1. (a) Find $y(x)$ if $\frac{dy}{dx} = x^2 y \sec^2(x^3)$, and $y(0) = -3$.

   (b) Find a function $y(x)$ such that $y(1) = -1$ and $xy' + y = y^2$.

2. When a cake is removed from an oven, the temperature of the cake is 210 F. The cake is left to cool at room temperature (70 F) and after 30 minutes the temperature of the cake is 140 F. According to Newton's law of cooling, the rate of change of temperature of a body is proportional to the temperature difference between the body and the environment. Set up and solve a differential equation to find when the temperature of the cake will be 100 F.

3. At time $t = 0$ a 5000 liter tank is full of pure water. Starting at that moment salt is added to it at a steady rate of 40 grams per hour. Assume that the salt is thoroughly mixed in the water. Meanwhile, pure water is entering the tank at 50 liters per hour and the salty water in the tank is leaving at the same rate. Let $y(t)$ be the amount of salt in grams in the water in the tank after t hours. Compute $\lim_{t\to\infty} y(t)$.

4. A vat contains 100 liters of yogurt. Pure yogurt flows in at 5 L/min. A bacteria colony is growing inside the vat (assume it is mixed uniformly at all times). Mixed yogurt flows out at 5 L/min. Initially there are 1000 bacteria. Let $P(t)$ represent the number of bacteria at time t. Left alone, the bacteria grow at a rate proportional to P (i.e. kP for some constant k).

   (a) Find an equation for $\frac{dP}{dt}$ in terms of k and t.

   (b) Solve for $P(t)$ in terms of k and t.

   (c) Suppose that left alone the bacteria population doubles every hour. Compute $\lim_{t\to\infty} P(t)$.

   (d) Let k be as in (c). Instead of pure yogurt, say we let bacteria filled yogurt flow in at 5 L/min. What concentration of bacteria flowing in (call it b bacteria/liter) will result in $\lim_{t\to\infty} P(t) = 1000$?