1. Compute the following integrals

   (a) $\displaystyle\int \frac{\cos x}{\sin^2 x - 4\sin x + 7}dx.$

   Let $u = \sin x$. The above becomes

   $$\int \frac{1}{u^2 - 4u + 7}du.$$

   We need to complete the square to rewrite as

   $$\int \frac{1}{(u-2)^2 + 3}du$$

   Let $t = u - 2$. The integral becomes

   $$\int \frac{1}{t^2+3}dt = \frac{1}{\sqrt{3}}\arctan\left(\frac{t}{\sqrt{3}}\right).$$

   So the answer is

   $$\frac{1}{\sqrt{3}}\arctan\left(\frac{\sin x - 2}{\sqrt{3}}\right) + C.$$

   (b) $\displaystyle\int x\sin^2 x\cos x dx$

   Do integration by parts with $u = x$ and $dv = \sin^2 x \cos x$.

   $$\frac{1}{3}x\sin^3 x + \frac{1}{3}\cos(x) - \frac{1}{9}\cos^3 x + C.$$

2. In 2000 there are 5000 wolves. In the absence of hunting the wolf population would increase at a rate of %1 per year. However, hunters are killing wolves at a steady rate of 100 wolves per year. When will the wolf population die out? (*answer* = 2069)

   Let $W(t)$ be the wolf population at time t. We know $\frac{dW}{dt}$ = INCREASE/yr - DECREASE/yr. We have

   $$\text{INCREASE/yr} = (.01)W(t)$$

   $$\text{DECREASE/yr} = 100$$

   So

   $$\frac{dW}{dt} = (.01)W - 100.$$

   We can integrate this out as

   $$\int \frac{dW}{.01W - 100} = \int dt$$

   Being careful (i.e. taking $u = .01W - 100$) we get

   $$\frac{1}{.01}\ln(.01W - 100) = t + C.$$

   This has solution

   $$W(t) = Ae^{\frac{t}{100}} + \frac{100}{.01} = Ae^{\frac{t}{100}} + 10000.$$

Using $W(0) = 5000$ we have $A = -5000$. So

$$W(t) = -5000e^{\frac{t}{100}} + 10000.$$

We solve $0 = W(t)$ and get

$$t = 100\ln(2) \approx 69.3 \text{ years}.$$

3. The region between the curve $y = e^{x^3}$ and the lines $y = 0$, $x = 1$ and $x = t$ is rotated about the vertical line $x = 1$. Here $t > 1$ is not further specified.

   (a) Write (but DO NOT evaluate) the integral for the volume of the resulting solid.

   $$V(t) = \int_0^t 2\pi x e^{x^3}\, dx.$$

   (b) Let $V(t)$ denote the volume found in (a). Compute $V'(2)$.

   $$V'(t) = 2\pi t e^{t^3} \implies V'(2) = 4\pi e^8.$$