LAKSHYA JEE

LAKSHYA KO HAR HAAL ME PAANA HAI

Electric Potential & Capacitance

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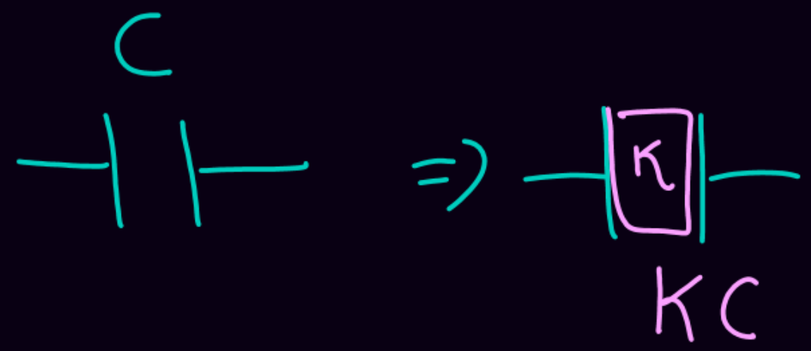
Today's GOALS!

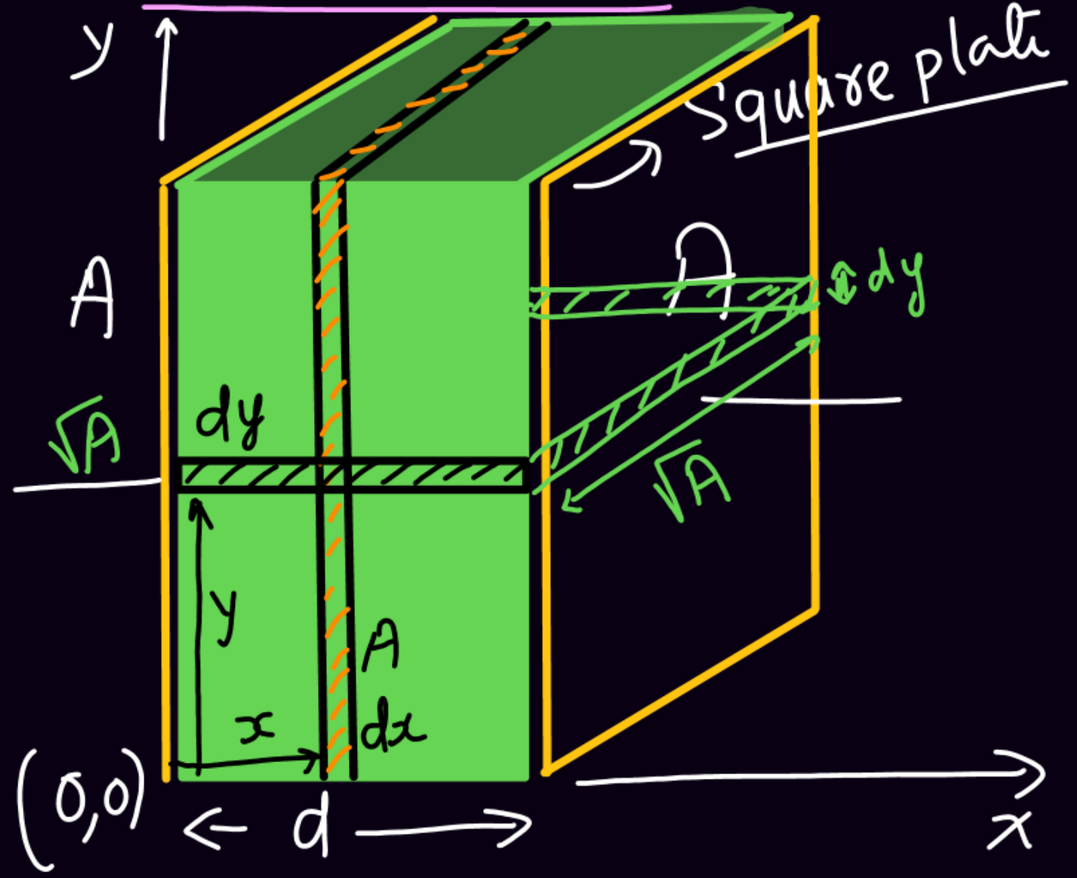
Capacitors

&

Dielectrics


C
K
KC

Variable K.



① $K = K_0 + K_1 x$

find C (Series).

② $K = K_0 + K_1 y$

find C. (Parallel)

① $dC = (K_0 + k_1 x)\dfrac{\varepsilon_0 A}{dx}$

$\dfrac{1}{C_{eq}} = \dfrac{1}{dc_1} + \dfrac{1}{dc_2} + \dfrac{1}{dc_3} + \cdots$

$\dfrac{1}{C_{eq}} = \int \dfrac{1}{dC} = \int_0^d \dfrac{dx}{\varepsilon_0 A (k_0 + k_1 x)}$

$$\frac{1}{C_{eq}} = \int \frac{1}{dC} = \int_0^d \frac{dx}{(K_0 + K_1 x)\,\varepsilon_0 A}$$

$$= \frac{1}{\varepsilon_0 A} \cdot \frac{\ln(K_0 + K_1 x)}{K_1}\Bigg|_0^d$$

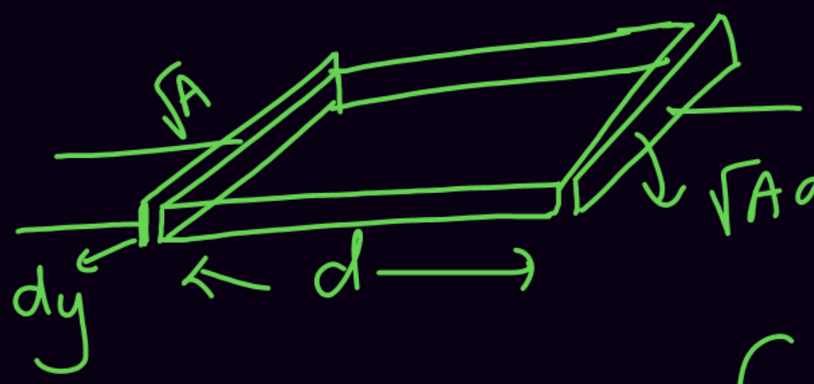
$$= \frac{1}{K_1 \varepsilon_0 A}\left[\ln(K_0 + K_1 d) - \ln(K_0 + K_1 \cdot 0)\right]$$

$$\frac{1}{C_{eq}} = \frac{1}{K_1 \varepsilon_0 A}\left(\ln\left(\frac{K_0 + K_1 d}{K_0}\right)\right)$$

$$\Rightarrow C_{eq} = \frac{K_1 \varepsilon_0 A}{\ln\left(\frac{K_0 + K_1 d}{K_0}\right)}$$

(2) $dC = \dfrac{(K_0 + K_1 y)\,\varepsilon_0\,(\sqrt{A}\,dy)}{(d)}$

$\dfrac{\varepsilon_0 A}{d}\left[K_0 + \dfrac{K_1\sqrt{A}}{2}\right]$

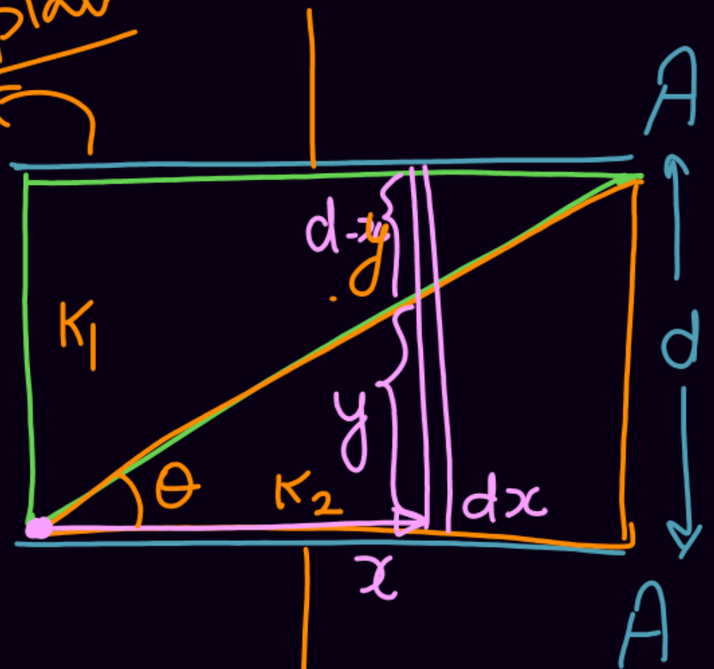


$$C_{eq} = \int dC = \int_0^{\sqrt{A}} (K_0 + K_1 y)\left(\frac{\varepsilon_0\sqrt{A}}{d}\right) dy$$

$$= \frac{\varepsilon_0\sqrt{A}}{d}\left[\int_0^{\sqrt{A}} K_0\,dy + \int_0^{\sqrt{A}} K_1 y\,dy\right]$$

$$= \frac{\varepsilon_0\sqrt{A}}{d}\left[K_0 y\Big|_0^{\sqrt{A}} + K_1\frac{y^2}{2}\Big|_0^{\sqrt{A}}\right] = \frac{\varepsilon_0\sqrt{A}}{d}\left[K_0\sqrt{A} + \frac{K_1}{2}(\sqrt{A})^2\right]$$

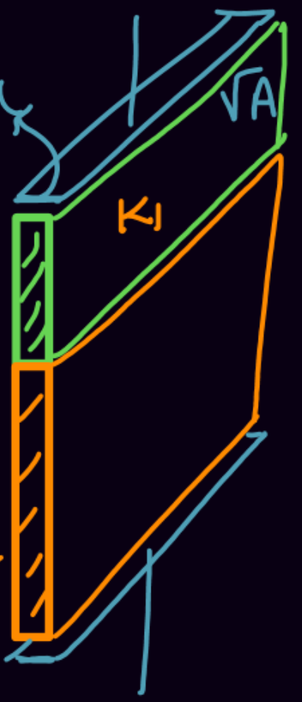
Square plate



$$\tan\theta = \frac{y}{x} = \frac{d}{\sqrt{A}}$$

$$y = \frac{d}{\sqrt{A}}\,x$$

Find C?



$$dC_1 = \frac{K_1 \varepsilon_0 \sqrt{A}\,dx}{d-y}$$

$$dC_2 = \frac{K_2 \varepsilon_0 \sqrt{A}\,dx}{y}$$

$$\frac{1}{dC} = \frac{1}{dC_1} + \frac{1}{dC_2} = \frac{d - \frac{dx}{\sqrt{A}}}{K_1 \varepsilon_0 \sqrt{A}\,dx} + \frac{\frac{dx}{\sqrt{A}}}{K_2 \varepsilon_0 \sqrt{A}\,dx}$$

$$\frac{1}{dC} = \frac{d}{\varepsilon_0 \sqrt{A}\,dx}\left[\frac{\frac{\sqrt{A}-x}{\sqrt{A}}}{K_1} + \frac{\frac{x}{\sqrt{A}}}{K_2}\right]$$

$$\frac{1}{dC} = \frac{d}{\varepsilon_0 A\,dx\,K_1 K_2}\left[K_2\sqrt{A} + (K_1 - K_2)x\right]$$

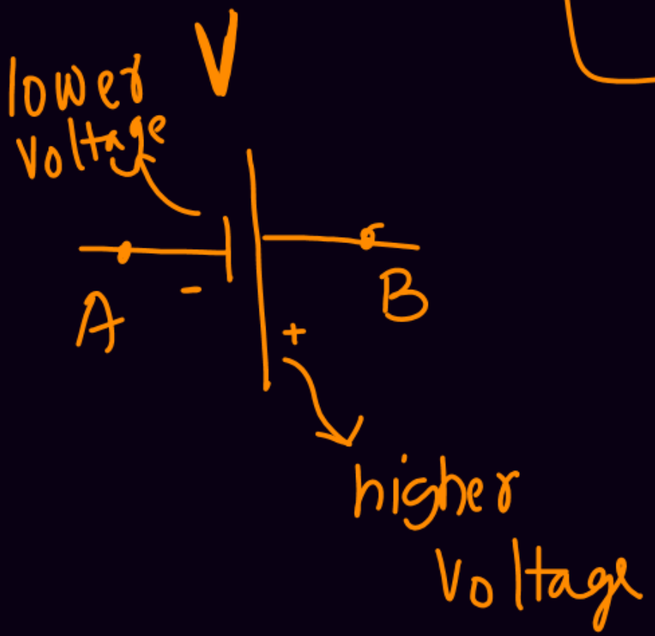
$$\int dC = \int_0^{\sqrt{A}} \frac{\varepsilon_0 A K_1 K_2 \, dx}{d\left(K_2\sqrt{A} + (K_1 - K_2)x\right)}$$

$$= \frac{\varepsilon_0 A K_1 K_2}{d} \frac{\ln\left(K_2\sqrt{A} + (K_1 - K_2)x\right)}{(K_1 - K_2)} \Bigg|_0^{\sqrt{A}}$$

$$= \frac{\varepsilon_0 A K_1 K_2}{d(K_1 - K_2)} \ln\left(\frac{K_2\sqrt{A} + (K_1 - K_2)\sqrt{A}}{K_2\sqrt{A}}\right)$$

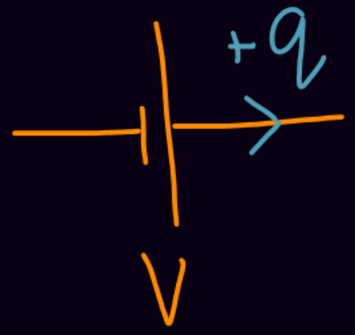
Battery

$$V_B - V_A = V$$



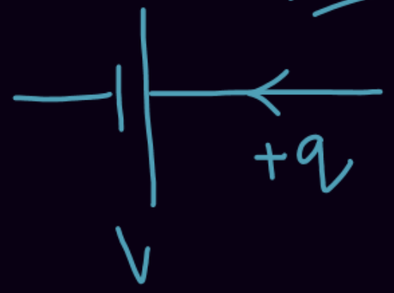
Work done by battery

Discharging of battery

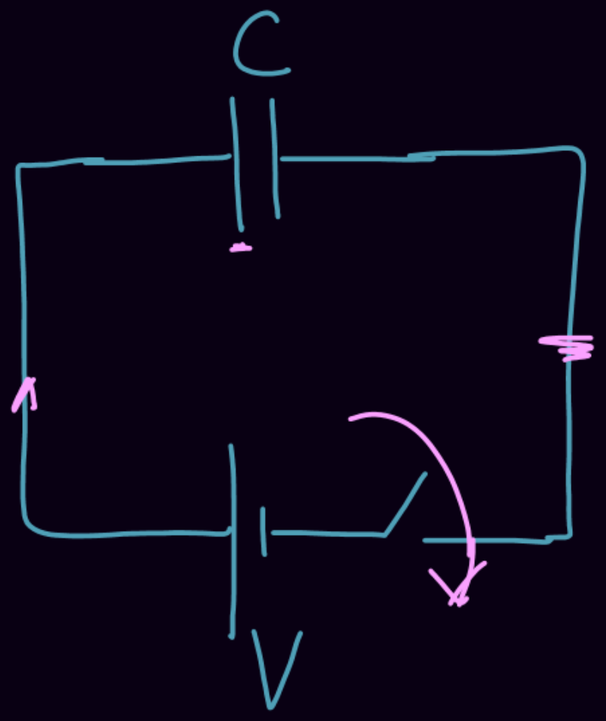


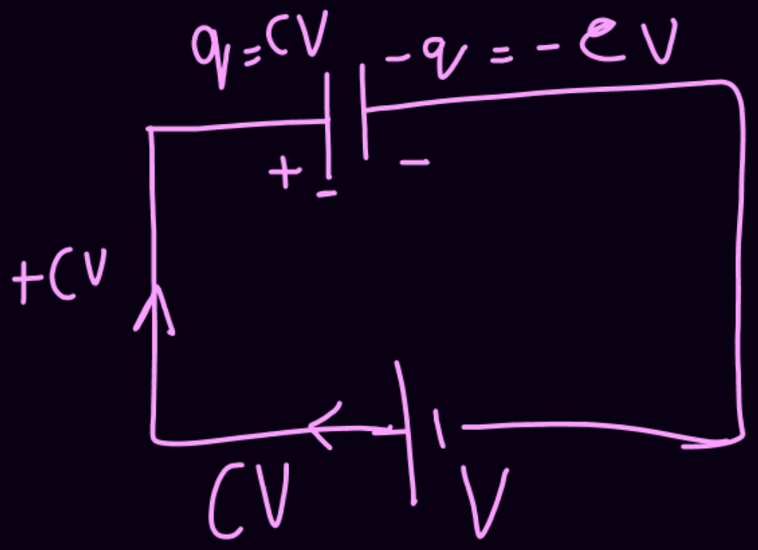
$W_{battery} = qV$

Charging



$W_{battery} = -qV$



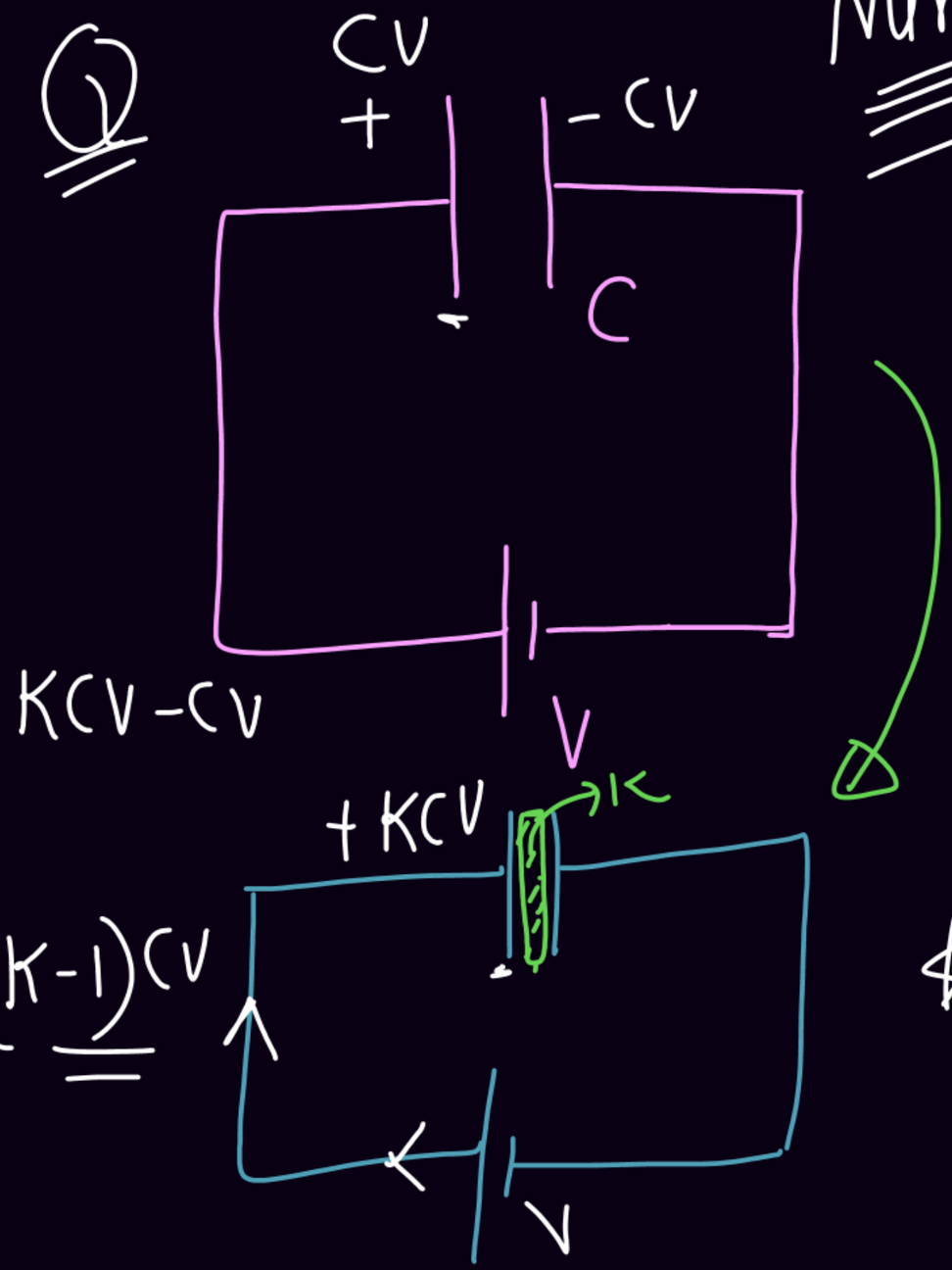


① $W_{battery} = +(CV)V = +CV^2$

② $U_{capacitor} = \frac{1}{2} C V^2$

$$U_{cap} = \frac{1}{2} W_{battery}.$$

Numerical



Find $W_{battery}$ when dielectric of dielectric constant k is inserted in the capacitor.

$$W_{battery} = (K-1)CV \times V$$
$$= (K-1)CV^2$$

Find the heat loss?

☆ Initial energy + $W_{by\ battery}$ = Final energy + Δ Heat loss

$$\frac{1}{2}CV^2 + (K-1)CV^2 = \frac{1}{2}(KC)V^2 + \Delta U + \Delta \text{Heat loss}$$

$$\frac{1}{2}cv^2 + (k-1)cv^2 = \frac{1}{2}(k+1)v^2 + \Delta n.$$

$$cv^2\left[\frac{1}{2} + k - 1 - \frac{k}{2}\right] = \Delta H.$$

$$cv^2\left(\frac{k}{2} - \frac{1}{2}\right)$$

$$\frac{1}{2}(k-1)cv^2 = \Delta n$$

Two condensers of capacities C_1 and C_2 are connected in parallel and charged. Then the ratio of charge on C_1 to charge on C_2 is

1) $\frac{C_2}{C_1}$

2) $\left(\frac{C_2}{C_1}\right)^2$

3) $\frac{C_1}{C_2}$

4) $\left(\frac{C_1}{C_2}\right)^2$

Two identical parallel plate capacitors are joined in series to 100 V battery. Now a dielectric constant (K = 4) is introduced between the plates of second capacitor. The potential differences on capacitors are

1) 60 V, 40 V 2) 70 V, 30 V 3) 75 V, 25 V 4) 80 V, 20 V

The work done in increasing the potential difference across the plates of a capacitor from 4 V to 6 V is W. The further work done in increasing the potential difference from 6 V to 8 V is

1) W 2) 5W/7 3) 7W/5 4) 2W/5

Three condensers of capacities 3 μF, 4 μF and 5 μF are connected in series and a constant potential is applied between the ends of the combination. Their potentials are in the ratio of

1) 5 : 4 : 3 2) 3 : 4 : 5 3) 4 : 5 : 3 4) 20 : 15 : 12

The radius of the circular plates of a parallel plate condenser is r_1. Air is there as dielectric. The distance between the plates if its capacitance is equal to that of an isolated sphere of radius r is

1) $r_1^2 / 4r$ 2) r^2 / r_1 3) r / r_1 4) $r^2 / 4$

Three condensers each of capacity 6 μF are available. The effective capacity that cannot be obtained by combining them all is

1) 18 μF 2) 9 μF 3) 4 μF 4) 8 μF

The capacity of a parallel plate condenser with air medium is 60 μF having distance of separation d. If the space between the plates is filled with two slabs each of thickness d/2 and dielectric constants 4 and 8, the effective capacity becomes

1) 160 μF 2) 320 μF 3) 640 μF 4) 360 μF

Two metal spheres of radii 16 cm and 20 cm having charges 10 nC and 6 nC respectively are connected by a wire. Then the common potential is

1) 400 V 2) 200 V 3) 300 V 4) 100 V

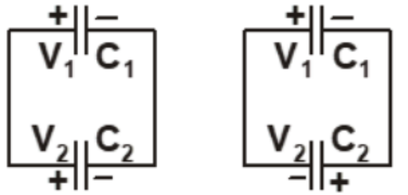
Two condensers of capacities C_1 and C_2 charged to potentials V_1 and V_2 are connected in two ways as shown in the figure. The ratio of loss of energies in the case (i) to case (ii) is

1) $\frac{V_1 - V_2}{V_1 + V_2}$

2) $\frac{(V_1 - V_2)^2}{(V_1 + V_2)^2}$

3) $\frac{V_1^2 - V_2^2}{V_1^2 + V_2^2}$

4) $\frac{(V_1 + V_2)^2}{(V_1 - V_2)^2}$



case(i)

case(ii)

A parallel plate capacitor with air as medium between the plates has a capacitance 10 μF. The area of the capacitor is divided into two equal halves and filled with two media having dielectric constants 2 and 4. The capacitance of the system will be

1) 10 μF 2) 20 μF 3) 30 μF 4) 40 μF

Thank You Lakshyians