 135°$

$OA = 20$ cm

Force, $P = 9.81$ N

$OB = 50$ cm

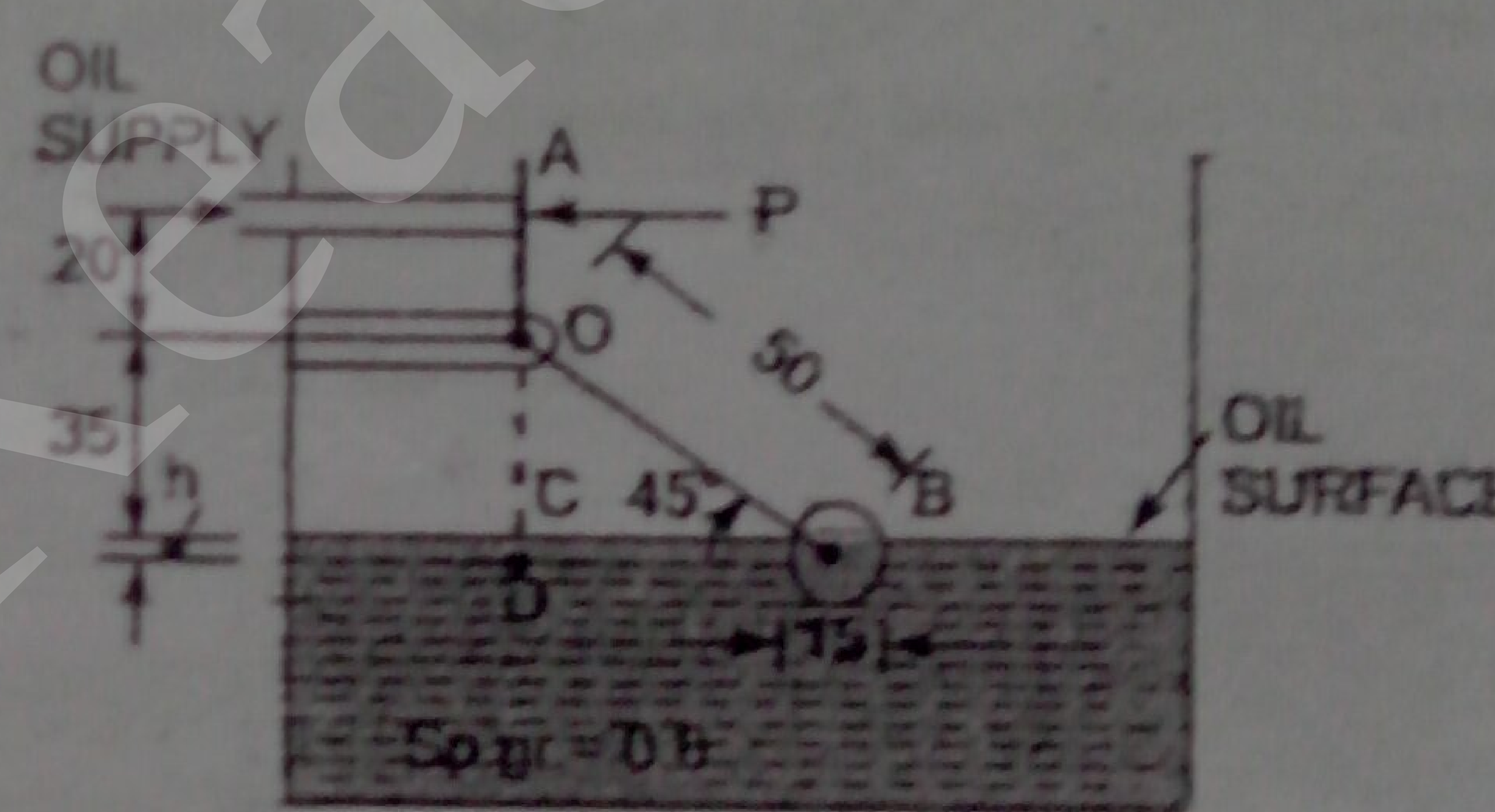


Fig. 4.4

Find the weight of the float. Let it is equal to W.

When the flow of oil is stopped, the centre of float is shown in Fig. 4.4

The level of oil is also shown. The centre of float is below the level of oil, by a depth 'h'.

From ΔBOD, $$\sin 45° = \frac{OD}{OB} = \frac{OC + CD}{OB} = \frac{35 + h}{50}$$

$\therefore$ $$50 \times \sin 45° = 35 + h$$

or $$h = 50 \times \frac{1}{\sqrt{2}} - 35 = 35.355 - 35 = 0.355 \text{ cm} = .00355 \text{ m.}$$

The weight of float is acting through B, but the upward buoyant force is acting through the centre of weight of oil displaced.

Volume of oil displaced $= \frac{2}{3}\pi r^3 + h \times \pi r^2$ $\left\{ r = \frac{D}{2} = \frac{15}{2} = 7.5 \text{ cm} \right\}$

$$= \frac{2}{3} \times \pi \times (.075)^3 + .00355 \times \pi \times (.075)^2 = 0.000945 \text{ m}^3$$

$\therefore$ Buoyant force = Weight of oil displaced

$$= \rho_0 \times g \times \text{Volume of oil}$$

$$= 800 \times 9.81 \times .000945 = 7.416 \text{ N}$$

The buoyant force and weight of the float passes through the same vertical line, passing through B. Let the weight of float is W. Then net vertical force on float

$$= \text{Buoyant force} - \text{Weight of float} = (7.416 - W)$$

Taking moments about the hinge O, we get

$$P \times 20 = (7.416 - W) \times BD = (7.416 - W) \times 50 \times \cos 45°$$

or

$$9.81 \times 20 = (7.416 - W) \times 35.355$$

$\therefore$

$$W = 7.416 - \frac{20 \times 9.81}{35.355} = 7.416 - 5.55 = \mathbf{1.866\ N.\ Ans.}$$

4.4 META-CENTRE

It is defined as the point about which a body starts oscillating when the body is tilted by a small angle. The meta-centre may also be defined as the point at which the line of action of the force of buoyancy will meet the normal axis of the body when the body is given a small angular displacement.

Consider a body floating in a liquid as shown in Fig. 4.5 (*a*). Let the body is in equilibrium and G is the centre of gravity and B the centre of buoyancy. For equilibrium, both the points lie on the normal axis, which is vertical.

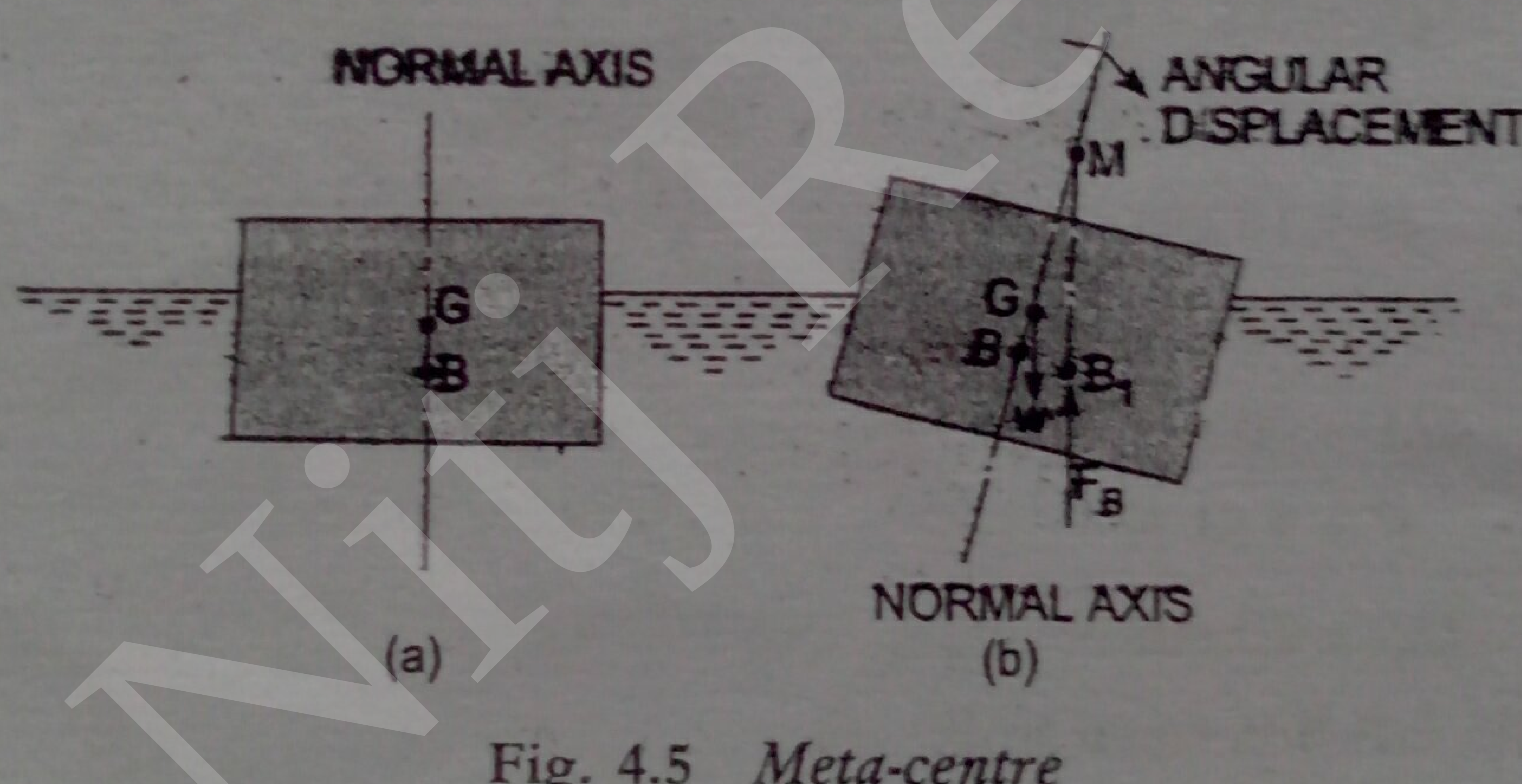


Fig. 4.5 *Meta-centre*

Let the body is given a small angular displacement in the clockwise direction as shown in Fig. 4.5 (*b*). The centre of buoyancy, which is the centre of gravity of the displaced liquid or centre of gravity of the portion of the body sub-merged in liquid, will now be shifted towards right from the normal axis. Let it is at B_1 as shown in Fig. 4.5 (*b*). The line of action of the force of buoyancy in this new position, will intersect the normal axis of the body at some point say M. This point M is called **Meta-centre.**

4.5 META-CENTRIC HEIGHT

The distance MG, *i.e.*, the distance between the meta-centre of a floating body and the centre of gravity of the body is called meta-centric height.

▸ 4.6 ANALYTICAL METHOD FOR META-CENTRE HEIGHT

Fig. 4.6 (*a*) shows the position of a floating body in equilibrium. The location of centre of gravity and centre of buoyancy in this position is at *G* and *B*. The floating body is given a small angular displacement in the clockwise direction. This is shown in Fig. 4.6 (*b*). The new centre of buoyancy is at B_1. The vertical line through B_1 cuts the normal axis at *M*. Hence *M* is the meta-centre and *GM* is meta-centric height.

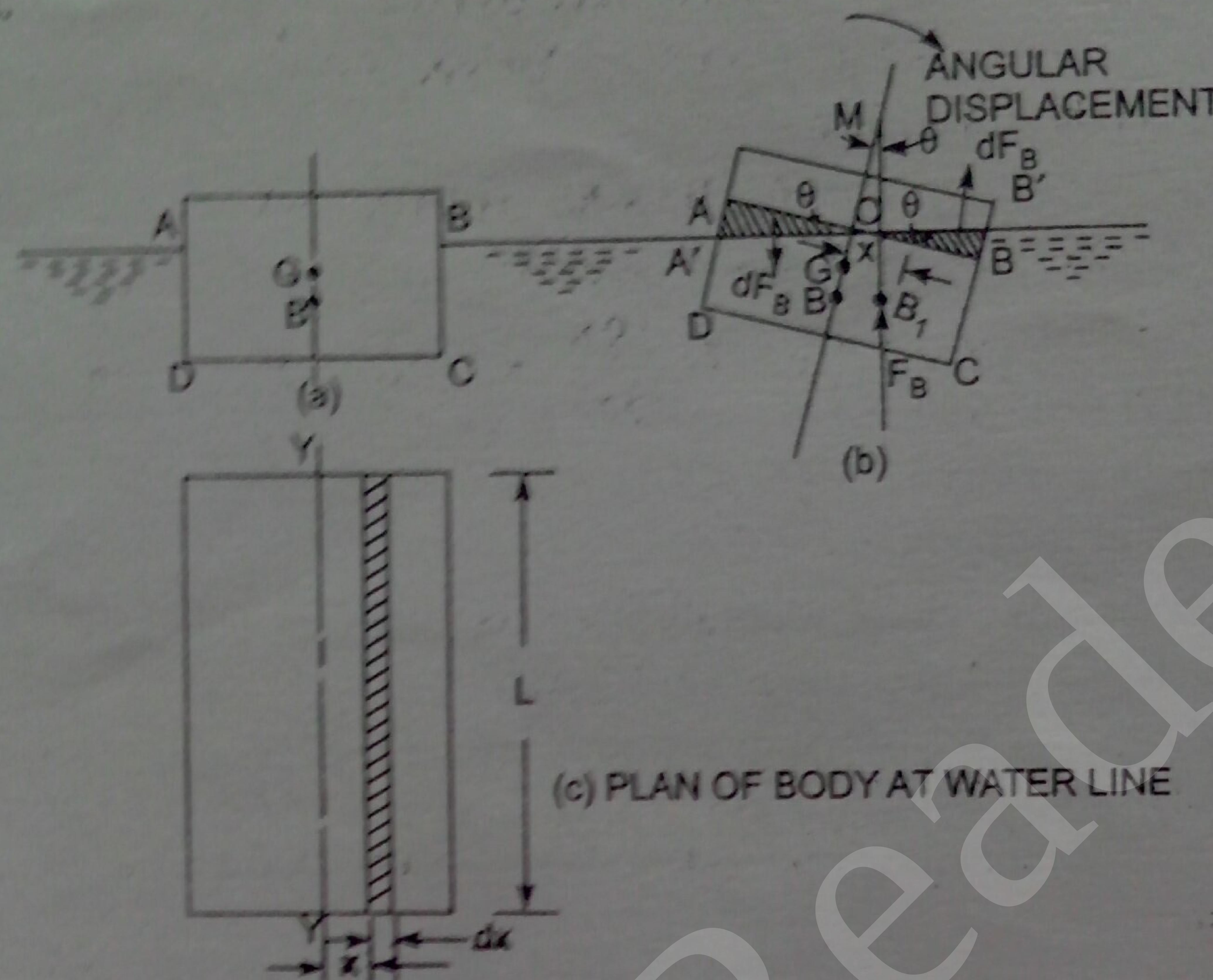


Fig. 4.6 Meta-centre height of floating body.

The angular displacement of the body in the clockwise direction causes the wedge-shaped prism *BOB′* on the right of the axis to go inside the water while the identical wedge-shaped prism represented by *AOA′* emerges out of the water on the left of the axis. These wedges represent a gain in buoyant force on the right side and a corresponding loss of buoyant force on the left side. The gain is represented by a vertical force dF_B acting through the C.G. of the prism *BOB′* while the loss is represented by an equal and opposite force dF_B acting vertically downward through the centroid of *AOA′*. The couple due to these buoyant forces dF_B tends to rotate the ship in the counterclockwise direction. Also the moment caused by the displacement of the centre of buoyancy from *B* to B_1 is also in the counterclockwise direction. Thus these two couples must be equal.

Couple Due to Wedges. Consider towards the right of the axis a small strip of thickness *dx* at a distance *x* from *O* as shown in Fig. 4.5 (*b*). The height of strip $x \times \angle BOB' = x \times \theta$.

$$\{\because \angle BOB' = \angle AOA' = BMB_1' = \theta\}$$

$\therefore$ Area of strip $= \text{Height} \times \text{Thickness} = x \times \theta \times dx$

If *L* is the length of the floating body, then

Volume of strip $= \text{Area} \times L$

$= x \times \theta \times L \times dx$

$\therefore$ Weight of strip $= \rho g \times \text{Volume} = \rho g x\, \theta L\, dx$

Similarly, if a small strip of thickness *dx* at a distance *x* from *O* towards the left of the axis is considered, the weight of strip will be $\rho g x \theta\, L\, dx$. The two weights are acting in the opposite direction and hence constitute a couple.

Moment of this couple = Weight of each strip × Distance between these two weights

$$= \rho g x \,\theta L\, dx\, [x + x]$$

$$= \rho g x\, \theta L\, dx \times 2x = 2\rho g x^2\, \theta L\, dx$$

∴ Moment of the couple for the whole wedge

$$= \int 2\rho g x^2\, \theta L\, dx \quad \text{...(4.1)}$$

Moment of couple due to shifting of centre of buoyancy from B to B_1

$$= F_B \times BB_1$$

$$= F_B \times BM \times \theta \quad \{\because BB_1 = BM \times \theta \text{ if } \theta \text{ is very small}\}$$

$$= W \times BM \times \theta \quad \{\because F_B = W\} \text{ ...(4.2)}$$

But these two couples are the same. Hence equating equations (4.1) and (4.2), we get

$$W \times BM \times \theta = \int 2\rho g x^2\, \theta\, L dx$$

$$W \times BM \times \theta = 2\rho g \theta \int x^2 L dx$$

$$W \times BM = 2\rho g \int x^2 L dx$$

Now Ldx = Elemental area on the water line shown in Fig. 4.6 (c) and = dA

∴ $$W \times BM = 2\rho g \int x^2 dA.$$

But from Fig. 4.5 (c) it is clear that $2\int x^2\, dA$ is the second moment of area of the plan of the body at water surface about the axis Y-Y. Therefore

$$W \times BM = \rho g I \quad \{\text{where } I = 2\int x^2\, dA\}$$

∴ $$BM = \frac{\rho g I}{W}$$

But **W = Weight of the body**

= Weight of the fluid displaced by the body

= ρg × Volume of the fluid displaced by the body

= ρg × Volume of the body sub-merged in water

$$= \rho g \times \forall$$

∴ $$BM = \frac{\rho g \times I}{\rho g \times \forall} = \frac{I}{\forall} \quad \text{...(4.3)}$$

$$GM = BM - BG = \frac{I}{\forall} - BG$$

∴ Meta-centric height $$= GM = \frac{I}{\forall} - BG. \quad \text{...(4.4)}$$

Problem 4.7 *A rectangular pontoon is 5 m long, 3 m wide and 1.20 m high. The depth of immersion of the pontoon is 0.80 m in sea water. If the centre of gravity is 0.6 m above the bottom of the pontoon, determine the meta-centric height. The density for sea water = 1025 kg/m^3.*

Solution. Given :

Dimension of pontoon = 5 m × 3 m × 1.20 m

Depth of immersion = 0.8 m

Distance $AG = 0.6$ m

Distance $AB = \frac{1}{2} \times$ Depth of immersion

$= \frac{1}{2} \times .8 = 0.4$ m

Density for sea water $= 1025$ kg/m^3

Meta-centre height GM, given by equation (4.4) is

$$GM = \frac{I}{\forall} - BG$$

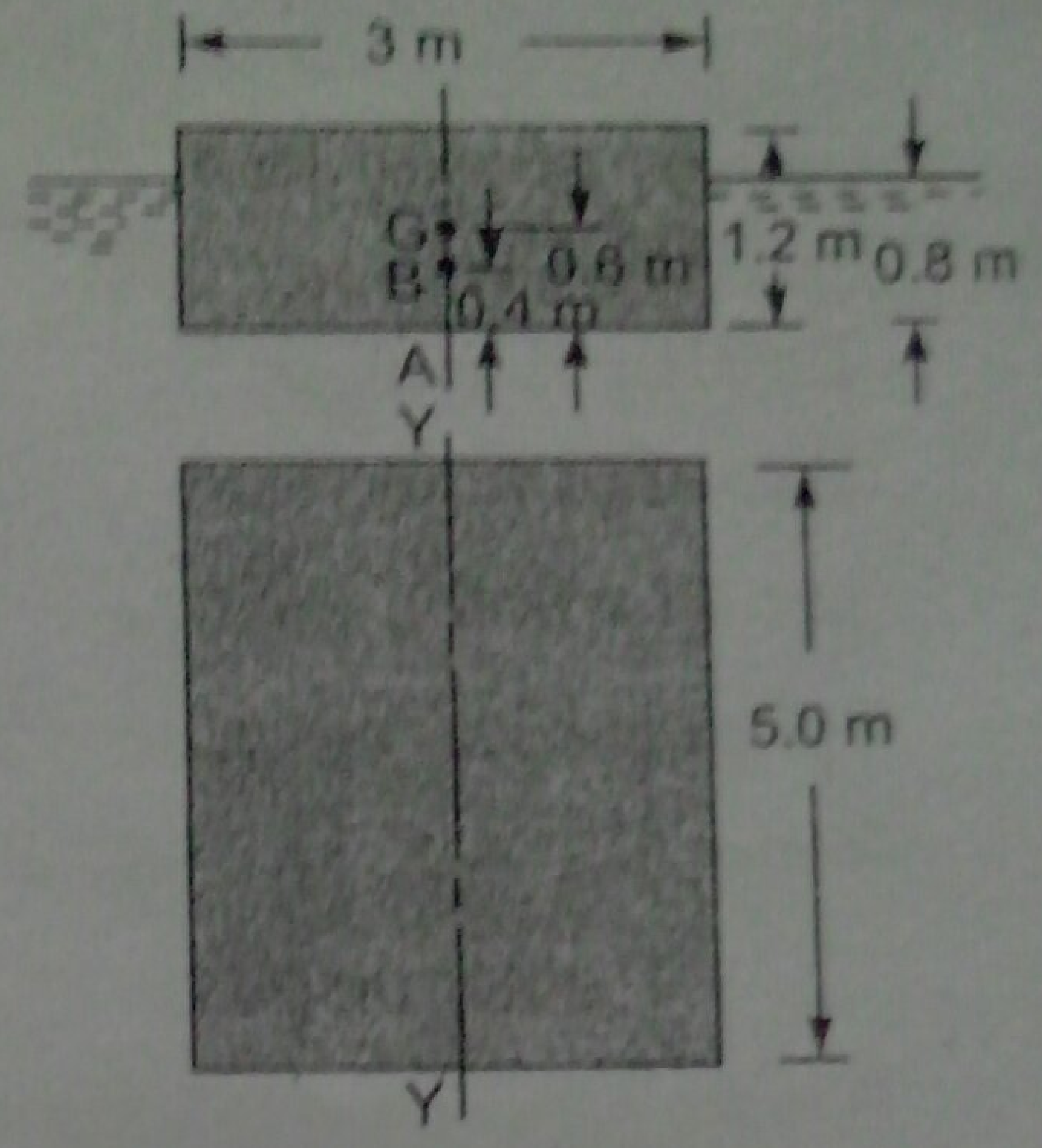
where I = M.O. Inertia of the plan of the pontoon about Y-Y axis

$$= \frac{1}{12} \times 5 \times 3^3 \text{ m}^4 = \frac{45}{4} \text{ m}^4$$

$\forall$ = Volume of the body sub-merged in water

$= 3 \times 0.8 \times 5.0 = 12.0$ m^3

$BG = AG - AB = 0.6 - 0.4 = 0.2$ m



PLAN AT WATER SURFACE

Fig. 4.7

$$\therefore \quad GM = \frac{45}{4} \times \frac{1}{12.0} - 0.2 = \frac{45}{48} - 0.2 = 0.9375 - 0.2 = \mathbf{0.7375 \text{ m.}} \quad \textbf{Ans.}$$

Problem 4.8 *A uniform body of size 3 m long × 2 m wide × 1 m deep floats in water. What is the weight of the body if depth of immersion is 0.8 m ? Determine the meta-centric height also.*

Solution. Given :

Dimension of body $= 3 \times 2 \times 1$

Depth of immersion $= 0.8$ m

Find (*i*) Weight of body, W

(*ii*) Meta-centric height, GM

(*i*) Weight of Body, W

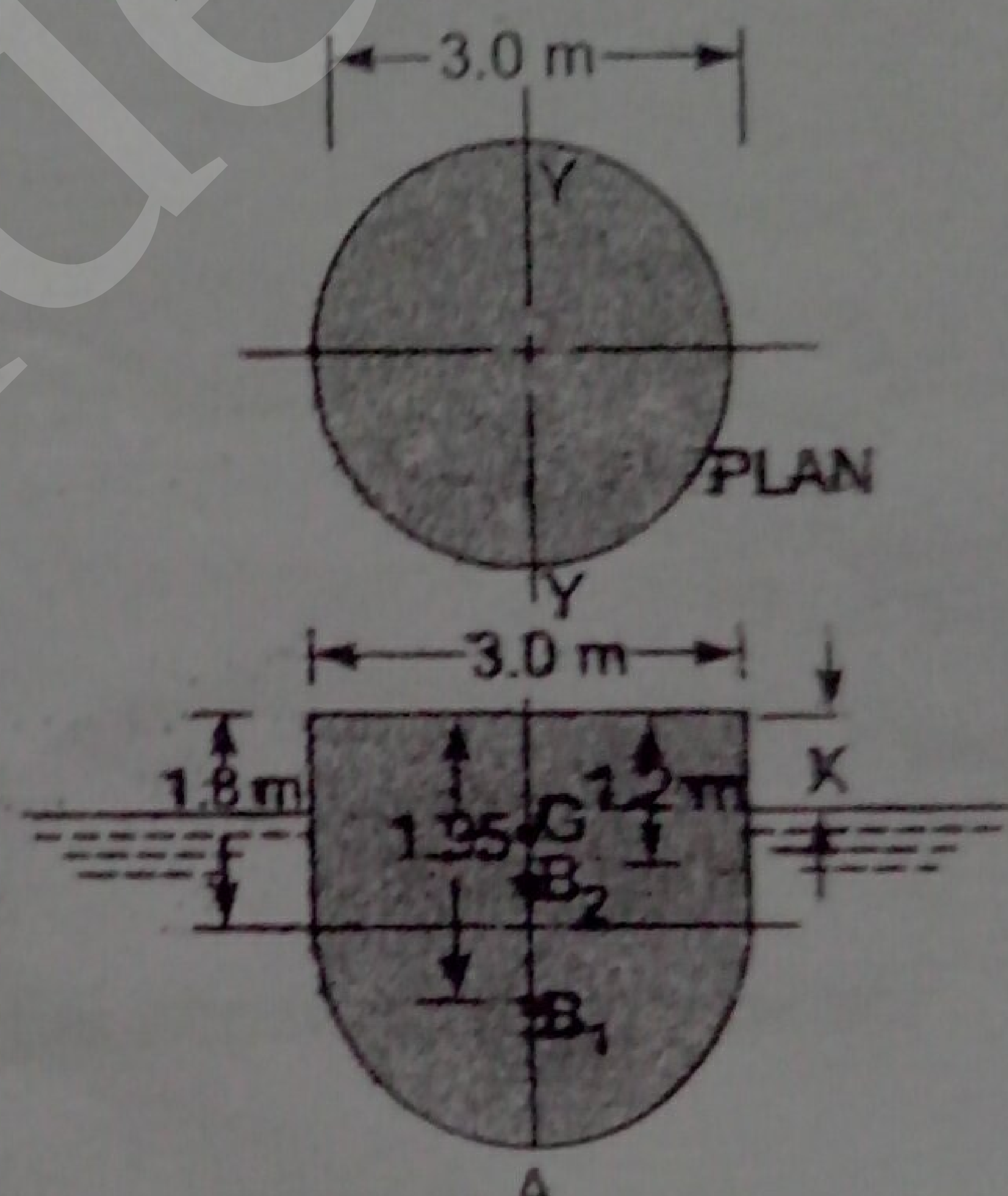
$= $ Weight of water displaced

$= \rho g \times$ Volume of water displaced

$= 1000 \times 9.81 \times$ Volume of body in water

$= 1000 \times 9.81 \times 3 \times 2 \times 0.8$ N

$=$ **47088 N. Ans.**



ELEVATION

Fig. 4.8

(*ii*) **Meta-centric Height, GM**

Using equation (4.4), we get

$$GM = \frac{I}{\forall} - BG$$

where I = M.O.I about Y-Y axis of the plan of the body

$$= \frac{1}{12} \times 3 \times 2^3 = \frac{3 \times 2^3}{12} = 2.0 \text{ m}^4$$

$\forall$ = Volume of body in water

$= 3 \times 2 \times 0.8 = 4.8$ m^3

$$BG = AG - AB = \frac{1.0}{2} - \frac{0.8}{2} = 0.5 - 0.4 = 0.1$$

$$\therefore \quad GM = \frac{2.0}{4.8} - 0.1 = 0.4167 - 0.1 = \mathbf{0.3167 \text{ m.}} \quad \textbf{Ans.}$$

Problem 4.9 *A block of wood of specific gravity 0.7 floats in water. Determine the meta-centric height of the block if its size is 2 m × 1 m × 0.8 m.*

Solution. Given :

Dimension of block $= 2 \times 1 \times 0.8$

Let depth of immersion $= h$ m

Sp. gr. of wood $= 0.7$

Weight of wooden piece = Weight density of wood* × Volume

$= 0.7 \times 1000 \times 9.81 \times 2 \times 1 \times 0.8$ N

Weight of water displaced = Weight density of water × Volume of the wood sub-merged in water

$= 1000 \times 9.81 \times 2 \times 1 \times h$ N

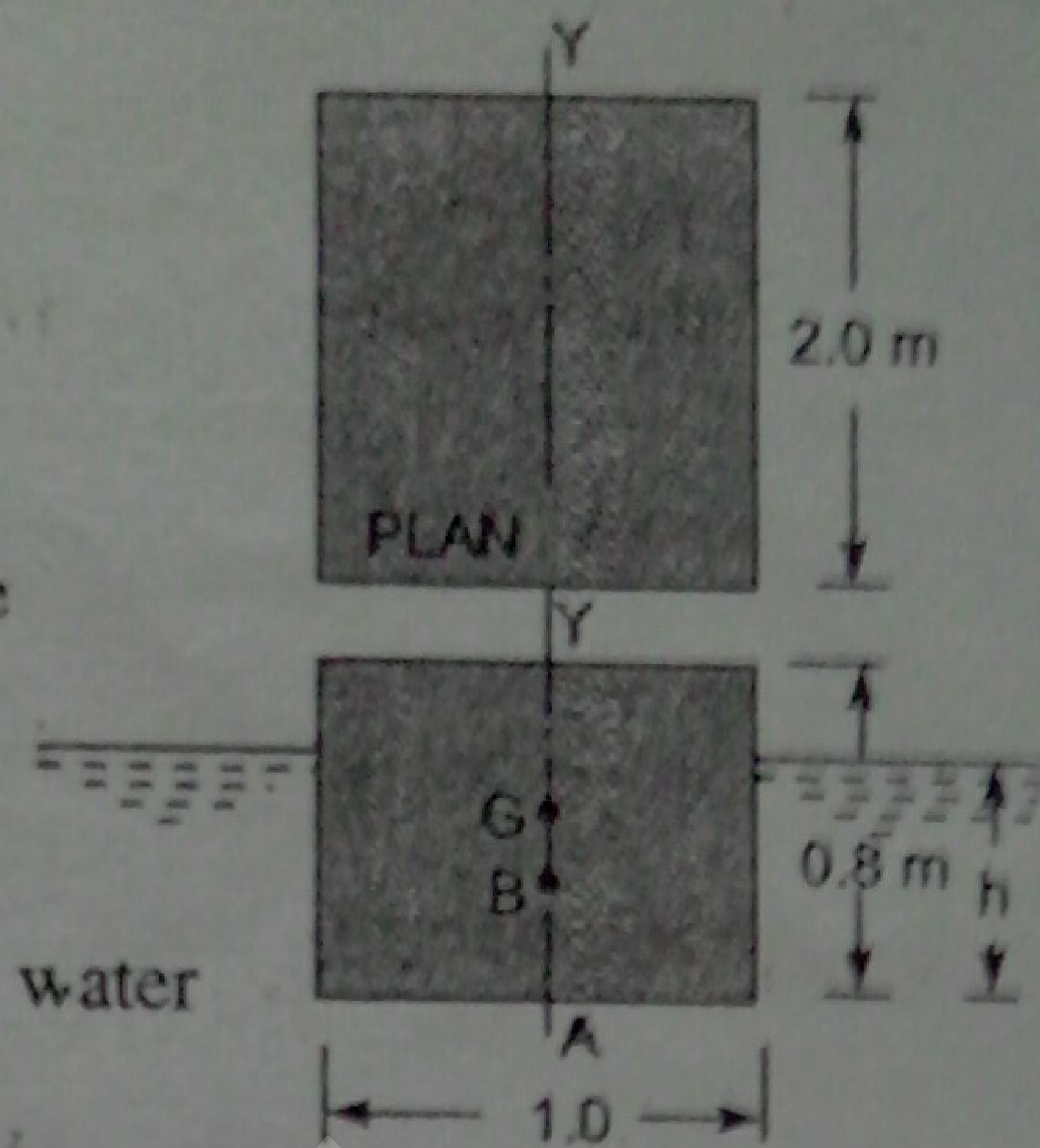


Fig. 4.9

For equilibrium,

Weight of wooden piece = Weight of water displaced

$$\therefore \quad 700 \times 9.81 \times 2 \times 1 \times 0.8 = 1000 \times 9.81 \times 2 \times 1 \times h$$

$$\therefore \quad h = \frac{700 \times 9.81 \times 2 \times 1 \times 0.8}{1000 \times 9.81 \times 2 \times 1} = 0.7 \times 0.8 = 0.56 \text{ m}$$

∴ Distance of centre of Buoyancy from bottom, *i.e.*,

$$AB = \frac{h}{2} = \frac{0.56}{2} = 0.28 \text{ m}$$

and

$$AG = 0.8/2.0 = 0.4 \text{ m}$$

$$\therefore \quad BG = AG - AB = 0.4 - 0.28 = \mathbf{0.12 \text{ m}}$$

The meta-centric height is given by equation (4.4) or

$$GM = \frac{I}{\forall} - BG$$

where $I = \frac{1}{12} \times 2 \times 1.0^3 = \frac{1}{6} \text{ m}^4$

$\forall$ = Volume of wood in water

$= 2 \times 1 \times h = 2 \times 1 \times .56 = 1.12 \text{ m}^3$

$$\therefore \quad GM = \frac{1}{6} \times \frac{1}{1.12} - 0.12 = 0.1488 - 0.12 = \mathbf{0.0288 \text{ m.}} \textbf{ Ans.}$$

Problem 4.10 *A solid cylinder of diameter 4.0 m has a height of 3 metres. Find the meta-centric height of the cylinder when it is floating in water with its axis vertical. The sp. gr. of the cylinder = 0.6.*

Solution. Given :

Dia. of cylinder, $D = 4.0$ m

Height of cylinder, $h = 3.0$ m

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* Weight density of wood $= \rho \times g$, where ρ = density of wood

$= 0.7 \times 1000 = 700 \text{ kg/m}^3$. Hence w for wood $= 700 \times 9.81 \text{ N/m}^3$.

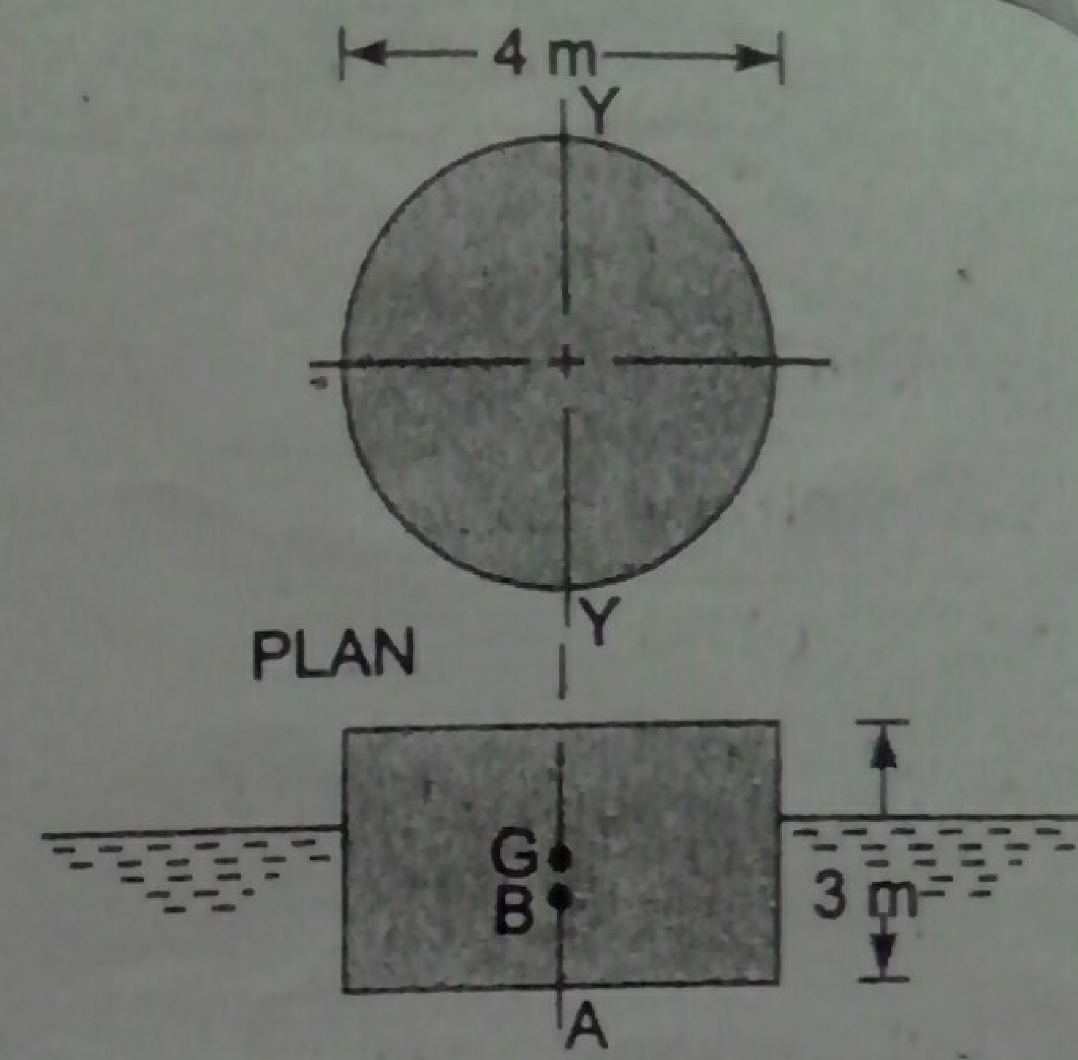


Fig. 4.10

Sp. gr. of cylinder $= 0.6$

Depth of immersion of cylinder

$$= 0.6 \times 3.0 = 1.8 \text{ m}$$

$$\therefore \quad AB = \frac{1.8}{2} = 0.9 \text{ m}$$

and $$AG = \frac{3}{2} = 1.5 \text{ m}$$

$$\therefore \quad BG = AG - AB$$
$$= 1.5 - 0.9 = 0.6 \text{ m}$$

Now the meta-centric height *GM* is given by equation (4.4)

$$GM = \frac{I}{\forall} - BG$$

But I = M.O.I. about *Y-Y* axis of the plan of the body

$$= \frac{\pi}{64} D^4 = \frac{\pi}{64} \times (4.0)^4$$

and $\forall$ = Volume of cylinder in water

$$= \frac{\pi}{4} D^2 \times \text{Depth of immersion}$$

$$= \frac{\pi}{4} (4)^2 \times 1.8 \text{ m}^3$$

$$\therefore \quad GM = \frac{\frac{\pi}{64} \times (4.0)^4}{\frac{\pi}{4} \times (4.0)^2 \times 1.8} - 0.6$$

$$= \frac{1}{16} \times \frac{4.0^2}{1.8} - 0.6 = \frac{1}{1.8} - 0.6 = 0.55 - 0.6 = \mathbf{-0.05 \text{ m. Ans.}}$$

– ve sign means that meta-centre, (*M*) is below the centre of gravity (*G*).

Problem 4.11 *A body has the cylindrical upper portion of 3 m diameter and 1.8 m deep. The lower portion is a curved one, which displaces a volume of 0.6 m³ of water. The centre of buoyancy of the curved portion is at a distance of 1.95 m below the top of the cylinder. The centre of gravity of the whole body is 1.20 m below the top of the cylinder. The total displacement of water is 3.9 tonnes. Find the meta-centric height of the body.*

Solution. Given :

Dia. of body $= 3.0$ m

Depth of body $= 1.8$ m

Volume displaced by curved portion

$$= 0.6 \text{ m}^3 \text{ of water.}$$

Let B_1 is the centre of buoyancy of the curved surface and *G* is the centre of gravity of the whole body.

Then $CB_1 = 1.95$ m

$CG = 1.20$ m

Total weight of water displaced by body = 3.9 tonnes

$= 3.9 \times 1000 = 3900$ kgf

$= 3900 \times 9.81$ N $\doteq 38259$ N

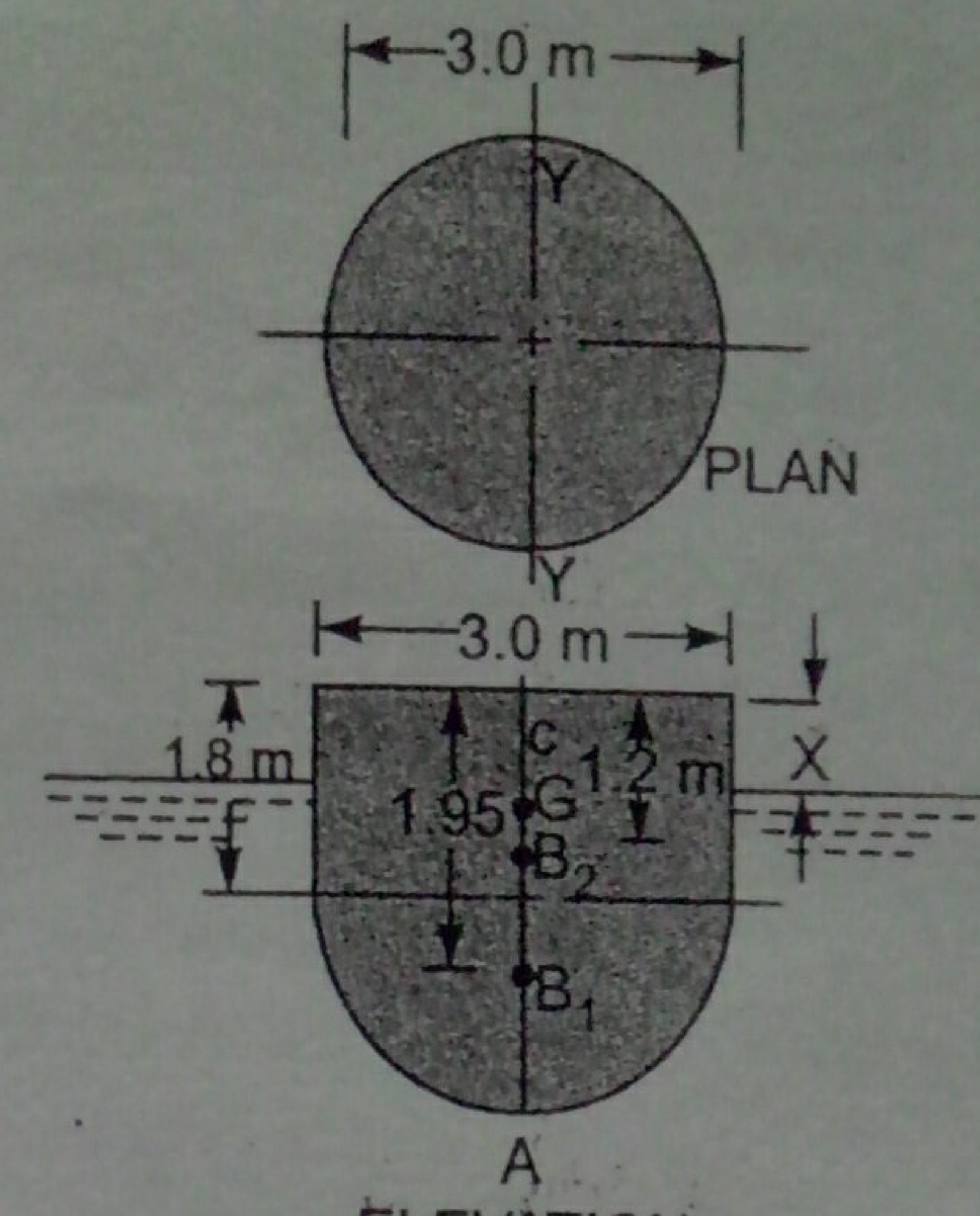


Fig. 4.11

Find **meta-centric** height of the body.

Let the height of the body above the water surface x m. Total weight of water displaced by body

= Weight density of water × [Volume of water displaced]

= 1000 × 9.81 × [Volume of the body in water]

= 9810 [Volume of cylindrical part in water + Volume of curved portion]

$$= 9810 \left[\frac{\pi}{4} \times D^2 \times \text{Depth of cylindrical part in water} + \text{Volume displaced by curved portion}\right]$$

or
$$38259 = 9810 \left[\frac{\pi}{4}(3)^2 \times (1.8 - x) + 0.6\right]$$

$$\therefore \quad \frac{\pi}{4}(3)^2 \times (1.8 - x) + 0.6 = \frac{38259}{9810} = 3.9$$

$$\therefore \quad \frac{\pi}{4} \times 3^2 \times (1.8 - x) = 3.9 - 0.6 = 3.3$$

or
$$1.8 - x = \frac{3.3 \times 4}{\pi \times 3 \times 3} = 0.4668$$

$$\therefore \quad x = 1.8 - .4668 = 1.33 \text{ m}$$

Let B_2 is the centre of buoyancy of cylindrical part and B is the centre of buoyancy of the whole body.

Then depth of cylindrical part in water $= 1.8 - x = 0.467$ m

$$\therefore \quad CB_2 = x + \frac{.467}{2} = 1.33 + .2335 = 1.5635 \text{ m.}$$

The distance of the centre of buoyancy of the whole body from the top of the cylindrical part is given as

CB = (Volume of curved portion × CB_1 + Volume of cylindrical part in water × CB_2) ÷ (Total volume of water displaced)

$$= \frac{0.6 \times 1.95 + 3.3 \times 1.5635}{(0.6 + 3.3)} = \frac{1.17 + 5.159}{3.9} = 1.623 \text{ m.}$$

Then $BG = CB - CG = 1.623 - 1.20 = .423$ m.

Meta-centric height, GM, is given by

$$GM = \frac{I}{\forall} - BG$$

where I = M.O.I. of the plan of the body at water surface about Y-Y axis

$$= \frac{\pi}{64} \times D^4 = \frac{\pi}{64} \times 3^4 \text{ m}^4$$

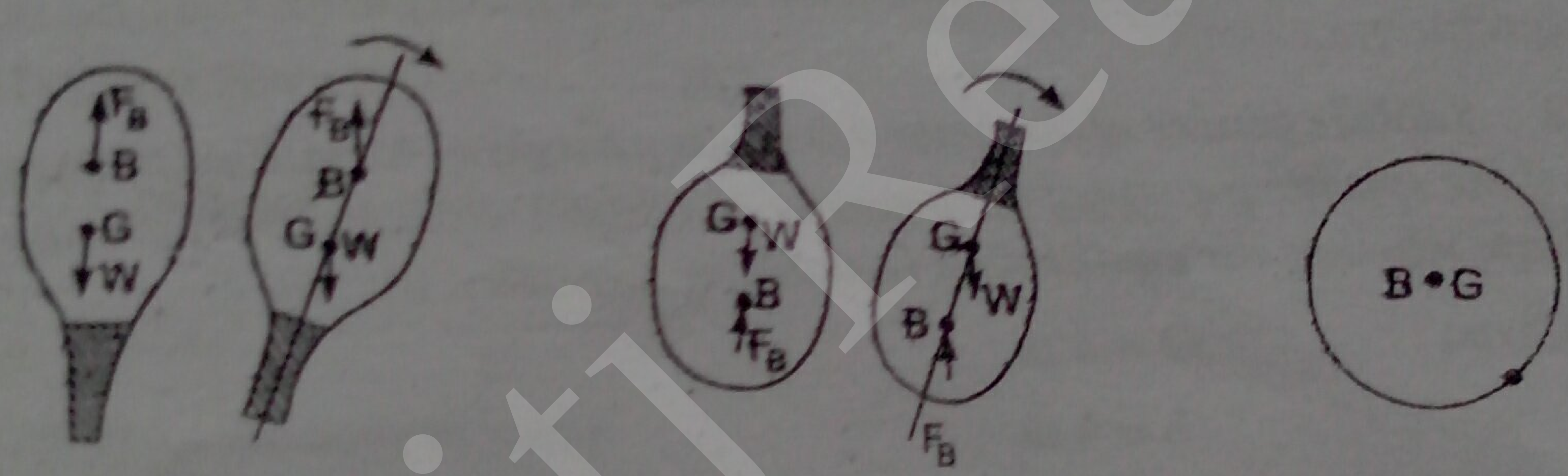
$\forall$ = Volume of the body in water = 3.9 m^3

$$\therefore \quad GM = \frac{\pi}{64} \times \frac{3^4}{3.9} - .423 = 1.019 - .423 = \mathbf{0.596 \text{ m. Ans.}}$$

▶ 4.7 CONDITIONS OF EQUILIBRIUM OF A FLOATING AND SUB-MERGED BODIES

A sub-merged or a floating body is said to be stable if it comes back to its original position after a slight disturbance. The relative position of the centre of gravity (G) and centre of buoyancy (B_1) of a body determines the stability of a sub-merged body.

4.7.1 Stability of a Sub-merged Body. The position of centre of gravity and centre of buoyancy in case of a completely sub-merged body are fixed. Consider a balloon, which is completely sub-merged in air. Let the lower portion of the balloon contains heavier material, so that its centre of gravity is lower than its centre of buoyancy as shown in Fig. 4.12 (*a*). Let the weight of the balloon is W. The weight W is acting through G, vertically in the downward direction, while the buoyant force F_B is acting vertically up, through B. For the equilibrium of the balloon $W = F_B$. If the balloon is given an angular displacement in the clockwise direction as shown in Fig. 4.12 (*a*), then W and F_B constitute a couple acting in the anti-clockwise direction and brings the balloon in the original position. Thus the balloon in the position, shown by Fig. 4.12 (*a*) is in stable equilibrium.



(a) STABLE EQUILIBRIUM (b) UNSTABLE EQUILIBRIUM (c) NEUTRAL EQUILIBRIUM

Fig. 4.12 *Stabilities of sub-merged bodies.*

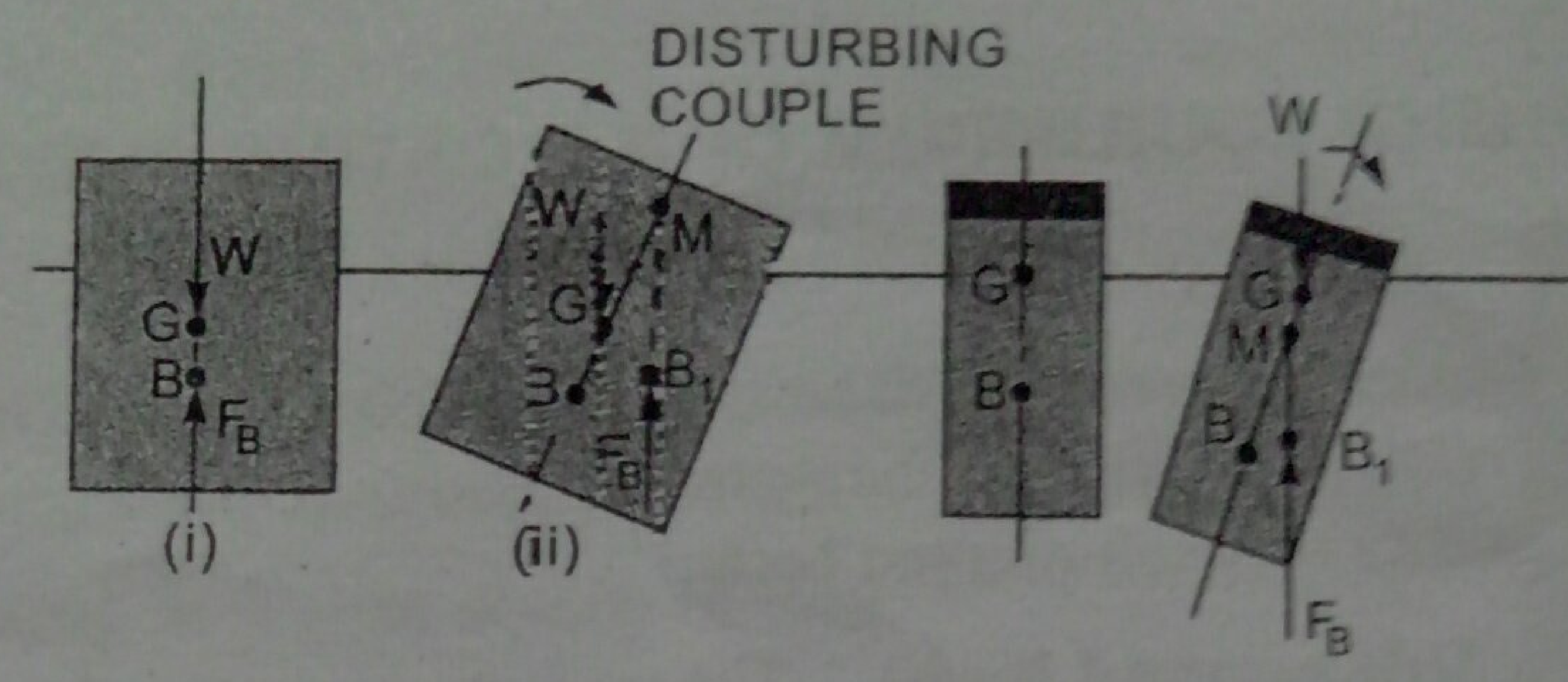
(*a*) **Stable Equilibrium.** When $W = F_B$ and point B is above G, the body is said to be in stable equilibrium.

(*b*) **Unstable Equilibrium.** If $W = F_B$, but the centre of buoyancy (B) is below centre of gravity (G), the body is in unstable equilibrium as shown in Fig. 4.12 (*b*). A slight displacement to the body, in the clockwise direction, gives the couple due to W and F_B also in the clockwise direction. Thus the body does not return to its original position and hence the body is in unstable equilibrium.

(*c*) **Neutral Equilibrium.** If $F_B = W$ and B and G are at the same point, as shown in Fig. 4.12 (*c*), the body is said to be in neutral equilibrium.

4.7.2 Stability of Floating Body. The stability of a floating body is determined from the position of Meta-centre (M). In case of floating body, the weight of the body is equal to the weight of liquid displaced.

(*a*) **Stable Equilibrium.** If the point M is above G, the floating body will be in stable equilibrium as shown in Fig. 4.13 (*a*). If a slight angular displacement is given to the floating body in the clockwise direction, the centre of buoyancy shifts from B to B_1 such that the vertical line through B_1 cuts at M. Then the buoyant force F_B through B_1 and weight W through G constitute a couple acting in the anti-clockwise direction and thus bringing the floating body in the original position.



(a) Stable equilibrium M is above G (b) Unstable equilibrium M is below G.

Fig. 4.13 *Stability of floating bodies.*

(*b*) **Unstable Equilibrium.** If the point M is below G, the floating body will be in unstable equilibrium as shown in Fig. 4.13 (*b*). The disturbing couple is acting in the clockwise direction. The couple due to buoyant force F_B and W is also acting in the clockwise direction and thus overturning the floating body.

(*c*) **Neutral Equilibrium.** If the point M is at the centre of gravity of the body, the floating body will be in neutral equilibrium.

Problem 4.12 *A solid cylinder of diameter 4.0 m has a height of 4.0 m. Find the meta-centric height of the cylinder if the specific gravity of the material of cylinder = 0.6 and it is floating in water with its axis vertical. State whether the equilibrium is stable or unstable.*

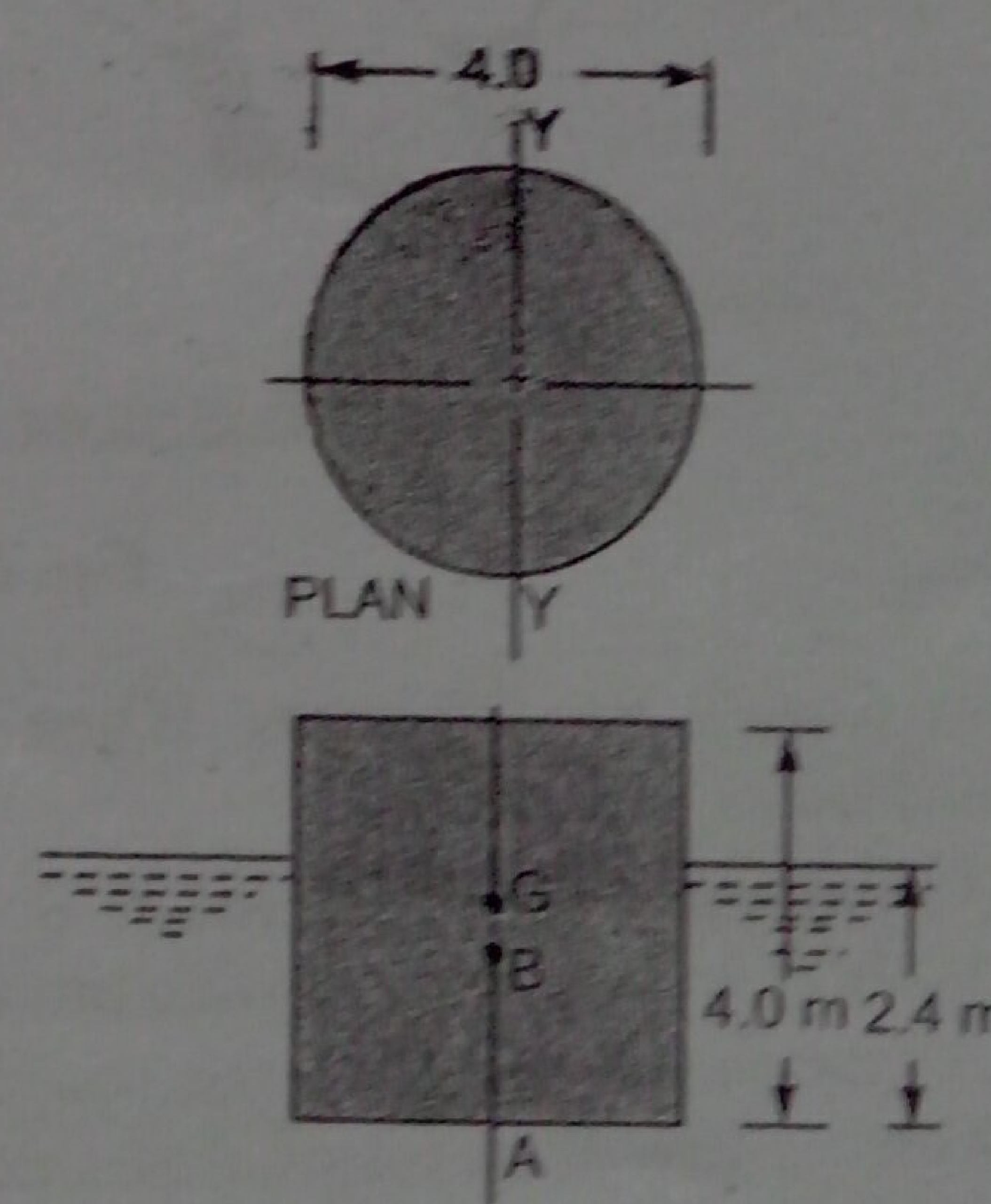


Fig. 4.14

Solution. Given : $D = 4$ m

Height, $h = 4$ m

Sp. gr. $= 0.6$

Depth of cylinder in water $=$ Sp. gr $\times h$

$= 0.6 \times 4.0 = 2.4$ m

$\therefore$ Distance of centre of buoyancy (B) from A

or $$AB = \frac{2.4}{2} = 1.2 \text{ m}$$

Distance of centre of gravity (G) from A

or $$AG = \frac{h}{2} = \frac{4.0}{2} = 2.0 \text{ m}$$

$\therefore$ $$BG = AG - AB = 2.0 - 1.2 = 0.8 \text{ m}$$

Now the meta-centric height GM is given by

$$GM = \frac{I}{\forall} - BG$$

where I = M.O.I. of the plan of the body about Y-Y axis

$$= \frac{\pi}{64} D^4 = \frac{\pi}{64} \times (4.0)^4$$

$\forall$ = Volume of cylinder in water

$$= \frac{\pi}{4.0} \times D^2 \times \text{Depth of cylinder in water} = \frac{\pi}{4} \times 4^2 \times 2.4 \text{ m}^3$$

$$\therefore \quad \frac{I}{\forall} = \frac{\frac{\pi}{64} \times 4^4}{\frac{\pi}{4} \times 4^2 \times 2.4} = \frac{1}{16} \times \frac{4^2}{2.4} = \frac{1}{2.4} = 0.4167 \text{ m}$$

$$\therefore \quad GM = \frac{I}{\forall} - BG = 0.4167 - 0.8 = -\mathbf{0.3833 \text{ m. Ans.}}$$

–ve sign means that the meta-centre (M) is below the centre of gravity (G). Thus the cylinder is in unstable equilibrium. **Ans.**

Problem 4.13 *A solid cylinder of 10 cm diameter and 40 cm long, consists of two parts made of different materials. The first part at the base is 1.0 cm long and of specific gravity = 6.0. The other part of the cylinder is made of the material having specific gravity 0.6. State, if it can float vertically in water.*

Solution. Given :

	$D = 10$ cm
Length,	$L = 40$ cm
Length of 1st part,	$l_1 = 1.0$ cm
Sp. gr.,	$S_1 = 6.0$
Density of 1st part,	$\rho_1 = 6 \times 1000 = 6000$ kg/m^3
Length of 2nd part,	$l_2 = 40 - 1.0 = 39.0$ cm
Sp. gr.,	$S_2 = 0.6$
Density of 2nd part,	$\rho_2 = 0.6 \times 1000 = 600$ kg/m^3

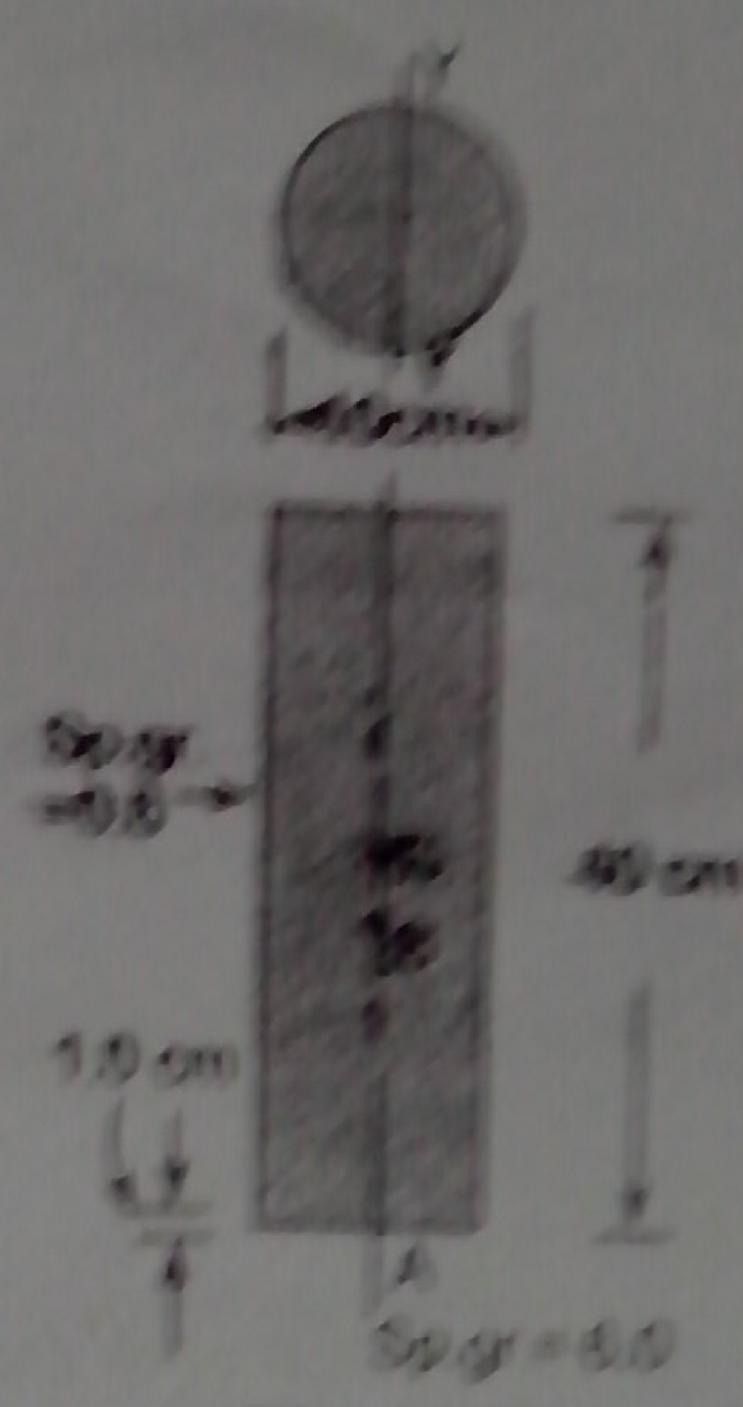


Fig. 4.15

The cylinder will float vertically in water if its meta-centric height GM is positive. To find meta-centric height, find the location of centre of gravity (G) and centre of buoyancy (B) of the combined solid cylinder. The distance of the centre of gravity of the solid cylinder from A is given as

AG = [(Weight of 1st part × Distance of C.G. of 1st part from A)
+ (Weight of 2nd part of cylinder
× Distance of C.G. of 2nd part from A)]
÷ [Weight of 1st part + weight of 2nd part]

$$= \frac{\left(\frac{\pi}{4} D^2 \times 1.0 \times 6.0 \times 0.5\right) + \left(\frac{\pi}{4} D^2 \times 39.0 \times 0.6 \times (1.0 + 39/2)\right)}{\left(\frac{\pi}{4} D^2 \times 1.0 \times 6.0 + \frac{\pi}{4} D^2 \times 39 \times 0.6\right)}$$

$$= \frac{1.0 \times 6.0 \times 0.5 + 39.0 \times .6 \times (20.5)}{1.0 \times 6.0 + 39.0 \times 0.6}$$

Cancel $\frac{\pi}{4} D^2$ in the Numerator and Denominator $= \frac{3.0 + 479.7}{6.0 + 23.4} = \frac{482.7}{29.4} = 16.42$

To find the centre of buoyancy of the combined two parts or of the cylinder, determine the depth of immersion of the cylinder. Let the depth of immersion of the cylinder is h. Then

Weight of the cylinder = Weight of water displaced

$$\frac{\pi}{4} \times (1)^2 \times \frac{39.0}{100} \times 600 \times 9.81 + \frac{\pi}{4} (1)^2 \times \frac{1.0}{100} \times 6000 \times 9.81 = \frac{\pi}{4} (1)^2 \times \frac{h}{100} \times 1000 \times 9.81$$

[$\because$ h is in cm]

or cancelling $\frac{\pi}{4} (1)^2 \times \frac{1000 \times 9.81}{100}$ throughout, we get

$$39.0 \times 0.6 + 1.0 \times 6.0 = h \quad \text{or} \quad h = 23.4 + 6.0 = 29.4$$

$\therefore$ The distance of the centre of the buoyancy B, of the cylinder from A is

$$AB = h/2 = \frac{29.4}{2} = 14.7$$

$$\therefore \quad BG = AG - AB = 16.42 - 14.70 = 1.72 \text{ cm}$$

Meta-centric height GM is given by

$$GM = \frac{I}{\forall} - BG$$

where I = M.O.I. of plan of the body about Y-Y axis

$$= \frac{\pi}{64} D^4 = \frac{\pi}{64} (10)^4 \text{ cm}^4$$

$\forall$ = Volume of cylinder in water

$$= \frac{\pi}{4} D^2 \times h = \frac{\pi}{4} (10)^2 \times 29.4 \text{ m}^3$$

$$\therefore \quad \frac{I}{\forall} = \frac{\pi}{64} (10)^4 \Big/ \frac{\pi}{4} (10)^2 \times 29.4 = \frac{1}{16} \times \frac{10^2}{29.4} = \frac{100}{16 \times 29.4} = 0.212$$

$$\therefore \quad GM = 0.212 - 1.72 = -1.508 \text{ cm}$$

As GM is – ve, it means that the Meta-centre M is below the centre of gravity (G). Thus the cylinder is in unstable equilibrium and so it cannot float vertically in water. **Ans.**

Problem 4.14 *A rectangular pontoon 10.0 m long, 7 m broad and 2.5 m deep weighs 686.7 kN. It carries on its upper deck an empty boiler of 5.0 m diameter weighing 588.6 kN. The centre of gravity of the boiler and the pontoon are at their respective centres along a vertical line. Find the meta-centric height. Weight density of sea water is 10.104 kN/m³.*

Solution. Given : Dimension of pontoon = $10 \times 7 \times 2.5$

Weight of pontoon, $W_1 = 686.7$ kN

Dia. of boiler, $D = 5.0$ m

Weight of boiler, $W_2 = 588.6$ kN

w for sea water $= 10.104$ kN/m³

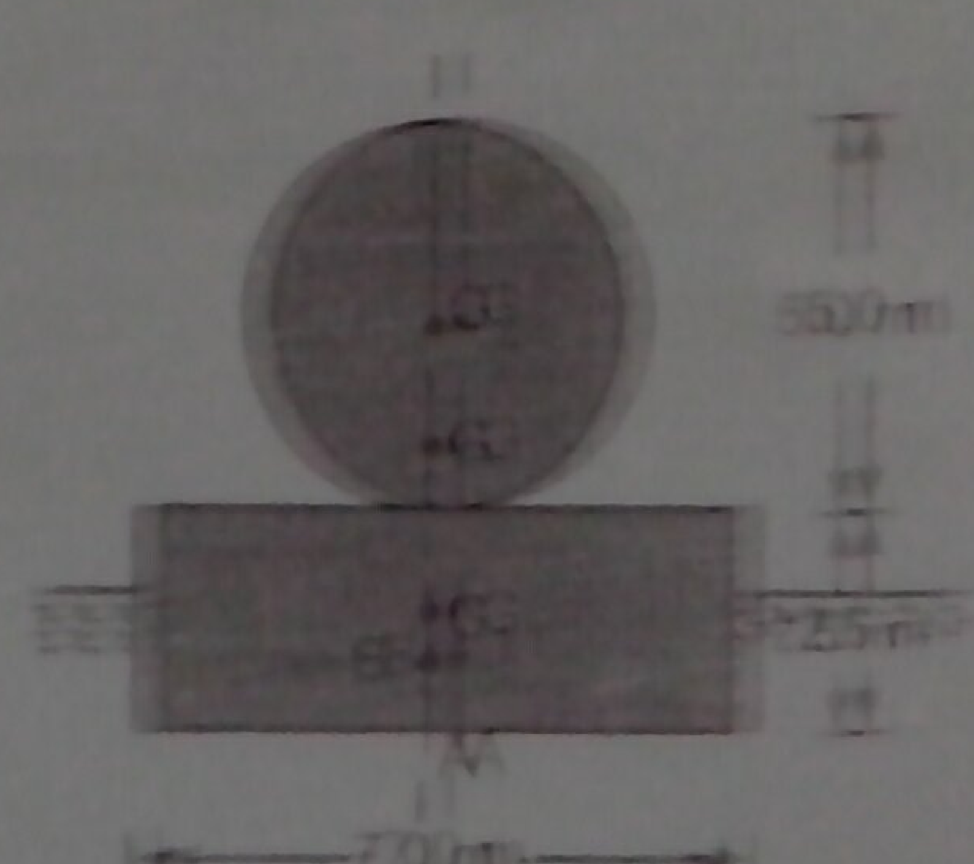


Fig. 4.16

To find the meta-centric height, first determine the common centre of gravity G and common centre of buoyancy B of the boiler and pontoon. Let G_1 and G_2 are the centre of gravities of pontoon and boiler respectively. Then

$$AG_1 = \frac{2.5}{2} = 1.25 \text{ m}$$

$$AG_2 = 2.5 + \frac{5.0}{2} = 2.5 + 2.5 = 5.0 \text{ m}$$

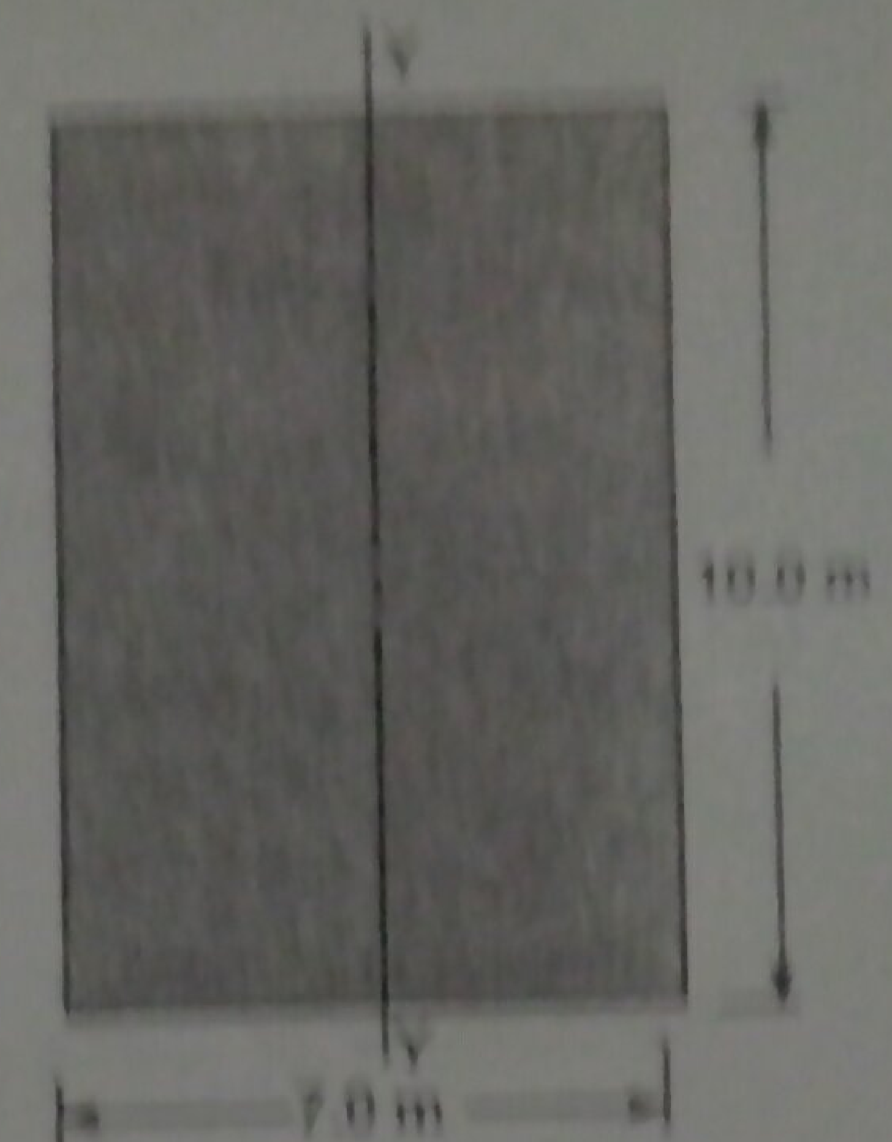


Fig. 4.17 *Plan of the body at water-line*

The distance of common centre of gravity G from A is given as

$$AG = \frac{W_1 \times AG_1 + W_2 \times AG_2}{W_1 + W_2}$$

$$= \frac{686.7 \times 1.25 + 588.6 \times 5.0}{(686.7 + 588.6)} = 2.98 \text{ m.}$$

Let h is the depth of immersion. Then

Total weight of pontoon and boiler = Weight of sea water displaced

or $(686.7 + 588.6) = w \times$ Volume of the pontoon in water

$= 10.104 \times L \times b \times$ Depth of immersion

$\therefore \quad 1275.3 = 10.104 \times 10 \times 7 \times h$

$$h = \frac{1275.3}{10 \times 7 \times 10.104} = 1.803 \text{ m}$$

$\therefore$ The distance of the common centre of buoyancy B from A is

$$AB = \frac{h}{2} = \frac{1.803}{2} = .9015 \text{ m}$$

$\therefore \quad$ **$BG = AG - AB = 2.98 - .9015 = 2.0785 \text{ m} = 2.078 \text{ m}$**

Meta-centric height is given by $GM = \frac{I}{\forall} - BG$

where $\quad I$ = M.O.I. of the plan of the body at the water level along Y-Y

$$= \frac{1}{12} \times 10.0 \times 7^3 = \frac{10 \times 49 \times 7}{12} \text{ m}^4$$

$\forall$ = Volume of the body in water

$= L \times b \times h = 10.0 \times 7 \times 1.857$

$$\therefore \quad \frac{I}{\forall} = \frac{10 \times 49 \times 7}{12 \times 10 \times 7 \times 1.857} = \frac{49}{12 \times 1.857} = 2.198 \text{ m}$$

$$\therefore \quad GM = \frac{I}{\forall} - BG = 2.198 - 2.078 = 0.12 \text{ m.}$$

$\therefore$ Meta-centric height of both the pontoon and boiler = **0.12 m. Ans.**

Problem 4.15 *A wooden cylinder of sp. gr. = 0.6 and circular in cross-section is required to float in oil (sp. gr. = 0.90). Find the L/D ratio for the cylinder to float with its longitudinal axis vertical in oil, where L is the height of cylinder and D is its diameter.*

Solution. Given :

Dia. of cylinder $= D$

Height of cylinder $= L$

Sp. gr. of cylinder, $S_1 = 0.6$

Sp. gr. of oil $S_2 = 0.9$

Let the depth of cylinder immersed in oil = h

For the principle of buoyancy

Weight of cylinder = wt. of oil displaced

$$\frac{\pi}{4} D^2 \times L \times 0.6 \times 1000 \times 9.81 = \frac{\pi}{4} D^2 \times h \times 0.9 \times 1000 \times 9.81$$

or $$L \times 0.6 = h \times 0.9$$

$\therefore$ $$h = \frac{0.6 \times L}{0.9} = \frac{2}{3} L.$$

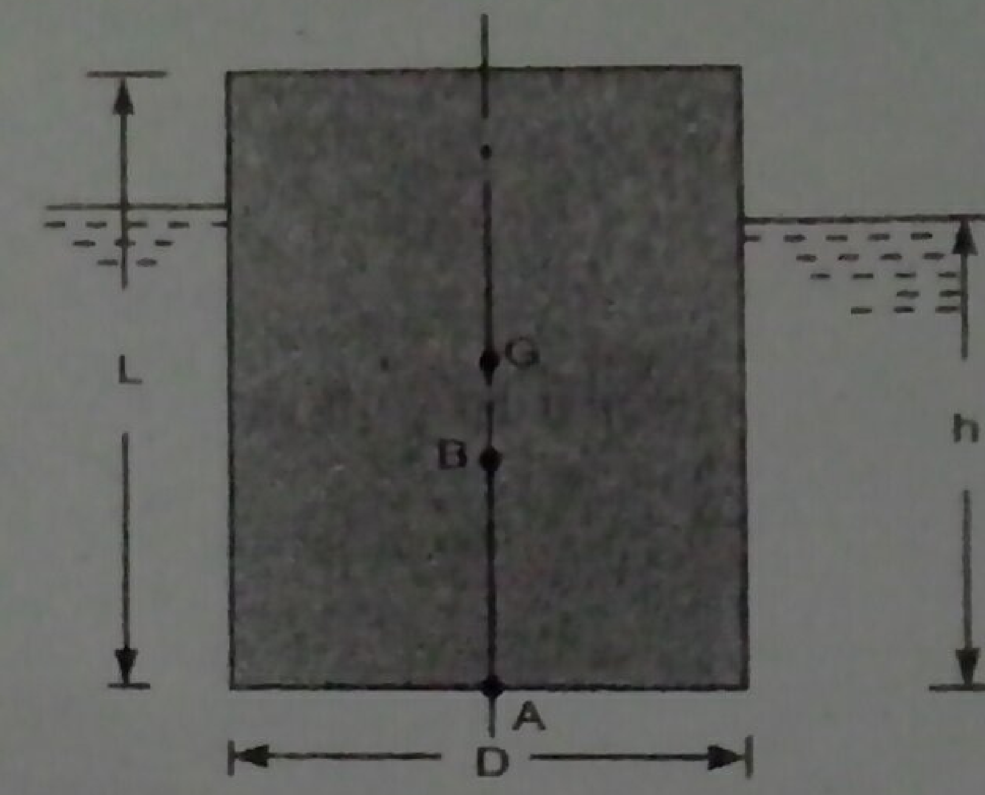


Fig. 4.18

The distance of centre of gravity G from A, $AG = \frac{L}{2}$

The distance of centre of buoyancy B from A,

$$AB = \frac{h}{2} = \frac{1}{2}\left[\frac{2}{3}L\right] = \frac{L}{3}$$

$\therefore$ $$BG = AG - AB = \frac{L}{2} - \frac{L}{3} = \frac{3L - 2L}{6} = \frac{L}{6}$$

The meta-centric height GM is given by

$$GM = \frac{I}{\forall} - BG$$

where $I = \frac{\pi}{64} D^4$ and $\forall$ = Volume of cylinder in oil = $\frac{\pi}{4} D^2 \times h$

$\therefore$ $$\frac{I}{\forall} = \left(\frac{\pi}{64} D^4 \Big/ \frac{\pi}{4} D^2 h\right) = \frac{1}{16}\frac{D^2}{h} = \frac{D^2}{16 \times \frac{2}{3} L} = \frac{3D^2}{32L} \qquad \left\{\because h = \frac{2}{3}L\right\}$$

$\therefore$ $$GM = \frac{3D^2}{32L} - \frac{L}{6}$$

For stable equilibrium, GM should be +ve or

$$GM > 0 \quad \text{or} \quad \frac{3D^2}{32L} - \frac{L}{6} > 0$$

or $$\frac{3D^2}{32L} > \frac{L}{6} \quad \text{or} \quad \frac{3 \times 6}{32} > \frac{L^2}{D^2}$$

or $$\frac{L^2}{D^2} < \frac{18}{32} \quad \text{or} \quad \frac{9}{16}$$

$\therefore$ $$\frac{L}{D} < \sqrt{\frac{9}{16}} = \frac{3}{4}$$

$\therefore$ **L/D < 3/4. Ans.**

Problem 4.16 *Show that a cylindrical buoy of 1 m diameter and 2.0 m height weighing 7.848 kN will not float vertically in sea water of density 1030 kg/m^3. Find the force necessary in a vertical chain attached at the centre of base of the buoy that will keep it vertical.*

Solution. Given : Dia. of buoy, $D = 1$ m

Height, $H = 2.0$ m

Weight, $W = 7.848$ kN

$= 7.848 \times 1000 = 7848$ N

Density, $\rho = 1030$ kg/m^3

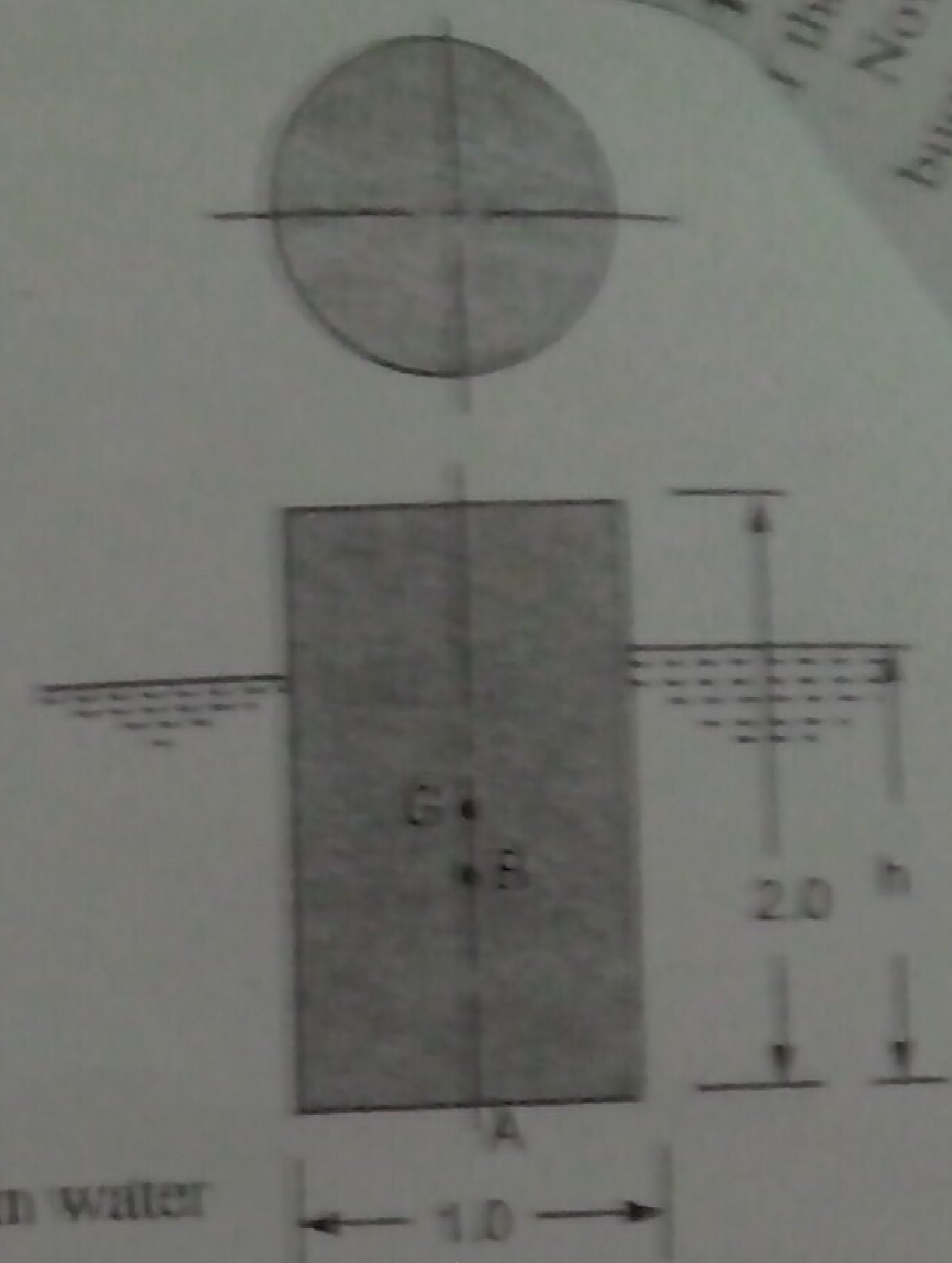


Fig. 4.19

(*i*) Show the cylinder will not float vertically.

(*ii*) Find the force in the chain.

Part I. The cylinder will not float if meta-centric height is –ve.

Let the depth of immersion be h

Then for equilibrium, Weight of cylinder

= Weight of water displaced

= Density × g × Volume of cylinder in water

$$\therefore \quad 7848 = 1030 \times 9.81 \times \frac{\pi}{4} D^2 \times h$$

$$= 10104.3 \times \frac{\pi}{4} (1)^2 \times h$$

$$\therefore \quad h = \frac{4 \times 7848}{10104.3 \times \pi} = 0.989 \text{ m.}$$

∴ The distance of centre of buoyancy B from A,

$$AB = \frac{h}{2} = \frac{0.989}{2} = 0.494 \text{ m.}$$

And the distance of centre of gravity G, from A is $AG = \frac{2.0}{2} = 1.0$ m

$$\therefore \quad BG = AG - AB = 1.0 - .494 = .506 \text{ m.}$$

Now meta-centric height GM is given by $GM = \frac{I}{\forall} - BG$

where $I = \frac{\pi}{64} D^4 = \frac{\pi}{64} \times (1)^4$ m^4

and $\forall$ = Volume of cylinder in water $= \frac{\pi}{4} D^2 \times h = \frac{\pi}{4} 1^2 \times .989$

$$\therefore \quad \frac{I}{\forall} = \frac{\frac{\pi}{64} \times 1^4}{\frac{\pi}{4} D^2 \times h} = \frac{\frac{\pi}{64} \times 1^4}{\frac{\pi}{4} \times 1^2 \times .989}$$

$$= \frac{1}{16} \times 1^2 \times \frac{1}{.989} = \frac{1}{16 \times .989} = 0.063 \text{ m}$$

$$\therefore \quad GM = .063 - .506 = \mathbf{-0.443 \text{ m. Ans.}}$$

As the meta-centric height is –ve, the point M lies below G and hence the cylinder will be in unstable equilibrium and hence cylinder will not float vertically.

Part II. Let the force applied in a vertical chain attached at the centre of the base of the buoy is T to keep the buoy vertical.

Now find the combined position of centre of gravity (G') and centre of buoyancy (B'). For the combined centre of buoyancy, let h' = depth of immersion when the force T is applied. Then

Total downward force = Weight of water displaced

or $(7848 + T)$ = Density of water × g × Volume of cylinder in water

$$= 1030 \times 9.81 \times \frac{\pi}{4} D^2 \times h' \text{ [where } h' = \text{depth of immersion]}$$

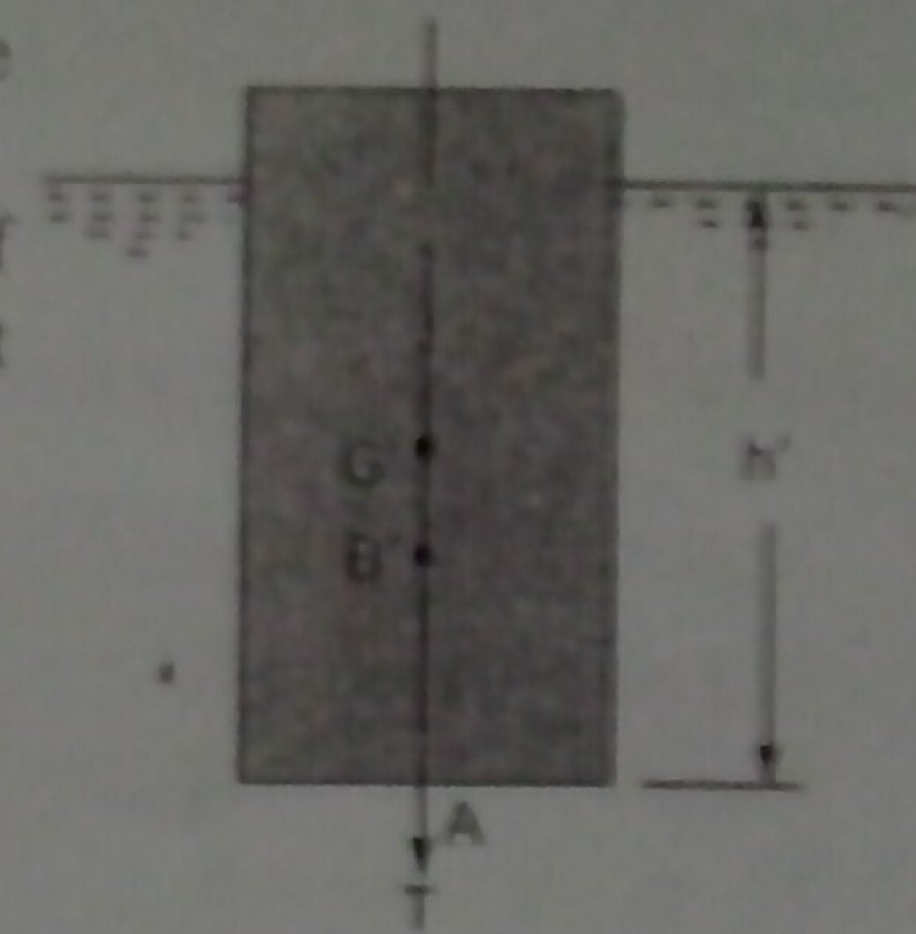


Fig. 4.20

$$\therefore \quad h' = \frac{7848 + T}{10104.3 \times \frac{\pi}{4} \times D^2} = \frac{7848 + T}{10104.3 \times \frac{\pi}{4} \times 1^2} = \frac{10104.3 + T}{7935.9} \text{ m}$$

$$\therefore \quad AB' = \frac{h'}{2} = \frac{1}{2}\left[\frac{7848 + T}{7935.9}\right] = \frac{7848 + T}{15871.8} \text{ m.}$$

The combined centre of gravity (G') due to weight of cylinder and due to tension T in the chain from A is

$$AG' = [\text{Wt. of cylinder} \times \text{Distance of C.G. of cylinder from } A$$
$$+ T \times \text{Distance of C.G. of } T \text{ from } A] \div [\text{Weight of cylinder} + T]$$
$$= \left(7848 \times \frac{2}{2} + T \times 0\right) \div [7848 + T] = \frac{7848}{7848 + T} \text{ m}$$

$$\therefore \quad B'G' = AG' - AB' = \frac{7848}{(7848 + T)} - \frac{(7848 + T)}{15871.8}$$

The meta-centric height GM is given by $GM = \frac{I}{\forall} - B'G'$

where $= \frac{\pi}{64} \times D^4 = \frac{\pi}{64} \times 1^4 = \frac{\pi}{64} \text{ m}^4$

and $\forall = \frac{\pi}{4} D^2 \times h' = \frac{\pi}{4} \times 1^2 \times \frac{(7848 + T)}{7935.9} = \frac{\pi}{4} \times \frac{7848 + T}{7935.9}$

$$\therefore \quad \frac{I}{\forall} = \frac{\frac{\pi}{64}}{\frac{\pi}{4}\frac{(7848 + T)}{7935.9}} = \frac{1}{16} \times \frac{7935.9}{(7848 + T)}$$

$$\therefore \quad GM = \frac{7935.9}{16\,(7848 + T)} - \left[\frac{7848}{(7848 + T)} - \frac{(7848 + T)}{15871.8}\right]$$

For stable equilibrium GM should be positive

or $$GM > 0$$

or $$\frac{7935.9}{16\,(7848 + T)} - \left[\frac{7848}{(7848 + T)} - \frac{(7848 + T)}{15871.8}\right] > 0$$

or $\dfrac{7935.9}{16(7848+T)} - \dfrac{7848}{(7848+T)} + \dfrac{7848+T}{15871.8} > 0$

or $\dfrac{7935.9 - 16 \times 7848}{16(7848+T)} + \dfrac{(7848+T)}{15871.8} > 0$

or $\dfrac{-117632}{16(7848+T)} + \dfrac{(7848+T)}{15871.8} > 0$

or $\dfrac{(7848+T)}{15871.8} > \dfrac{117632}{16(7848+T)}$

or $(7848+T)^2 > \dfrac{117632}{16.0} \times 15871.8$

> 116689473.5

$> (10802.3)^2$

$\therefore \quad 7848 + T > 10802.3$

$\therefore \quad T > 10802.3 - 7848$

$> $ **2954.3 N. Ans.**

$\therefore$ The force in the chain must be at least 2954.3 N so that the cylindrical buoy can be kept in vertical position. **Ans.**

Problem 4.17 *A solid cone floats in water with its apex downwards. Determine the least apex angle of cone for stable equilibrium. The specific gravity of the material of the cone is given 0.8.*

Solution. Given :

Sp. gr. of cone $= 0.8$

Density of cone, $\rho = 0.8 \times 1000 = 800$ kg/m^3

Let $D =$ Dia. of the cone

$d =$ Dia. of cone at water level

$2\theta =$ Apex angle of cone

$H =$ Height of cone

$h =$ Depth of cone in water

$G =$ Centre of gravity of the cone

$B =$ Centre of buoyancy of the cone

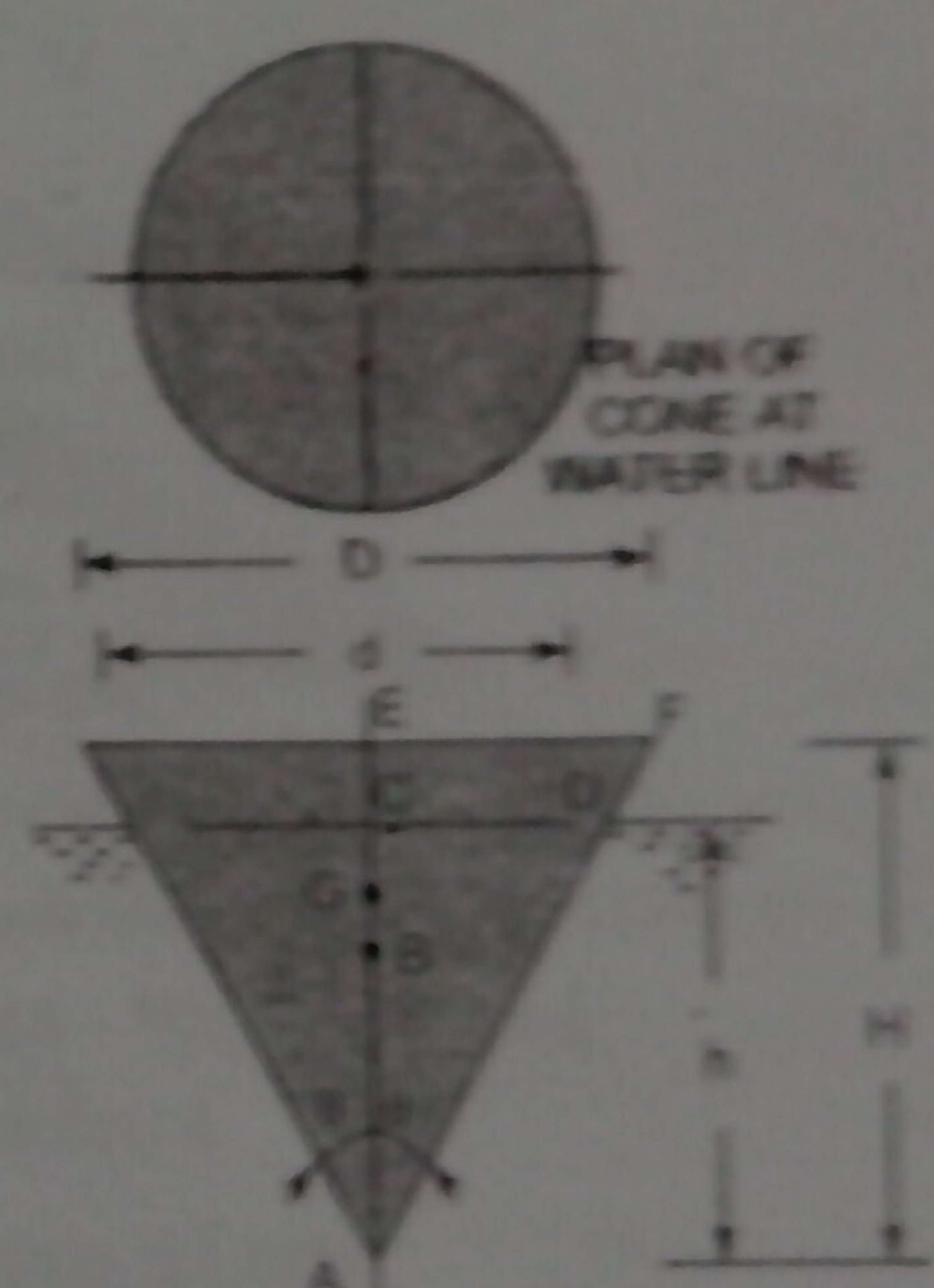


Fig. 4.21

For the cone, the distance of centre of gravity from the apex A is

$AG = \frac{3}{4}$ height of cone $= \frac{3}{4} H$

also $AB = \frac{3}{4}$ depth of cone in water $= \frac{3}{4} h$

Volume of water displaced $= \frac{1}{3} \pi r^2 \times h$

Volume of cone $= \frac{1}{3} \times \pi R^2 \times h$

$\therefore$ Weight of cone $= 800 \times g \times \frac{1}{3} \times \pi R^2 \times H$

Now from ΔAEF, $\tan \theta = \dfrac{EF}{EA} = \dfrac{R}{H}$

$\therefore \quad R = H \tan \theta$

Similarly, $r = h \tan \theta$

$\therefore$ Weight of cone $= 800 \times g \times \frac{1}{3} \times \pi \times (H \tan \theta)^2 \times H = \dfrac{800 \times g \times \pi \times H^3 \tan^2 \theta}{3}$

$\therefore$ Weight of water displaced $= 1000 \times g \times \frac{1}{3} \times \pi r^2 \times h$

$$= 1000 \times g \times \tfrac{1}{3} \times \pi (h \tan \theta)^2 \times h = \frac{1000 \times g \times \pi \times h^3 \tan^2 \theta}{3.0}$$

For equilibrium

Weight of cone = Weight of water displaced

or $$\frac{800 \times g \times \pi \times H^3 \tan^2 \theta}{3.0} = \frac{1000 \times 9.81 \times \pi \times h^3 \times \tan^2 \theta}{3.0}$$

or $$800 \times H^3 = 1000 \times h^3$$

$\therefore$ $$H^3 = \frac{1000}{800} \times h^3 \text{ or } \frac{H}{h} = \left(\frac{1000}{800}\right)^{1/3}$$

For stable equilibrium, Meta-centric height GM should be positive. But GM is given by

$$GM = \frac{I}{\forall} - BG$$

where I = M.O.I. of cone at water-line $= \dfrac{\pi}{64} d^4$

$\forall$ = Volume of cone in water $= \dfrac{1}{3} \dfrac{\pi}{4} d^2 \times h$

$\therefore$ $$\frac{I}{\forall} = \frac{\pi}{64} d^4 \Big/ \frac{1}{3} \times \frac{\pi}{4} d^2 \times h$$

$$= \frac{1 \times 3}{16} \times \frac{d^2}{h} = \frac{3d^2}{16h} = \frac{3}{16h} \times (2r)^2 = \frac{3}{4} \frac{r^2}{h}$$

$$= \frac{3}{4} \frac{(h \tan \theta)^2}{h} \qquad \{\because \; r = h \tan \theta\}$$

$$= \tfrac{3}{4} h \tan^2 \theta$$

and $$BG = AG - AB = \tfrac{3}{4} H - \tfrac{3}{4} h = \tfrac{3}{4}(H - h)$$

$\therefore$ $$GM = \tfrac{3}{4} h \tan^2 \theta - \tfrac{3}{4}(H - h)$$

For stable equilibrium GM should be positive or

$\frac{3}{4} h \tan^2 \theta - \frac{3}{4}(H - h) > 0$ or $h \tan^2 \theta - (H - h) > 0$

or $h \tan^2 \theta > (H - h)$ or $h \tan^2 \theta + h > H$

or $h[\tan^2 \theta + 1] > H$ or $1 + \tan^2 \theta > H/h$ or $\sec^2 \theta > \dfrac{H}{h}$

But $$\frac{H}{h} = \left(\frac{1000}{800}\right)^{1/3} = 1.077$$

$\therefore$ $$\sec^2 \theta > 1.077 \text{ or } \cos^2 \theta > \frac{1}{1.077} = 0.9285$$

$\therefore$ $$\cos \theta > 0.9635$$

$\therefore$ $$\theta > 15° \, 30' \quad \text{or} \quad 2\theta > 31°$$

$\therefore$ Apex angle (2θ) should be at least 31°. **Ans.**

Problem 4.18 *A cone of specific gravity S, is floating in water with its apex downwards. It has diameter D and vertical height H. Show that for stable equilibrium of the cone* $H < \frac{1}{2}\left[\frac{D^2 . S^{1/3}}{2 - S^{1/3}}\right]$.

Solution. Given :

Dia. of cone $= D$

Height of cone $= H$

Sp. gr. of cone $= S$

Let G = Centre of gravity of cone

B = Centre of buoyancy

2θ = Apex angle

A = Apex of the cone

h = Depth of immersion

d = Dia. of cone at water surface

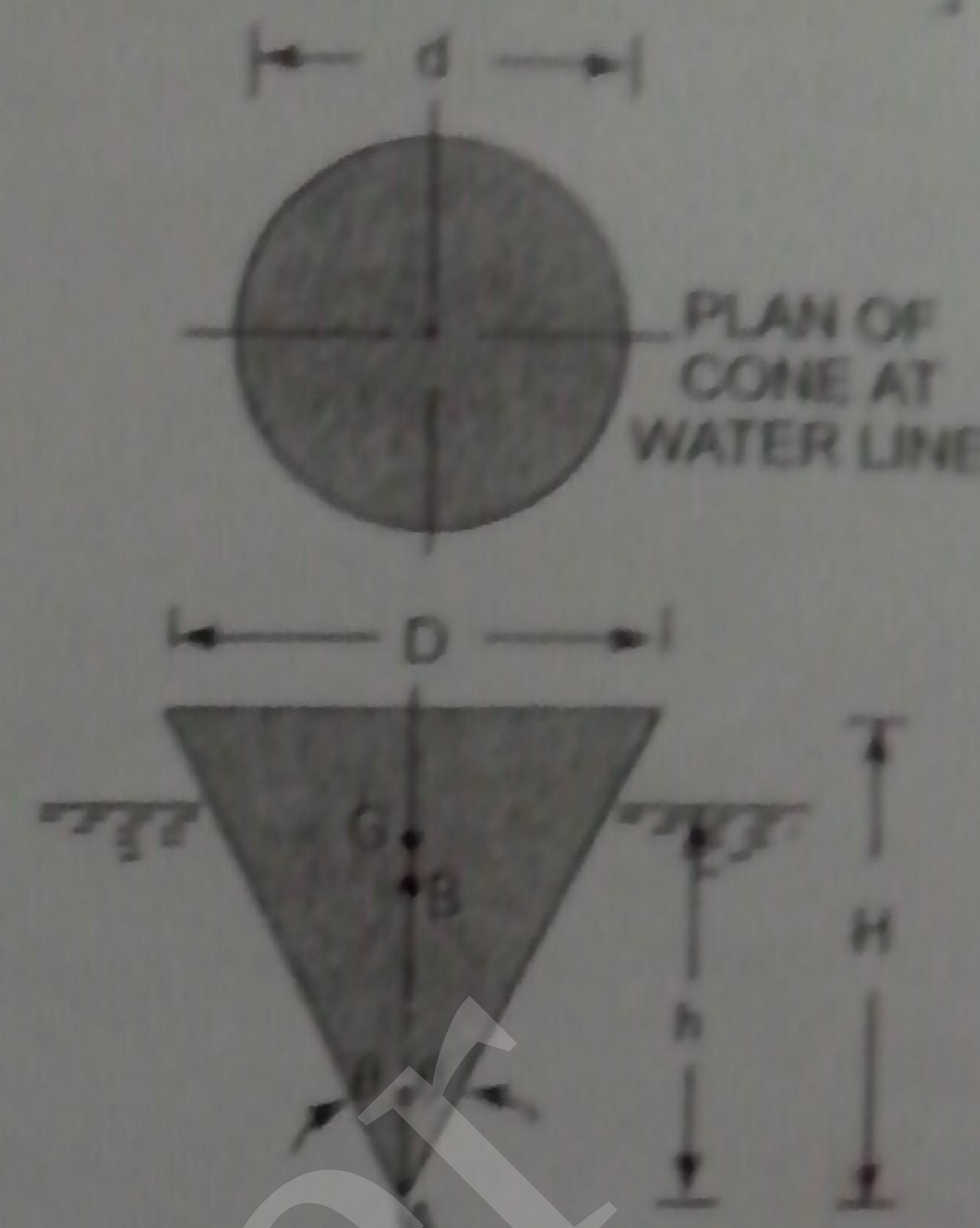


Fig. 4.22

Then $$AG = \frac{3}{4} H$$

$$AB = \frac{3}{4} h$$

Also weight of cone =Weight of water displaced.

$$1000\, S \times g \times \tfrac{1}{3} \pi R^2 \times \text{H} = 1000 \times g \times \frac{1}{3} \pi r^2 \times h \quad \text{or} \quad SR^2 H = r^2 h$$

$\therefore$ $$h = \frac{SR^2 H}{r^2}$$

But $$\tan\theta = \frac{R}{H} = \frac{r}{h}$$

$\therefore$ $$R = H \tan\theta,\ r = h \tan\theta$$

$\therefore$ $$h = \frac{S \times (H \tan\theta)^2 \times H}{(h \tan\theta)^2}$$

$$h = \frac{S \times H^2 \times \tan^2\theta \times H}{h^2 \tan^2\theta} = \frac{SH^3}{h^2} \quad \text{or} \quad h^3 = SH^3$$

or $$h = (SH^3)^{1/3} = S^{1/3} H \qquad \text{...(1)}$$

Distance, $$BG = AG - AB$$

$$= \frac{3}{4} H - \frac{3}{4} h = \frac{3}{4}(H - h) = \frac{3}{4}(H - S^{1/3} H) \qquad \{\because h = S^{1/3} H\}$$

$$= \frac{3}{4} H[1 - S^{1/3}] \qquad \text{...(2)}$$

Also I = M.O. Inertia of the plan of body at water surface

$$= \frac{\pi}{64} d^4$$

$$\forall = \text{Volume of cone in water} = \frac{1}{3} \times \frac{\pi}{4} \times d^2 \times h = \frac{1}{3}\frac{\pi}{4} d^2\, [H.S^{1/3}]$$

$$\therefore \quad \frac{I}{\forall} = \frac{\frac{\pi}{64} d^4}{\frac{1}{3} \times \frac{\pi}{4} d^2\, H.S^{1/3}} = \frac{3d^2}{16.H.S^{1/3}}$$

Now Meta-centric height GM is given as

$$GM = \frac{I}{\forall} - BG = \frac{3d^2}{16.H.S^{1/3}} - \frac{3H}{4}\,[1 - S^{1/3}]$$

GM should be +ve for stable equilibrium or $GM > 0$

or $$\frac{3d^2}{16.H.S^{1/3}} - \frac{3H}{4}(1 - S^{1/3}) > 0$$

or $$\frac{3d^2}{16.H.S^{1/3}} > \frac{3H}{4}(1 - S^{1/3}) \qquad ...(3)$$

Also we know $R = H \tan\theta$ and $r = h \tan\theta$

$$\therefore \quad \frac{R}{r} = \frac{H}{h} = \frac{D}{d}$$

$$\therefore \quad d = \frac{Dh}{H} = \frac{D}{H} \times HS^{1/3} = DS^{1/3}$$

Substituting the value of d in equation (3), we get

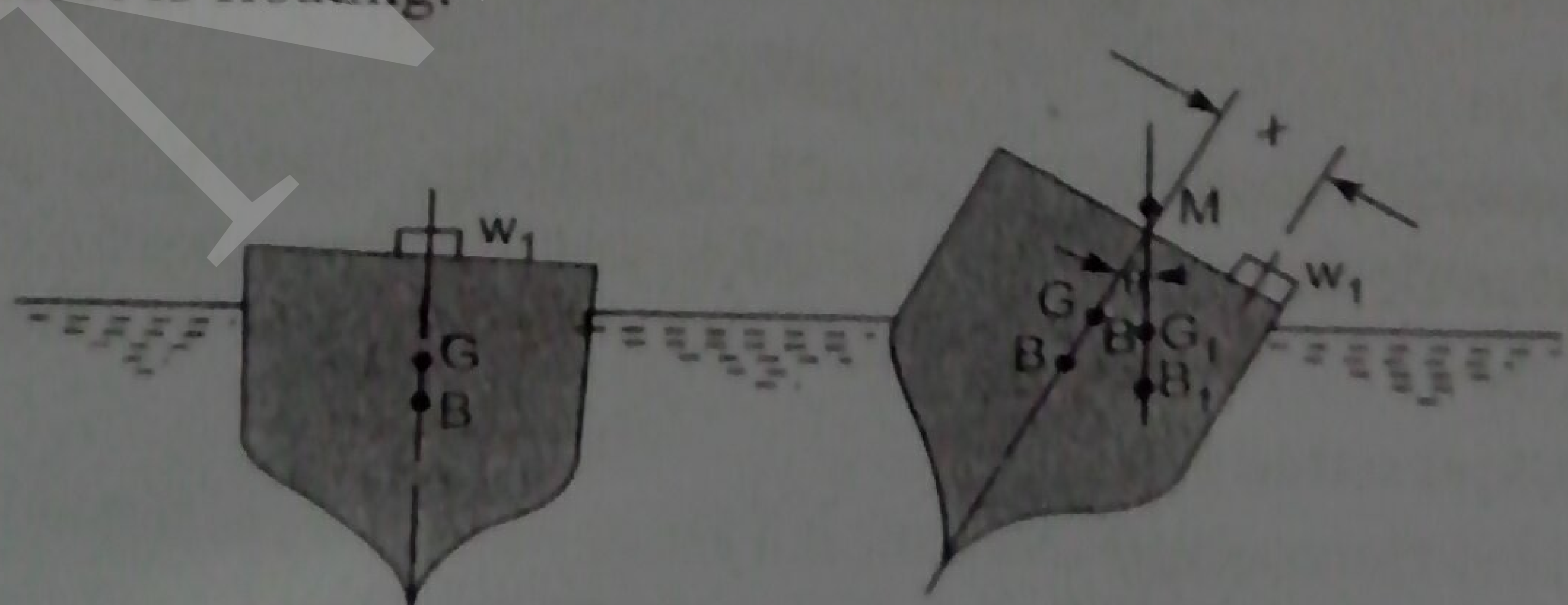
$$\frac{3\left(DS^{1/3}\right)^2}{16.H.S^{1/3}} > \frac{3H}{4}(1 - S^{1/3}) \quad \text{or} \quad \frac{D^2.S^{1/3}}{4.H} > H(1 - S^{1/3})$$

or $$\frac{D^2.S^{1/3}}{4\left(1 - S^{1/2}\right)} > H^2 \quad \text{or} \quad H^2 < \frac{D^2.S^{1/3}}{4\left(1 - S^{1/3}\right)}$$

or $$H < \frac{1}{2}\left[\frac{D^2.S^{1/3}}{1 - S^{1/3}}\right]^{1/2} \quad \textbf{Ans.}$$

▶ 4.8 EXPERIMENTAL METHOD OF DETERMINATION OF META-CENTRIC HEIGHT

The meta-centric height of a floating vessel can be determined, provided we know the centre of gravity of the floating vessel. Let w_1 is a known weight placed over the centre of the vessel as shown in Fig. 4.23 (*a*) and the vessel is floating.



(a) Floating body (b) Tilted body

Fig. 4.23 *Meta-centric height.*

Let W = Weight of vessel including w_1

G = Centre of gravity of the vessel

B = Centre of buoyancy of the vessel

The weight w_1 is moved across the vessel towards right through a distance x as shown in Fig. 4.23 (b). The vessel will be tilted. The angle of heel θ is measured by means of a plumbline and a protractor attached on the vessel. The new centre of gravity of the vessel will shift to G_1 as the weight w_1 has been moved towards the right. Also the centre of buoyancy will change to B_1 as the vessel has tilted. Under equilibrium, the moment caused by the movement of the load w_1 through a distance x must be equal to the moment caused by the shift of the centre of gravity from G to G_1. Thus

The moment due to change of $G = GG_1 \times W = W \times GM \tan\theta$

The moment due to movement of $w_1 = w_1 \times x$

$$\therefore \quad w_1 x = W GM \tan\theta$$

Hence

$$GM = \frac{w_1 x}{W \tan\theta} \quad \ldots(4.5)$$

Problem 4.19 *A ship 70 m long and 10 m broad has a displacement of 19620 kN. A weight of 343.35 kN is moved across the deck through a distance of 6 m. The ship is tilted through 6°. The moment of inertia of the ship at water-line about its fore and aft axis is 75% of M.O.I. of the circumscribing rectangle. The centre of buoyancy is 2.25 m below water-line. Find the meta-centric height and position of centre of gravity of ship. Specific weight of sea water is 10104 N/m³.*

Solution. Given :

Length of ship, $L = 70$ m

Breadth of ship, $b = 10$ m

Displacement, $W = 19620$ kN

Angle of heel, $\theta = 6°$

M.O.I. of ship at water-line = 75% of M.O.I. of circumscribing rectangle

w for sea-water $= 10104\ N/m^3 = 10.104\ kN/m^3$

Movable weight, $w_1 = 343.35$ kN

Distance moved by w_1, $x = 6$ m

Centre of buoyancy = 2.25 m below water surface

Find (i) Meta-centric height, GM

(ii) Position of centre of gravity, G.

(i) **Meta-centric height, GM** is given by equation (4.5)

$$\therefore \quad GM = \frac{w_1 x}{W \tan\theta} = \frac{343.35\ kN \times 6.0}{19620\ kN \times \tan 6°}$$

$$= \frac{343.35\ kN \times 6.0}{19620\ kN \times .1051} = \mathbf{0.999\ m.\ Ans.}$$

(ii) **Position of Centre of Gravity, G**

$$GM = \frac{I}{\forall} - BG$$

where I = M.O.I. of the ship at water-line about Y-Y

water line about its force and aft axis is 75% of moment of inertia the circumscribing rectangle. The centre of buoyancy is 2.75 m below water line. Find the meta-centric height and position of centre of gravity of ship. Take specific weight of sea water = 10104 N/m^3. [Ans. 1.1145 m, 0.53 m below water surface]

19. A pontoon of 1500 tonnes displacement is floating in water. A weight of 20 tonnes is moved through a distance of 6 m across the deck of pontoon, which tilts the pontoon through an angle of 5°. Find meta-centric height of the pontoon. [Ans. 0.9145 m]

20. Find the time period of rolling of a solid circular cylinder of radius 2.5 m and 5.0 m long. The specific gravity of the cylinder is 0.9 and is floating in water with its axis vertical. [Ans. 6.35 sec]