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Practical Implicit Geological Modelling

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ABSTRACT

Traditional method of explicitly defining three-dimensional (3D) ore-waste and geological boundaries relies heavily on a time-consuming process of manual digitisation. This method of modelling can be best described as surface modelling, as complex surface geometry is built up by digitising points that lie on the surface. With the advent of fast 3D interpolation methods, however, construction of geological surfaces using volume functions is now a practical alternative to explicit modelling of surfaces. Unlike explicit modelling, surfaces contained in volume functions are not explicitly defined or digitised. Instead the existence of surfaces in the volume function is implicit, thus the process of modelling surfaces from volume function is called 'implicit modelling'.

Based on recent advances in fast scattered data interpolation methods, implicit modelling first defines a continuous three-dimensional function that describes the grade or rock distribution. This volumetric function is interrogated for a grade value, or a geological surface, thus allowing the extraction of the 3D object to be automated and eliminating the need to manually digitise surfaces. Since the function is continuous throughout space and does not depend on a mesh or grid for its definition, the extracted geological or grade wireframes can be constructed at any desired resolution in the specific volume of interest.

The volume modelling method can work on scattered drillhole data of any data density, including processing combined information from dense grade control data as well as sparse resource drilling. It offers distinct advantages over surface modelling, including: being able to wireframe geological objects that may take days to digitise in tens of minutes; substantial improvement in modelling accuracy; and the ability to generate conditional models rapidly allowing mining risks inherent in geological modelling to be examined.

INTRODUCTION

The construction of grade and lithological wireframe models from drillhole data is a time-consuming and challenging task performed routinely by mine geologists. Generating accurate grade and geological boundary models is clearly necessary, as the quality of the model will influence various downstream mining practices. These include grade estimation and mine planning that ultimately impact on mine economics.

The traditional 'explicit' method of 3D modelling—implemented in all major general mining software packages (GMPs)—relies heavily on manual digitisation. In this paper, we introduce the 'implicit' modelling methodology, which can speed up modelling processes significantly. A digitisation task that may take tens of hours with the traditional method, for example, could be compressed down to tens of minutes using the implicit modelling method. We illustrate this new method with examples, ranging from grade value wireframing to geological boundary modelling.

THE TRADITIONAL METHOD OF SURFACE MODELLING

Traditional solid modelling methods use a patchwork of triangles to define a complex-shaped surface. This is termed an 'explicit' model of the solid because the surface is defined by the surface elements and their arrangement. There is nothing else—the coordinates of the triangles are explicit and can be immediately rendered to a computer screen.

The explicit modelling method requires digitising the outlines of complex three-dimensional bodies by viewing the drillhole data in serial sections. These 2D polylines are then joined using tie-lines to define the three-dimensional connectivity between the polylines. The tied polylines are then triangulated to produce the 3D solid-body triangulation model. During this modelling process, the geologist must firstly ensure that lithological boundary positions sampled by the drillholes are honoured, and secondly, incorporate any local directional bias the deposit may have during the digitisation process, such as interpreted geological or grade continuity trends.

The manual joining of the 2D polylines is the most time-consuming part of the workflow and requires an experienced modeller to construct complex geometries with digitisation tools. Geometries of ore bodies often have to be simplified in order to get a model constructed in a practical timeframe, but another disadvantage of manual digitisation is the fact that the model produced is unique to each individual geologist's interpretation and cannot be exactly replicated by others. This is because the directional bias and smoothing that the geologist introduced during the modelling process is unique to the individual; therefore interpretations are written into the solid model during its manual construction.

For most mines, only a single working model is maintained because of time constraints. Rarely is there an opportunity to model alternative interpretations and compare resource estimations based on the alternative models. This misses the opportunity to assess mining risk that is inherent in geological modelling. Further, it is uncommon for an operating mine to drastically alter their working models on a regular basis as new drillhole data becomes available due to the time-consuming nature of the modelling methodology. However, it is unavoidable that geological interpretations change as more data becomes available, but the surface-based modelling procedure does not allow wholesale rapid changes to the models, or semiautomatic construction of models as new data becomes available. The result of this technological limitation is that geological modelling is conducted in campaigns.

Geological shapes of any geometry can be manually digitised, but the limitations of this methodology are as follows:

- manual digitisation is time consuming if complex shapes are being modelled;

- models consisting of explicit surface triangulations cannot be automatically updated as more data becomes available;

- any edits or additions involve complex manipulation of the model and thus is approached on campaigns rather than on a continual basis; and

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- interpretations of the geologist are written into the model, therefore the model cannot be easily replicated by other geologists, placing an unknown risk to any downstream mining procedures.

These issues related to time are considerable and an alternative modelling methodology would clearly be desirable. We propose implicit modelling as an alternative modelling methodology to manual digitisation.

IMPLICIT MODELLING

Implicit surfaces

An 'implicit' model of a solid is given by a function defined throughout space[†]. This volume function is modelled from spatially interpolating sampled drillhole data and the surface of the solid is extracted as triangulations from this function. The surfaces to be modelled are therefore not constructed directly, as done in the explicit method, but instead are finite approximation of surfaces with infinite detail. These surfaces are implied to exist in a continuous volume function, therefore are referred to as 'implicit surfaces'.

A simple illustrative example of such a function would be that of a sphere with an unit radius: $x^2 + y^2 + z^2 - 1 = 0$ (which is in the form $f(x,y,z) = C$, where C is a constant). This equation describes the infinite number of (x,y,z) coordinates that lie on the surface of the sphere. Note that the surface of the sphere is only implied in the equation, as the coordinates are functional arguments. The actual coordinate position of the sphere therefore cannot be directly determined from the equation. In order to determine the position of the sphere in space, various (x,y,z) coordinates are inserted into the sphere equation and the scalar values returned will indicate whether the point is inside (< 0) or outside (> 0) of the sphere surface (= 0). This is conventionally done on a three-dimensional grid. By using grid-based evaluation methodologies, one can spatially converge to coordinate positions where the function value approaches zero and the approximate position of the sphere surface can be determined (eg Boomenthal, 1998).

It seems unnecessarily complicated to propose such a computer-intensive sampling technique to construct geological surfaces. However, there is one important characteristic of implicit functions that cannot be replicated with automated methods based on explicit equations: that is, the ability of implicit functions to describe shapes of any geometry, such as complex overturned folds, or enclosed surfaces such as spheres.

Consider an explicit function that might express the z coordinate in terms of x and y coordinates; that is, $z = f(x,y)$. Such a surface is called a height field function, and are useful for describing data such as topography. However, the different treatment of z to x and y coordinates does not allow multiple z values to occur at an unique x, y coordinate position and this inherently limits the types of shapes that can be described by such a surface function. For example, a height field cannot contain vertical slope or overfolding of a surface, but such geometries are commonly represented in geological bodies. Research into automated processes to generate shapes of any geometry using surface function methods have not resulted in trivial solutions (eg Sirakov *et al*, 2002; Xu and Dowd, 2003). Geologists therefore have had limited choice other than to hand digitise complex geological surfaces and grade boundaries.

Such geometrical restrictions do not exist when implicit surfaces are modeled with volume functions. In addition, since geological data is inherently volumetric, the implicit representation of surfaces is an ideal one. Grade distribution, for example, can be defined as a volumetric function and the grade isosurfaces evaluated at any resolution in the volume of interest. Such use of volume functions and its implicit surfaces to represent grade envelopes and geological boundary surfaces is herein termed 'implicit modelling'.

Implicit modelling of real world objects is not widely attempted, even in the field of computer graphics, as it has been considered computationally impractical. Research into the representation of real world objects with implicit surfaces can be traced back to the pioneering work of Savchenko *et al* (1995). More recently, papers discussing similar techniques have appeared in the computer graphics and geological literature (Turk and O'Brien, 1999, 2002; Carr *et al* 2001; Ledez, 2001) but implicit modelling is generally unknown in the resource industry. Most published papers on implicit modelling describe working with small number of datapoints, but the technique outlined by Carr *et al* (2001) is the only published methodology that allows for the processing of very large datasets that typify drillhole databases.

From manual wireframing to implicit models

In order to illuminate the path from explicit modelling to implicit modelling, consider a two dimensional drillhole fence, from a three-dimensional drillhole database, illustrated in Figure 1a.

The traditional explicit modelling method would have the modeller simply join up the contact points manually, producing the angular overturned contact shape shown as the red line (Figure 1b). It is entirely the choice of the modeller to include extra points between the observed contact locations to smooth the surface. It should be also noted that the positions of the additional contact points is arbitrary.

To construct an implicit model of the contact, a volume function with an isosurface that includes the contact points and points connecting them must be created. A function is defined throughout space by specifying the function values at selected points and interpolating through the rest of the space. To ensure that all the contact points are included in the isosurface, the same values are assigned to these points. Conventionally a value of zero is assigned. This is not sufficient to give the isosurface we require because an interpolation from these points will yield zero values everywhere. To produce a locus of zero points that are confined to a contour that include the contact points, regions must be specified where the function becomes positive and regions where it is negative. By choosing one of the lithologies in Figure 1a to represent positive values of the function and the other negative leads to Figure 1c.

Following this, the lithology data sampled by the drillholes are now converted from lithological code to numerical values, with the surface contact intercepts attributed with a value of zero. The contact surface can be now treated as a scattered data interpolation problem. Once the data is interpolated in space the zero isosurface can be extracted from the function as the contact surface between the two lithologies at any resolution (Figure 1d). Although the example in Figure 1 is two dimensional to emphasise clarity, the principle extends to the third dimension and the contact surface is extracted as a 3D triangulated isosurface mesh of value zero.

A major advantage of implicit modelling is that it requires little or no manual digitisation, as the surfacing is defined by attributing the lithological data with numerical values and the rest is taken care of by three-dimensional interpolation. Though not detailed here, the method does require careful consideration of gradients imposed on the value attribution for off-surface values for lithological modelling, but this step can also be automated in most situations.

[†] This function is variously referred to as 'field', 'scalar field', 'potential' or 'implicit' functions in the computer graphics literature. The implicit surfaces of this function are commonly referred to as 'level set', or an 'isosurface' of the volume function.
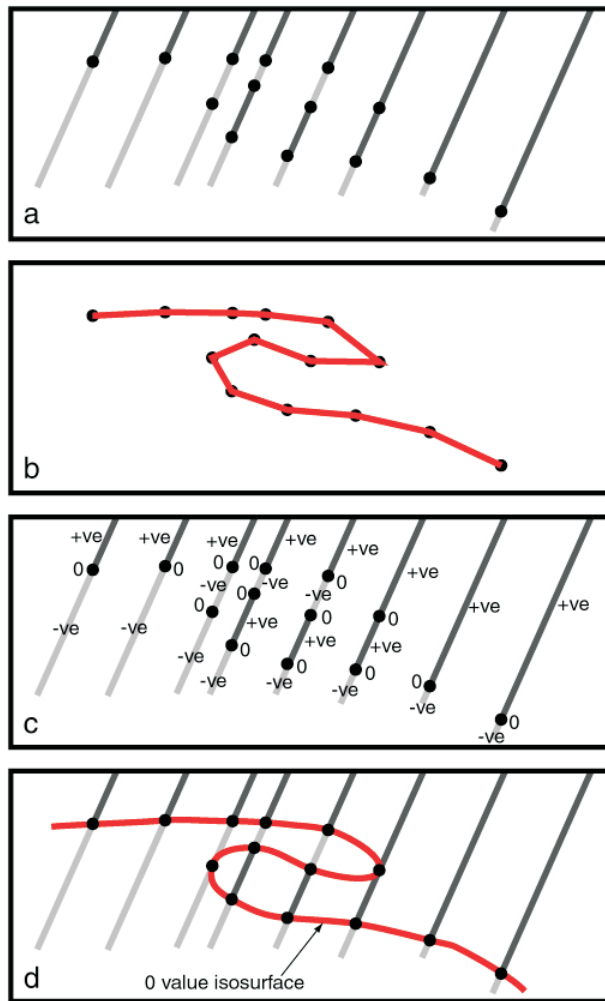
FIG 1 - (**a**) Hypothetical 2D drillhole fence from a three dimensional drillhole dataset. Black and grey colours represent two lithologies and the contact surface is interpreted to be an overturned fold. (**b**) An angular polyline represents manual joining of the contact positions (black dots). (**c**) The drillhole data can be coded according to their relative position from the contact surface. The on-surface points are attributed as zero and the off-surface regions are assigned positive and negative values. (**d**) Spatial interpolation of the data values and subsequent evaluation of the zero value isocontour (isosurface in 3D), yields the contact surface between the two lithologies. The 3D function is continuous and differentiable, therefore the contact surface can be meshed at any desired resolution.

Radial basis function

One essential ingredient of implicit modelling is that a practical three-dimensional interpolation method must be used to construct the volume function. We use radial basis function (RBF) interpolation to model grade and lithological data in 3D space. Radial basis functions are a family of interpolation functions that were first introduced in the geological literature by Hardy (1971). Hardy's multiquadrics and related RBF interpolants (Franke, 1982; Hardy, 1990; Dubrule, 2003) were developed in parallel, but independently of, the theory of regionalised variables (Matheron, 1963).

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‡   Data source of some of these examples are confidential.

RBF interpolation represents the function as a sum of so-called basic functions, with linear weights in exactly the same way as the dual formulation of kriging (cf. Chilès and Delfiner, 1999). The difference between kriging and RBF interpolation is that kriging uses the covariance function obtained from the data (a variogram) and RBF uses a basic function that is chosen from a standard set. This set includes a function that corresponds to a linear variogram without a sill: the biharmonic function. In situations where the variogram cannot be obtained or where the assumptions of stationarity are not necessarily valid (eg interpolating across different domains; sparse sampling), RBF interpolation is a valuable technique. The technique is suitable for lithological boundary modelling, as there is no formal mathematical description of how a geological surface must behave, other than the fact that the sampled contacts should be honoured. The biharmonic function (analogous to thin-plate spline in 2D) minimises the volumetric energy of interpolation function to describe geological surfaces. This results in smooth surfaces between the sample points, which is ideal, given that when most geological surfaces are manually constructed and the surfaces between the drillholes are smoothed out manually by digitising intervening nodes.

RBF interpolation, being a global interpolation method, requires all the data points to be used to calculate the coefficients (ie the weights assigned to each value). One of the limitations of RBF is the fact that large datasets result in data storage problems (eg a modest 20 000 points appears to require 1.5 GB RAM). Also calculation of the RBF coefficients with a database of 40 000 data points appears to require hours on current workstations. Even if such modest numbers of data could be interpolated, datapoints of 40 000 falls far short of numbers of assay data of large mines where it is not uncommon to have more than half a million assay values in its database.

Because of this severe limitation of RBF to process large datasets (ie >40 000 3D scattered data values), the application of RBF to geological problems has only seen very limited use and scope (eg Saunderson, 1994; Ahmed and Murthy, 1997). We instead use a fast form of RBF interpolation, as this method has managed to overcome limitations of the direct RBF interpolation method (Beatson *et al*, 1999). Full description of the mathematics of this method is beyond the scope of this paper, although it is suffice to say that the methodology works on the fact that when RBF computations are performed, infinite precision is neither required nor expected (Beatson *et al*, 1999; Carr *et al*, 2001; Billings *et al*, 2002). The use of approximation is therefore allowed to simplify the computation of the coefficients to within a predetermined accuracy. This accuracy, in the case of assay data, is set to a fraction of the detection limit. In addition to being able to globally interpolate very large datasets the fast numerical method does not require storage of the data matrix at once, thus eliminating the memory storage problem. The fast RBF method also sees a dramatic increase in the speed of computation and very large datasets, such as entire mine drillhole data can be processed within hours on a modern personal computer. This is a feat previously thought unattainable with RBF (Sibson and Stone, 1991), making this the first practical 3D interpolation method available to process large datasets.

APPLICATIONS OF IMPLICIT GEOLOGICAL MODELLING

Below we outline six applications of implicit modelling conducted using real geological drillhole data‡. The examples highlight the varied use of implicit modelling but they all share the same property – that the surfaces constructed are generated from a continuous volumetric function obtained from interpolating the original drillhole data with RBF.

Example 1: Near mine exploration and targeting – Cosmo Howley gold deposit

The Proterozoic Cosmo Howley gold deposit (Alexander *et al*, 1990; Matthäi *et al*, 1995) has produced a total of 14.77 Mt (475 000 oz) of gold between 1987 and 1994 from an open pit operated by Dominion Mining Ltd. The resource comprises several steeply dipping stratabound lenses that are developed on the limbs and axial zone of a prominent regional-scale, overturned, northwest plunging anticline.

Recent acquisition of a 50 per cent interest in Cosmo Howley by Harmony Gold Operations Ltd prompted an internal review of the deposit to identify drillhole targets and determine the resource under the current 155 m deep pit. The main data available to accomplish this task were the resource and grade control drillhole database that were reconstructed and validated from the original database of Dominion Mining Ltd. It was considered important to derive as much information on the within-pit distribution of mineralisation as possible so that informed geological assessment can be made of ore shoot continuity below the grade control data into the sparsely drilled region below the pit.

In order to determine the major trends in mineralisation below the pit, 3D solids of grade cut-off meshes constructed from the resource and grade control data were regarded as essential information. Applying the explicit modelling methodology, polylines of 1 g/t and 4 g/t grade contours were digitised in 2.5 m or 5 m benches and an attempt was made to join these contours in the third dimension with tie lines. Joining these contours vertically proved frustratingly difficult as grade continuity from one level to the next was difficult to assess. The main hindrance to the interpretation was the high nugget effect of the assay data, which contributes significantly to discontinuous spatial behaviour of gold grade.

Three days were spent manually digitising these two grade cut-off values and the resulting volume wireframed was very small in relation to the scale of the pit (Figure 2a). The time allocated for this modelling exercise was ten working days. The manual digitisation was therefore deemed inappropriate and was abandoned in favour of implicit modelling of the grade boundaries.

The fold-hinge setting of the mineralisation clearly indicated that the assumption of stationarity is not appropriate at the scale of the deposit. The combined grade control and resource grade data was therefore modelled with a RBF function, which did not require time-consuming domaining and inputs of 3D variogram parameters. The only pre-processing step that was required was to composite the grade control and resource data to the same interval length so that sample support is not compromised. A total of 293 690 desurveyed and spatially scattered composite grade values were then interpolated with the fast RBF method. Five grade cut-off values (0.25, 0.5, 1, 2, 4 g/t) were meshed at 5 m resolution from the volume function. The entire processing from functional fitting to isosurface extraction took two hours and 40 minutes[§] and the resulting meshes were imported into a GMP software product for assessment and validation (Figure 2b).

The seamless grade isosurfaces generated with the implicit modelling method, assisted in the identification of approximately 4Mt of resource below the present pit. This time saving allowed the underground mine scoping study at Cosmo Howley to be completed in a timely manner. Apart from the valuable insights into the grade trends, the grade envelopes were used extensively for the presentation purposes as they helped to communicate an otherwise complex mineralisation pattern.

[§] All processing times are based on Athlon 1.2 Mhz processor with 1.5 GB DDR RAM.

Example 2: Ore boundary definition for resource estimation – Saratoga gold deposit

The Saratoga gold deposit, located 280 km north-east of Perth, is part of the Mt Gibson Project acquired by Oroya Mining in 2002. Base metal exploration in the early-1980s identified gold in drill holes intersecting the surface laterite. Mining took place from 1986 to 1998, initially from shallow laterites and later from deeper ore. Gold mineralisation in the primary ore is hosted in multiple quartz-pyrite lodes within and aligned along the major regional Mt Gibson-Meekatharra Shear. Drilling to date has defined mineralisation over a strike in excess of 1400 m and width of 200 m and remains open in all directions. There is an absence of clear lithological boundaries that control the gold mineralisation at Saratoga. The ore-waste boundary can be therefore defined on the basis of a gold mineralisation envelope defined by a cut-off grade.

Oroya geological staff, with the aid of surface modelling methods, digitised the primary mineralisation outlines at a nominal sample cut-off grade of 0.5 g/t Au. During this process the mineralisation was interpreted to have a planar trend that was dipping steeply to the East. This trend was confirmed with 3D variography with the range ellipsoid to have the following principal directions: 165 m plunging 10/010, 110 m plunging 68/125 and 37 m plunging 20/276. That is, the major axis plunges 10° to the north in the plane of largest continuity with a dip/dip-azimuth of 70/100. The manual wireframing of this grade envelope was a protracted process. It took more than a month to model this single grade cut-off and complete geological validation. The partially completed wireframe of the cut-off grade of 0.5 g/t Au is shown in Figure 3a.

It was recognised that the manual wireframing procedure can be replicated with implicit modelling. During the implicit modelling process the grade anisotropy determined from 3D variography of the data was used to transform the geographic coordinates to allow anisotropic interpolation (cf. Kushnir and Yarus, 1992, for discussion applied to 2D data). This effectively results in a grade continuity model that is identical in structure to local interpolation methods that are applied with search ellipsoids. The resulting wireframes at 0.25 g/t and 0.5 g/t are shown in Figure 3b.

The Saratoga data, consisting of 12 133 composite gold assay values, took 1.2 hours processing on a desktop PC. This represents a 140-fold saving in time, given that the manual wireframing took a month. The fact that the implicit method generated two grade cut-off meshes during the 1.2 hours, effectively represents a 300-fold time saving. A further two days were spent validating the meshes with geological information, but this still represents an enormous time saving as two contrasting ore boundaries were considered. The 0.25 g/t constraint was eventually chosen as the basis for the construction of a three-dimensional block model which has subsequently been used in grade estimation.

Without the need for manual digitising, the entire process from ore boundary definition to resource estimation using Uniform Conditioning took approximately two weeks. In practical terms the implicit modelling method effectively reduced the workflow from nearly two months down to two weeks.

Example 3: Geological modelling – kimberlite pipe delineation

For kimberlite pipes, as is the case for all mineral deposits, geologists must contend with limited volumetric and grade data when attempting to estimate the mineral resource. The outer boundary of the kimberlite pipe may be poorly defined, with only very few drillholes that pierce the contact surface especially during the early stages of evaluation. Compounding this
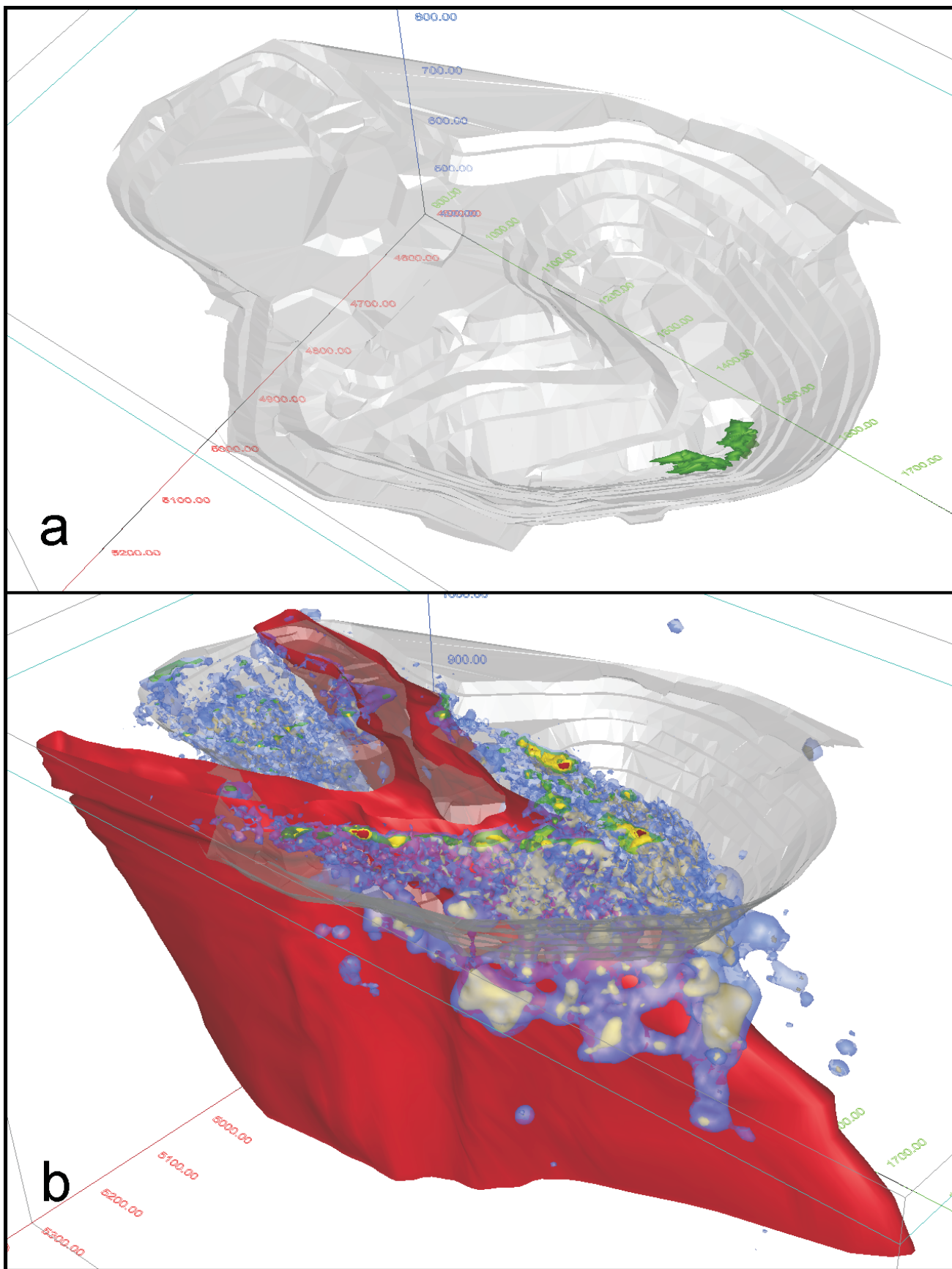
FIG 2 - (**a**) Cosmo Howley pit, and the 1 g/t modelled wireframe mesh (at lower right in pit) accomplished by traditional manual digitisation of grade control data. (**b**) Four grade wireframes modelled with implicit modelling method. Four cut-off grades are shown, along with a folded dolerite sill in red. Drillhole data are not shown for purposes of clarity. The pit and dolerite sill were generated in GMP software.
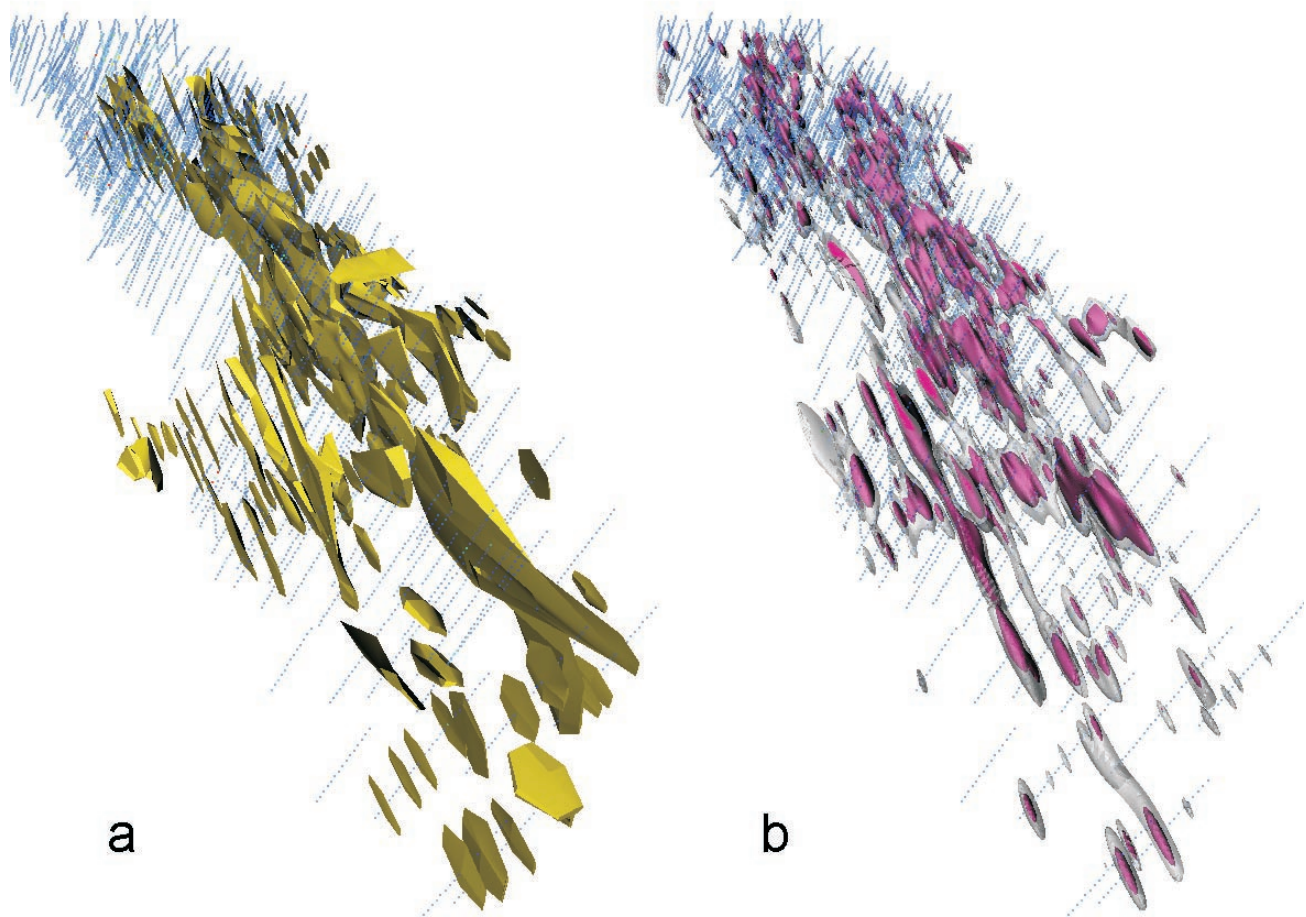
FIG 3 - (**a**) Manually digitised Saratoga ore-waste boundary generated with the traditional explicit modelling methodology. The wireframe is incomplete, and had taken over a month to digitise. (**b**) Wireframes of 0.5 g/t (inside) and 0.25 g/t (transparent) meshed at 3 m resolution generated with the implicit modelling method.

modelling problem, the diamond grade distribution within the pipe can be problematic due to the high nugget effect. Implicit modelling with biharmonic RBF, however, ensures that the large gaps between the drillhole intercepts of the pipe boundary are filled in smoothly regardless of the low numbers of intercepts. Due of the sparsity of data the processing time with the implicit method is very rapid, which enables the geologist to concentrate on the more important task of grade estimation within the delineated pipe.

At BHP Billiton's EKATI Diamond Mine, the geological modelling process begins with the delineation of the pipe outline. In the greenfields stage the perimeter of the subcropping kimberlite pipe, which is usually under a small lake and overburden, is determined by geophysical techniques. The pipe is intercepted by a combination of diamond drill core and reverse circulation holes. Geologists digitise polygons in plan at 10m to 15m vertical spacing using a thick section view to assist in visually positioning the outlines from one level to the next. Following the explicit modelling method, the polygons are the tied vertically and the solid mesh is created. An example of a kimberlite pipe model created with manual digitisation is shown in Figure 4a.

Several campaigns of drilling are undertaken as a pipe moves along through pre-feasibility and feasibility prior to mining. After each campaign the old pipe model is refined using the above steps. Each iteration of modelling takes one to two days of manual digitisation and editing using traditional editing tools found in GMP software products.

The results of explicit modelling, however, can be replicated without any need for manual digitisation using the implicit method. First step involves classification of the drillhole data into kimberlite facies or country rock. This data is extracted by querying the database and the sampled contact points between the two lithologies and positions within the two lithologies are appropriately converted to numerical values. This ensures that none of the boundary points to be surfaces will be missed, as all boundary points are automatically honoured with RBF interpolation.

The kimberlite pipe mesh extracted from the volume function is shown in Figure 4b. Comparisons with the manually digitised model indicate that there are only minor differences, which are not likely to drastically influence volume of the pipe or grade estimates.

Drillhole data preparation may take several minutes depending on the complexity of the pipe geometry and the processing time for implicit modelling for this data was 3.5 minutes on a desktop PC. The entire modelling can therefore be completed within ten minutes rather than hours. This represents a 50-fold saving on modelling time assuming the manual modelling process takes a day to complete for each iteration.

The fast delineation of kimberlite pipe boundary will save perhaps weeks of manual digitisation for each kimberlite deposit as the process moves from greenfields exploration, feasibility, to mining. The implicit modelling method simply remodels the kimberlite pipe using new drillhole data that becomes available, thus saving many hours of editing that is required with the explicit method.
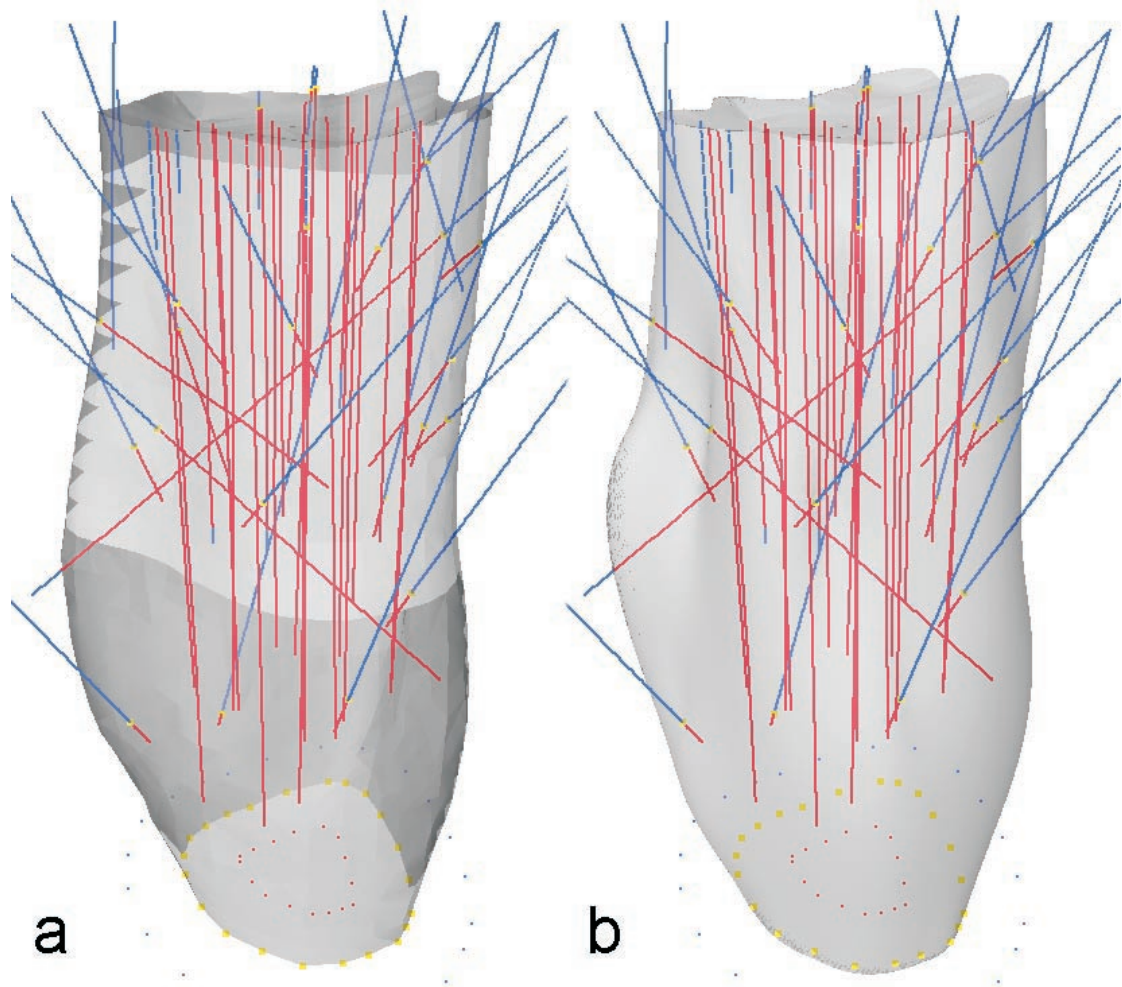
FIG 4 - (**a**) Kimberlite pipe model generated with traditional modelling method by digitising plan outlines. (**b**) Implicit kimberlite boundary surface meshed at 2 m resolution, generated from RBF interpolation of lithology data. Kimberlite, country rock, and contact data are coloured red, blue and yellow respectively.

Example 4: Contour modelling without tie-lines

The principle aim of this paper is to introduce the technique of direct 3D modelling from the drillhole database without any digitisation. There are, nevertheless, many mine operations that have already digitised numerous level and sectional polylines of inferred ore-waste boundaries or geological contacts. In some large operations, the number of contours can be in the hundreds, making it a time-consuming, or impossible, exercise to tie all of these polylines to build an explicit solid model. In addition, the construction of solids that bifurcate from one section to another, can be a time-consuming process. In these cases, the continuous polyline contour in one section must first be split and tied separately to the bifurcating polylines in the adjacent section. In some situations trial and error process must be used to test various scenarios, which can be costly.

The implicit modelling technique can however use these polylines to build 3D surface models without having to digitise tie-lines.

Firstly, it is necessary to convert the polyline data to a data suitable for implicit modelling. The initial step is to create 'off-surface' polylines on either side of the 'on-surface' polyline in the plane of digitisation. This can be done using a GMP software by copying and off-setting the polylines on both sides equidistant from each other. The nodes of the polylines are saved as a text file and the nodes of the original contours are attributed as zero. One set of off-set contours is attributed as a positive value of the displacement distance and the other set is saved as a negative distance value.

Creating the signed off-surface polylines can take some time using GMP products and in extremely large datasets and it is not efficient to do this manually. The process, however, can be replaced with computer code, which automates the file preparation procedure.

Once the polylines are correctly attributed and saved as a text file, the data is interpolated with RBF. The zero isosurface wireframe mesh is then evaluated at a desired resolution from the 3D function. The sectional data are interpolated in 3D orthogonal to the plane of digitisation and this effectively serves as joining the surface between the cross-section data, thus eliminating any need for tie-lines.

As seen in the example in Figure 5, surfaces that are overturned or bifurcating can easily be modelled with a volume function and implicit surface extracted. In addition, any other three-dimensional attribute (eg rock density, assay value, geotechnical data, alteration index) can be interpolated in space and evaluated on the surface. The example in Figure 5f shows polyline node density on the implicit surface. Surface patches without any polyline data, are smoothly interpolated between areas with data, thus the coloured data density serves as a 'confidence' map of the surface so that uncertainty of the interpolation can be visually evaluated.
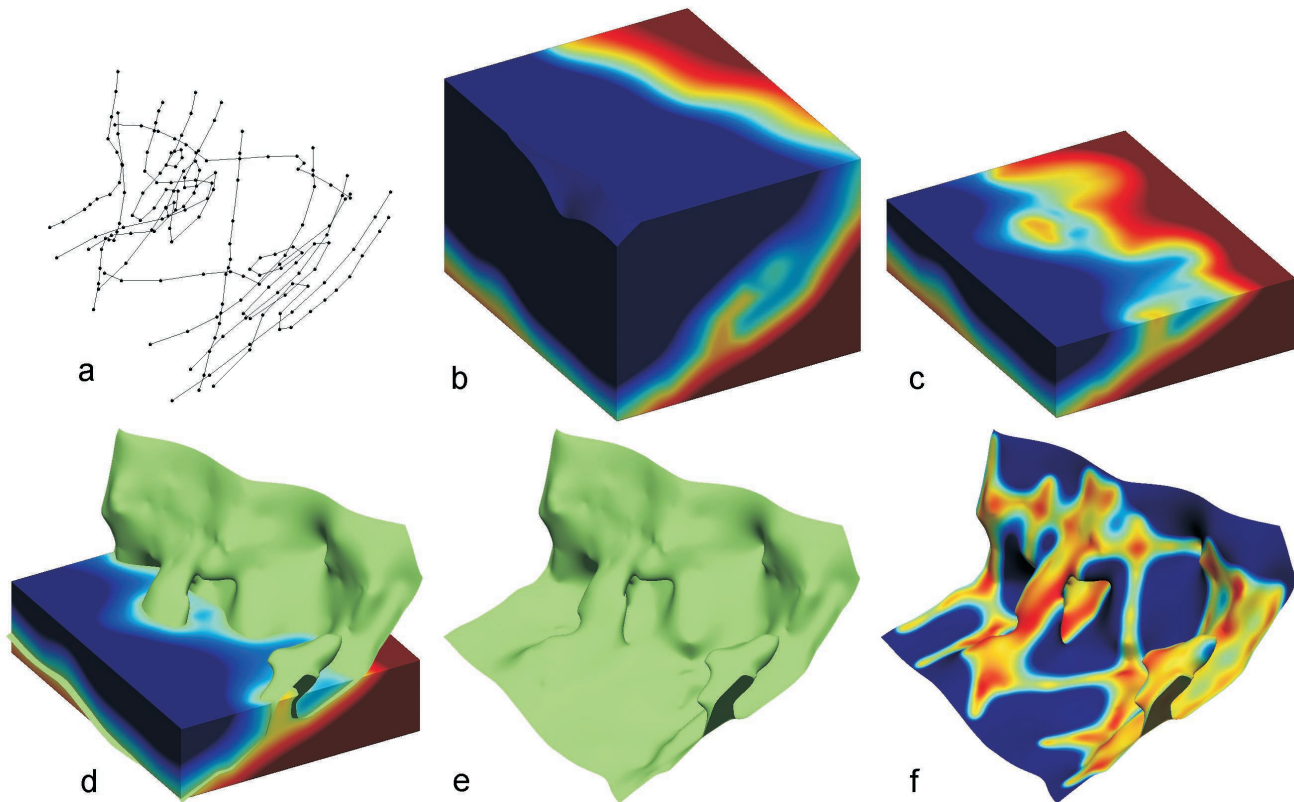
FIG 5 - (**a**) Sectional polylines digitised in 2D sections. (**b**) Volume function constructed from the procedure outlined in text. Red and blue regions are respectively positive and negative distances from the zero isosurface. (**c**) Lower half of the volume function shows complex gradient patterns. (**d**) Zero-value isosurface evaluated from the volumetric function is the surface that fits the data shown in (**a**). (**e**) Extracted triangulation surface at a fixed resolution. (**f**) Polyline node density interpolated, and evaluated on the zero-value surface. Blue regions are where there are no data values, but the surface is smoothly interpolated by the RBF interpolation.

Example 5: Single domain geological modelling: granite-porphyry unit

Regardless of how much time a geologist may spend on digitising geological boundaries, most lithological intercept patterns seen in drillhole datasets are too noisy to hand digitise and form solids. This is especially the case when drilling data is not in regular directions such as in underground situations or where multiple drill fence directions have been employed. Bypassing the hand digitisation process would be desirable in most practical situations.

Lithologies that exhibit a consistent orientation can be modelled easily by assuming a continuity pattern. East-dipping, sheet-like intrusion patterns of granite-porphyry seen in an anonymous gold deposit (from Western Australian goldfields) can be modelled with a consistent anisotropic trend where strike of the lithology is continuous. The most likely dip of the intrusive unit is a steep dip to the East (Figure 6a). Note that the apparent thicknesses of the granite-porphyry unit changes significantly through space and it is difficult to join one section from another because of its bifurcations and pinch-outs. It therefore takes careful and time-consuming work to construct the solid by manual digitisation. The implicit modelling method, however, is straight forward, as outlined above (Figure 1).

The granite-porphyry data, which consisted of 1594 drillholes (representing an average of 81 m in length with a total of 129 681 m), was processed and meshed within 20 minutes (only a tenth of the data shown in the thick slice of Figure 6). Manual digitisation of granite-porphyry would take many days of digitisation.

Another clear advantage of the implicit method is that other continuity directions can be tested and a completely different geological model can be generated but still being conditional to the lithological boundaries sampled in the drillholes (Figure 6b). Generating multiple conditional models with manual digitisation is impractical therefore we see the implicit modelling being the only practical way to generate many conditional geological wireframe models. These multiple models can be used for testing 'what-if' scenarios that may influence grade estimations and downstream mining procedures thereby allowing assessment of mining risk that are associated with each geological model.

Example 6: Modelling with user-defined geological morphology – Wallaby gold deposit

Three-dimensional contouring is a useful tool for visualisation of grade data; however geologists also need to ensure the grade continuity interpolated in space honours the grade geometry envisaged from the geological interpretation. User-defined grade continuity, or lithological boundaries, can be incorporated into the modelling technique to realistically model drillhole data.

The three iso-grade wireframes of the Wallaby gold deposit (Nielson and Currie, 1999) shown in Figure 7a illustrate the unconstrained isotropic modelling of grade from composited grade control and resource drilling gold assays. The disjointed high grades can be more realistically interpreted as being continuous along convex-up planes (Figure 7b). The planes are saved as a 'geological morphology' data file that can be used to constrain the grade interpolation (Cowan *et al*, 2002). This information enables constraining of the local direction in which grades are connected in space.
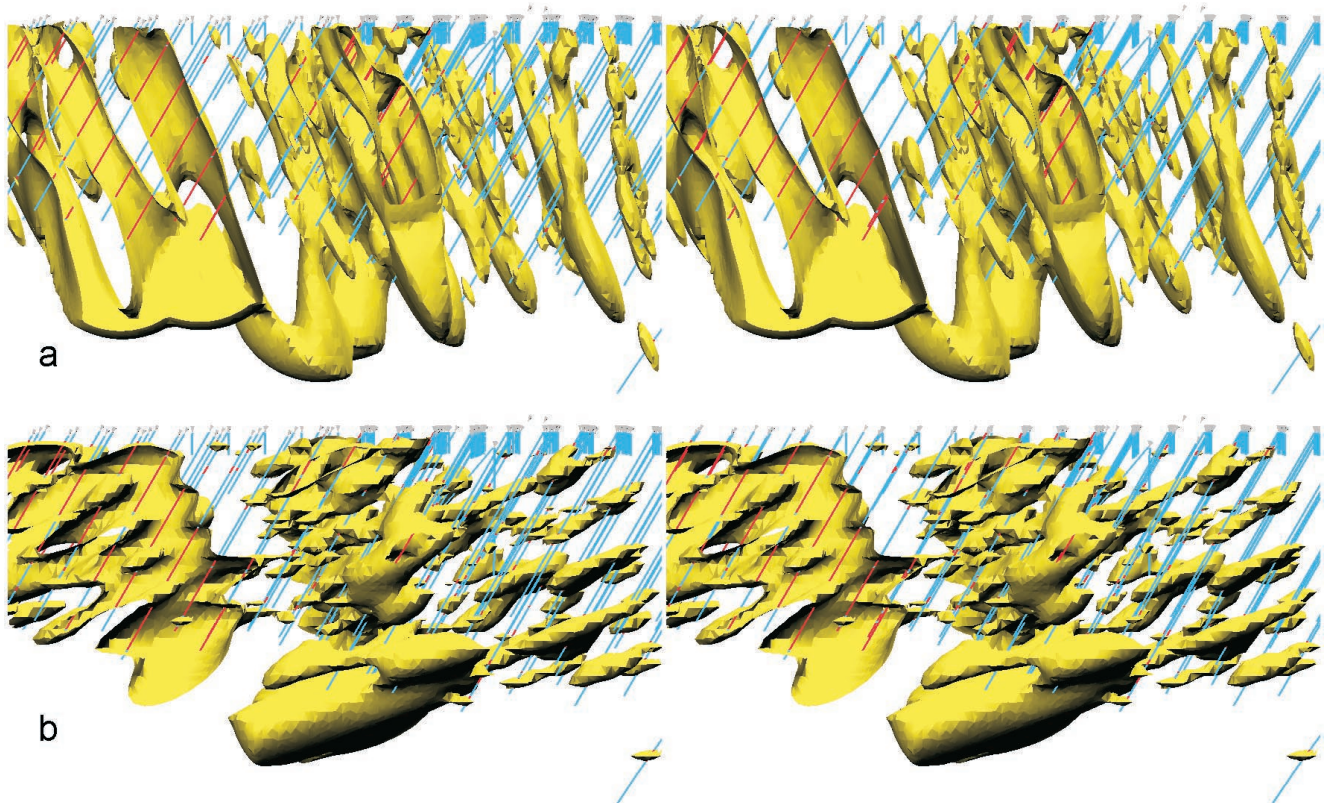
FIG 6 - Cross-eyed stereo pairs of granite-porphyry lithology wireframe modelled with the implicit method. (To view in stereo, right and left eyes view the left and right images, respectively.) Red intervals represent the positions of the granite-porphyry unit, and light blue represents all other lithologies. View is to the north. (**a**) Model with lithological continuity steeply dipping to the East. (**b**) Model assuming lithological continuity shallowly dipping to the West. Geological validation is needed, but both geological models are conditional to the data. That is, the lithological boundaries are honoured in both models. Some missed boundaries are due to the fact that the meshing is done at a fixed resolution; therefore intervals that are shorter than the evaluated mesh dimension are not surfaced.

The constrained interpolation technique results in grade boundary meshes which honour the trends envisaged by the geologist (Figure 7b), thereby bypassing the need for time-consuming manual digitising. Various geological morphologies can be defined and then used to generate contrasting conditional models that honour the data.

The most significant benefit of this user-dependent contouring method is that it not only saves time to produce a geologically realistic model, but also allows multiple grade distribution or lithological models to be constructed directly from sparse drillhole data without the necessity for gridding the data. A range of geological ideas can therefore be used to generate 'what if?' scenarios for testing.

CONCLUSIONS

Previously considered impractical or impossible, the implicit modelling of geological surfaces from drillhole data is now a practical reality with the advent of fast 3D interpolation methods. What was thought only possible by slow hand digitisation, modelling of complex shaped geological surfaces can now be semiautomatically generated on a desktop PC. The case studies presented herein demonstrate the truly practical nature of this new modelling technique.

Advantages offered by this method over the traditional manual digitisation include:

- The ability to model complex geological objects of any shape.
- The ability of model iso-grade wireframes rapidly directly

from drillhole data without the need for time-consuming domaining or variography.

- Being able to process sparse to very large datasets on a personal computer.
- The rapid speed in which the modelling can be accomplished represents one to three orders of magnitude in time savings over manual digitisation.
- Being able to identify grade trends that aid the identification of drillhole targets, directly from processing of non-gridded data.
- Unlike manual digitisation, where geological interpretation is written into the models, implicit modelling allows the separation of interpretation from the process of surface generation. The ability to separate geological intuition from the modelling process allows multiple models to be constructed that are all conditional to the drillhole data.
- Implicit models can be rapidly updated as new drillhole data becomes available. This keeps the geological models dynamic, as the modelling methodology can now keep up with speed of data acquisition.

The shift from explicit surface modelling to volume modelling also allows the representation of both surfaces and volumetric data to be conveniently treated within a single modelling framework. This opens up modelling techniques and applications that were not possible within the surface-based modelling paradigm. The development of software tools that takes advantages of these new opportunities is being further explored.
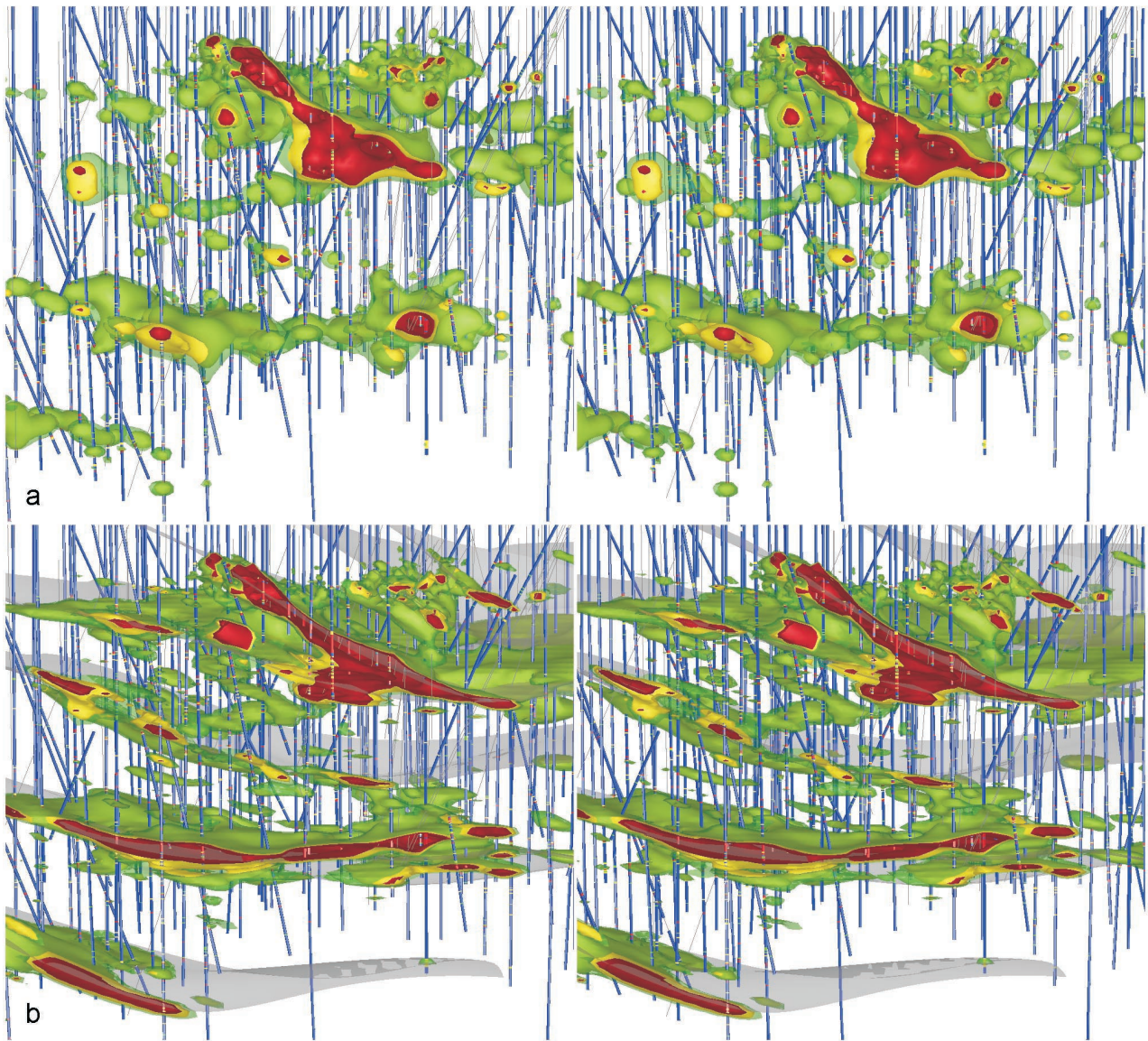
FIG 7 - Cross-eyed stereo pairs of iso-grade wireframes of Wallaby gold deposit processed from combined grade control (not shown) and resource drilling (shown). (**a**) unconstrained interpolation of gold values. (**b**) Iso-grade values of gold constrained by the interpretive curvi-planar trends shown as grey planes. Both sets of wireframes are conditional to the drillhole grade data.

Processing methods illustrated in Figures 1, 2 and 4 can now be accomplished in the version 1 of Leapfrog software¤. Lithological and grade modelling methods, as exemplified in Figures 3, 5, 6 and 7 are being developed for future upgrades of Leapfrog.

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¤ Leapfrog software is available from Zaparo Ltd.
For more information see www.leapfrog3d.com

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