Name: **Quiz 1 Solutions** Student #

1. Compute the following limits. Showing no work will receive **zero** points.

   (a) (3 points)

$$\lim_{h\to 0}\frac{\sqrt{(2+h)^2+1}-\sqrt{5}}{h} = \lim_{h\to 0}\frac{(2+h)^2+1-5}{h(\sqrt{(2+h)^2+1}+\sqrt{5})}$$
$$= \lim_{h\to 0}\frac{h^2+4h+4+1-5}{h(\sqrt{(2+h)^2+1}+\sqrt{5})}$$
$$= \lim_{h\to 0}\frac{h(h+4)}{h(\sqrt{(2+h)^2+1}+\sqrt{5})}$$
$$= \lim_{h\to 0}\frac{h+4}{\sqrt{(2+h)^2+1}+\sqrt{5}}$$
$$= \frac{4}{2\sqrt{5}}$$
$$= \frac{2}{\sqrt{5}}$$

   (b) (3 points) $\lim_{x\to\infty}\frac{4x^3+5x^2}{\sqrt[3]{8x^9+3x}}$

   Multiply top and bottom by $\frac{\frac{1}{x^3}}{\frac{1}{x^3}}$, simplify so it looks like

$$\lim_{x\to\infty}\frac{4+\frac{5}{x}}{\sqrt[3]{8+\frac{3}{x^8}}}$$

   and then take the limit and get 2.

2. (a) (1 point) Find a number c so that the function below is continuous for all x.

$$f(x) = \begin{cases} cx^2+1, & x<1 \\ x^3+4, & x\geq 1 \end{cases}$$

   $c+1 = 1+4$ when $c=4$.

   (b) (3 points) Recall that a function is continuous at $x=a$ if

   (i) a is in the domain

   (ii) $\lim_{x\to a} g(x) = g(a)$.

   Explain why the c you found in Part (a) guarantees that f is continuous **for all** x.

(1 point) Polynomials are continuous so the only discontinuity could be at $x=1$.

(1 point) $x=1$ is in the domain since $f(1)=5$.

(1 point) $\lim_{x\to 1} f(x) = f(1)$ if and only if the left and right hand limits exist. When $c=4$, $\lim_{x\to 1^-} f(x) = 5 = \lim_{x\to 1^+ f(x)} = f(1)$.