A Theory of Liquidity Spillover Between Bond and CDS Markets

Batchimeg Sambalaibat*

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I build a dynamic search model of bond and CDS markets and show that allowing short positions through CDS contracts increases liquidity of the underlying bond market. This result contrasts with existing theories on derivatives, which show that derivatives fragment traders across the derivative and underlying markets and thereby decrease liquidity in the underlying cash market. I reach the opposite conclusion by endogenizing the aggregate number of investors. My results help explain how sovereign bond markets reacted to a naked CDS ban.

During the last European debt crisis, the controversy surrounding credit default swaps (CDS) culminated in bans on "naked" purchases of CDS. A credit default swap is a financial derivative instrument that resembles an insurance protection against a firm or a government default. It allows investors to trade the credit risk of the bond issuer without trading the bonds themselves. A naked CDS purchase refers to a CDS purchase where the protection buyer does not own the underlying bonds. As they are a short and, possibly, a speculative position against the bond issuer, regulators blamed them for exacerbating the European debt crisis. Ultimately, in October 2011, the European Union voted to permanently ban naked CDS contracts referencing EU sovereigns. What is interesting about the ban is how the underlying bond market reacted. Following the ban, liquidity in the underlying bond market decreased (as measured by bond bid-ask spreads). This pattern is documented in Sambalaibat (2014).

*Indiana University, Kelley School of Business. E-mail: bsambala@iu.edu. I thank Laurence Ales, Patrick Augustin, Luca Benzoni, Maria Chaderina, Peter Feldhutter, Rick Green, Burton Hollifield, Lars Kuehn, Pete Kyle, Artem Neklyudov, Nicolas Petrosky-Nadeau, Guillaume Rocheteau, Bryan Routledge, Stefano Sacchetto, Laurence Lescourret, Chester Spatt, Chris Telmer, Pierre-Olivier Weill, Ariel Zetlin-Jones, Haoxiang Zhu, and participants at numerous conferences and seminars. All errors are my own. The current version of the paper can be found at https://sites.google.com/site/sambalaibat/.

This bond market reaction challenges existing theories. Models on the interaction between the derivative and the underlying markets show that introducing derivatives fragments traders across the derivative and the underlying markets and thereby attracts liquidity away from the underlying market.[1] Since this result implies that banning derivatives reverses the fragmentation and improves liquidity in the underlying, it does not line up with the observed decrease in liquidity. Existing results on short-selling also do not help rationalize the observed decrease in liquidity. They are specific to short-selling and do not generalize to shorting through CDS contracts.[2] Thus, we still need to understand how CDS positions that do not require trading the underlying bonds affect liquidity of the underlying bonds.

In this paper, I build a dynamic search model of bond and CDS markets and show that relaxing a key assumption present in a variety of models reconciles the fact. The assumption is fixed aggregate number of traders. Existing theories keep the aggregate number of investors fixed, and, as a result, introducing derivatives, by construction, fragments traders across multiple markets and attracts liquidity away from the underlying market.[3] I show that when the aggregate number of investors is instead endogenous, the effect is the opposite: Introducing CDS increases liquidity in the bond market.

I refer to this effect as a liquidity spillover effect, and it works as follows. Introducing short positions through CDS contracts attracts into credit markets not only investors who want to short the underlying credit risk and buy CDS but also investors who want to take the opposite side and long the underlying credit risk. In turn, long investors—for whom buying bonds and selling CDS are economically similar positions—search and trade at the same time as bond buyers. They do this to increase their trading opportunities

[1]Subrahmanyam (1991) and Gorton and Pennacchi (1993) using Kyle (1985) framework show that stock index futures market and security baskets, respectively, lower liquidity in the underlying stock market as some traders migrate to the derivative markets. John, Koticha, Subrahmanyam, and Narayanan (2003) show a similar effect of options on stock market liquidity using Glosten and Milgrom (1985) framework.

[2]For example, Vayanos and Weill (2008) show that an asset with short-selling activity has a higher spot market liquidity. They show this is because short-selling requires trading in the spot market (first as a seller to establish the short position, then as a buyer to unwind the position). Duffie, Garleanu, and Pedersen (2002) show that asset prices are higher in the presence of short-sales. They show this is because bond holders earn an additional cash flow from lending their asset to those who want to borrow and short-sell. Both mechanisms are thus specific to short-selling and do not generalize to shorting via naked CDS purchases. In a CDS contract, neither the seller nor the buyer has to trade the underlying the asset to achieve a long or a short position with respect to the underlying. Thus, a priori, whether similar results would arise with CDS is not obvious.

[3]The fragmentation arises not only in models of an underlying and derivative markets but also in any model with multiple markets and a fixed total number of investors. For example, results in information-based frameworks of Admati and Pfleiderer (1988), Pagano (1989), and Chowdhry and Nanda (1991) and search-theoretic frameworks of Vayanos and Wang (2007), Vayanos and Weill (2008), and Weill (2008) imply that traders endogenously concentrate in one market and trading in the other market either deteriorates or disappears.

and alleviate their search frictions. The result is an increase in bond market liquidity: a greater number of bond buyers, a shorter expected search time for bond sellers, a larger bond turn over, and a larger volume of trade. The bond price increases also. This is the spillover effect and is the main insight of the paper.

The spillover effect works in reverse also: Banning naked CDS positions decreases bond market liquidity. Investors can no longer sell CDS because their counterparties, the naked CDS buyers, are banned from buying CDS. Long investors exit the CDS market, but by exiting the CDS market they pull out from the bond market also. The result is a decrease in bond market liquidity, consistent with the bond market reaction after the EU ban. Thus, preventing investors from shorting ultimately ends up banning investors who want to take the opposite side and long the asset.

I show the liquidity spillover effect with a model that builds on Duffie, Garleanu, and Pedersen (2005, 2007) and, in particular, on Vayanos and Weill (2008). A fraction of bond owners, upon a liquidity shock, try to sell their bonds. Finding a buyer, however, involves search. When a seller finds a buyer, the difficulty of finding another buyer forces the seller to accept a discounted price. Search frictions thus create an illiquidity discount in the bond price. The illiquidity discount, the length of the search process, and the volume of trade all depend on the relative number of sellers and buyers. The number of buyers and sellers are, in turn, endogenous.

I add to this environment, first, CDS contracts. CDSs pay when the underlying bond defaults. A CDS buyer—who stands to benefit if the bond defaults—has a short exposure to the underlying credit risk. The CDS seller has the opposite long exposure. CDS are in zero net supply; bonds are in fixed supply. CDS and bonds are effectively inside and outside money, respectively.[4] Trading CDS contracts, as with trading bonds, involves search and bilateral bargaining.

Second, I endogenize entry and, consequently, the aggregate number of

[4]Outside money refers to assets that are in positive fixed supply within a specific sector of the economy and that serve as money (e.g. circulate as a medium of exchange or serve as store of value, etc.). Inside money refers to claims that are created within that sector (hence, are in zero net supply) and that serve as money. The credit market economy in my model is the relevant private sector of the economy. Bonds and CDSs are outside and inside money in the sense that bonds are in fixed supply, while CDSs are created within the economy and are in zero net supply. Both are money in the sense that they serve as store of value and expand the set of feasible allocations. For discussion on financial innovation as inside money see, for example, Gennaioli, Shleifer, and Vishny (2012) and Brunnermeier and Sannikov (2016). For discussion on inside money (particularly, as a medium of exchange) see, for example, Cavalcanti and Wallace (1999a), Cavalcanti and Wallace (1999b), and references in Lagos (2010). The latter literature addresses, for example, when inside and outside money co-exist and whether they are complements versus substitutes. I instead shed light on how the creation of inside money (CDS) affects liquidity of outside money (bonds).

investors.[5] The total mass of long investors has a natural interpretation as funding liquidity because long investors, in practice, take the capital intensive side of bond and CDS trades and, as a result, supply capital into credit markets. The model thereby features distinct notions of funding liquidity and market liquidity as in Brunnermeier and Pedersen (2009), and both are endogenous.

With this environment, the paper offers three contributions. First, I show the spillover effect and thereby a novel insight on how financial derivatives affects the market for the underlying asset. This insight applies to any new financial innovation (be it a tradable instrument, a trading mechanism, or a market place).

Second, I provide, to my knowledge, the first theoretical framework of OTC trading in both the underlying and derivative markets. In existing microstructure models of derivatives, illiquidity arises from asymmetric information.[6] We thus lack models of derivatives where illiquidity arises from a key friction in trading assets over-the-counter: search costs. In the context of OTC traded assets, search models are the current workhorse environment of endogenous liquidity frictions and asset prices. But, so far, they feature either a single asset or multiple assets with identical cash flows.[7] In contrast, I model the endogenous interaction between multiple OTC traded assets where one asset is a derivative of the other. I thereby shed light on agents' incentive to search and trade the underlying versus the derivative asset, the amount of long and short interest in both assets, market liquidity of both assets, and terms of trade negotiated between counterparties in an environment where these variables are interdependent.

[5] Afonso (2011) and Lagos and Rocheteau (2009) also endogenize entry and the aggregate number of investors in a search framework but do so in a single market setting.

[6] They include Subrahmanyam (1991), Gorton and Pennacchi (1993), and John, Koticha, Subrahmanyam, and Narayanan (2003) that I mentioned previously. Additionally, Back (1993) develops a framework based on Kyle (1985) to study the effect of options on price volatility. Biais and Hillion (1994) provide another information-based model of options and study their effect on price informativeness of the underlying asset. For information-based frameworks dealing with themes of complementarity versus substitutability, see Goldstein, Li, and Yang (2013) in the context of multiple markets and Goldstein and Yang (2015) in the context of multiple dimensions of information. Similar themes are present in my paper because we can interpret my results as: Bonds and CDS are complements when investors face entry costs and adjust their participation rate but are substitutes when they do not.

[7] Single asset frameworks include Duffie, Garleanu, and Pedersen (2005, 2007), Lagos and Rocheteau (2009), Neklyudov (2012), Hugonnier, Lester, and Weill (2014), Shen, Wei, and Yan (2015), Sambalaibat and Neklyudov (2016), and Uslu (2016). Feldhütter (2012) provides a quantitive treatment of search models. Multiple asset frameworks include Vayanos and Wang (2007), Vayanos and Weill (2008), and Weill (2008). Atkeson, Eisfeldt, and Weill (2015) model using a static search framework how a bank's CDS exposure depends on the size of the bank and the bank's ex-ante exposure to the underlying credit risk. They focus on CDS trading and abstract from bond trading. I instead allow trade in both the bond and the CDS market and thereby shed light on the endogenous feedback from the CDS market into the underlying bond market.

Third, I shed light on naked CDS purchases and thereby fill a gap in the CDS literature that focuses on covered CDS purchases (where investors buy CDS protection on bonds they own).[8] Allowing investors to trade the issuer's credit risk without trading or owning the bonds is what defines CDS, why they proliferated, and why they were controversial.

The paper is organized as follows. Section 1 presents the model environment, Section 2 characterizes the equilibrium, prices, and measures of liquidity, and Section 3 derives the main result. Section 4 provides additional results: First, I model short-selling and contrast its effect to that of CDS; second, I show how the main result changes when investors can also short-sell; and, third, I endogenize investors' search intensities. An Appendix contains some of the proofs, while an online Appendix contains the longer proofs and the proofs to results in Section 4.

1 Model Environment

Time is continuous and goes from zero to infinity. Agents are risk-averse, live infinitely, have idiosyncratic stochastic endowments, and can invest in a risk-free asset with return $r > 0$. They hold and trade bilaterally a risky bond and a derivative "CDS" contract with a cash flow based on the risky bond. Finding someone to trade with involves search. Agents enter the economy if it makes them better off than their outside option. This is the model in a nutshell; the rest of this section elaborates.

Assets

The bond is a perpetual asset that occasionally comes short of its promised cash flow. I define such occasions as default. In particular, the bond has supply S, trades at price p_b, and has a cumulative cash flow process $D_{b,t}$ satisfying:

$$dD_{b,t} = \delta dt - J dN_t, \tag{1}$$

where $\delta > 0$ is the promised rate of the coupon flow, $\{N_t, t \geq 0\}$ is a Poisson counting process with intensity parameter $\eta > 0$, and $J > 0$ is a constant and is the default size.[9] The process N_t counts the number of defaults in

[8] Oehmke and Zawadowski (2015) explore how CDS trading (both covered or naked) affects bond prices in Amihud and Mendelson (1986) type framework with exogenous trading frictions. In contrast, my model features endogenous trading costs and, thereby, an endogenous interaction and spillover between the underlying and derivative markets. For models on issues surrounding *covered* CDS purchases specifically, see, for example, Thompson (2007), Arping (2014), Bolton and Oehmke (2011), Sambalaibat (2012), and Parlour and Winton (2013).

[9] Default risk in the model is exogenous; I focus, instead, on changes in asset prices through changes in asset liquidity. See He and Milbradt (2014), for example, for a model of endogenous feedback loop between credit risk and liquidity.

$[0, t]$, and its increment, dN_t, is 0 or 1. Thus, (1) says, in a small interval $[t, t+dt]$, with probability ηdt, the bond defaults, and its cash flow decreases by J. Otherwise, it pays the coupon at the promised rate. I restrict agents' bond position to $\theta_b \in \{0, 1\}$ units and assume, for now, that agents cannot short bonds. In Section 4, I relax this assumption.

In a CDS contract, the buyer of the contract pays a premium flow p_c to the seller of the contract; the seller, in turn, pays the buyer J if the bond defaults. The CDS buyer's cumulative cash flow $D_{c,t}$, as a result, follows:

$$dD_{c,t} = JdN_t. \tag{2}$$

Comparing (2) with (1), $D_{c,t}$ and $D_{b,t}$ are perfectly negatively correlated. The CDS buyer thus has a short exposure to the underlying credit risk. Conversely, the CDS seller has a cash flow that is positively correlated with the bond ($-JdN_t$) and is thus long credit risk. From hereon, when I refer to a long or a short position, I will mean with respect to the underlying credit risk.[10] I denote an agent's CDS position with $\theta_c \in \{-1, 0, 1\}$, where each denotes a short, a neutral, and a long position, respectively. I rule out simultaneous long positions in both assets.

An investor terminates a CDS contract by paying the other party a fee. The fee is endogenous and makes the nonterminating party indifferent between (a) continuing the contract and (b) accepting the fee, searching for a new counterparty, and, upon a match, entering a new position. I assume that when the non-terminating side is indifferent, she accepts the fee and starts the process again. I denote with T_{S} and T_{B} the fees the seller and the buyer pay their counterparty, respectively.

Agents

Agents have time preference rate β and CARA utility preferences with risk aversion parameter α: $u(C) = -e^{-\alpha C}$. Agent i's cumulative endowment process $e_{i,t}$ follows:

$$de_{i,t} = \mu_e \rho_{i,t} dt + \rho_{i,t} \sigma_e (-dN_t) + \sqrt{1 - \rho_{i,t}^2} \sigma_e dZ_t, \tag{3}$$

where $\mu_e > 0$ and $\sigma_e > 0$ are constants, Z_t is a standard Brownian motion, and $\rho_{i,t}$ is the instantaneous correlation process between the bond cash flow and the agent's endowment process. The processes $\{Z_t, \rho_{i,t}, N_t\}$ are pairwise independent. The correlation process $\rho_{i,t}$ is independent across agents and is a three-state Markov chain with states $\rho_{i,t} \in \{-\rho, 0, \rho\}$ where $\rho > 0$. Agents switch from negative and positive correlation states to an uncorrelated state

[10] Thus, a long position through the CDS market, for example, does not mean an investor has bought CDS but means she has sold CDS and, hence, is (long-) exposed to the underlying default risk.

with Poisson intensities γ_d and γ_u, respectively. The intensity of switching from the uncorrelated state to either the positive or negative correlation state is zero (the uncorrelated state is thus an absorbing state).[11]

The different correlation realizations across agents generate heterogenous private valuations for the underlying credit risk. As I show later, an investor whose endowment is currently negatively correlated with the bond ($\rho_{i,t} = -\rho$) has the highest private valuation for the bond (hence, the most willing to buy it), those with an uncorrelated endowment ($\rho_{i,t} = 0$) have an intermediate valuation, and those with a positively correlated endowment ($\rho_{i,t} = \rho$) have the lowest valuation. This difference in valuations introduces a motive for trade. In particular, a random change in an agent's valuation (due to a random change in her correlation) generates a need to trade and rebalance her portfolio. From hereon, I refer to an agent with $\rho_{i,t} = -\rho$ as a high-valuation type or "h" for short, with $\rho_{i,t} = 0$ as an average-valuation ("a") type, and with $\rho_{i,t} = \rho$ as a low-valuation ("l") type. Where necessary, I will denote the valuation types in subscripts with i where $i \in \{h, a, l\}$. Referring to agents according to their valuations is simpler than referring to their correlations.

Agents' Decisions

Agents make two sets of decisions. First, they decide whether to enter the economy. At any point, fixed flows of agents F_h and F_l are born as high- and low-valuation agents, respectively. An agent $i \in \{h, l\}$ enters if the value of entering, $V_{i[0,0]}$, is at least greater than her exogenous entry cost, O_i, where $V_{i[0,0]}$ denotes her continuation value upon entry and the subscript $[0, 0]$ captures the fact that investors enter without an existing position. An endogenous fraction ν_i of i-valuation investors, as a result, enter according to:

$$\nu_i = \begin{cases} 1 & V_{i[0,0]} > O_i \\ [0,1] \text{ if} & V_{i[0,0]} = O_i \\ 0 & V_{i[0,0]} < O_i \end{cases} \tag{4}$$

for $i \in \{h, l\}$, the flows of high and low-type entrants are $\nu_h F_h$ and $\nu_l F_l$, and their steady state measures are $\frac{\nu_h F_h}{\gamma_d}$ and $\frac{\nu_l F_l}{\gamma_u}$, respectively.[12] The entry cost

[11]In Section 2, as I describe how different agents trade in equilibrium, I explain why I model three types where one of them is an absorbing type. In short, I do so to model both short positions and entry and exit.

[12]We can ignore the entry decision of average type agents because, in equilibrium as a result of an additional parameter condition, the expected utility of participating in the economy as an average type agent is zero: $V_{a[0,0]} = 0$. Thus, for any positive outside option ($O_a > 0$ even if it is small), their entry rate is zero. Moreover, the results in the paper depend not on the absolute levels of O_h and O_l but on their magnitudes relative to O_a (i.e. the model can be recast in terms of $O_h - O_a$ and $O_l - O_a$). Thus, without loss

of high type investors, O_h, in particular, can be interpreted as funding cost because, in practice, long investors take the capital intensive side of bond and CDS trades.

Second, once in the economy, agents choose their consumption, C, and their bond and CDS portfolio $[\theta_b, \theta_c]$. The feasible asset positions are: $[\theta_b, \theta_c] \in \{[1,0], [0,1], [0,0], [0,\text{-}1], [1,\text{-}1]\}$.[13] An agent's optimal asset position depends only on her current valuation type (high, average, or low) and her current asset position. So I categorize agents into types $\tau \in \mathcal{T}$—where a type $\tau = i[\theta_b, \theta_c]$ specifies the agent's valuation $i \in \{h, a, l\}$ and asset position $[\theta_b, \theta_c]$—and recast their choice over asset positions as choice over types. Moreover, the positions feasible to an agent depend on, for example, whether the agent finds a counterparty. So I summarize the events affecting a type τ_t agent with a counting process $\hat{N}_t(\tau_t)$ and denote its dimension with $K(\tau_t)$ and the intensity associated with dimension k with $\gamma(k, \tau_t)$.[14] Then, when an event associated with dimension k arrives, the agent chooses between types $\tau_t^{'} \in \mathcal{T}(\tau_t, k) \subset \mathcal{T}$.

The agent's optimization problem is thus:

$$U(W_0, \tau_0) = \max_{\{C_t \in \mathbb{R}, \tau_t^{'} \in \mathcal{T}(\tau_t, k)\}} \mathbb{E}\left[\int_0^\infty e^{-\beta t} u(C_t) dt\right] \tag{5}$$

where $U(W_t, \tau_t)$ denotes the indirect utility of type τ_t agent with wealth W_t at time t. The agent optimizes subject to her wealth process, W_t:

$$dW_t = (rW_t - C_t)\,dt + de_t + dD_t^b \theta_{b,t} - p_b d\theta_{b,t} + (p_c dt - dD_t^c)\theta_{c,t}, \tag{6}$$

and the transversality condition $\lim_{T \to \infty} \mathbb{E}[e^{-\beta T} e^{-\alpha r W_T}] = 0$.

The Bond and the CDS Market

Establishing new asset positions or rebalancing existing ones involves search. A market prescribes which asset an investor can trade given a match with a counterparty. In market $m \in \{b, c\}$, where b stands for the bond market and c for the CDS market, buyers and sellers meet at a total rate

$$M_m \equiv \lambda_m \mu_{m,\mathrm{B}} \mu_{m,\mathrm{S}}, \tag{7}$$

of generality, I set $O_a = 0$. As for the outside options of high and low types, my main results hold for any O_h and O_l including for the special case where $O_h = O_l$. I denote them separately to be general and to later show where the effects come from.

[13] The position where investors buy bonds and buy CDS ($[\theta_b, \theta_c] = [1, \text{-}1]$) can be referred to as a covered CDS position. Eventhough it is feasible, I show in online Appendix L that it does not arise in equiliribrium.

[14] Appendix A explains the counting process in detail. It is simply a notation that helps characterize agents' optimization problem. Later, as I characterize the equilibrium via a conjecture-and-verify method, I incorporate the events affecting an agent and, hence, this process directly into the equilibrium conditions.

where λ_m is the exogenous matching efficiency of market $m \in \{b, c\}$, and $\mu_{m,\text{B}}$ and $\mu_{m,\text{S}}$ are the masses of B–uyers and S-ellers in market $m \in \{b, c\}$, respectively.[15] Given the total meeting rate, buyers find a seller with intensity $\frac{M_m}{\mu_{m,\text{B}}} = \lambda_m \mu_{m,\text{S}}$, and sellers find a buyer with intensity $\frac{M_m}{\mu_{m,\text{S}}} = \lambda_m \mu_{m,\text{B}}$. For now, the matching efficiencies λ_b and λ_c are exogenous, but I endogenize them in Section 4.3.

2 Equilibrium, Prices, and Liquidity

2.1 Characterization of Equilibrium

In this section, I start by characterizing the equilibrium objects—agents' continuation values, the distribution of agent types, prices, and entry rates—and the equilibrium conditions they satisfy. I then define the steady state equilibrium and prove its existence in Proposition 2.

(i) Value Functions

The first equilibrium object is value functions, V_τ, defined by (8).[16] As Proposition 1 shows, they arise from agents' optimization problem (5).

Proposition 1. *Solutions for $U(W, \tau)$ are of the form:*

$$U(W, \tau) = -e^{-r\alpha(W+V_\tau+\bar{a})} \tag{8}$$

where $\bar{a}$ is:

$$\bar{a} \equiv \frac{1}{r}\left(\frac{log\,(r)}{\alpha} - \frac{r-\beta}{r\alpha} - \frac{1}{2}r\alpha\sigma_e^2\right). \tag{9}$$

The term V_τ is given by

$$\begin{aligned} rV_\tau = {} & (\delta - \eta J - x_\tau)\,\theta_b - y|\theta_b| + (p_c - \eta J - x_\tau)\,\theta_c - y|\theta_c| \\ & + \sum_{k=1}^{K(\tau)} \gamma(k, \tau) \max_{\tau' \in \mathcal{T}(\tau,k)} \frac{1}{r\alpha}\left(1 - e^{-r\alpha(V_{\tau'} - V_\tau + P(\tau,\tau'))}\right) \end{aligned} \tag{10}$$

where $x_\tau = -x$ for high, $x_\tau = 0$ for average, and $x_\tau = x$ for low-valuation investors,

$$x \equiv r\alpha\rho\sigma_e\eta J, \tag{11}$$

[15] I denote the search efficiencies in the bond and the CDS market, λ_b and λ_c, separately to be general. The distinction allows for, in a reduced form, potential institutional and regulatory differences between trading bonds vs. CDS contracts (e.g. unlike bonds, CDS trading involves setting up ISDA agreements between counterparties). The main results do not depend on their difference but hold for any λ_b and λ_c including for the special case where $\lambda_c = \lambda_b$. Later, the distinction helps me to shut down frictions in the bond and the CDS market one by one and to tease out various effects.

[16] As (8) shows, the indirect utilities $U(W, \tau)$ are monotone transformations of V_τ's. So I will work directly with V_τ's and refer to V_τ's as value functions.

$$y \equiv \frac{r\alpha}{2}\eta J^2, \tag{12}$$

and $P(\tau, \tau')$, given in (B8), is the instantaneous payoff of switching from τ to τ'.

From (10), private valuations are endogenously heterogenous as follows. The flow utility of a long position through the bond market (i.e. for a bond owner: $[\theta_b, \theta_c] = [1, 0]$) is $\delta - \eta J + x - y$ for a high type, $\delta - \eta J - y$ for an average type, and $\delta - \eta J - x - y$ for a low type. Thus, the high type derives an additional utility of x and $2x$ compared to the average and the low type, respectively. It is analogous for a long position through CDS. The flow utility from a short position (i.e. for a CDS buyer, $[\theta_b, \theta_c] = [0, -1]$) is $-p_c + \eta J - x - y$ for a high type, $-p_c + \eta J - y$ for an average type, and $-p_c + \eta J + x - y$ for a low type. Thus, the low type derives the most utility from a short position; the high type derives the least.

The difference in valuations, $x = r\alpha\rho\sigma_e\eta J$, captures the benefit of sharing endowment risk. It increases with agents' risk aversion, the correlation between the agents' endowment and the bond, the volatility of agents' endowment, σ_e, and the bond default risk (both the default intensity, η, and the size of the default, J).

The term $y = \frac{r\alpha}{2}\eta J^2$ affects both long and short exposures and in the same direction; thus, it captures a holding cost. It increases with agents' risk aversion and default risk (again, both the default intensity and the size of default).

To simplify derivations and to work with linearized versions of (10), I assume the risk aversion parameter α is small. See Duffie, Garleanu, Pedersen (2007) and Vayanos and Weill (2008) for similar approximations.

(ii) The Distribution of Agents and Inflow-Outflow Equations

Next, I characterize the population masses of the different agent types. I start by conjecturing the equilibrium agent types and their optimal trading strategies.[17] See Figure 1 to follow along the next discussion.

For high-valuation investors, the optimal exposure is long credit risk. Since an investor goes long by either buying a bond or selling CDS, at any point, a high-valuation investor is one of three asset positions: (1) owns a bond: $[\theta_b, \theta_c] = [1, 0]$, (2) sold CDS: $[\theta_b, \theta_c] = [0, 1]$, or (3) without a position: $[\theta_b, \theta_c] = [0, 0]$. High-valuation investors with the first two positions are inactive: They have reached their terminal optimal position. The ones without a position, on the other hand, seek a long exposure and do so by searching for a counterparty in both the bond and the CDS market at the

[17] The proof of the equilibrium existence then shows that the conjectured strategies are indeed optimal.

same time. They, as a result, make up the mass of bond buyers and CDS sellers: $\mu_{b,\text{B}} = \mu_{h[0,0]}$ and $\mu_{c,\text{S}} = \mu_{h[0,0]}$, where μ_τ denotes the mass of type τ agents.

Low-valuation investors short credit risk. Since they achieve this by buying CDS, they are the naked CDS buyers in the model. At any point, they are one of two asset positions: (1) bought CDS: $[\theta_b, \theta_c] = [0,\text{-}1]$ or (2) looking to buy CDS: $[\theta_b, \theta_c] = [0, 0]$. The latter make up the mass of CDS buyers: $\mu_{c,\text{B}} = \mu_{l[0,0]}$.

Average-valuation investors in equilibrium prefer to remain out of the markets and exist in the model to provide an exit point for high and low types when they get a valuation shock.[18] See Vayanos and Wang (2007), Vayanos and Weill (2008), Rocheteau and Weill (2011), and Afonso (2011) for similar setups. To ensure that exiting is optimal, first, the optimal position for an average type has to be no position: $[\theta_b, \theta_c] = [0, 0]$ (investors cannot exit with an existing position).[19] Assumption 1 ensures this in equilibrium.[20] Second, the valuation type has to be an absorbing state to rule out any incentive to wait to switch to a different valuation type. Hence, this assumption earlier. An investor with a CDS position can unwind her position immediately upon reverting to an average type. So, in equilibrium, the only average types seeking to rebalance their portfolio are bond owners, and they are the bond sellers in the economy: $\mu_{b,\text{S}} = \mu_{a[1,0]}$.

Assumption 1. $2y > x - (r + \gamma_d)O_h > -(x - 2y - (r + \gamma_u)O_l) > 0.$

Put together, the equilibrium agent types are $\mathcal{T} \equiv \{h[0, 0], h[1, 0], h[0, 1], a[1, 0], l[0, 0], l[0, \text{-}1]\}$.

Given the above trading strategies, the steady state masses are such that the flow of agents switching into a type equals the flow of agents switching out of that type. For example, the mass of $h[0, 0]$ agents evolves as:

$$\frac{\partial \mu_{h[0,0]}}{\partial t} = \overbrace{\nu_h F_h + \gamma_u \mu_{h[0,1]}}^{\text{inflow}} - \overbrace{\left(\gamma_d \mu_{h[0,0]} + \left(\lambda_b \mu_{a[1,0]} + \lambda_c \mu_{l[0,0]}\right) \mu_{h[0,0]}\right)}^{\text{outflow}}. \quad (13)$$

In (13), the flow of agents turning into $h[0, 0]$-type are (1) new high-valuation entrants, $\nu_h F_h$, and (2) long investors who had previously sold CDS, but are

[18] In Appendix C, I elaborate further why I need three types.

[19] This also means, as I show in online Appendix L, that covered CDS positions do not arise in equilibrium.

[20] Appendix D discusses Assumption 1 in detail and provides more intuition. In short, what the assumption implies about the underlying parameters is as follows. If O_h and O_l are small, for example, then the assumption bounds the default size, J, between 1 and 2 units of $\rho\sigma_e$, which is the part of the endowment risk that can be hedged by trading bond or CDS. Otherwise, if default risk is too small, then even average types also want to hold CDS positions; but if it is too large, then none of the investors want to trade CDS with each other. General values of O_h and O_l, however, (especially, if they differ) help relax this parameter space restriction.

searching again because their counterparty terminated the contract, $\gamma_u \mu_{h[0,1]}$. The agents switching out of type $h[0,0]$ are those (1) hit by a valuation shock, $\gamma_d \mu_{h[0,0]}$, and (2) matched with either a bond seller or a CDS buyer. The steady state mass is characterized by $\frac{\partial \mu_{h[0,0]}}{\partial t} = 0$; that is, $\mu_{h[0,0]}$ is constant, and the inflow equals the outflow. The inflow-outflow equations for the other agent types are analogous and are in Appendix B.

(*iii*) Market Clearing

For the bond market to clear, the total mass of bond owners has to equal the bond supply:

$$\mu_{h[1,0]} + \mu_{a[1,0]} = S. \tag{14}$$

For CDS market clearing, the number of CDSs sold has to equal the number of CDSs purchased:

$$\mu_{h[0,1]} = \mu_{l[0,\text{-}1]}. \tag{15}$$

(*iv*) Prices and Premiums

The bond price arises from Nash-bargaining between buyers and sellers. The marginal benefit of buying a bond (i.e. the buyer's reservation value) is the difference in the expected utility of owning vs. not owning the bond: $V_{h[1,0]} - V_{h[0,0]}$. The buyer's gains from trade is thus $V_{h[1,0]} - V_{h[0,0]} - p_b$. Similarly, the seller's reservation value is $V_{a[1,0]}$, and her gains from trade is $p_b - V_{a[1,0]}$. A buyer and a seller bargain over the price so that each gets half of the total surplus: $V_{h[1,0]} - V_{h[0,0]} - V_{a[1,0]}$. The bond price, as a result, is the average between the buyer and seller's reservation values:

$$p_b = \frac{1}{2} V_{a[1,0]} + \frac{1}{2}(V_{h[1,0]} - V_{h[0,0]}). \tag{16}$$

I characterize the CDS premium (or, equivalently, the CDS spread), p_c, analogously. A CDS buyer's surplus is $V_{l[0,\text{-}1]} - V_{l[0,0]}$, while the seller's is $V_{h[0,1]} - V_{h[0,0]}$. Thus, the CDS premium is implicitly defined by

$$V_{h[0,1]} - V_{h[0,0]} = \frac{1}{2}\left(V_{l[0,\text{-}1]} - V_{l[0,0]} + V_{h[0,1]} - V_{h[0,0]}\right). \tag{17}$$

CDS termination fees are characterized similarly in Appendix B, and the entry conditions were given earlier.

Put together, the bond price and the CDS spread depend on value functions. Value functions depend on the expected search times that, in turn, depend on the masses of buyers and sellers in each market. The masses of buyers and sellers and the entire distribution of agents types depend on high and low types' entry rates. Entry rates, in turn, depend on value functions. I focus my analysis on the steady state equilibrium defined as follows:

Definition 1. *A steady state equilibrium is value functions* $\{V_\tau\}_{\tau\in\mathcal{T}}$*, population measures* $\{\mu_\tau\}_{\tau\in\mathcal{T}}$*, prices* $\{p_b, p_c\}$*, termination fees* $\{T_{\text{B}}, T_{\text{S}}\}$*, and entry rates* $\{\nu_h, \nu_l\}$ *such that (i) agents' value functions* $\{V_\tau\}_{\tau\in\mathcal{T}}$ *solve their optimization problem (5), (ii) population masses equate the flow of agents switching into type* $\tau \in \mathcal{T}$ *to the flow of agents switching out of* τ *and solve (B9)-(B14), (iii) market clearing conditions (14) and (15) hold, (iv) bond and CDS prices* $\{p_b, p_c\}$ *arise from bargaining and solve (16) and (17), (v) entry decisions* $\{\nu_h, \nu_l\}$ *solve (4), and (vi) termination fees* $\{T_{\text{B}}, T_{\text{S}}\}$ *solve (B15) and (B16).*

Proposition 2. *Under conditions (E5) and (E8), a steady state equilibrium exists where the entry rates are given by an interior solution:* $\nu_h \in (0, 1)$ *and* $\nu_l \in (0, 1)$.

I outline the proof in Appendix E and provide the full proof in online Appendix F. Condition (E8) ensures the existence of an interior solution for ν_l. Whenever an interior solution for ν_l exists, two corner solutions ($\nu_l = 0$ and $\nu_l = 1$) exist also. Thus, multiple equilibria exist each with a different entry rate of low-valuation investors: a unique interior solution $\nu_l \in (0, 1)$ and two corner solutions ($\nu_l = 0$, $\nu_l = 1$). For any ν_l, the solution for the entry rate of high-type investors, however, is unique. Condition (E5) ensures that it is, in particular, an interior solution. The fact that $\nu_l = 0$ is one of the solutions shows that even if CDS trading is feasible, investors may not trade CDS in equilibrium. Since the paper is about the effect of CDS, I contrast the equilibria with CDS (i.e. $\nu_l > 0$ be it an interior or a corner solution) to the environment in which I shut down the CDS market (or, equivalently, to the equilibrium with $\nu_l = 0$). The marginal effect of CDS I highlight below is qualitatively the same for both the interior, $\nu_l \in (0, 1)$, and the corner, $\nu_l = 1$, levels of the entry rate. So the equilibrium multiplicity due to the different entry rates of low types is unimportant.

2.2 Characterization of Liquidity and Prices

I measure bond market liquidity, first, with the trading volume defined in (7) and, second, with the illiquidity discount in the bond price. The latter arises from search frictions and is the difference between the bond price with bond market search frictions (Proposition 4) versus without (Proposition 3).

Proposition 3. *Absent bond market search frictions (*$\lambda_b \to \infty$*), the bond price is*

$$p_b = \frac{(\delta - \eta J) + x - y}{r} - \frac{(r + \gamma_d)O_h}{r}. \tag{18}$$

Eq. (18) shows that the bond is priced by high-valuation agents. The absence of search frictions allows a bond owner to sell her bond to another high-valuation trader the moment she gets a valuation shock. Only high-valuation agents, as a result, own the bond.[21] Since the bond price is the average between the marginal valuations of different bond owners, and high-valuation investors are the only bond owners, the bond price depends on their valuation only. In particular, $\delta - \eta J$ is the expected cash flow of the bond, and $\delta - \eta J + x - y$ is long investors' utility valuation of this cash flow. The additional discount in the bond price, $(r + \gamma_d)O_h$, arises as a compensation for a long investor for her entry cost (or, equivalently, her funding cost).

Proposition 4. *With bond market search frictions ($\lambda_b < \infty$), the bond price is given by*

$$p_b = \frac{(\delta - \eta J) + x - y}{r} - \underbrace{\frac{(r + \gamma_d)O_h}{r}}_{\substack{\textit{discount due to} \\ \textit{funding cost}}} - \underbrace{\frac{(r + 2\gamma_d)}{r}\frac{1}{2}\frac{(x - (r + \gamma_d)O_h)}{r + \gamma_d + \lambda_b \mu_{h[0,0]}\frac{1}{2}}}_{\textit{illiquidity discount}}, \quad (19)$$

Comparing Propositions 3 and 4, search costs depress the bond price below the frictionless price in (18). Upon a sudden decrease in her valuation, a bond owner tries to sell her bond but because of search frictions is unsuccessful for some time. When she does find a buyer, she sells it at a discounted price accounting for the difficulty of locating another buyer. Similarly, a potential buyer anticipating the difficulty of reversing positions negotiates a discounted price. Thus, search costs create an illiquidity discount in the bond price: the difference between the price with frictions (19) versus without (18).

Definition 2. *The illiquidity discount, d_b, in the bond price is*:

$$d_b \equiv \frac{(r + 2\gamma_d)}{r}\frac{1}{2}\frac{(x - (r + \gamma_d)O_h)}{r + \gamma_d + \lambda_b \mu_{h[0,0]}\frac{1}{2}}. \quad (20)$$

Proposition 5. *The CDS premium (or, equivalently, the CDS spread) can be characterized as:*

$$p_c = (\eta J - x + y) + \underbrace{(r + \gamma_d)O_h}_{\substack{\textit{premium due to} \\ \textit{funding cost}}} + \underbrace{(r + 2\gamma_d)\frac{1}{2}\frac{2x - 2y - (r + \gamma_d)O_h}{r + \gamma_d + \gamma_u + \lambda_c \mu_{h[0,0]}\frac{1}{2}}}_{\textit{premium due to CDS market illiquidity}}. \quad (21)$$

Eq. (21) expresses the CDS spread from the perspective of a CDS seller, who is a high type investor. It reflects the cost of selling CDS.[22] It increases in default risk—both the default intensity, η, and the size of the default, J—and, hence, with the expected default payment. The entire term $\eta J - x + y$ is a long

[21] In a frictionless environment and in the absence of the CDS market, the steady state mass of long investors ($\frac{\nu_h F_h}{\gamma_d}$)—which captures the total demand for credit risk—converges to the bond supply, S. In an environment with CDS where the CDS market is also frictionless, it converges to the bond supply plus the total short interest, $S + \frac{\nu_l F_l}{\gamma_u}$.

[22] It can expressed symmetrically as the value of buying CDS from the perspective of CDS buyers.

investor's utility valuation of the expected payment. The second term, $(r + \gamma_d)O_h$, shows that long investors' entry cost (or, equivalently, their funding cost) gets passed on to buyers as a higher CDS premium. The third term is zero if the CDS market is frictionless ($\lambda_c \to \infty$) but is positive otherwise. It thus captures CDS market illiquidity, and it increases the premium.

3 The Main Result

The next proposition gives the main result of the paper. It shows that shorting bonds through naked CDS purchases increases bond market liquidity.

Proposition 6 (The Spillover Effect)**.** *Suppose $\lambda_b < \infty$ and $\lambda_c < \infty$. In the equilibria of Proposition 2 in which investors trade CDS, the bond market has fewer sellers ($\mu_{a[1,0]}$) and more buyers ($\mu_{h[0,0]}$), the illiquidity discount (d_b) is smaller, the bond price (p_b) is higher, the volume of trade (M_b) is larger, and high-valuation investors enter at a higher rate than in the environment without the CDS market.*

I give the intuition in the next paragraphs. With the introduction of CDS, long investors have additional counterparties that they can search for as they search for bond sellers. Searching for both counterparties at the same time benefits a long investor in two ways. First, by expanding the investor's pool of potential counterparties, searching for both shortens the investor's expected search time. She values this because she is impatient: She (a) discounts and (b) risks getting hit by a valuation shock (in which case, she loses altogether future trading opportunities and the gains they yield). Second, the ease of finding a counterparty improves the investor's bargaining power. So when she does find a counterparty, she extracts a larger surplus.[23] Put together, the introduction of CDS increases a long investor's continuation value.[24]

The additional and simultaneous trading opportunity relaxes long investors' participation constraint: The benefit of entering and trading as a long investor, $V_{h[0,0]}$, now exceeds the cost: $V_{h[0,0]} > O_h$. Long investors respond by entering at a higher rate and do so until the increase in $V_{h[0,0]}$ is

[23] Recall that search frictions preclude competition and that once a match is found, each side has a bargaining power against the other side. If an investor lets a counterparty go without trading with him, she (a) postpones hedging benefits a trade would have yielded and (b) while waiting for another one, risks getting hit by a valuation shock and losing altogether all future trading opportunities and the gains they yield. But if she trades, she foregoes other counterparties she could have potentially traded with. The price the two sides negotiate—and hence the gains from trade each side realizes—balances these incentives to reach a deal.

[24] In the main setup, I took as given that long investors search in both markets. However, as Lemma 2 in online Appendix H formalizes, searching in both markets at the same time is indeed the optimal thing to do if long investors have an explicit choice between searching in just the bond market, just the CDS market, or in both markets at the same time.

reversed and the marginal entrant is again indifferent: $V_{h[0,0]} = O_h$.[25] The result is an increase in the aggregate number of long investors in the economy.

The increase in the number of long investors, importantly, increases the mass of bond buyers and, thereby, bond market liquidity. After they enter, the additional long investors spend time searching for a counterparty and, as a result, expand the mass of long investors looking for a counterparty, $h[0, 0]$. Since they search for both bond sellers and CDS buyers, for bond sellers, a larger mass of long investors searching for a counterparty means a larger number of potential counterparties. The result is an increase in bond market liquidity: a shorter search time for bond sellers, fewer bond sellers (hence, less misallocation), a larger bond turn over, and a larger volume of trade. If the model had dealers, the increase in liquidity would manifest as a decrease in the bond bid-ask spread also.

The increase in bond market liquidity increases the bond price. Sellers' reservation value for the bond increases because they can find a buyer quickly. Buyers are also willing to pay a higher price because they know reversing positions has become easier. Put together, they negotiate and trade at a higher price.

The spillover effect works also in reverse: Banning CDS decreases bond market liquidity.[26] Investors can no longer sell CDS because their counterparties, the naked CDS buyers, are banned from buying CDS. Long investors exit the CDS market, but by exiting the CDS market they pull out from the bond market also. The result is a decrease in bond market liquidity, consistent with the reaction after the permanent ban. This suggests that investors respond to a permanent CDS ban by scaling back their credit market operations: They fire their CDS and bond traders, shut down their credit trading desks, and switch into equity or currency markets. Thus, preventing investors from shorting ultimately ends up banning investors who want to take the opposite side and long the asset.

3.1 Key Ingredients

The liquidity spillover effect relies on four key ingredients. The first is endogenous entry. Suppose the entry rate (hence, the mass) of long investors is fixed. Then, naked CDS buyers attract investors who would have otherwise

[25] The increase in the number of long investors reverses the two effects that increased long investors' utility in the first place. First, the number of counterparties for long investors decreases as more of them match up with other long investors. Second, long investors lose their bargaining power when there is more of them.

[26] A CDS ban is equivalent to either setting the CDS matching efficiency (λ_c) to zero or decreasing the flow of low-valuation investors (F_l) to zero because, except for bond owners, the ban made entering and buying CDS infinitely costly. Moreover, the actual ban prevented CDS purchases used to both speculate and to hedge long positions correlated with the sovereign. In the model, consistent with the actual ban, both would be considered naked CDS purchases because the CDS buyer does not hold the underlying bonds.

bought bonds away from the bond market. Naked CDS buyers, as a result, reduce the number of counterparties for bond sellers and worsen their search costs. Thus, introducing CDS but keeping the entry rate of long investors fixed reverses the spillover effect: Bond market liquidity deteriorates.

The second ingredient is condition (E5). It is related to the first ingredient. Recall that it ensures the existence of an interior solution for the entry rate of high type investors, ν_h, before and after CDS is introduced.[27] It thereby ensures that a sufficient number of long investors exist on the sideline that can then enter and absorb the short interest. Otherwise, the demand for short positions (captured by the total mass of low types, $\frac{\nu_l F_l}{\gamma_u}$) is too large relative to the supply of long capital (which is at most $\frac{F_h}{\gamma_d}$), and introducing short positions crowds out bond sellers.[28] In turn, the demand for short positions is large if either short investors have poor outside option, O_l, the holding cost y is small, the intensity of getting a valuation shock, or if γ_u, is small.[29]

Proposition 7. *Let hats denote the counterfactual environment without CDS, and their absence the environment with CDS. If either the bond or the CDS market is frictionless ($\lambda_b \to \infty$ or $\lambda_c \to \infty$), then $d_b = \hat{d}_b$, $p_b = \hat{p}_b$, and $M_b = \hat{M}_b$.*

As Proposition 7 shows, the third ingredient for the spillover effect is search frictions in both the CDS and the bond market. If the CDS market is frictionless ($\lambda_c \to \infty$), CDS attracts additional long investors as before, but the increase in the aggregate mass of long investors does not translate to an increase in the mass of bond buyers, $h[0,0]$. This is because the additional entrants sell CDS immediately upon entry.[30] CDS contracts, as a result, do not affect bond market liquidity and are thus redundant. Similarly, CDS is redundant if the bond market is frictionless ($\lambda_b \to \infty$). In this case, even

[27] Condition (E5) simplifies the proof of the main result. It is, however, stronger than I need, and I elaborate why in Appendix E.

[28] This occurs if an interior solution for ν_h does not exist (hence, the entry rate is given by a corner solution, $\nu_h = 1$), and the entry rate at which the bond price would have started to exceed the price without CDS is larger than this corner value. That is, if the equilibrium entry rate of high type investors hits the corner value $\nu_h = 1$ before a sufficient number of long investors can enter and absorb the short interest.

[29] The intuition is as follows. If O_l is small, the value of entering as a short investor, captured by $V_{l[0,0]}$, is more likely to exceed the outside option implying a larger entry of short investors. A small holding cost implies a large gains from CDS trade and, hence, a larger flow of short investors. If γ_u is small, short investors expect to hold their position longer, derive a larger gains from trade, and, hence, enter at a higher rate. A larger flow of short investors, in turn, means that it is more likely for ν_h to hit the corner value before a sufficient number of long investors can enter to take the opposite side.

[30] In particular, the increase in the equilibrium number of high-valuation investors, $(\nu_h - \hat{\nu}_h)\frac{F_h}{\gamma_d}$, equals the total demand for CDS (the measure of all low-valuation investors, including those who have purchased CDS: $\frac{\nu_l F_l}{\gamma_u} = \mu_{l[0,0]} + \mu_{l[0,-1]}$). That is, the new high-valuation entrants replace one-to-one the bond buyers that migrate to the CDS market and sell CDS instead.

without the CDS market, the illiquidity discount is zero, and the bond volume is the maximum possible.

Proposition 8. *Let hats denote the counterfactual environment without CDS, and their absence the environment with CDS. If long investors cannot search at the same time in bond and CDS markets but, upon entering, have to choose one of them to search in (i.e. direct their search effort to), then* $d_b = \hat{d}_b$*,* $p_b = \hat{p}_b$*, and* $M_b = \hat{M}_b$*.*

The last ingredient is investors' ability to search in both markets at the same time. Proposition 8 shows that if we segment bond and CDS markets by shutting down the ability to search in both markets, the introduction of CDS is again redundant. Recall that the ability to also trade with short investors increased the probability of trade and the bargaining power of long investors. Removing the ability to search simultaneously—by removing the substitutability between bond and CDS trades—cancels these effects and, with them, the reasons long investors increased their entry rate in the first place. The same number of long investors enter the bond market as without the CDS market, and the spillover effect does not arise.

4 Additional Results

To qualify the spillover effect of CDS and to understand it further, I consider three extensions in this section. First, in Section 4.1, I model short-selling and compare its effect with that of shorting through CDS contracts. I also contrast the spillover effect with existing results on short-sales. In Section 4.2, I relax the assumption that investors cannot short-sell directly and analyze how the main result changes. Finally, in Section 4.3, I endogenize the intensities with which investors search in the two markets and show that the spillover effect remains intact.

4.1 Short Selling Directly

I model bond shorting following Vayanos and Weill (2008). The model works as follows. After purchasing the bond, long investors now lend the bond in a repo (i.e. a lending) market and, as a result, earn a lending fee. On the other side of the repo transaction, short investors ($l[0, 0]$) borrow the bond to sell it in the spot market. Meetings in both the spot and repo markets occur through search. I denote with λ_r the exogenous search intensity in the repo market. Parties negotiate over the bond price in spot transactions and over the lending fee in repo transactions. An investor unwinds a short position by first buying the bond in the spot market and then delivering it back to the bond lender. To unwind a bond loan, if her counterparty has not yet (short-) sold the bond, the lender recalls the bond, sells it, and exits. If the counterparty has already sold the bond, the lender walks away with the collateral that the short seller has put aside. The full model is in online Appendix I.

Below results on how short selling affects bond market liquidity are new relative to Vayanos and Well (2008). First, they keep the entry of high and low-valuation investors fixed, while I endogenize them. Second, they compare bond market liquidity in the cross section across multiple bonds. In contrast, I focus on how liquidity of a single bond differs with versus without short selling.

Proposition 9. *The bond price in the presence of short-sales is given by:*

$$p_b = \frac{(\delta - \eta J) + x - y - (r+\gamma_d)O_h}{r} - \underbrace{\frac{(r+2\gamma_d)}{r}\frac{1}{2}\frac{(x-(r+\gamma_d)O_h)}{r+\gamma_d+\lambda_b\mu_{b,\mathrm{B}}\frac{1}{2}}}_{\text{illiquidity discount}} + \underbrace{\frac{(r+\lambda_b\mu_{b,\mathrm{B}})}{r}\frac{1}{2}\frac{\lambda_r\mu_{l[0,0]}\frac{1}{2}\omega_r}{r+\gamma_d+\lambda_b\mu_{b,\mathrm{B}}\frac{1}{2}}}_{\text{lending fee effect}}, \tag{22}$$

where ω_r, defined by (I23), is the total gains from trade from a repo contract.

Short-selling has two effects on the bond. The first is a lending fee effect and is captured by the third term in (22). Bond owners by lending their bond to short investors and charging them for the loan earn an extra cash flow. The prospect of an additional cash flow raises long investors' reservation value for the bond and, as a result, the price negotiated between bond buyers and sellers. Proposition 10 (a) formalizes this effect by shutting down spot market search frictions and, thereby, any confounding effects of short selling on the bond price through its effect on bond market liquidity.[31]

Proposition 10. *Let hats denote the counterfactual environment without short-selling, and their absence the environment with short-selling. (a) Suppose the spot market is frictionless ($\lambda_b \to \infty$). Then, $p_b > \hat{p}_b$ and $M_b > \hat{M}_b$. (b) Suppose the repo market is Walrasian ($\lambda_r \to \infty$). Then, $p_b > \hat{p}_b$, $d_b = \hat{d}_b = 0$, and $M_b > \hat{M}_b$.*

The second effect is a liquidity effect. It is the analogue of the spillover effect of CDS. Allowing short positions introduces into the spot market an

[31]In particular, in the limit as $\lambda_b \to \infty$, the second term in (22) is zero with or without short-selling; thus, short-selling affects the bond price only through the third term. Short-selling also increases the volume of bond trade by increasing both the number of sellers (due to short-sellers who are first establishing the short position and selling) and the number of buyers (due to short-sellers who are looking to back the bond in the bond market). But when the bond market is Walrasian, this increase in volume has no effect on the illiquidity component of the bond price.

additional mass of sellers—the short sellers. These short sellers have the same effect on the bond that naked CDS buyers had. They attract additional long investors into the economy who, in turn, trade with both types of bond sellers. The result is a decrease in the illiquidity discount, the second term in (22), and an increase in the bond trading volume.[32] The only difference is short and long investors meet in the spot market, whereas, in the CDS environment, they met in the CDS market. As in the case of CDS, this effect relies on bond market search frictions. Proposition 10 (b) formalizes this effect by shutting down search frictions in the repo market and, thereby, the lending fee effect.[33]

The above results highlight three differences between the lending fee and the spillover effects. First, even though both effects increase the bond price, the lending fee effect does so by changing the cash flow the bond yields its holders, while the spillover effect does so by improving its liquidity. Second, the lending fee effect is specific to direct short selling and does not generalize to shorting via derivatives. The spillover effect is, instead, general: It arises with both short-selling and shorting through CDS contracts. Lastly, while the lending fee effect has been shown before, for example, by Duffie, Garleanu, and Pedersen (2002), the spillover effect is a novel channel on how shorting affects the underlying asset.

The spillover effect also contrasts with a related result by Vayanos and Weill (2008). They show that an asset with short-selling activity has a higher spot market liquidity. They show this is because short-selling requires trading in the spot market (first as a bond seller to establish the short position, then as a bond buyer to unwind the position). This mechanism is therefore specific to short-selling and does not generalize to derivatives. In the mechanism I show, in contrast, CDS increases bond market liquidity even though it does not require trading the underlying bond to achieve a long or a short exposure to the underlying bond. Thus, relative to Vayanos and Weill (2008), the channel I show in the paper is both novel and general (it applies to shorting both directly and through CDS contracts).

The spillover effect even differs across the two contexts of shorting, directly and via CDS. Recall that long investors, $h[0,0]$, search for two sets of counterparties: the regular bond sellers and the low type short investors. In the context of CDS, the latter are the CDS buyers, and, in the context

[32] Above with the lending fee effect, bond borrowers in the repo market attract additional long investors into the economy (since they eventually trade as bond lenders). Although similar, here it is instead additional bond sellers in the spot market (the short sellers) that attract long investors as bond buyers.

[33] Repo market search frictions ($\lambda_r < \infty$) preclude competition and result in bond lenders charging a lending fee. As $\lambda_r \to \infty$ and competition on both sides of the market intensify, the lending fee converges to zero. As a result, the third term in (22) is zero, and the effect of short-selling on the bond price is due to its effect on the illiquidity discount (the second term in (22)).

of short-selling, they are the low type investors who have borrowed a bond and are now looking to (short-)sell it in the bond market. For the spillover effect to arise, meetings with this second set of counterparties have to involve search frictions. In the case of short-selling, since the long and the short sides meet directly in the bond market, frictions in the bond market are sufficient to generate the spillover effect. Repo market frictions are unnecessary. In turn, the additional trading activity in the spot market due to short-selling automatically benefits existing bond traders. In the case of CDS, in contrast, bond market frictions are insufficient. The effect also needs CDS market frictions; otherwise, CDS is redundant. This difference in necessary conditions illustrates the derivative nature of CDS contracts.

Lastly, the spillover effect contrasts with the larger short-sales literature. A large strand of the theoretical literature on short-sales shows that short-sale constraints bias prices up by keeping low-valuation investors out of the markets (see Miller (1977), Harrison and Kreps (1978), Jarrow (1980), Chen, Hong, and Stein (2002), Scheinkman and Xiong (2003), and Hong, Scheinkman, and Xiong (2006)).[34] I show that this well known result reverses if we allow one side of the market to respond endogenously to the introduction or ban of the other side. This insight is the novelty of the paper. In particular, I show that preventing short-sales keeps out not only investors who want to short but also investors who want to take the opposite side and long the underlying asset. Trading volume decreases as a result, and, when the bond market is not Walrasian, the decrease in volume lowers asset prices.

4.2 The Effect of CDS If Investors Can Short Sell

In the previous section, I compared the effects of direct short selling with that of shorting via CDS. In this section, I relax the assumption that investors cannot short-sell directly and numerically analyze the marginal effect of CDS relative to a benchmark environment in which investors already short-sell.[35] I present the model in online Appendix J. Figure 2 in online Appendix J illustrates the results I highlight in this section.

[34] In these environments, investors agree to disagree about asset fundamentals. Heterogenous beliefs, in turn, generate heterogenous private valuations for the asset, and hence, a motive for trade. Another strand analyzes how short sale constraints affect price informativeness in Glosten and Milgrom (1985) type environment (see Diamond and Verrecchia (1987), Bai, Chang, and Wang (2006), Cornelli and Yılmaz (2013)). They show that prices are not necessarily biased upwards. Since in my environment investors trade due to difference in private valuations, my results are more comparable to the results with heterogeneous beliefs. I thank an anonymous referee for pointing this out.

[35] The model with both CDS and short-selling is complicated and involves solving, at minimum, a system of 23 equations and variables (10 value functions, 9 population masses, 2 entry rates, the cds premium, and the lending fee). Analytically showing any results, as a result, is intractable.

When investors already short-sell the underlying bond, the introduction of CDS affects the bond in two ways. First, it creates the liquidity spillover effect as it did when investors could not short-sell. The intuition is as follows. Faced with the probability of finding a counterparty in the CDS market, long investors put more weight on the gains from trading in the CDS market than on the gains from trading in the bond market.[36] The gains from trading with a CDS counterparty, moreover, is larger than with a bond counterparty. Put together, the value of entering the economy as a long investor increases, and long investors enter at a higher rate. Once they enter, they search simultaneously for a bond seller creating the spillover of long investors and liquidity into the bond market. The result is an increase in the bond price and trading volume. This result arises even if the entry rate of low type investors were to remain fixed. In addition, CDS changes the participation incentives of low-valuation investors also and, for the same reasons as on the long side, attracts additional low-valuation investors. The increase in the mass of low types creates another layer of the spillover effect: Long investors react to the increase in the mass of low types and enter at an even higher rate than if the entry rate of low types remained fixed. The result is a further increase in the bond price and trading volume.

Second, CDS affects the bond price by changing the lending fee cash flow the bond generates. The direction of the change depends on the matching efficiency of the CDS market, λ_c, relative to that of bond and repo markets. If λ_c is small, the additional inflow of low types translates to a large increase in the number of investors looking for a short position, $\mu_{l[0,0]}$. Since they search to borrow a bond at the same time, this means an increase in the demand to borrow a bond, the lending fee, and, thereby, the bond price. As λ_c increases, this result starts to reverse. As investors start to buy CDS quickly, there are fewer of those who are still searching for a short position (hence, those searching to also borrow a bond). As a result, there is less demand to borrow a bond and a greater supply of lendable bonds. This drives down the lending fee, the cash flow the bond generates through the lending fee, buyers' reservation value for the bond, and, hence, the bond price.

The net impact on the bond price depends on the above two effects. For a relatively large λ_c, the lending fee decreases sufficiently that the resulting downward pressure on the bond price dominates the opposite pressure from the spillover effect. The net effect is a decrease in the bond price. For a

[36] Specifically, the expected utility of high type investors, $V_{h[0,0]}$, is given by $(r + \gamma_d)V_{h[0,0]} = \lambda_b \mu_{b,\text{S}} \frac{1}{2}\omega_b + \lambda_c \mu_{c,\text{B}} \frac{1}{2}\omega_c$. It is a weighted average between the gains from a bond transaction, ω_b, and the gains from a CDS transaction, ω_c, where the weights are proportional to the masses of potential counterparties in the bond ($\mu_{b,\text{S}}$) and the CDS market ($\mu_{c,\text{B}}$). Without the CDS market, their expected utility depends just on the gains from trading in the bond market: $(r + \gamma_d)\hat{V}_{h[0,0]} = \lambda_b \hat{\mu}_{b,\text{S}} \frac{1}{2}\hat{\omega}_b$.

relatively small λ_c, the lending fee either increases or, if it decreases, the decrease does not dominate the spillover effect. The net effect is an increase in the bond price. Thus, when investors can short sell, the net effect of CDS on the bond price is ambiguous. However, it still creates the liquidity spillover effect, and the effect of this channel on the bond price is unambiguous.

4.3 Endogenous Search Intensities

In the main environment, search intensities in the bond and the CDS market, λ_b and λ_c, were fixed. In this section, I endogenize them and show that the liquidity spillover effect arises even if investors react to a CDS introduction by adjusting their search efforts in the two markets.[37]

I endogenize investors' search intensities as follows. A high-valuation investor, $h[0,0]$, searches for a counterparty in market $m \in \{c, b\}$ with search effort λ_m. As a result, she meets a counterparty from market m at Poisson arrival times with an intensity equal to her search effort in that market times the mass of potential counterparties in that market (that is, with total intensity $\lambda_b \mu_{a[1,0]}$ in the bond market and $\lambda_c \mu_{l[0,0]}$ in the CDS market). Since a total mass $\mu_{h[0,0]}$ of long investors does the same thing, the total volumes of matches are $M_b = \lambda_b \mu_{a[1,0]} \mu_{h[0,0]}$ in the bond market and $M_c = \lambda_c \mu_{l[0,0]} \mu_{h[0,0]}$ in the CDS market. For simplicity, I endogenize the search effort of long investors only and set the search effort of investors on the opposite side (i.e. of bond sellers and CDS buyers) to zero.[38]

Investors incur a flow search cost

$$c(\lambda_b, \lambda_c) \equiv c_0 \left(\lambda_b\right)^2 + c_0 \left(\lambda_c\right)^2 \tag{23}$$

as a function of their search efforts $\{\lambda_b, \lambda_c\}$, where $c_0 > 0$ is a constant.[39]

[37] The environment of this section, the environment of Proposition 8, and the main environment can all be interpreted as environments with endogenous search effort but with different feasible regions for the search effort. The feasible regions are: $\{\lambda_b, \lambda_c\} \in [0, \infty) \times [0, \infty)$ in the endogenous search environment, $\{\lambda_b, \lambda_c\} \in \{\{\bar{\lambda}_b, 0\}, \{0, \bar{\lambda}_c\}\}$ in the segmented environment for some values $\bar{\lambda}_b$ and $\bar{\lambda}_c$ (that is, the search effort vector $\{\lambda_b, \lambda_c\}$ can be either $\{\bar{\lambda}_b, 0\}$ or $\{0, \bar{\lambda}_c\}$), and $\{\lambda_b, \lambda_c\} = \{\bar{\lambda}_b, \bar{\lambda}_c\}$ in the baseline environment.

[38] In this section, I focus on the environment with only the long side choosing search efforts because endogenizing search efforts on both sides of the market makes the model intractable. However, in online Appendix K, I endogenize search effort of all investors and characterize the parameter conditions under which the spillover effect still arises. The additional complication does not yield any additional benefit. Numerically, the results with one- versus two-sided endogenous search intensities are analogous.

[39] The convex cost specification implies that if an investor splits her search effort and searches with, say, 50 and 50 units of effort in each market, her total search cost is smaller than if she searches in one market with 100 units. I abstract from micro-foundations that would result in such a functional form. The specification captures, for example, complementarities of trading in multiple related markets. The complementarity could be in the form of, for example, information (e.g., resources expended on pricing individual bonds help an investor price CDS relatively quickly and vice versa). Or it could be in the form of trading relationships (e.g., networks formed in the bond market help form trading

Investors therefore internalize the cost of searching in multiple markets at the same time. If they were to search in just one market m, their total search cost would be the cost of searching in that market only, $c_0 (\lambda_m)^2$.

Investors choose $\{\lambda_b, \lambda_c\}$ to maximize their expected utility. The first order condition with respect to the search effort in the bond market, λ_b, is

$$r\alpha \frac{\partial c(\lambda_b, \lambda_c)}{\partial \lambda_b} = \mu_{a[1,0]} \left(1 - e^{-r\alpha\left(-p_b + V_{h[1,0]} - V_{h[0,0]}\right)}\right), \tag{24}$$

while the first order condition with respect to the search effort in the CDS market, λ_c, is

$$r\alpha \frac{\partial c(\lambda_b, \lambda_c)}{\partial \lambda_c} = \mu_{l[0,0]} \left(1 - e^{-r\alpha\left(V_{h[0,1]} - V_{h[0,0]}\right)}\right). \tag{25}$$

In (24) and (25), the optimal search effort equates the marginal cost (the left-hand side) with the marginal benefit (the right-hand side) of an additional unit of search effort. The marginal benefit is the product of the mass of potential matches and the gains from trade upon a match. The steady state equilibrium now includes the optimal search efforts as additional endogenous variables that satisfy the first order conditions, (24) and (25).

Lemma 1. *The characterization of the bond price and the illiquidity discount are the same as (19) and (20).*

Proposition 11. *Let hats denote the counterfactual environment without CDS, and their absence the environment with CDS. With the introduction of CDS, bond market liquidity increases ($d_b < \hat{d}_b$ and $M_b > \hat{M}_b$), and long investors lower their search effort in the bond market ($\lambda_b < \hat{\lambda}_b$).*

In Proposition (11), the reduced search effort of long investors and greater bond market liquidity are byproducts of the same underlying change in the bond market. Long investors search in the bond market with an effort proportional to the rents they expect to extract from trading in it. The rents they expect to extract, in turn, is proportional to the number of bond sellers (relative to the number of buyers).[40] Naked CDS buyers expand the pool of long investors in the economy who, in turn, trade in the bond market due

relationships in the CDS market).

We can specify the total search cost more generally as $c(\lambda_b, \lambda_c) = (c_0 (\lambda_b)^g + c_0 (\lambda_c)^g)^a$ for some constants g and a. Proving the liquidity spillover effect for a general functional form is impossible, but numerical results suggest that the main effect arises as long as the cost function is a convex function of λ_b and λ_c, and investors can search simultaneously in both markets.

[40] Specifically, for small α, (24) simplifies to $\lambda_b = \frac{1}{2c_0}\mu_{a[1,0]} \left(-p_b + V_{h[1,0]} - V_{h[0,0]}\right)$, where the right-hand-side is the rents a long investor expects to extract. It is the product of the mass of potential matches (i.e. bond sellers) and the gains from trade upon a match. The gains from trade itself is an implicit function of market illiquidity: It increases in the number of sellers, $\mu_{a[1,0]}$. Thus, the whole right-hand-side increases in $\mu_{a[1,0]}$.

to the substitutability between bond and CDS positions. The result is an increase in the number of bond buyers and a decrease in the number of sellers. This change in the bond market, on the one hand, means higher bond market liquidity. On the other hand, the decrease in the number of sellers (and the resulting decrease in the expected rents) reduces long investors' incentive to search in the bond market. Thus, long investors reallocate some of their search effort from the bond to the CDS market, but they do so precisely because the bond market is more liquid.

These results with endogenous search efforts highlight what are the crucial ingredients in the main environment and what are not. In the main environment, I implicitly assumed that search intensities come for free. This assumption is not crucial: The spillover effect arises even if investors internalize the cost of searching in multiple markets and adjust, at the intensive margin, their search intensities. The crucial ingredients are instead (1) the substitutability between bond and CDS trades, (2) search frictions, and (3) costly and endogenous entry. Each ingredient matters because of the other ingredients. Participating at the same time in the CDS market alleviates long investors' search frictions and increases their bargaining power. These effects on search frictions and bargaining power, in turn, relax long investors' ex-ante participation constraints and give them more incentive to trade in the bond market.

5 Conclusion

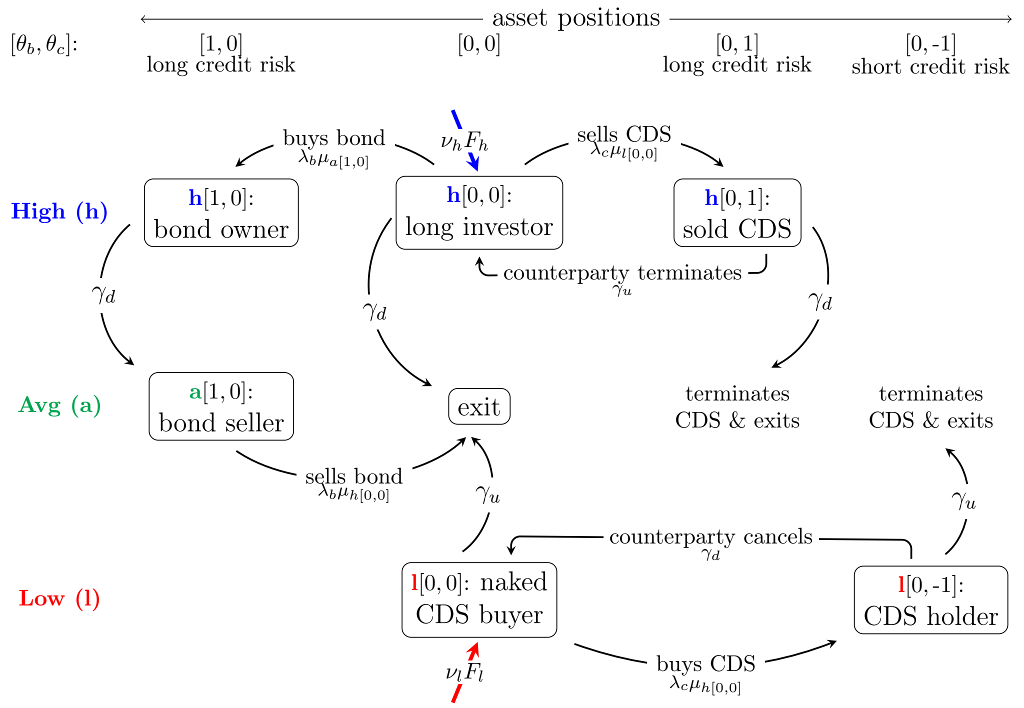
The point I make in this paper is simple: If we want to model and understand the effect of new financial instruments on existing instruments, the number of investors that could potentially trade and use the instruments should be endogenous.

I make this point in the context of CDS and bond markets. I build a continuous time, dynamic search model of bond and CDS trading and show that introducing short positions through CDS contracts attracts into credit markets not only investors who want to short the underlying credit risk but also investors who want to take the opposite side and long the underlying. In turn, long investors—for whom bond and CDS positions are economically similar positions—search and trade at the same time as buyers in the bond market because doing so increases their trading opportunities and alleviates their search frictions. The result is an increase in the number of bond buyers, bond market liquidity, and the bond price. I refer to this effect as a liquidity spillover effect. This insight applies not just to derivative instruments but also to any mechanism that expands the set of feasible allocations (e.g. tradeable securities and contracts, trading mechanisms and venues, private currencies, etc.).

The spillover effect works in reverse also: Shutting down naked CDS positions decreases bond market liquidity. This suggests that by banning single-name naked CDS positions referencing sovereign bonds, regulators in Europe inadvertently decreased bond market liquidity, reduced bond prices, and, thereby, increased sovereigns' borrowing cost exactly when they were trying to achieve the opposite and quell a sovereign debt crisis.

Figure 1: A Snapshot of Transitions Between Agent Types

The figure shows the transitions between agent types. Flows of $\nu_h F_h$ and $\nu_l F_l$ agents enter the economy as new high- and low-valuation investors. High- and low-valuation agents switch to an average-valuation with intensities γ_d and γ_u, respectively. A trader seeking a long position ($h[0,0]$) finds a counterparty in the bond and CDS markets with intensities $\lambda_b \mu_{a[1,0]}$ and $\lambda_c \mu_{l[0,0]}$, respectively. A bond seller ($a[1,0]$) finds a buyer with intensity $\lambda_b \mu_{h[0,0]}$. A trader seeking a short position by buying CDS ($l[0,0]$) finds a counterparty with intensity $\lambda_c \mu_{h[0,0]}$.



Appendix

A The Counting Process $\hat{N}_t(\tau)$

The counting process $\hat{N}_t(\tau)$ embeds changes in an agent's type due to both a valuation shock and an endogenous change in an asset position. Consider, for example, agent type $\tau = h[0,0]$. The dimension of $\hat{N}_t(\tau)$ is $K(\tau) = 3$, where the three possible events are: (1) the agent's valuation changes, (2) the agent finds a counterparty in the bond market, and (3) the agent finds a counterparty in the CDS market. Intensities of these events are $\gamma(1,\tau) = \gamma_d$, $\gamma(2,\tau) = \lambda_b \mu_{a[1,0]}$, and $\gamma(3,\tau) = \lambda_c \mu_{l[0,0]}$, respectively. Similarly, consider agent $\tau = h[0,1]$ (an agent who has sold CDS). Then, $K(\tau) = 2$, and the two possible events are (1) the agent himself gets a valuation shock or (2) his counterparty's valuation changes. The intensities are: $\gamma(1,\tau) = \gamma_d$ and $\gamma(2,\tau) = \gamma_u$. It is analogous for the other agent types.

B Value Functions, Population Masses, Termination Fees

Substituting in (16) and (17), the value functions simplify to

$$
\begin{aligned}
rV_{l[0,0]} &= \gamma_u(0 - V_{l[0,0]}) + \frac{M_c}{\mu_{l[0,0]}}\frac{1}{2}\omega_c && \text{(B1)}\\
rV_{h[0,0]} &= \gamma_d(0 - V_{h[0,0]}) + \frac{M_b}{\mu_{h[0,0]}}\frac{1}{2}\omega_b + \frac{M_c}{\mu_{h[0,0]}}\frac{1}{2}\omega_c && \text{(B2)}\\
rV_{h[1,0]} &= (\delta - \eta J) + x - y + \gamma_d(V_{a[1,0]} - V_{h[1,0]}) && \text{(B3)}\\
rV_{a[1,0]} &= (\delta - \eta J) - y + \frac{M_b}{\mu_{a[1,0]}}\frac{1}{2}\omega_b && \text{(B4)}\\
rV_{h[0,1]} &= p_c - (\eta J - x) - y + \gamma_d(-T_{\text{S}} - V_{h[0,1]}) && \text{(B5)}\\
rV_{l[0,\text{-}1]} &= -p_c + (\eta J + x) - y + \gamma_u\left(-T_{\text{B}} - V_{l[0,\text{-}1]}\right). && \text{(B6)}
\end{aligned}
$$

where ω_c is the total gains from a CDS transaction:

$$\omega_c \equiv \left(V_{h[0,1]} - V_{h[0,0]}\right) + \left(V_{l[0,\text{-}1]} - V_{l[0,0]}\right). \tag{B7}$$

and ω_b is the total gains from a bond transaction:

$$\omega_b \equiv V_{h[1,0]} - V_{h[0,0]} - V_{a[1,0]}$$

The instantaneous payoff from a transition from τ to τ' is given by:

$$P(\tau,\tau') = \begin{cases} -p_b & \text{if } \tau = i[0,\theta_c] \text{ and } \tau' = i[1,\theta_c] \\ p_b & \text{if } \tau = i[1,\theta_c] \text{ and } \tau' = i[0,\theta_c] \\ 0 & \text{else.} \end{cases} \tag{B8}$$

In the steady state equilibrium, the masses of agent types are constant: The flow of agents turning into a particular type has to equal the flow of agents switching out of that type. Thus, the inflow-outflow equations are:

$$\begin{aligned}
\text{long investor } h[0,0]: &\quad \nu_h F_h + \gamma_u \mu_{h[0,1]} = \gamma_d \mu_{h[0,0]} + M_b + M_c && \text{(B9)}\\
\text{naked CDS buyer } l[0,0]: &\quad \nu_l F_l + \gamma_d \mu_{l[0,\text{-}1]} = \gamma_u \mu_{l[0,0]} + M_c && \text{(B10)}\\
\text{bond owner } h[1,0]: &\quad M_b = \gamma_d \mu_{h[1,0]} && \text{(B11)}\\
\text{bond seller } a[1,0]: &\quad \gamma_d \mu_{h[1,0]} = M_b && \text{(B12)}\\
\text{sold CDS } h[0,1]: &\quad M_c = \gamma_d \mu_{h[0,1]} + \gamma_u \mu_{h[0,1]} && \text{(B13)}\\
\text{bought CDS } l[0,\text{-}1]: &\quad M_c = (\gamma_u + \gamma_d) \mu_{l[0,\text{-}1]} && \text{(B14)}
\end{aligned}$$

Consider fees CDS counterparties pay each other to terminate the contract. If a buyer terminates, the seller goes from being a $h[0,1]$ type to $h[0,0]$, and the seller's utility decreases by $(V_{h[0,1]} - V_{h[0,0]})$. To make the seller indifferent then, the buyer has to pay a fee equal to the decrease in the seller's utility:

$$T_{\text{B}} = V_{h[0,1]} - V_{h[0,0]}, \tag{B15}$$

Analogously, a CDS seller (the long side) has to pay the short side:

$$T_{\text{S}} = V_{l[0,\text{-}1]} - V_{l[0,0]}, \tag{B16}$$

The right-hand sides coincide with the gains from trade to each side; hence, both equal $\frac{1}{2}\omega_c$.

C Why Three Valuation Types

Here, I explain further why I need three valuation types.

In an environment with just two types (say, high and low types) and no entry and exit, introducing CDS deteriorates bond market liquidity. In such environment in the absence of CDS, the optimal position for the low type is no position: Investors buy the bond as a high type and sell when they switch to a low type. When we introduce CDS, the optimal position for the low type turns into a short position. They, as a result, go one asset position further and buy CDS after they sell their bond. But because the number of investors of each type is fixed, allowing CDS and, hence, short positions deteriorates bond market liquidity.

The point of the paper is to, instead, show that investors' participation incentive changes in response to CDS. I thus need to endogenize the aggregate number of investors of each type. One way to do that is to endogenize their entry. But to model investors' entry, I have to model their exit also. Otherwise, entry without any exits results in infinite masses.

Exiting is optimal under two conditions. First, investors cannot exit with an existing position. So to ensure that investors unwind their existing positions, their valuation has to change to a type whose terminal optimal position is no position. Second, once their valuation changes, they cannot have an incentive to wait to switch to another valuation instead of exiting. That is, the valuation they switch to has to be an absorbing type.

A model with just two types but with entry and exit also does not generate the result I want to show. Suppose investors enter the economy as a high type, switch to a low type, and, once they switch, remain forever a low type. In the absence of short positions, this model works fine. Investors buy the bond after they enter; when they switch to a low type, they sell and exit forever. In the presence of CDS, high types still go long, but low types now want to short (as their terminal optimal position). This, however, implies that exiting is not optimal for the low types. They, instead, want to remain in the economy and short.

Thus, an environment with two types allows short positions in the absence of entry and exit, allows entry and exit in the absence of short positions, but not both short positions and entry and exit at the same time.

D Intuition for Assumption 1

The intuition for Assumption 1 is as follows. The gains from CDS trade between high and average types is proportional to $x - 2y - (r + \gamma_d)O_h$. This is negative by Assumption 1. Intuitively, the difference in their valuations—hence, the total hedging benefit $(x - 0)$—is too small relative to the holding cost both sides incur $(2y)$ and the implicit entry cost, $(r + \gamma_d)O_h$. The absence of the gains from trade ensures that (a) average types do not buy CDS from a high type, and (b) once a CDS buyer (initially, a low type) switches to an average type, she prefers to unwind her short position she has with a high-type than to remain a CDS buyer.[41] The gains from CDS trade exists only between high- and low-valuation investors: $2x - 2y - (r + \gamma_d)O_h - (r + \gamma_u)O_l > 0$. Their valuations are far apart enough that the total hedging benefit, $2x = x - (-x)$, outweighs the holding cost, $2y$, and the costs of entry, $(r + \gamma_d)O_h + (r + \gamma_u)O_l$.

On the bond side, the gains from trade between high- and average types is proportional to the difference in valuations, $x - 0$, minus the opportunity cost of entry: $x - (r + \gamma_d)O_h$.[42] This is positive by Assumption 1. Thus, a bond owner who switches to an average type prefers to unwind and sell her bond.

Consider how these parameter conditions relate to the original parameters. To gain intuition, let us ignore O_h and O_l.[43] Then, Assumption 1 simplifies to $2x > 2y > x$ or $x > y > \frac{1}{2}x$. Substituting in the definitions of x and y,

$$2r\alpha\rho\sigma_e\eta J > 2\frac{r\alpha}{2}\eta J^2 > r\alpha\rho\sigma_e\eta J$$

[41] It is analogous between average and low types. The gains from CDS trade between them is proportional to $x - 2y - (r + \gamma_u)O_l$, which is negative. This ensures that (a) average types do not sell CDS to a low type, and (b) once a CDS seller (a high type) switches to an average, she prefers to unwind her long position than to remain a CDS seller.

[42] When high and average types trade a bond, the total holding cost does not change because, unlike CDS transactions, investors do not create new positions. Only the ownership of the bond changes.

[43] If, for example, $x - y > 0$, $x - 2y < 0$, and $(x - 2y) + [x - (r + \gamma_d)O_h - (r + \gamma_u)O_l] > 0$, then Assumption 1 holds.

Canceling terms:

$$2\rho\sigma_e > J > \rho\sigma_e.$$

Thus, Assumption 1 bounds the default size, J, between 1 and 2 units of $\rho\sigma_e$, which is the part of the endowment risk that can be hedged via bond or CDS.

E Proofs

Because proofs of Propositions 1 and 2 are long, I relegate them to online Appendix F and G.

Here, I summarize some of the main steps of Proposition 2 proof. In step 1 of the proof, I show that the equilibrium conditions narrow down to a set of five equations and five unknowns $\{\mu_{h[0,0]}, V_{h[0,0]}, V_{l[0,0]}, \nu_h, \nu_l\}$:

$$(r+\gamma_d)V_{h[0,0]} - \lambda_b \frac{\gamma_d S}{\left(\gamma_d + \lambda_b \mu_{h[0,0]}\right)} \frac{1}{2} \frac{x - (r+\gamma_d)V_{h[0,0]}}{r + \gamma_d + \lambda_b \mu_{h[0,0]} \frac{1}{2}} - \lambda_c \frac{(\gamma_d+\gamma_u)\frac{\nu_l F_l}{\gamma_u}}{\gamma_d + \gamma_u + \lambda_c \mu_{h[0,0]}} \frac{1}{2} \frac{2x - 2y - (r+\gamma_d)V_{h[0,0]}}{r + \gamma_d + \gamma_u + \lambda_c \mu_{h[0,0]} \frac{1}{2}} = 0. \tag{E1}$$

$$V_{l[0,0]} = \frac{1}{r+\gamma_u} \lambda_c \mu_{h[0,0]} \frac{1}{2} \frac{2x - 2y - (r+\gamma_d)V_{h[0,0]}}{r + \gamma_d + \gamma_u + \lambda_c \mu_{h[0,0]} \frac{1}{2}}. \tag{E2}$$

$$\nu_h F_h = \gamma_d \mu_{h[0,0]} + \gamma_d \frac{\lambda_b \mu_{h[0,0]} S}{\left(\lambda_b \mu_{h[0,0]} + \gamma_d\right)} + \gamma_d \lambda_c \mu_{h[0,0]} \frac{\frac{\nu_l F_l}{\gamma_u}}{\gamma_u + \gamma_d + \lambda_c \mu_{h[0,0]}}. \tag{E3}$$

$$\nu_i = \begin{cases} 1 & V_{i[0,0]} > O_i \\ [0,1] \text{ if} & V_{i[0,0]} = O_i \\ 0 & V_{i[0,0]} < O_i. \end{cases} \qquad \text{for } i \in \{h, l\} \tag{E4}$$

In step 2, I show that $\frac{\partial V_{h[0,0]}(\nu_h,\nu_l)}{\partial \nu_h} < 0$, where $V_{h[0,0]}(\nu_h, \nu_l)$ defines $V_{h[0,0]}$ as an implicit function of ν_h and ν_l and is given by (E1) and (E3). This and the assumption that

$$\frac{Sx\gamma_u\left(r+\gamma_d+\gamma_u\right)\lambda_b + 2(x-y)\left(r+\gamma_d\right)\lambda_c\nu_l F_l}{\left(r+\gamma_d\right)\left(\gamma_u\left(r+\gamma_d+\gamma_u\right)\left(2\left(r+\gamma_d\right)+S\lambda_b\right)+\left(r+\gamma_d\right)\lambda_c\nu_l F_l\right)} > O_h \qquad \text{for } \nu_l \in [0,1] \tag{E5}$$

ensure that, taking ν_l as given, the solution for ν_h is unique, positive, and interior.

Ensuring that the entry rate of high-type investors is given by an interior solution both before and after CDS is introduced simplifies the analysis. In search models with exogenous entry, given the conjectured trading strategies, the system of equations characterizing the population masses does not depend on value functions and can be solved on its own. Then, value functions are a linear system of equations of the population masses. Thus, the conjecture-and-verify method simplifies the analysis and proofs by decoupling the system of equations into two sets. Endogenizing entry, however, reverses this decoupling. Population masses depend on the entry rates, but the entry rates depend on value fuctions, which, in turn, depend on population mases. So all three sets of variables have to be solved simultaneously. Thus, the model with endogenous entry is significantly more complicated. Focusing on the interior solution helps simplify the analysis.

In step 3, using the result from step 2 that ν_h is given by an interior solution, (E1) and (E2) become:

$$\begin{aligned}(r+\gamma_d)O_h - \lambda_b \frac{\gamma_d S}{\left(\gamma_d+\lambda_b\mu_{h[0,0]}\right)}\frac{1}{2}\frac{x-(r+\gamma_d)O_h}{r+\gamma_d+\lambda_b\mu_{h[0,0]}\frac{1}{2}} \\ -\lambda_c\frac{(\gamma_d+\gamma_u)\frac{\nu_l F_l}{\gamma_u}}{\gamma_d+\gamma_u+\lambda_c\mu_{h[0,0]}}\frac{1}{2}\frac{2x-2y-(r+\gamma_d)O_h}{r+\gamma_d+\gamma_u+\lambda_c\mu_{h[0,0]}\frac{1}{2}} = 0.\end{aligned} \tag{E6}$$

$$V_{l[0,0]} = \frac{1}{r+\gamma_u}\lambda_c\mu_{h[0,0]}\frac{1}{2}\frac{2x-2y-(r+\gamma_d)O_h}{r+\gamma_d+\gamma_u+\lambda_c\mu_{h[0,0]}\frac{1}{2}}. \tag{E7}$$

Equations (E6) and (E7) together define $V_{l[0,0]}$ as an implicit function of ν_l: $V_{l[0,0]}(\nu_l)$. I show that $V_{l[0,0]}(\nu_l)$ strictly increases in ν_l. This and the condition that

$$0 < \frac{2\gamma_u\left(r+\gamma_d+\gamma_u+\frac{1}{2}\lambda_c\mu_h^*\right)\left(\gamma_d+\gamma_u+\lambda_c\mu_h^*\right)\left[O_h\left(r+\gamma_d\right)-\frac{S\gamma_d(x-O_h(r+\gamma_d))\lambda_b}{\left(\gamma_d+\lambda_b\mu_h^*\right)\left(2(r+\gamma_d)+\lambda_b\mu_h^*\right)}\right]}{F_l\left(2x-2y-O_h\left(r+\gamma_d\right)\right)\left(\gamma_d+\gamma_u\right)\lambda_c} < 1 \tag{E8}$$

where

$$\mu_h^* \equiv \frac{2O_l\left(r+\gamma_u\right)\left(r+\gamma_d+\gamma_u\right)}{\lambda_c\left(2x-2y-\left(r+\gamma_d\right)O_h-\left(r+\gamma_u\right)O_l\right)}$$

ensure that a unique interior solution exists for ν_l. They also imply that two corner solutions exist: $\nu_l = 0$ and $\nu_l = 1$.

In step 4, I show that—taking the entry rates as given—the rest of the equilibrium variables are uniquely determined, and the population masses and the gains from trade are, in addition, positive. Finally, in step 5, I show all the conjectured optimal trading strategies are indeed optimal.

Proof of Proposition 3. I derive, first, the bond price with frictions. Substituting the value functions of $h[1,0]$, $h[0,0]$ and $a[1,0]$ into the bond price, (16), and simplifying:

$$rp_b = \delta - \eta J + \frac{1}{2}x - y - \frac{1}{2}\gamma_d\omega_b - \frac{1}{2}\left(\lambda_b\mu_{a[1,0]}\frac{1}{2}\omega_b + \lambda_c\mu_{l[0,0]}\frac{1}{2}\omega_c - \lambda_b\mu_{h[0,0]}\frac{1}{2}\omega_b\right). \tag{E9}$$

Combining the value functions for $h[0,0]$, $h[1,0]$, and $a[1,0]$, substituting in M_b and M_c, using $V_{h[0,0]} = O_h$, and simplifying, we get

$$(r+\gamma_d)\omega_b = x-(r+\gamma_d)O_h + \lambda_b\mu_{h[0,0]}\frac{1}{2}\omega_b \tag{E10}$$

Using (B2), $V_{h[0,0]} = O_h$, and (E10), (E9) becomes

$$\begin{aligned} rp_b &= \delta - \eta J + \frac{1}{2}x - y - \frac{1}{2}\gamma_d\omega_b - \frac{1}{2}\left((r+\gamma_d)O_h - x + (r+\gamma_d)O_h + (r+\gamma_d)\omega_b\right) \\ &= \delta - \eta J + x - y - (r+\gamma_d)O_h - \frac{1}{2}\left(r + 2\gamma_d\right)\omega_b. \end{aligned} \tag{E11}$$

From (E10),

$$\omega_b = \frac{x - (r+\gamma_d)O_h}{r + \gamma_d + \lambda_b \mu_{h[0,0]}\frac{1}{2}}. \tag{E12}$$

Substituting this into (E11), we get

$$p_b = \frac{\delta - \eta J + x - y - (r+\gamma_d)O_h}{r} - \frac{(r+2\gamma_d)}{2r}\frac{x - (r+\gamma_d)O_h}{r + \gamma_d + \lambda_b \mu_{h[0,0]}\frac{1}{2}}. \tag{E13}$$

The proof of Proposition 7 in online Appendix H shows that as $\lambda_b \to \infty$, $\omega_b \to 0$. Hence, the bond price absent search frictions in the bond market is given by the first term in (E13). □

Proof of Proposition 4. The derivation is shown in the proof of Proposition 3. □

Proof of Proposition 5. Combining (B2), (B5), and the termination fees, we get:

$$(r+\gamma_d)\left(V_{h[0,1]} - V_{h[0,0]}\right) = p_c - (\eta J - x) - y - \gamma_d \frac{1}{2}\omega_c - \frac{M_b}{\mu_{h[0,0]}}\frac{1}{2}\omega_b - \frac{M_c}{\mu_{h[0,0]}}\frac{1}{2}\omega_c.$$

Using the fact that $V_{h[0,1]} - V_{h[0,0]} = \frac{1}{2}\omega_c$, the CDS premium is given by:

$$\begin{aligned} p_c &= (\eta J - x) + y + \frac{1}{2}\left(r + 2\gamma_d\right)\omega_c + \frac{M_b}{\mu_{h[0,0]}}\frac{1}{2}\omega_b + \frac{M_c}{\mu_{h[0,0]}}\frac{1}{2}\omega_c \\ &= \eta J - x + y + \frac{1}{2}\left(r + 2\gamma_d\right)\omega_c + (r+\gamma_d)O_h. \end{aligned} \tag{E14}$$

where the second equality uses (B2) and $V_{h[0,0]} = O_h$. Combining value functions,

$$\omega_c = \frac{2x - 2y - (r+\gamma_d)V_{h[0,0]}}{r + \gamma_d + \gamma_u + \lambda_c \mu_{h[0,0]}\frac{1}{2}}. \tag{E15}$$

Substituting this into (E14), we get (21). □

Proof of Proposition 6. The expected rents a long investor extracts from trading in the bond market, $\lambda_b \mu_{a[1,0]}\frac{1}{2}\omega_b$, has to be smaller in the equilibrium with CDS than in the equilibrium without CDS. To see this, the value function of a long investor is given by

$$(r+\gamma_d)V_{h[0,0]} = \lambda_b \mu_{a[1,0]}\frac{1}{2}\omega_b + \lambda_c \mu_{l[0,0]}\frac{1}{2}\omega_c. \tag{E16}$$

The first term is the expected rents a long investor extracts from trading in the bond market: The probability of finding a counterparty in the bond market times the gains from trade from a bond transaction. The second term is the analogous expected gains from trade in the CDS market. Since high types' entry rate entry is an interior solution with and without CDS, (E16) with and without CDS is

$$(r+\gamma_d)O_h = \lambda_b \mu_{a[1,0]} \frac{1}{2}\omega_b + \lambda_c \mu_{l[0,0]} \frac{1}{2}\omega_c$$
$$(r+\gamma_d)O_h = \lambda_b \hat{\mu}_{a[1,0]} \frac{1}{2}\hat{\omega}_b,$$

respectively. Since $\lambda_c \mu_{l[0,0]} \frac{1}{2}\omega_c > 0$, and the left hand sides are the same, it has to be that: $\lambda_b \hat{\mu}_{a[1,0]} \frac{1}{2}\hat{\omega}_b > \lambda_b \mu_{a[1,0]} \frac{1}{2}\omega_b$.

Combining (14) and (B11), we get

$$\lambda_b \mu_{a[1,0]} \mu_{h[0,0]} = \gamma_d (S - \mu_{a[1,0]}) \tag{E17}$$

Equations (E12) and (E17) define $\mu_{h[0,0]}$ and ω_b as implicit functions of $\mu_{a[1,0]}$. Using (E12) and (E17), $\mu_{h[0,0]}$ and ω_b change with $\mu_{a[1,0]}$ as:

$$\frac{\partial \mu_{h[0,0]}}{\partial \mu_{a[1,0]}} = -\frac{\gamma_d + \lambda_b \mu_{h[0,0]}}{\lambda_b \mu_{a[1,0]}},$$

$$\frac{\partial \omega_b}{\partial \mu_{a[1,0]}} = \frac{(\gamma_d + \lambda_b \mu_{h[0,0]})\frac{1}{2}\omega_b}{\mu_{a[1,0]}(r + \gamma_d + \lambda_b \mu_{h[0,0]}\frac{1}{2})}.$$

Thus, $\mu_{h[0,0]}$ decreases with $\mu_{a[1,0]}$, while ω_b increases with $\mu_{a[1,0]}$. Then, the expected rents a long investor extracts from trading in the bond market, $\lambda_b \mu_{a[1,0]} \frac{1}{2}\omega_b$, as an implicit function of $\mu_{a[1,0]}$, increases with $\mu_{a[1,0]}$. As a result, $\mu_{a[1,0]}$ has to be smaller in the equilibrium with CDS than in the equilibrium without CDS: $\mu_{a[1,0]} < \hat{\mu}_{a[1,0]}$. In turn, this implies that: $\omega_b < \hat{\omega}_b$ and $\mu_{h[0,0]} > \hat{\mu}_{h[0,0]}$. Since the illiquidity discount d_b just depends on $\mu_{h[0,0]}$, we have: $d_b < \hat{d}_b$. From (E17), a decrease in $\mu_{a[1,0]}$ implies an increase in bond volume: $M_b > \hat{M}_b$. From (E3), an increase in $\mu_{h[0,0]}$ requires an increase in ν_h especially since ν_l changes from zero to a positive value in the presence of CDS. □

This proof relied on condition (E5), which ensured that the entry rate of high type investors is an interior solution with and without CDS. This condition is sufficient to show that CDS increases bond market liquidity, but it is not necessary. The effect of CDS introduction before the entry rate of long investors adjusts is a decrease in the mass of long investors searching for a counterparty, $\mu_{h[0,0]}$, and an increase in their expected utility, $V_{h[0,0]}$. Both effects affect the bond price in the same direction: The bond price decreases. As long investors start to respond to short investors and enter at a higher rate, these two effects start to reverse: $\mu_{h[0,0]}$ starts to increase (this drives the bond price up), and $V_{h[0,0]}$, in turn, starts to decrease (this also drives the price up). At some point, which I will denote with $\widetilde{\nu}_h$, enough long investors enter so that (a) $\mu_{h[0,0]}$ is now higher than in the environment without CDS, (b) even though the increase in $V_{h[0,0]}$ is not reversed yet (and $V_{h[0,0]}$ is still higher than without CDS), the increase in the bond price from the increase in $\mu_{h[0,0]}$ dominates the opposite pressure from the increase in $V_{h[0,0]}$. With a further entry of long investors, $\mu_{h[0,0]}$ increases so much so that the increase in $V_{h[0,0]}$ is fully reversed, and $V_{h[0,0]}$ is the same as in the absence of CDS. This is the new equilibrium entry rate of high-valuation investors. This level of entry rate is higher than necessary because, even at the lower entry rate, $\widetilde{\nu}_h$, the bond price and volume starts to surpass their levels in the absence of CDS.

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Online Appendix: A Theory of Liquidity Spillover Between Bond and CDS Markets

F Proof of Proposition 1

Proof of Proposition 1. I first derive the Hamilton-Jacobi-Bellman (HJB) equations.

Eq. (5) can be written recursively as:[44]

$$U(W_t, \tau_t) = \max_{C_t \in \mathbb{R}, \tau_{t+\Delta t} \in \mathcal{T}(\tau_t, k)} u(C_t)\Delta t + (1 - \beta \Delta t)\, \mathbb{E} U(W_{t+\Delta t}, \tau_{t+\Delta t}). \tag{F1}$$

Subtract $(1 - \beta\Delta t)\, U(W_t, \tau_t)$ from both sides and divide by Δt:

$$\beta U(W_t, \tau_t) = \max_{C_t \in \mathbb{R}, \tau_{t+\Delta t} \in \mathcal{T}(\tau_t, k)} u(C_t) + (1 - \beta \Delta t)\, \mathbb{E}\left[\frac{U(W_{t+\Delta t}, \tau_{t+\Delta t}) - U(W_t, \tau_t)}{\Delta t}\right]. \tag{F2}$$

In the limit as $\Delta t \to 0$, (F2) becomes

$$\beta U(W_t, \tau_t) = \max_{C_t \in \mathbb{R}, \tau_{t+dt} \in \mathcal{T}(\tau_t, k)} u(C_t) + \mathbb{E}\left[\frac{dU(W_t, \tau_t)}{dt}\right]. \tag{F3}$$

Consider the expectation of $dU(W_t, \tau_t)$. Applying a Taylor series expansion to $U(W_t, \tau_t)$ and taking its expectation, we get:[45]

$$\begin{aligned}\mathbb{E} dU(W_t, \tau_t) =& U_W(W_t, \tau_t)\mathbb{E}\left[dW_t\right] + \frac{1}{2}U_{WW}(W_t, \tau_t)\mathbb{E}\left[dW_t^2\right] \\ &+ \sum_{k=1}^{K} \gamma_t(\tau_t, k)dt\left[U(W_t + P(\tau_t, \tau_{t+dt}), \tau_{t+dt}) - U(W_t, \tau_t)\right].\end{aligned} \tag{F4}$$

Consider dW_t in the first term. Using (1), (2), (3), and (6) and rearranging, we get:

$$\begin{aligned}dW_t =& \left(rW_t - C_t + \mu_e \rho_t + \delta \theta_{b,t} + p_c \theta_{c,t}\right) dt + \left(\sigma_e \rho_t + J\left(\theta_{b,t} + \theta_{c,t}\right)\right)(-dN_t) \\ &+ \sqrt{1 - \rho_t^2}\sigma_e dZ_t - p_b d\theta_{b,t}.\end{aligned}$$

Using $E[dN] = \eta dt$,

$$\mathbb{E}[dW_t] = \left(rW_t - C_t + \left[\mu_e \rho_t - \sigma_e \rho_t \eta\right] + (\delta - \eta J)\theta_{b,t} + (p_c - \eta J)\theta_{c,t}\right) dt. \tag{F5}$$

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[44] This comes from observing that over a small time interval $[0, \Delta t]$, (5) can be written as:

$$U(W_0, \tau_0) = \mathbb{E}\int_0^\infty e^{-\beta t} u(c_t^*)dt = u(c_0^*)\Delta t + e^{-\beta\Delta t}\mathbb{E}\left[\int_{\Delta t}^\infty e^{-\beta(t-\Delta t)} u(c_t^*)dt\right]$$

where $\{c_t^*\}$ is the optimal consumption path. The term inside the expectations operation is $U(W_{\Delta t}, \tau_{\Delta t})$, thus $U(W_0, \tau_0) = \max_{c_0} u(c_0)\Delta t + e^{-\beta\Delta t}\mathbb{E}U\left(W_{\Delta t}, \tau_{\Delta t}\right)$. Similarly, if we start at $\{W_t, \tau_t\}$ and approximate $e^{-\beta\Delta t} \approx 1 - \beta\Delta t$, we get (F1).

[45] $dU(W_t, \tau_t) = U_W(W_t, \tau_t)dW_t + \frac{1}{2}U_{WW}(W_t, \tau_t)dW_t^2 + U_\tau(W_t, \tau_t)d\tau_t + \frac{1}{2}U_{\tau\tau}(W_t, \tau_t)d\tau_t^2$.

Using $E[dN^2] = \eta dt$,

$$\begin{aligned}\mathbb{E}[dW_t^2] &= \left(\sigma_e\rho_t + J\left(\theta_{b,t} + \theta_{c,t}\right)\right)^2 \eta dt + \left(1-\rho_t^2\right)\sigma_e^2 dt \\ &= \left(J^2\left(\theta_{b,t} + \theta_{c,t}\right)^2 \eta + 2\sigma_e\rho_t J\left(\theta_{b,t} + \theta_{c,t}\right)\eta + \left[\left(\sigma_e\rho_t\right)^2\eta + \left(1-\rho_t^2\right)\sigma_e^2\right]\right) dt.\end{aligned} \tag{F6}$$

Thus, substituting (F5) and (F6) back into (F4), we get

$$\begin{aligned}&\mathbb{E}dU(W_t,\tau_t) = \\ &U_W(W_t,\tau_t)\left[\left(rW_t - C_t + \left[\mu_e\rho_t - \sigma_e\rho_t\eta\right] + (\delta-\eta J)\theta_{b,t} + (p_c - \eta J)\theta_{c,t}\right)dt\right] \\ &+ \frac{1}{2}U_{WW}(W_t,\tau_t)\left(\left(J\theta_{b,t} + J\theta_{c,t}\right)^2\eta + 2\sigma_e\rho_t\eta\left(J\theta_{b,t} + J\theta_{c,t}\right) + \left[\left(\sigma_e\rho_t\right)^2\eta + \left(1-\rho_t^2\right)\sigma_e^2\right]\right)dt \\ &+ \sum_{k=1}^{K}\gamma_t(\tau_t,k)dt\left[U(W_t + P(\tau_t,\tau_{t+dt}),\tau_{t+dt}) - U(W_t,\tau_t)\right].\end{aligned} \tag{F7}$$

Substituting (F7) back into (F3), the HJB in the steady state is given by

$$\begin{aligned}&\beta U(W,\tau) = \\ &\max_{C\in\mathbb{R},\tau'\in\mathcal{T}(\tau,k)} u(C) + U_W(W,\tau)\left[rW - C + \left[\mu_e\rho_\tau - \sigma_e\rho_\tau\eta\right] + (\delta-\eta J)\theta_b + (p_c-\eta J)\theta_c\right] \\ &+ \frac{1}{2}U_{WW}(W,\tau)\left[\left(J\theta_b + J\theta_c\right)^2\eta + 2\sigma_e\rho_\tau\eta\left(J\theta_b + J\theta_c\right) + \left[\left(\sigma_e\rho_\tau\right)^2\eta + \left(1-\rho_\tau^2\right)\sigma_e^2\right]\right] \\ &+ \sum_{k=1}^{K}\gamma(\tau,k)\left[U(W + P(\tau,\tau'),\tau') - U(W,\tau)\right].\end{aligned} \tag{F8}$$

Using the guessed functional form (8) and the FOC of (F8) with respect to C, the optimal consumption rate for agent τ is:[46]

$$C_\tau = -\frac{log\left(r\right)}{\alpha} + r\left(W + V_\tau + \bar{a}\right). \tag{F9}$$

where $\bar{a}$ is defined in (9). Inserting (F9) back into (F8) and using (8), $U_W = r\alpha e^{-r\alpha(W+V_\tau+\bar{a})}$, and $U_{WW} = -r^2\alpha^2 e^{-r\alpha(W+V_\tau+\bar{a})}$, we get:

$$\begin{aligned}&-\beta e^{-r\alpha(W_t+V_\tau+\bar{a})} = -e^{log(r)-r\alpha(W+V_\tau+\bar{a})} + \\ &r\alpha e^{-r\alpha(W+V_\tau+\bar{a})}\left[\frac{log\left(r\right)}{\alpha} - r\left(V_\tau + \bar{a}\right) + \left[\mu_e\rho_\tau - \sigma_e\rho_\tau\eta\right] + (\delta-\eta J)\theta_b + (p_c - \eta J)\theta_c\right] \\ &- \frac{1}{2}r^2\alpha^2 e^{-r\alpha(W+V_\tau+\bar{a})}\left[\left(J\theta_b + J\theta_c\right)^2\eta + 2\sigma_e\rho_\tau\eta\left(J\theta_b + J\theta_c\right) + \left(\sigma_e\rho_\tau\right)^2\eta + \left(1-\rho_\tau^2\right)\sigma_e^2\right] \\ &+ \sum_{k=1}^{K}\gamma(\tau,k)\max_{\tau'\in\mathcal{T}(\tau,k)}\left[U(W + P(\tau,\tau'),\tau') - U(W,\tau)\right].\end{aligned} \tag{F10}$$

[46]The FOC with respect to C is: $0 = \alpha e^{-\alpha C} - U_W(W_t,\tau_t)$. Using $U_W = r\alpha e^{-r\alpha(W+V_\tau+\bar{a})}$, $re^{-r\alpha(W+V_\tau+\bar{a})} = e^{-\alpha C}$. Rewrite it as: $e^{log(r)}e^{-r\alpha(W+V_\tau+\bar{a})} = e^{-\alpha C}$.

Divide both sides of (F10) by $-\frac{1}{r\alpha}e^{-ra(W+V_\tau+\bar{a})}$ and rearrange to get:

$$\begin{aligned}0 = rV_\tau - e^{ra(W+V_\tau+\bar{a})}\frac{1}{r\alpha}\sum_{k=1}^{K}\gamma(\tau,k)\max_{\tau'\in\mathcal{T}(\tau,k)}\left[U(W+P(\tau,\tau'),\tau') - U(W,\tau)\right] + r\bar{a}\\ - r\frac{1}{r}\left[\frac{log\,(r)}{\alpha} - \frac{r-\beta}{r\alpha} - \frac{1}{2}r\alpha\sigma_e^2 + \left[\mu_e\rho_\tau - \sigma_e\rho_\tau\eta\right] - \frac{1}{2}r\alpha\left[(\sigma_e\rho_\tau)^2\eta - \sigma_e^2\rho_\tau^2\right]\right]\\ - \left[(\delta-\eta J)\theta_b - \frac{1}{2}r\alpha\left((J\theta_b + J\theta_c)^2\eta + 2\sigma_e\rho_\tau\eta\left(J\theta_b + J\theta_c\right)\right) + (p_c - \eta J)\theta_c\right].\end{aligned}$$

Using $\bar{a}$ defined in (9), $b_\tau \equiv \mu_e\rho_\tau - \sigma_e\rho_\tau\eta - \frac{1}{2}r\alpha\left[(\sigma_e\rho_\tau)^2\eta - \sigma_e^2\rho_\tau^2\right]$, and $\theta_b\theta_c = 0$, we get:

$$\begin{aligned}rV_\tau = b_\tau + \left[\delta - \eta J - r\alpha\sigma_e\rho_\tau\eta J\right]\theta_b - \frac{1}{2}r\alpha J^2\eta\left[(\theta_b)^2 + (\theta_c)^2\right] + \left[p_c - (\eta J + r\alpha\sigma_e\rho_\tau\eta J)\right]\theta_c\\ + e^{ra(W+V_\tau+\bar{a})}\frac{1}{r\alpha}\sum_{k=1}^{K}\gamma(\tau,k)\max_{\tau'\in\mathcal{T}(\tau,k)}\left[U(W+P(\tau,\tau'),\tau') - U(W,\tau)\right].\end{aligned}$$

I assume that $\mu_e\rho_\tau - \sigma_e\rho_\tau\eta - \frac{1}{2}r\alpha\left(\sigma_e\rho_\tau\right)^2(\eta-1) = 0$ so that $b_\tau = 0$. Using x_τ and y defined in (11) and (12) and the guessed functional form for $U(W,\tau)$

$$\begin{aligned}rV_\tau = \left((\delta-\eta J) - x_\tau\right)\theta_b - y|\theta_b| + \left[p_c - (\eta J + x_\tau)\right]\theta_c - y|\theta_c|.\\ + \frac{1}{r\alpha}\sum_{k=1}^{K}\gamma(\tau,k)\max_{\tau'\in\mathcal{T}(\tau,k)}\left[1 - e^{-r\alpha(P(\tau',\tau)+V_{\tau'}-V_\tau)}\right].\end{aligned} \tag{F11}$$

□

G Equilibrium Existence

Proof of Proposition 2. I prove in five steps. In step 1, I narrow down the equilibrium conditions into a set of five equations and five unknowns. In step 2, I show the solution for the entry rate of high type investors is unique and interior. In step 3, I characterize the solution for the entry rate of low type investors and show that three solutions exist. In step 4, I show that—taking the entry rates as given—population masses, value functions, gains from trade, and prices are uniquely determined. Population masses and gains from trade are also positive. Finally, in step 5, I show that the conjectured optimal trading strategies are indeed optimal. □

Step 1

Proof. From the market clearing conditions, the masses of agents who have reached their optimal asset position are:

$$\mu_{h[1,0]} = S - \mu_{a[1,0]} \tag{G1}$$

$$\mu_{l[0,\text{-}1]} = \mu_{h[0,1]} \tag{G2}$$

$$\mu_{h[0,1]} = \frac{1}{\gamma_d + \gamma_u} M_c. \tag{G3}$$

They depend only on the masses of active searchers.

I simplify the rest of the equilibrium conditions, first, into a set of nine equations of nine unknowns, $\mu_{a[1,0]}$, $\mu_{l[0,0]}$, $\mu_{h[0,0]}$, ω_b, ω_c, $V_{l[0,0]}$, $V_{h[0,0]}$, ν_h, and ν_l:

$$\lambda_b \mu_{a[1,0]} \mu_{h[0,0]} = \gamma_d (S - \mu_{a[1,0]}) \tag{G4}$$

$$(r + \gamma_d)\omega_b = x - \lambda_b \left[\mu_{a[1,0]} + \mu_{h[0,0]}\right] \frac{1}{2}\omega_b - \lambda_c \mu_{l[0,0]} \frac{1}{2}\omega_c \tag{G5}$$

$$(r + \gamma_d + \gamma_u)\omega_c = (2x - 2y) - \lambda_b \mu_{a[1,0]} \frac{1}{2}\omega_b - \lambda_c \left(\mu_{l[0,0]} + \mu_{h[0,0]}\right) \frac{1}{2}\omega_c \tag{G6}$$

$$\frac{\nu_l F_l}{\gamma_u} = \mu_{l[0,0]} + \frac{1}{\gamma_d + \gamma_u} M_c \tag{G7}$$

$$(r + \gamma_d) V_{h[0,0]} = \lambda_b \mu_{a[1,0]} \frac{1}{2}\omega_b + \lambda_c \mu_{l[0,0]} \frac{1}{2}\omega_c \tag{G8}$$

$$(r + \gamma_u) V_{l[0,0]} = \lambda_c \mu_{h[0,0]} \frac{1}{2}\omega_c \tag{G9}$$

$$\nu_h F_h + \gamma_u \frac{1}{\gamma_d + \gamma_u} M_c = \gamma_d \mu_{h[0,0]} + M_b + M_c \tag{G10}$$

$$\nu_i = \begin{cases} 1 & V_{i[0,0]} > O_i \\ [0,1] \text{ if } & V_{i[0,0]} = O_i \\ 0 & V_{i[0,0]} < O_i. \end{cases} \quad \text{for } i \in \{h, l\} \tag{G11}$$

Eq. (G4) comes from combining (14) and (B11). Eq. (G5) comes from combining the value functions for $h[0,0]$, $h[1,0]$, and $a[1,0]$, substituting in M_b and M_c, and simplifying. Combining the value functions for $h[0,0]$, $h[0,1]$, $l[0,0]$, and $l[0,\text{-}1]$ and substituting in M_b and M_c yields (G6). Eq. (G7) comes from (B10). Substituting M_b and M_c into (B2), we get (G8). Substituting M_b into (B1), we get (G9). Combining (B9) and (G3), we get (G10). Eq. (G11) is the entry condition for high and low types, given in (4).

Next, I simplify the nine equations further into a set of five equations of five unknowns. Combining (G5) and (G8),

$$\omega_b = \frac{x - (r + \gamma_d) V_{h[0,0]}}{r + \gamma_d + \lambda_b \mu_{h[0,0]} \frac{1}{2}}. \tag{G12}$$

Combining (G6) and (G8),

$$\omega_c = \frac{2x - 2y - (r + \gamma_d) V_{h[0,0]}}{r + \gamma_d + \gamma_u + \lambda_c \mu_{h[0,0]} \frac{1}{2}}. \tag{G13}$$

From (G4), solve for $\mu_{a[1,0]}$ as

$$\mu_{a[1,0]} = \frac{\gamma_d S}{\left(\lambda_b \mu_{h[0,0]} + \gamma_d\right)}. \tag{G14}$$

From (G7), solve for $\mu_{l[0,0]}$ as

$$\mu_{l[0,0]} = \frac{\frac{\nu_l F_l}{\gamma_u}}{1 + \frac{1}{\gamma_d+\gamma_u}\lambda_c \mu_{h[0,0]}}. \tag{G15}$$

Plugging these expressions back into (G8)-(G11) gives five equations of five unknowns $\{\mu_{h[0,0]}, V_{h[0,0]}, V_{l[0,0]}, \nu_h, \nu_l\}$:

$$\begin{aligned}(r+\gamma_d)V_{h[0,0]} - \lambda_b \frac{\gamma_d S}{\left(\gamma_d + \lambda_b \mu_{h[0,0]}\right)} \frac{1}{2} \frac{x - (r+\gamma_d)V_{h[0,0]}}{r + \gamma_d + \lambda_b \mu_{h[0,0]} \frac{1}{2}} \\ - \lambda_c \frac{(\gamma_d+\gamma_u)\frac{\nu_l F_l}{\gamma_u}}{\gamma_d + \gamma_u + \lambda_c \mu_{h[0,0]}} \frac{1}{2} \frac{2x - 2y - (r+\gamma_d)V_{h[0,0]}}{r + \gamma_d + \gamma_u + \lambda_c \mu_{h[0,0]} \frac{1}{2}} = 0.\end{aligned} \tag{G16}$$

$$V_{l[0,0]} = \frac{1}{r+\gamma_u} \lambda_c \mu_{h[0,0]} \frac{1}{2} \frac{2x - 2y - (r+\gamma_d)V_{h[0,0]}}{r + \gamma_d + \gamma_u + \lambda_c \mu_{h[0,0]} \frac{1}{2}}. \tag{G17}$$

$$\nu_h F_h = \gamma_d \mu_{h[0,0]} + \gamma_d \frac{\lambda_b \mu_{h[0,0]} S}{\left(\lambda_b \mu_{h[0,0]} + \gamma_d\right)} + \gamma_d \lambda_c \mu_{h[0,0]} \frac{\frac{\nu_l F_l}{\gamma_u}}{\gamma_u + \gamma_d + \lambda_c \mu_{h[0,0]}}. \tag{G18}$$

$$\nu_i = \begin{cases} 1 & V_{i[0,0]} > O_i \\ [0,1] \text{ if} & V_{i[0,0]} = O_i \\ 0 & V_{i[0,0]} < O_i. \end{cases} \qquad \text{for } i \in \{h, l\} \tag{G19}$$

□

Step 2

Proof. In the second step, I establish that, taking ν_l as given, a unique positive solution exists for ν_h.

I start by showing the sign of $\mu_{h[0,0]}$. The right-hand side of (G18) strictly increases in $\mu_{h[0,0]}$, is zero at $\mu_{h[0,0]} = 0$, and goes to ∞ as $\mu_{h[0,0]} \to \infty$. Thus, from (G18), $\mu_{h[0,0]}$ is positive and uniquely determined for any $\nu_h > 0$ and $\nu_l \geq 0$.

Next, consider how $V_{h[0,0]}$ changes with ν_h taking as given ν_l. Equations (G16) and (G18) together define $V_{h[0,0]}$ as an implicit function of ν_h and ν_l: $V_{h[0,0]}(\nu_h, \nu_l)$. Applying the Implicit Function Theorem to (G16) and using the fact that $\mu_{h[0,0]}$ is positive, $\frac{\partial V_{h[0,0]}}{\partial \mu_{h[0,0]}}$ evaluated at an interior solution for ν_h (i.e. at $V_{h[0,0]} = O_h$) is negative. Applying the Implicit Function Theorem to (G18) and using the fact that $\mu_{h[0,0]}$ is positive, $\mu_{h[0,0]}$

increases in ν_h for any $\nu_l \geq 0$. Put together, for ν_h^* such that $V_{h[0,0]} = O_h$,

$$\frac{\partial V_{h[0,0]}(\nu_h^*, \nu_l)}{\partial \nu_h} = \frac{\partial V_{h[0,0]}}{\partial \mu_{h[0,0]}} \frac{\partial \mu_{h[0,0]}(\nu_h^*, \nu_l)}{\partial \nu_h} < 0.$$

Next, I establish the existence of a solution. From (G18), $\mu_{h[0,0]} = 0$ when $\nu_h = 0$. Solving for $V_{h[0,0]}$ from (G16) and evaluating it at $\mu_{h[0,0]} = 0$ gives the left-hand-side of Assumption (E5). Thus, by Assumption (E5), $V_{h[0,0]}(0, \nu_l) > O_h$ for any $\nu_l \in [0, 1]$. I also assume that F_h is large. Then, for a given level of ν_l, from (G18) and the earlier result that $\mu_{h[0,0]}$ increases in ν_h, $\mu_{h[0,0]}$ becomes large as $\nu_h \to 1$. In turn, from (G16), $V_{h[0,0]} \to 0$ as $\mu_{h[0,0]}$ becomes large. Hence, $V_{h[0,0]}(1, \nu_l) < O_h$, and a positive solution for ν_h exists on the interval $(0, 1]$.

Finally, uniqueness of the solution follows from the result that $\frac{\partial V_{h[0,0]}(\nu_h,\nu_l)}{\partial \nu_h} < 0$ evaluated at ν_h^* such that $V_{h[0,0]}(\nu_h^*, \nu_l) = O_h$. □

Step 3

Proof. In this step, I characterize the solution for the entry rate of low types.

First, I establish that $V_{l[0,0]}$ strictly increases in ν_l. Using the result in step 2 that the solution for ν_h is an interior solution, (G16) and (G17) become

$$\begin{aligned}(r + \gamma_d)O_h - \lambda_b \frac{\gamma_d S}{\left(\gamma_d + \lambda_b \mu_{h[0,0]}\right)} \frac{1}{2} \frac{x - (r + \gamma_d)O_h}{r + \gamma_d + \lambda_b \mu_{h[0,0]} \frac{1}{2}} \\ - \lambda_c \frac{(\gamma_d + \gamma_u) \frac{\nu_l F_l}{\gamma_u}}{\gamma_d + \gamma_u + \lambda_c \mu_{h[0,0]}} \frac{1}{2} \frac{2x - 2y - (r + \gamma_d)O_h}{r + \gamma_d + \gamma_u + \lambda_c \mu_{h[0,0]} \frac{1}{2}} = 0.\end{aligned} \tag{G20}$$

$$V_{l[0,0]} = \frac{1}{r + \gamma_u} \lambda_c \mu_{h[0,0]} \frac{1}{2} \frac{2x - 2y - (r + \gamma_d)O_h}{r + \gamma_d + \gamma_u + \lambda_c \mu_{h[0,0]} \frac{1}{2}}. \tag{G21}$$

Equations (G20) and (G21) define $V_{l[0,0]}$ as an implicit function of ν_l: $V_{l[0,0]}(\nu_l)$. Applying the Implicit Function Theorem to (G20), $\mu_{h[0,0]}$ strictly increases in ν_l. The right-hand-side of (G21) increases with $\mu_{h[0,0]}$ and, hence, with ν_l. Put together, $V_{l[0,0]}$ strictly increases in ν_l.

Solving (G20) and (G21) for ν_l and $\mu_{h[0,0]}$ and evaluating them at $V_{l[0,0]} = O_h$, we get: □

$$\nu_l^* = \frac{2\gamma_u \left(r + \gamma_d + \gamma_u + \frac{1}{2}\lambda_c \mu_h^*\right) \left(\gamma_d + \gamma_u + \lambda_c \mu_h^*\right) \left[O_h \left(r + \gamma_d\right) - \frac{S\gamma_d (x - O_h(r+\gamma_d))\lambda_b}{\left(\gamma_d + \lambda_b \mu_h^*\right)\left(2(r+\gamma_d) + \lambda_b \mu_h^*\right)}\right]}{F_l \left(2x - 2y - O_h \left(r + \gamma_d\right)\right) \left(\gamma_d + \gamma_u\right) \lambda_c} \tag{G22}$$

where

$$\mu_h^* \equiv \frac{2O_l \left(r + \gamma_u\right) \left(r + \gamma_d + \gamma_u\right)}{\lambda_c \left(2x - 2y - \left(r + \gamma_d\right) O_h - \left(r + \gamma_u\right) O_l\right)}.$$

If $0 < \nu_l^* \leq 1$ (which is Assumption (E8)), an interior solution exists and is uniquely given by (G22). Since $V_{l[0,0]}$ as an implicit function of ν_l strictly increases in ν_l, two corners solutions ($\nu_l = 0$ and $\nu_l = 1$) also exist when $0 < \nu_l^* < 1$.

Table 1 characterizes solutions for ν_l for all possible parameter conditions, not just (E8). The last row, for example, says that if low type investors' outside option is too good, then in equilibrium none of them enter. By assuming (E8), I rule out such condition to ensure that the CDS market can exist in equilibrium.

Table 1:
ν_l^* is given by (G22).

Parameter condition	Solution
$\nu_l^* < 0$	$\nu_l = 1$
$\nu_l^* = 0$	$\nu_l = 0$, $\nu_l = 1$
$0 < \nu_l^* < 1$	$\nu_l = 0$, $\nu_l \in (0, 1)$, $\nu_l = 1$
$\nu_l^* = 1$	$\nu_l = 0$, $\nu_l = 1$
$\nu_l^* > 1$	$\nu_l = 0$

Step 4

Proof. Equations (G1)-(G3), (G14), (G15), and (G20) uniquely determine the masses of agent types. In turn, ω_b and ω_c are uniquely given by (G12) and (G13) where $V_{h[0,0]} = O_h$. Given the agent masses, (B1)-(B6), (16)-(17), (B15), and (B16) uniquely determine the value functions, prices, and termination fees.

The solution, moreover, is positive. Using $V_{h[0,0]} = O_h$, Assumption 1, and the fact that $\mu_{h[0,0]} > 0$, we have that $\omega_b > 0$ and $\omega_c > 0$. The result that $\mu_{h[0,0]} > 0$ implies that $\mu_{a[1,0]} > 0$ and $\mu_{l[0,0]} > 0$. Non-searcher masses are similarly positive. □

Step 5

Proof. In this step, I verify that the conjectured trading strategies are in fact optimal.

From step 4, the gains from a bond transaction is positive, $\omega_b > 0$. The bond buyer and seller's respective trading strategies, as a result, are optimal. This also shows that an average-valuation investor prefers to not hold a bond.

Analogously, since the total gains from a CDS transaction is positive: $\omega_c > 0$, it is optimal for $l[0, 0]$ type agent to buy CDS, and for $h[0, 0]$ type agent to sell CDS as conjectured.

Consider a CDS seller's decision to terminate her contract if she reverts to an average type. Upon reverting to an average-type, if she pays the termination fee and terminates the CDS contract, her utility changes by $-T_{\text{S}} + 0 - V_{h[0,1]}$. If she instead remains a CDS seller, her utility changes by $V_{a[0,1]} - V_{h[0,1]}$, where

$$rV_{a[0,1]} = p_c - \eta J - y + \gamma_u(T_{\text{B}} + 0 - V_{a[0,1]}).$$

She prefers to terminate if $V_{a[0,1]} < -T_{\text{S}}$, that is, if

$$\begin{aligned} V_{a[0,1]} + T_{\text{S}} &= V_{a[0,1]} + \frac{1}{2}\omega_c \\ &= \frac{p_c - \eta J - y + \gamma_u T_{\text{B}}}{(r+\gamma_u)} + \frac{1}{2}\omega_c \\ &= \frac{(r+\gamma_u+\gamma_d)(x-2y) - (x-(r+\gamma_d)O_h)\lambda_c \mu_{h[0,0]}\frac{1}{2}}{(r+\gamma_u)\left(r+\gamma_d+\gamma_u+\lambda_c\mu_{h[0,0]}\frac{1}{2}\right)} \end{aligned} \tag{G23}$$

is negative. The third equality uses (21) and (G13). For an interior solution of ν_l (i.e. using $V_l[0,0] = O_l$ and (G21)), the numerator of (G23) simplifies to:

$$\frac{[r+\gamma_d+\gamma_u]\,[2x-2y-(r+\gamma_d)O_h]\,[x-2y-(r+\gamma_u)O_l]}{2x-2y-(r+\gamma_d)O_h-(r+\gamma_u)O_l}$$

By Assumption 1, the second squared bracket is positive, the third bracket is negative, and the denominator is positive. The whole term, as a result, is negative. For a corner solution $\nu_l = 1$, since $\mu_{h[0,0]}$ increases in ν_l and the numerator of (G23) decreases in $\mu_{h[0,0]}$, the numerator of (G23) will remain negative. Thus, $V_{a[0,1]} < -T_{\text{S}}$: once a CDS seller switches to an average type, she prefers to pay the fee and exit the market than to remain a CDS seller and wait until her counterparty terminates the contract. This shows that an average type prefers no position than a long position through the CDS market.

Consider now a CDS buyer's decision to terminate. Upon reverting to an average type, if she pays the termination fee and exits, her utility changes by: $-T_{\text{B}} + 0 - V_{l[0,\text{-}1]}$. If she remains a CDS buyer, her utility changes by: $V_{a[0,\text{-}1]} - V_{l[0,\text{-}1]}$, where

$$rV_{a[0,\text{-}1]} = -p_c + \eta J - y + \gamma_d(T_{\text{S}} + 0 - V_{a[0,\text{-}1]}).$$

She prefers to pay the fee if

$$V_{a[0,\text{-}1]} < -T_{\text{B}},$$

where $T_{\text{B}} = \frac{1}{2}\omega_c$. The difference is

$$V_{a[0,\text{-}1]} + T_{\text{B}} = \frac{x-2y-(r+\gamma_d)O_h}{r+\gamma_d}.$$

The right-hand-side is negative by Assumption 1. Thus, a CDS buyer, upon a valuation shock, prefers to pay the fee and exit the market than to remain a CDS buyer. That is, an average type prefers no position than a short position through the CDS market. □

H Proofs of Lemma 2 and Propositions 7-8

Lemma 2. *Given a choice between searching in just the bond market, just the CDS market, or in both markets at the same time, long investors, $h[0,0]$, optimally choose to search in both markets at the same time.*

Proof of Lemma 2. I denote with $V^m_{h[0,0]}$ a long investor's expected utility associated with market choice $m \in \{b, c, bc\}$ and with ν_h^m the fraction of long investors that choose m, where b, c, and bc stand for entering just the bond market, just the CDS market, and both markets, respectively. The ability to search simultaneously affects long investors only because the other investors (bond sellers and CDS buyers) can establish their optimal positions through only one of the markets. The equilibrium entry rates $\{\nu_h^m\}_{m\in\{b,c,bc\}}$ solve:

$$\nu_h^m = \begin{cases} 1 & V_{h[0,0]}^m > \left\{V_{h[0,0]}^b, V_{h[0,0]}^c, V_{h[0,0]}^{bc}, O_h\right\} / V_{h[0,0]}^m \\ [0,1] \text{ if} & V_{h[0,0]}^m = \left\{V_{h[0,0]}^b, V_{h[0,0]}^c, V_{h[0,0]}^{bc}, O_h\right\} / V_{h[0,0]}^m \\ 0 & V_{h[0,0]}^m < \left\{V_{h[0,0]}^b, V_{h[0,0]}^c, V_{h[0,0]}^{bc}, O_h\right\} / V_{h[0,0]}^m \end{cases} \tag{H1}$$

Where the distinction is necessary, I denote in superscripts the investor's market choice: $m \in \{b, c, bc\}$.

The value functions are:

$$rV_{h[1,0]} = \delta - \eta J + x - y + \gamma_d \left(V_{a[1,0]} - V_{h[1,0]}\right) \tag{H2}$$

$$\begin{aligned} rV_{a[1,0]} = {} & \delta - \eta J - y + \lambda_b \mu^b_{\text{h}[0,0]} \frac{1}{2} \left(V_{h[1,0]} - V_{a[1,0]} - V^b_{h[0,0]}\right) \\ & + \lambda_b \mu^{\text{bc}}_{\text{h}[0,0]} \frac{1}{2} \left(V_{h[1,0]} - V_{a[1,0]} - V^{bc}_{h[0,0]}\right) \end{aligned} \tag{H3}$$

$$rV^b_{h[0,0]} = \gamma_d \left(0 - V^b_{h[0,0]}\right) + \lambda_b \mu_{\text{a}[1,0]} \frac{1}{2} \left(V_{h[1,0]} - V_{a[1,0]} - V^b_{h[0,0]}\right) \tag{H4}$$

$$rV^{bc}_{h[0,0]} = -\gamma_d V^{bc}_{h[0,0]} + \lambda_b \mu_{\text{a}[1,0]} \frac{1}{2} \left(V_{h[1,0]} - V_{a[1,0]} - V^{bc}_{h[0,0]}\right) + \lambda_c \mu_{l[0,0]} \left(V^{bc}_{h[0,1]} - V^{bc}_{h[0,0]}\right) \tag{H5}$$

$$rV^{bc}_{h[0,1]} = p^{bc}_{\text{c}} - \eta J + x - y + \gamma_d \left(-T^{bc} - V^{bc}_{h[0,1]}\right) \tag{H6}$$

$$rV^{\text{bc}}_{l[0,\text{-}1]} = -p^{bc}_{\text{c}} + \eta J + x - y + \gamma_u \left(-T^{bc} - V^{\text{bc}}_{l[0,\text{-}1]}\right) \tag{H7}$$

$$rV_{l[0,0]} = \gamma_u \left(0 - V_{l[0,0]}\right) + \lambda_c \mu^{\text{bc}}_{\text{h}[0,0]} \left(V^{\text{bc}}_{l[0,\text{-}1]} - V_{l[0,0]}\right) + \lambda_c \mu^c_{\text{h}[0,0]} \left(V^{\text{c}}_{l[0,\text{-}1]} - V_{l[0,0]}\right) \tag{H8}$$

$$rV^c_{\text{h}[0,0]} = \gamma_d \left(0 - V^c_{\text{h}[0,0]}\right) + \lambda_c \mu_{l[0,0]} \left(V^c_{h[0,1]} - V^c_{\text{h}[0,0]}\right) \tag{H9}$$

$$rV^c_{h[0,1]} = p^c_c - \eta J + x - y + \gamma_d \left(-T_c - V^c_{h[0,1]}\right) \tag{H10}$$

$$rV^{\text{c}}_{l[0,\text{-}1]} = -p^c_c + \eta J + x - y + \gamma_u \left(-T_c - V^{\text{c}}_{l[0,\text{-}1]}\right) \tag{H11}$$

A CDS buyer and a long investor negotiate a CDS premium characterized by

$$V^{bc}_{h[0,1]} - V^{bc}_{h[0,0]} = \frac{1}{2} \left(V^{bc}_{h[0,1]} + V^{\text{bc}}_{l[0,\text{-}1]} - V^{bc}_{h[0,0]} - V_{l[0,0]}\right)$$

when the long investor searches in both markets and by

$$V^c_{h[0,1]} - V^c_{\text{h}[0,0]} = \frac{1}{2} \left(V^c_{h[0,1]} + V^{\text{c}}_{l[0,\text{-}1]} - V^c_{\text{h}[0,0]} - V_{l[0,0]}\right)$$

when the long investor searches in just the CDS market. Bond prices are given by

$$p^b_b = \frac{1}{2}(V_{h[1,0]} - V^b_{h[0,0]}) + \frac{1}{2} V_{a[1,0]}$$

when the buyer searches in just the bond market and by

$$p_b^{bc} = \frac{1}{2}(V_{h[1,0]} - V_{h[0,0]}^{bc}) + \frac{1}{2}V_{a[1,0]}$$

when the buyer searches in both markets simultaneously. Termination fees are given by

$$T^{bc} = \frac{1}{2}\left(V_{h[0,1]}^{bc} + V_{l[0,\text{-}1]}^{\text{bc}} - V_{h[0,0]}^{bc} - V_{l[0,0]}\right)$$

$$T^{c} = \frac{1}{2}\left(V_{h[0,1]}^{c} + V_{l[0,\text{-}1]}^{\text{c}} - V_{\text{h}[0,0]}^{c} - V_{l[0,0]}\right)$$

We can express the value function of a long investor searching in both markets as

$$\begin{aligned} V_{h[0,0]}^{bc} = & \frac{(r+\gamma_d)}{r+\gamma_d+\lambda_b\mu_{\text{a}[1,0]}+\lambda_c\mu_{l[0,0]}}0 + \frac{\lambda_b\mu_{\text{a}[1,0]}}{r+\gamma_d+\lambda_b\mu_{\text{a}[1,0]}+\lambda_c\mu_{l[0,0]}}\left(V_{h[1,0]} - p_b^{bc}\right) \\ & + \frac{\lambda_c\mu_{l[0,0]}}{r+\gamma_d+\lambda_b\mu_{\text{a}[1,0]}+\lambda_c\mu_{l[0,0]}}V_{h[0,1]}^{bc} \end{aligned}$$

It is a weighted average between three different outcomes: (1) getting a valuation shock and exiting in which case her utility is zero, (2) finding a counterparty in the bond market in which case her utility is the utility of a bond owner minus the cost of becoming a bond owner, and (3) finding a counterparty in the CDS market in which case her utility is that of a CDS seller. The weights capture the probabilities of these outcomes.

Consider how $V_{h[0,0]}^{bc}$ and $V_{\text{h}[0,0]}^{\text{b}}$ relate to each other. Express $V_{h[0,0]}^{bc}$ as:

$$\begin{aligned} V_{h[0,0]}^{bc} = & \frac{(r+\gamma_d)}{r+\gamma_d+\frac{1}{2}\lambda_b\mu_{\text{a}[1,0]}+\lambda_c\mu_{l[0,0]}}0 + \frac{\lambda_b\mu_{\text{a}[1,0]}}{r+\gamma_d+\frac{1}{2}\lambda_b\mu_{\text{a}[1,0]}+\lambda_c\mu_{l[0,0]}}\frac{1}{2}\left(V_{h[1,0]} - V_{a[1,0]}\right) \\ & + \frac{\lambda_c\mu_{l[0,0]}}{r+\gamma_d+\frac{1}{2}\lambda_b\mu_{\text{a}[1,0]}+\lambda_c\mu_{l[0,0]}}V_{h[0,1]}^{bc} \end{aligned}$$

Express the value function of a long investor searching in only the bond market as

$$V_{\text{h}[0,0]}^{\text{b}} = \frac{(r+\gamma_d)}{r+\gamma_d+\frac{1}{2}\lambda_b\mu_{\text{a}[1,0]}+\lambda_c\mu_{l[0,0]}}0 + \frac{\lambda_b\mu_{\text{a}[1,0]}}{r+\gamma_d+\frac{1}{2}\lambda_b\mu_{\text{a}[1,0]}+\lambda_c\mu_{l[0,0]}}\frac{1}{2}\left(V_{h[1,0]} - V_{a[1,0]}\right)$$

Combining the two, we get:

$$V_{h[0,0]}^{bc} = \frac{r+\gamma_d+\frac{1}{2}\lambda_b\mu_{\text{a}[1,0]}}{r+\gamma_d+\frac{1}{2}\lambda_b\mu_{\text{a}[1,0]}+\lambda_c\mu_{l[0,0]}}V_{h[0,0]}^{b} + \frac{\lambda_c\mu_{l[0,0]}}{r+\gamma_d+\frac{1}{2}\lambda_b\mu_{\text{a}[1,0]}+\lambda_c\mu_{l[0,0]}}V_{h[0,1]}^{bc} \quad \text{(H12)}$$

$$\frac{(r+\gamma_d+\frac{1}{2}\lambda_b\mu_{\text{a}[1,0]}+\lambda_c\mu_{l[0,0]})V_{h[0,0]}^{bc} - \lambda_c\mu_{l[0,0]}V_{h[0,1]}^{bc}}{(r+\gamma_d+\frac{1}{2}\lambda_b\mu_{\text{a}[1,0]})} = V_{h[0,0]}^{b} \quad \text{(H13)}$$

$$(r+\gamma_d+\frac{1}{2}\lambda_b\mu_{\text{a}[1,0]}+\lambda_c\mu_{l[0,0]})V_{h[0,1]}^{bc} > (r+\gamma_d+\frac{1}{2}\lambda_b\mu_{\text{a}[1,0]}+\lambda_c\mu_{l[0,0]})V_{h[0,0]}^{bc}$$

From (H12), $V^{bc}_{h[0,0]}$ is a weighted average between the utility of an investor who searches in just the bond market and the utility of a CDS seller. The weight on $V^{b}_{h[0,0]}$ reflects both the probability of getting a valuation shock and the probability finding a bond seller. Thus, the utility of a long investor searching in both markets embeds the utility of the counterfactual market choice of searching in just the bond market, $V^{b}_{h[0,0]}$. This is because the outcomes that affect the investor searching in just the bond market—getting a valuation shock and finding a bond seller—also affect the investor searching simultaneously.

The relationship between $V^{bc}_{h[0,0]}$ and $V^{b}_{h[0,0]}$ narrows down the set of equilibrium entry rates. From (H12), a long investor prefers to search in both markets than in just the bond market (i.e. $V^{bc}_{h[0,0]} > V^{b}_{h[0,0]}$) if $V^{bc}_{h[0,1]} > V^{b}_{h[0,0]}$. For $V^{bc}_{h[0,1]} > V^{b}_{h[0,0]}$ to hold, using (H12) it has to be that $V^{bc}_{h[0,1]} > V^{bc}_{h[0,0]}$ (to see this solve for $V^{b}_{h[0,0]}$ from (H12) and substitute it into the right-hand side of $V^{bc}_{h[0,1]} > V^{b}_{h[0,0]}$). In turn, $V^{bc}_{h[0,1]} > V^{bc}_{h[0,0]}$ if there is gains from trade from selling CDS. The argument is analogous for an investor who searches in just the CDS market: If there is gains from a bond transaction, she will prefer to search in both markets, not just the CDS market. Then, in an equilibrium in which some investors find it optimal to search in both markets $\nu^{bc} > 0$, it cannot be the case that others choose to search in just one of the markets. That is, if $\nu^{bc} > 0$, it has to be that $\nu^{b} = 0$ and $\nu^{c} = 0$. The possible equilibrium entry rates, as a result, are $\{\nu^{bc} > 0,\, \nu^{b} = 0,\, \nu^{c} = 0\}$ and $\{\nu^{bc} = 0,\, \nu^{b} \geq 0,\, \nu^{c} \geq 0\}$.

In the rest of the proof, I show that $\{\nu^{bc} = 0,\, \nu^{b} \geq 0,\, \nu^{c} \geq 0\}$ cannot be an equilibrium. I show this by supposing that $\{\nu^{bc} = 0,\, \nu^{b} \geq 0,\, \nu^{c} \geq 0\}$ is the equilibrium (i.e. everyone chooses to search in just one of the markets). I then check whether investors have an incentive to deviate and become an investor who searches in both markets at the same time (i.e. whether $V^{bc}_{h[0,0]} > V^{b}_{h[0,0]}$ or $V^{bc}_{h[0,0]} > V^{c}_{h[0,0]}$). Per above discussion, checking $V^{bc}_{h[0,0]} > V^{b}_{h[0,0]}$, for example, is equivalent to checking whether there is gains from a CDS transaction from the perspective of investors who deviate and search in both markets at the same time.[47]

If $\{\nu^{bc} = 0,\, \nu^{b} \geq 0,\, \nu^{c} \geq 0\}$ is the equilibrium, $\mu^{bc}_{\mathrm{h}[0,0]} = 0$. Moreover, combining (H2)-(H4), the entry condition $V^{b}_{h[0,0]} = O_h$, and $M_b = \gamma_d(S - \mu_{a[1,0]})$, we get:

$$\omega^{b}_{b} = \frac{(r+\gamma_d)O_h\,(-2r-\gamma_d)+\sqrt{E}}{S\gamma_d\lambda_b} \tag{H14}$$

$$\mu_{a[1,0]} = \frac{(r+\gamma_d)O_h\,(2r+\gamma_d)+\sqrt{E}}{2\,(x-(r+\gamma_d)O_h)\,\lambda_b} \tag{H15}$$

$$\mu^{b}_{\mathrm{h}[0,0]} = \frac{-(r+\gamma_d)O_h\,(2r+3\gamma_d)+\sqrt{E}}{2(r+\gamma_d)O_h\lambda_b} \tag{H16}$$

where $E \equiv (r+\gamma_d)O_h\left((r+\gamma_d)O_h\,(2r+\gamma_d)^{2} + 4S\,(x-(r+\gamma_d)O_h)\,\gamma_d\lambda_b\right)$. Combining

[47] In the proof of equilibrium existence, I showed that the gains from trade are positive, but I did so by assuming ν^{b}_{h} and ν^{c}_{h} are zero. For simplicity, let us focus on the interior solutions for ν^{b} and ν^{c} so that $V^{b}_{h[0,0]} = O_h$ and $V_{\mathrm{l}[0,0]} = O_l$.

(H8)-(H11), termination fees, and the entry condition $V^c_{\text{h}[0,0]} = O_h$, we get:

$$\omega^c_c = \frac{2x - 2y - (r+\gamma_d)O_h - (r+\gamma_u)O_l}{r+\gamma_d+\gamma_u} \tag{H17}$$

$$\mu^c_{\text{h}[0,0]} = \frac{2O_l(r+\gamma_u)\,(r+\gamma_d+\gamma_u)}{\lambda_c\,(2x - 2y - (r+\gamma_d)O_h - (r+\gamma_u)O_l)} \tag{H18}$$

$$\mu_{\text{l}[0,0]} = \frac{2(r+\gamma_d)O_h\,(r+\gamma_d+\gamma_u)}{\lambda_c\,(2x - 2y - (r+\gamma_d)O_h - (r+\gamma_u)O_l)} \tag{H19}$$

Consider the gains from a bond transaction if an investor where to deviate and search in both markets at the same time. Combining (H2), (H3), (H5), and setting $\mu^{bc}_{\text{h}[0,0]} = 0$, ω^{bc}_b is

$$\omega^{\text{bc}}_b = \frac{4x\,(r+\gamma_d+\gamma_u) - 2\,(r+\gamma_d+\gamma_u)\,\lambda_b\mu^b_{\text{h}[0,0]}\omega^b_b + \lambda_c\mu_{l[0,0]}\left(4y - 2x - \lambda_b\mu^b_{\text{h}[0,0]}\omega^b_b + \lambda_c\mu^c_{\text{h}[0,0]}\omega^c_c\right)}{2\,(r+\gamma_d+\gamma_u)\left(2(r+\gamma_d) + \lambda_b\mu_{\text{a}[1,0]}\right) + 2(r+\gamma_d)\lambda_c\mu_{l[0,0]}}$$

The sign depends on the sign of the numerator. Substituting in (H14) and (H16)-(H19), the numerator is:

$$\frac{4\,[2x - 2y - O_l(r+\gamma_u)]\,(r+\gamma_d)\,(r+\gamma_d+\gamma_u)\left[-O_h\,(r+\gamma_d)\,(2r+\gamma_d) + \sqrt{A}\right]}{S\gamma_d\,(2x - 2y - (r+\gamma_d)O_h - O_l(r+\gamma_u))\,\lambda_b} \tag{H20}$$

where $A \equiv O_h r_d\,(4Sx\gamma_d\lambda_b + (r+\gamma_d)O_h\,((2r+\gamma_d)^{\,2} - 4S\gamma_d\lambda_b))$. The term in the second square brackets is positive because $x - (r+\gamma_d)O_h > 0$ from Assumption 1 implies that $A > ((r+\gamma_d)O_h\,(2r+\gamma_d))^2$. The first square bracket term in the numerator and the denominator are both positive by Assumption 1. Thus, $\omega^{\text{bc}}_b > 0$ implying $V^{bc}_{h[0,0]} > V^c_{h[0,0]}$. This contradicts that searching in the CDS market only is the optimal choice.

Similarly, consider the gains from a CDS transaction if an investor where to deviate and search in both markets at the same time. Combine (H5)-(H8) to get:

$$\omega^{\text{bc}}_c = \frac{8(r+\gamma_d)(x-y) + (2x-4y)\lambda_b\mu_{\text{a}[1,0]} + \lambda_b^2\mu_{\text{a}[1,0]}\mu^b_{\text{h}[0,0]}\omega^b_b - \lambda_c\left(2(r+\gamma_d) + \lambda_b\mu_{\text{a}[1,0]}\right)\mu^c_{\text{h}[0,0]}\omega^c_c}{2\,(r+\gamma_d+\gamma_u)\left(2\,(r+\gamma_d) + \lambda_b\mu_{\text{a}[1,0]}\right) + 2\,(r+\gamma_d)\,\lambda_c\mu_{l[0,0]}}$$

Substituting in (H14)-(H18), the sign of ω^{bc}_c depends on:

$$\frac{(2x - 2y - (r+\gamma_d)O_h - (r+\gamma_u)O_l)\left((r+\gamma_d)\,(2\,(x - (r+\gamma_d)O_h) + x + (x - O_h\gamma_d)) + \sqrt{A}\right)}{x - (r+\gamma_d)O_h}$$

By arguments analogous to the above discussion, $\omega^{\text{bc}}_c > 0$ implying $V^{bc}_{h[0,0]} > V^b_{h[0,0]}$. This shows that searching in only the bond market is not optimal.

Put together, searching in just one of the markets is not optimal. Everyone wants search in the other market at the same time. Thus, $\{\nu^{bc} = 0,\ \nu^b \geq 0,\ \nu^c \geq 0\}$ cannot be an equilibrium. Instead, in equilibrium, all long investors choose to search in both markets at the same time: $\{\nu^{bc} > 0,\ \nu^b = 0,\ \nu^c = 0\}$. □

Proof of Proposition 7. Condition (E5)—which ensures that ν_h is given by an interior solution and that $V_{h[0,0]} = O_h$—applies for any λ_b and λ_c including at ∞ limits. I use this condition for below results.

Consider, first, $\lambda_b \to \infty$. I start by analyzing what $\lambda_b \mu_{h[0,0]}$ limits to. First, suppose ν_l limits to a corner solution: $\lim_{\lambda_b \to \infty} \nu_l = 0$, then (G20) evaluated at $\nu_l = 0$ is

$$(r + \gamma_d)O_h - \lambda_b \frac{\gamma_d S}{(\gamma_d + \lambda_b \mu_{h[0,0]})} \frac{1}{2} \frac{x - (r + \gamma_d)O_h}{r + \gamma_d + \lambda_b \mu_{h[0,0]} \frac{1}{2}} = 0. \tag{H21}$$

If $\lim_{\lambda_b \to \infty} \lambda_b \mu_{h[0,0]} < \infty$, the left-hand side is negative and, hence, a contradiction. It has to be that $\lim_{\lambda_b \to \infty} \lambda_b \mu_{h[0,0]} = \infty$. Next, suppose ν_l limits to either an interior positive solution or to a positive corner solution. Then, (G21) and (G19) together imply

$$\frac{1}{r + \gamma_u} \lambda_c \mu_{h[0,0]} \frac{1}{2} \frac{2x - 2y - (r + \gamma_d)O_h}{r + \gamma_d + \gamma_u + \lambda_c \mu_{h[0,0]} \frac{1}{2}} \geq O_l. \tag{H22}$$

If $\mu_{h[0,0]}$ limits to zero, (H22) does not hold for $\lambda_c < \infty$ and, hence, a contradiction. It has to be that $\infty > \lim_{\lambda_b \to \infty} \mu_{h[0,0]} > 0$ and, hence, $\infty > \lim_{\lambda_b \to \infty} \lambda_b \mu_{h[0,0]} > 0$. Put together, regardless of what the entry rate of low types converges to, it has to be that $\lim_{\lambda_b \to \infty} \lambda_b \mu_{h[0,0]} = \infty$.

Using (G12) and $V_{h[0,0]} = O_h$, $\lim_{\lambda_b \to \infty} \lambda_b \mu_{h[0,0]} = \infty$ implies that $\lim_{\lambda_b \to \infty} \omega_b = 0$ and that introducing CDS market does not affect the bond price. From (G14), $\mu_{a[1,0]} \to 0$ as $\lambda_b \mu_{h[0,0]} \to \infty$. Thus, bond volume, given in (G4), approaches $\gamma_d S$ and is unaffected by the introduction of the CDS market.

Next, consider $\lambda_c \to \infty$. Suppose the limit of ν_l is given by a positive solution (interior or corner): $\nu_l > 0$. If $\lim_{\lambda_c \to \infty} \lambda_c \mu_{h[0,0]} < \infty$, the left-hand side of (G20) limits to $-\infty$, which is a contradiction. Thus, it has to be that $\lim_{\lambda_c \to \infty} \lambda_c \mu_{h[0,0]} = \infty$. Eq. (G20) at this limit is

$$(r + \gamma_d)O_h - \lambda_b \frac{\gamma_d S}{(\gamma_d + \lambda_b \mu_{h[0,0]})} \frac{1}{2} \frac{x - (r + \gamma_d)O_h}{r + \gamma_d + \lambda_b \mu_{h[0,0]} \frac{1}{2}} = 0. \tag{H23}$$

In the absence of CDS, $\mu_{h[0,0]}$ is characterized by the same equation as (H23). The mass of long investors, $\mu_{h[0,0]}$, as a result, is the same with or without CDS. In turn, the trading surplus (ω_b), the bond illiquidity discount (d_b), the mass of sellers ($\mu_{a[1,0]}$), and bond volume are the same with or without CDS.

If the entry rate of low types limits to zero as $\lambda_c \to \infty$, then the environment with CDS converges to the environment without CDS. Hence, the introduction of CDS again does not affect the bond market. □

Proof of Proposition 8. The value functions of investors participating in the bond market are characterized by:

$$
\begin{aligned}
rV^b_{h[0,0]} &= \gamma_d(0 - V^b_{h[0,0]}) + \lambda\mu_{a[1,0]}\frac{1}{2}\omega_b && \text{(H24)}\\
rV_{h[1,0]} &= (\delta - \eta J) + x - y + \gamma_d(V_{a[1,0]} - V_{h[1,0]}) && \text{(H25)}\\
rV_{a[1,0]} &= (\delta - \eta J) - y + \frac{M_b}{\mu_{a[1,0]}}\frac{1}{2}\omega_b. && \text{(H26)}
\end{aligned}
$$

Population masses $\{\mu_{h[1,0]},\, \mu^b_{h[0,0]},\, \mu_{a[1,0]}\}$ are given by

$$
\nu^b_h F_h = \gamma_d \mu^b_{h[0,0]} + M_b
$$

$$
M_b = \gamma_d \mu_{h[1,0]} \tag{H27}
$$

$$
\mu_{h[1,0]} + \mu_{a[1,0]} = S. \tag{H28}
$$

The entry rate into the bond market, ν^b_h, solves (H1).

The value functions of investors in the CDS market are characterized by:

$$
\begin{aligned}
rV^c_{h[0,0]} &= \gamma_d(0 - V^c_{h[0,0]}) + \frac{M_c}{\mu_{h[0,0]}}\frac{1}{2}\omega_c && \text{(H29)}\\
rV_{l[0,0]} &= \gamma_u(0 - V_{l[0,0]}) + \frac{M_c}{\mu_{l[0,0]}}\frac{1}{2}\omega_c && \text{(H30)}\\
rV_{h[0,1]} &= p_c - (\eta J - x) - y + \gamma_d(-T_{\text{S}} - V_{h[0,1]}) && \text{(H31)}\\
rV_{l[0,\text{-}1]} &= -p_c + (\eta J + x) - y + \gamma_u\left(-T_{\text{B}} - V_{l[0,\text{-}1]}\right). && \text{(H32)}
\end{aligned}
$$

Population masses $\{\mu_{h[0,1]},\, \mu^c_{h[0,0]},\, \mu_{l[0,\text{-}1]},\, \mu_{l[0,0]}\}$ of investors in the CDS market are characterized by:

$$
\begin{aligned}
\nu^c_h F_h + \gamma_u \mu_{h[0,1]} &= \gamma_d \mu^c_{h[0,0]} + M_c\\
\nu_l F_l + \gamma_d \mu_{l[0,\text{-}1]} &= \gamma_u \mu_{l[0,0]} + M_c && \text{(H33)}\\
M_c &= \gamma_d \mu_{h[0,1]} + \gamma_u \mu_{h[0,1]}
\end{aligned}
$$

$$
\mu_{h[0,1]} = \mu_{l[0,\text{-}1]}.
$$

The entry rates into the CDS market, ν^c_h and ν_l, solve (H1) and (4), respectively.

Combining the equations characterizing population masses, we get

$$
\frac{\nu^b_h F_h}{\gamma_d} = \mu^b_{h[0,0]} + \frac{\lambda_b \mu^b_{h[0,0]}}{\left(\lambda_b \mu^b_{h[0,0]} + \gamma_d\right)} S.
$$

Applying the Implicit Function Theorem to it, $\mu^b_{h[0,0]}$ increases with ν^b_h.

I describe next how bond market liquidity depends on the mass and, hence, the entry rate of high valuation investors. Combining the value functions for $h[0,0]$, $h[1,0]$, $a[1,0]$

, the bond price is

$$p_b = \frac{\delta - \eta J - y}{r} + \frac{1}{r}\frac{(r+\lambda_b\mu^b_{h[0,0]})\frac{1}{2}x}{r+\gamma_d+\lambda_b\mu^b_{h[0,0]}\frac{1}{2}+\lambda_b\mu_{a[1,0]}\frac{1}{2}}. \tag{H34}$$

$$= \frac{\delta - \eta J - y}{r} + \frac{1}{r}\frac{x(r+\lambda_b\mu^b_{h[0,0]})(\gamma_d+\lambda_b\mu^b_{h[0,0]})}{2\gamma_d(r+\gamma_d)+\gamma_d\lambda_b S+(2r+3\gamma_d+\lambda_b\mu_{h[0,0]})\lambda_b\mu_{h[0,0]}} \tag{H35}$$

where the last equality uses (H27) and (H28). The right-hand side and, thus, the bond price is increasing in $\mu_{h[0,0]}$. Combining the equations characterizing population masses,

$$M_b = \gamma_d\mu_{h[1,0]} \tag{H36}$$

$$= \gamma_d(S - \mu_{a[1,0]}) \tag{H37}$$

$$= \gamma_d(S - \frac{\gamma_d S}{(\lambda_b\mu_{h[0,0]}+\gamma_d)}) \tag{H38}$$

$$= S\frac{\lambda_b\mu_{h[0,0]}\gamma_d}{\lambda_b\mu_{h[0,0]}+\gamma_d}. \tag{H39}$$

Bond volume, as a result, increases in $\mu_{h[0,0]}$. Thus, the bond market is more liquid if high-valuation investors enter at a higher rate.

Combining the value functions and the population masses of the bond market investors:

$$(r+\gamma_d)V^b_{h[0,0]} = \frac{\lambda_b\gamma_d S}{\lambda_b\gamma_d S + 2(r+\gamma_d+\lambda_b\mu^b_{h[0,0]}\frac{1}{2})\left(\lambda_b\mu^b_{h[0,0]}+\gamma_d\right)}x.$$

Thus, $V^b_{h[0,0]}$ is decreasing in $\mu^b_{h[0,0]}$ and, hence, in ν^b_h. To simplify the analysis I assume that, prior to CDS introduction, the entry rate into the bond market, ν^b_h, is given by an interior solution.

When CDS trading is feasible, we can rule out $V^b_{h[0,0]} < O_h$ as an equilibrium outcome. $V^b_{h[0,0]} < O_h$ implies $\nu^b_h = 0$. But a decrease in ν^b_h due to CDS implies $V^b_{h[0,0]} > O_h$, not $V^b_{h[0,0]} < O_h$. Thus, in equilibrium when CDS is feasible: $V^b_{h[0,0]} \geq O_h$.

Table 2:

				Combined
1			$V^b_{h[0,0]} > V^c_{h[0,0]}$	$V^b_{h[0,0]} > V^c_{h[0,0]}$
2		$V^c_{h[0,0]} > O_h$	$V^b_{h[0,0]} = V^c_{h[0,0]}$	$V^b_{h[0,0]} = V^c_{h[0,0]} > O_h$
3	$V^b_{h[0,0]} > O_h$		$V^c_{h[0,0]} > V^b_{h[0,0]}$	$V^c_{h[0,0]} > V^b_{h[0,0]} > O_h$
4		$V^c_{h[0,0]} = O_h$	$V^b_{h[0,0]} > V^c_{h[0,0]}$	$V^b_{h[0,0]} > O_h = V^c_{h[0,0]}$
5		$V^c_{h[0,0]} < O_h$	$V^b_{h[0,0]} > V^c_{h[0,0]}$	$V^b_{h[0,0]} > O_h > V^c_{h[0,0]}$
6		$V^c_{h[0,0]} > O_h$	$V^c_{h[0,0]} > V^b_{h[0,0]}$	$V^c_{h[0,0]} > V^b_{h[0,0]} = O_h$
7	$V^b_{h[0,0]} = O_h$	$V^c_{h[0,0]} = O_h$	$V^c_{h[0,0]} = V^b_{h[0,0]}$	$V^c_{h[0,0]} = V^b_{h[0,0]} = O_h$
8		$V^c_{h[0,0]} < O_h$	$V^b_{h[0,0]} > V^c_{h[0,0]}$	$V^b_{h[0,0]} = O_h > V^c_{h[0,0]}$

To figure out the equilibrium outcomes, Table 2 lists possible orderings of $V^b_{h[0,0]}$, $V^c_{h[0,0]}$, and O_h. The first row cannot be an equilibrium. $V^b_{h[0,0]} > V^c_{h[0,0]}$ would mean that $\nu^b_h = 1$ and $\nu^c_h = 0$. At $\nu^c_h = 0$, however, low types do not enter either ($\nu_l = 0$) and hence $V^c_{h[0,0]} = 0$. This contradicts $V^c_{h[0,0]} > O_h$. The fourth and fifth rows cannot be an equilibrium. $V^b_{h[0,0]} > O_h \geq V^c_{h[0,0]}$ implies that $\nu^c_h = 0$ and $\nu^b_h = 1$, but since $V^b_{h[0,0]}$ is decreasing in ν^b_h, $V^b_{h[0,0]}$ evaluated at $\nu^b_h = 1$ should be lower than O_h. Hence, a contradiction. The sixth row is not an equilibrium. $V^c_{h[0,0]} > V^b_{h[0,0]} = O_h$ implies that $\nu^c_h = 1$ and $\nu^b_h = 0$. However, $V^b_{h[0,0]}$ evaluated at $\nu^b_h = 0$ does not equal O_h.

Four equilibria exist. In two of them, the entry rate into the bond market, ν^b_h, stays the same as in the environment without CDS. In one of these, the entry into the CDS market is positive: $\nu^c_h + \nu^b_h \leq 1$ and $V^c_{h[0,0]} = V^b_{h[0,0]} = O_h$, while in the other, none of the high type investors enter the CDS market, and $V^b_{h[0,0]} = O_h > V^c_{h[0,0]} = 0$. In the other two equilibria, the entry rate into the bond market, ν^b_h, decreases. In one of these, shown in row 3, $V^c_{h[0,0]} > V^b_{h[0,0]} > O_h$ implying that $\nu^b_h = 0$ and $\nu^c_h = 1$. This means all high type investors enter the CDS, and trading activity in the bond market disappears. In the other, shown in row 2, $V^b_{h[0,0]} = V^c_{h[0,0]} > O_h$ implying that the entry rate into the bond market decreases (so that $V^b_{h[0,0]}$ is higher).[48]

Thus, with the introduction of the CDS market, high valuation investors enter the bond market either at the same or at a lower rate. If the entry rate remains the same, the illiquidity discount and bond volume also remain unaffected by CDS introduction. If the entry rate decreases, bond market liquidity deteriorates: Bond volume decreases, and the illiquidity discount increases. □

I Short-Selling

I use the short-selling framework of Vayanos and Weill (2008). In contrast to Vayanos and Weill (2008), I endogenize the entry rate of long and short investors. To be self-containing, I describe the main features and mechanism of the Vayanos and Weill (2008) framework, but for more details I refer the reader to their paper.

Agents' asset positions, $[\theta_b, \theta_r]$, denoted in subscripts on population masses and value functions, are: $\theta_b = 1$ if the investor holds the bond, $\theta_b = 0$ if the investor does not hold the bond, $\theta_r = 1$ if the investor has the lent the bond, and $\theta_r = -1$ if the investor has borrowed the bond. Combined, we have five feasible asset positions: $[1, 0]$ is a bond owner, $[0, 1]$ is an investor who has lent out her bond, $[1,$-1$]$ is an investor who has borrowed a bond but has not yet sold it short, $[0,$-1$]$ is an investor who has borrowed a bond and has sold it short, and $[0, 0]$ is an investor with no asset position.

The conjectured equilibrium agent types and their optimal trading strategies are as follows. In equilibrium, a bond owner, $[1, 0]$, can be either a high- or average-valuation type. The high-valuation type bond owner searches for a bond borrower in the repo market (i.e. for a short investor) to lend her bond to. The average type bond owner prefers neither a long nor a short position and, hence, searches for a bond buyer in the

[48]The entry rate could decrease to another, smaller, rate $\nu^b_h > 0$, or all the way to zero, $\nu^b_h = 0$.

bond market to sell her bond to. An investor who has lent her bond, $[0, 1]$, is a high type investor. If she reverts to an average-valuation while her counterparty has not yet sold the bond, she demands back the bond and becomes a bond seller ($a[1, 0]$). If she reverts to an average-valuation after her counterparty has already (short-) sold the bond, she seizes the collateral her counterparty has put aside and exits the market. An investor who has borrowed but has not yet sold the bond, $[1,\text{-}1]$, is a low type investor. She is the short seller who searches for a bond buyer in the bond market to sell her bond to. If she reverts to an average valuation (that is, before she could short sell the bond), she immediately delivers back the bond and exits the market. An investor with position $[0,\text{-}1]$ can be either a low or an average type. A low type investor with position $[0,\text{-}1]$ is the short investor who has reached her optimal asset position. An average type with position $[0,\text{-}1]$ is an investor who was previously a short investor that reverted to an average valuation. She searches to buy back the bond to deliver the bond back to the lender and unwind her short position. Investors with no position can be high or low type: If they are a high type, they seek a long position by buying a bond. If they are a low type, they seek a short position by borrowing a bond.

Define the total mass of bond sellers as

$$\mu_{b,\text{S}} \equiv \mu_{l[1,\text{-}1]} + \mu_{a[1,0]},$$

where $\mu_{l[1,\text{-}1]}$ is the mass of short sellers and $\mu_{a[1,0]}$ is the mass of regular bond sellers (that is, those with a long position looking to unwind their long position). Analogously, the total mass of bond buyers is

$$\mu_{b,\text{B}} \equiv \mu_{h[0,0]} + \mu_{a[0,\text{-}1]},$$

where $h[0, 0]$ are the long investors seeking a long position by buying a bond, and $a[0,\text{-}1]$ are the investors looking to buy back the bond in order to deliver it to its repo counterparty and unwind their short position. In the repo market, the active searchers are the bond borrowers, $l[0, 0]$, and bond lenders, $h[1, 0]$.

Define with q's meeting intensities:

$$q_{\text{B}} \equiv \lambda_b \mu_{b,\text{B}},$$

$$q_{\text{S}} \equiv \lambda_b \mu_{b,\text{S}},$$

$$q_{\text{LE}} \equiv \lambda_r \mu_{h[1,0]},$$

$$q_{\text{BO}} \equiv \lambda_r \mu_{l[0,0]}.$$

To distinguish the environments with and without short-selling, I denote the environment without short-selling with hats.

The inflow-outflow equations are:

$$\nu_h F_h = \gamma_d \mu_{h[0,0]} + q_{\text{S}} \mu_{h[0,0]} \tag{I1}$$

$$q_{\text{S}}\mu_{h[0,0]} + \gamma_u\mu_{l[1,\text{-}1]} + q_{\text{S}}\mu_{a[0,\text{-}1]} = q_{\text{BO}}\mu_{h[1,0]} + \gamma_d\mu_{h[1,0]} \tag{I2}$$

$$q_{\text{BO}}\mu_{h[1,0]} = \gamma_d\mu_{h[0,1]} + \gamma_u\mu_{l[1,\text{-}1]} + q_{\text{S}}\mu_{a[0,\text{-}1]} \tag{I3}$$

$$\nu_l F_l + \gamma_d\mu_{l[1,\text{-}1]} + \gamma_d\mu_{l[0,\text{-}1]} = \gamma_u\mu_{l[0,0]} + q_{\text{LE}}\mu_{l[0,0]} \tag{I4}$$

$$q_{\text{LE}}\mu_{l[0,0]} = \gamma_u\mu_{l[1,\text{-}1]} + q_{\text{B}}\mu_{l[1,\text{-}1]} + \gamma_d\mu_{l[1,\text{-}1]} \tag{I5}$$

$$q_{\text{B}}\mu_{l[1,\text{-}1]} = (\gamma_u + \gamma_d)\,\mu_{l[0,\text{-}1]} \tag{I6}$$

$$\gamma_u\mu_{l[0,\text{-}1]} = q_{\text{S}}\mu_{a[0,\text{-}1]} + \gamma_d\mu_{a[0,\text{-}1]} \tag{I7}$$

$$\gamma_d\mu_{h[1,0]} + \gamma_d\mu_{l[1,\text{-}1]} = q_{\text{B}}\mu_{a[1,0]} \tag{I8}$$

Market clearing conditions are:

$$\mu_{h[1,0]} + \mu_{a[1,0]} + \mu_{l[1,\text{-}1]} = S \tag{I9}$$

$$\mu_{h[0,1]} = \mu_{l[1,\text{-}1]} + \mu_{l[0,\text{-}1]} + \mu_{a[0,\text{-}1]}. \tag{I10}$$

To characterize the value functions of agents who have lent their bond, $h[0,1]$, we have to keep track of their counterparty. The subscript on the value functions of an $h[0,1]$ investor, as a result, denotes both the agent's own type and the type of her counterparty. For example, $V_{h[0,1]l[1,\text{-}1]}$ denotes the value function of an investor who has lent her bond ($h[0,1]$) whose counterparty is $l[1,\text{-}1]$ type.

The value functions are:

$$rV_{h[0,0]} = \gamma_d\left(0 - V_{h[0,0]}\right) + q_{\text{S}}\left(V_{h[1,0]} - V_{h[0,0]} - p_b\right) \tag{I11}$$

$$rV_{h[1,0]} = \delta - \eta J + x - y + q_{\text{BO}}\left(V_{h[0,1]l[1,\text{-}1]} - V_{h[1,0]}\right) + \gamma_d\left(V_{a[1,0]} - V_{h[1,0]}\right) \tag{I12}$$

$$\begin{aligned} rV_{h[0,1]l[1,\text{-}1]} = {} & \delta - \eta J + x - y + fee + \gamma_d\left(V_{a[1,0]} - V_{h[0,1]l[1,\text{-}1]}\right) + \gamma_u\left(V_{h[1,0]} - V_{h[0,1]l[1,\text{-}1]}\right) \\ & + q_{\text{B}}\left(V_{h[0,1]l[0,\text{-}1]} - V_{h[0,1]l[1,\text{-}1]}\right) \end{aligned} \tag{I13}$$

$$rV_{h[0,1]l[0,\text{-}1]} = \delta - \eta J + x - y + fee + \gamma_d\left(z - V_{h[0,1]l[0,\text{-}1]}\right) + \gamma_u\left(V_{h[0,1]a[0,\text{-}1]} - V_{h[0,1]l[0,\text{-}1]}\right) \tag{I14}$$

$$rV_{h[0,1]a[0,\text{-}1]} = \delta - \eta J + x - y + fee + \gamma_d\left(z - V_{h[0,1]a[0,\text{-}1]}\right) + q_{\text{S}}\left(V_{h[1,0]} - V_{h[0,1]a[0,\text{-}1]}\right) \tag{I15}$$

$$rV_{l[0,0]} = \gamma_u\left(0 - V_{l[0,0]}\right) + q_{\text{LE}}\left(V_{l[1,\text{-}1]} - V_{l[0,0]}\right) \tag{I16}$$

$$rV_{l[1,\text{-}1]} = -fee + \gamma_u\left(0 - V_{l[1,\text{-}1]}\right) + q_{\text{B}}\left(V_{l[0,\text{-}1]} - V_{l[1,\text{-}1]} + p_b\right) + \gamma_d\left(V_{l[0,0]} - V_{l[1,\text{-}1]}\right) \tag{I17}$$

$$rV_{l[0,\text{-}1]} = -fee - (\delta - \eta J - x) - y + \gamma_u\left(V_{a[0,\text{-}1]} - V_{l[0,\text{-}1]}\right) + \gamma_d\left(V_{l[0,0]} - z - V_{l[0,\text{-}1]}\right) \tag{I18}$$

$$rV_{a[0,\text{-}1]} = -fee - (\delta - \eta J) - y + q_{\text{S}}\left(0 - p_b - V_{a[0,\text{-}1]}\right) + \gamma_d\left(0 - z - V_{a[0,\text{-}1]}\right) \tag{I19}$$

$$rV_{a[1,0]} = \delta - \eta J - y + q_{\text{B}}\left(p_b + 0 - V_{a[1,0]}\right), \tag{I20}$$

where z is the collateral that the bond lender seizes from the bond borrower if the bond borrower cannot deliver the bond. Following Vayanos and Weill (2008), I set $z = V_{a[1,0]}$.

The entry decisions of high and low-type valuation investors are given by (4).

In a bond transaction, the bond buyer $h[0,0]$ and the seller $a[1,0]$ bargain over price. The bond price, as a result, is characterized by:

$$p_b = \frac{1}{2}\left(V_{h[1,0]} - V_{h[0,0]}\right) + \frac{1}{2}V_{a[1,0]} \tag{I21}$$

I assume, following Vayanos and Weill (2008), that the other bond buyer ($a[0,\text{-}1]$) and seller ($l[1,\text{-}1]$) transact at the same price and verify later it is optimal to do so.

The variable, fee, is the flow security lending fee the bond borrower pays the bond lender throughout the repo contract. The bond lender and the borrower negotiate over the fee so that each gets half of the total gains from trade:

$$V_{h[0,1]l[1,\text{-}1]} - V_{h[1,0]} = \frac{1}{2}\left(V_{h[0,1]l[1,\text{-}1]} - V_{h[1,0]} + V_{l[1,\text{-}1]} - V_{l[0,0]}\right). \tag{I22}$$

Analogous to the environment with CDS trading, define the gains from a bond transaction as

$$\omega_b \equiv V_{h[1,0]} - V_{h[0,0]} - V_{a[1,0]}.$$

and from a repo transaction as:

$$\omega_r \equiv \left(V_{h[0,1]l[1,\text{-}1]} - V_{h[1,0]}\right) + V_{l[1,\text{-}1]} - V_{l[0,0]}. \tag{I23}$$

Proof of Proposition 9. Take the difference between (I12) and (I11) and get

$$r\left(V_{h[1,0]} - V_{h[0,0]}\right) = \delta - \eta J + x - y + q_{\text{BO}}\left(V_{h[0,1]l[1,\text{-}1]} - V_{h[1,0]}\right) - \gamma_d\omega_b - q_{\text{S}}\frac{1}{2}\omega_b. \tag{I24}$$

Combine this with (I20) and solve for ω_b as:

$$\omega_b = \frac{x + q_{\text{BO}}\frac{1}{2}\omega_r}{\left(r + \gamma_d + q_{\text{S}}\frac{1}{2} + q_{\text{B}}\frac{1}{2}\right)}. \tag{I25}$$

Combine (I20) and (I24) and simplify:

$$rp_b = \delta - \eta J - y + \frac{1}{2}x + \frac{1}{2}\left(-\gamma_d\omega_b - (r+\gamma_d)V_{h[0,0]} + q_{\text{B}}\frac{1}{2}\omega_b\right) + \frac{1}{4}q_{\text{BO}}\omega_r. \tag{I26}$$

From (I25), $q_{\text{B}}\frac{1}{2}\omega_b = x + q_{\text{BO}}\frac{1}{2}\omega_r - \omega_b\left(r + \gamma_d + q_{\text{S}}\frac{1}{2}\right)$. Substitute this and $V_{h[0,0]} = O_h$ into (I26) and simplify to get

$$rp_b = \delta - \eta J + x - y - (r+\gamma_d)\,O_h - \left(\frac{r + 2\gamma_d}{2}\right)\omega_b + q_{\text{BO}}\frac{1}{2}\omega_r. \tag{I27}$$

Expressing ω_b as

$$\omega_b = \frac{x - (r+\gamma_d)O_h + q_{\text{BO}}\frac{1}{2}\omega_r}{\left(r + \gamma_d + q_{\text{B}}\frac{1}{2}\right)},$$

plugging it into (I27), and simplifying, we get (22).

The bond price without short-selling is

$$r\hat{p}_b = \delta - \eta J + x - y - (r+\gamma_d)\,O_h - \left(\frac{r+2\gamma_d}{2}\right)\frac{x-(r+\gamma_d)O_h}{\left(r+\gamma_d+\hat{q}_{\text{B}}\frac{1}{2}\right)}. \tag{I28}$$

□

Before I prove Proposition 10, I first show some intermediate results in Lemmas 3-5.

Lemma 3. *The inflow-outflow and market clearing equations boil down to a set of four equations of four variables $\mu_{h[0,0]}$, $\mu_{l[0,0]}$, $\mu_{b,\text{s}}$, and $\mu_{b,\text{B}}$:*

$$\nu_h F_h = \gamma_d \mu_{h[0,0]} + q_{\text{S}}\mu_{h[0,0]} \tag{I29}$$

$$\nu_l F_l = \frac{\gamma_u}{\gamma_u+\gamma_d}\lambda_r\left(S-\mu_{b,\text{s}}\right)\mu_{l[0,0]} + \gamma_u\mu_{l[0,0]} \tag{I30}$$

$$\mu_{b,\text{B}} = \mu_{h[0,0]} + \frac{\gamma_u}{\lambda_b\mu_{b,\text{s}}+\gamma_d}\frac{\lambda_b\mu_{b,\text{B}}}{\gamma_u+\gamma_d}\frac{\lambda_r\mu_{l[0,0]}\left(S-\mu_{b,\text{s}}\right)}{\gamma_u+\gamma_d+\lambda_b\mu_{b,\text{B}}} \tag{I31}$$

$$\mu_{b,\text{s}} = \frac{\gamma_d S}{\gamma_d+\lambda_b\mu_{b,\text{B}}} + \frac{\lambda_r\left(S-\mu_{b,\text{s}}\right)\mu_{l[0,0]}}{\gamma_u+\gamma_d+\lambda_b\mu_{b,\text{B}}}. \tag{I32}$$

Proof. The first equation (I29) is given by (I1) and characterizes $\mu_{h[0,0]}$.

The second equation (I30) characterizes $\mu_{l[0,0]}$. Substituting $q_{\text{B}}\mu_{l[1,\text{-}1]}$ from (I6) into (I5) and solving for $\mu_{l[1,\text{-}1]}+\mu_{l[0,\text{-}1]}$, we get

$$\mu_{l[1,\text{-}1]}+\mu_{l[0,\text{-}1]} = \frac{q_{\text{LE}}\mu_{l[0,0]}}{\gamma_u+\gamma_d}. \tag{I33}$$

Substituting it into (I4) and simplifying yields:

$$\nu_l F_l + \gamma_d\frac{q_{\text{LE}}\mu_{l[0,0]}}{\gamma_u+\gamma_d} = \gamma_u\mu_{l[0,0]} + q_{\text{LE}}\mu_{l[0,0]}. \tag{I34}$$

Then, using $q_{\text{LE}} = \lambda_r\mu_{h[1,0]}$ and the bond market clearing condition, we get:

$$\nu_l F_l = \frac{\gamma_u}{\gamma_u+\gamma_d}\lambda_r\left(S-\mu_{b,\text{s}}\right)\mu_{l[0,0]} + \gamma_u\mu_{l[0,0]}. \tag{I35}$$

The third equation (I31) characterizes $\mu_{b,\text{B}}$. Solving for $\mu_{l[1,\text{-}1]}$ from (I5) and using $\mu_{h[1,0]} = S-\mu_{b,\text{s}}$:

$$\mu_{l[1,\text{-}1]} = \frac{\lambda_r\left(S-\mu_{b,\text{s}}\right)\mu_{l[0,0]}}{\gamma_u+\gamma_d+\lambda_b\mu_{b,\text{B}}}. \tag{I36}$$

Substituting it into (I6) and using $q_{\text{B}} = \lambda_b\mu_{b,\text{B}}$, we get

$$\mu_{l[0,\text{-}1]} = \frac{\lambda_b\mu_{b,\text{B}}}{\gamma_u+\gamma_d}\frac{\lambda_r\left(S-\mu_{b,\text{s}}\right)\mu_{l[0,0]}}{\gamma_u+\gamma_d+\lambda_b\mu_{b,\text{B}}}. \tag{I37}$$

Plugging (I37) into (I7) gives

$$\mu_{a[0,\text{-}1]} = \frac{\gamma_u}{q_{\rm S} + \gamma_d} \frac{\lambda_b \mu_{b,\rm B}}{\gamma_u + \gamma_d} \frac{\lambda_r \left(S - \mu_{b,\rm S}\right) \mu_{l[0,0]}}{\gamma_u + \gamma_d + \lambda_b \mu_{b,\rm B}}. \tag{I38}$$

Plugging (I38) into $\mu_{b,\rm B} = \mu_{h[0,0]} + \mu_{a[0,\text{-}1]}$, we get

$$\mu_{b,\rm B} = \mu_{h[0,0]} + \frac{\gamma_u}{\lambda_b \mu_{b,\rm S} + \gamma_d} \frac{\lambda_b \mu_{b,\rm B}}{\gamma_u + \gamma_d} \frac{\lambda_r \mu_{l[0,0]} \left(S - \mu_{b,\rm S}\right)}{\gamma_u + \gamma_d + \lambda_b \mu_{b,\rm B}}. \tag{I39}$$

The fourth equation (I32) characterizes $\mu_{b,\rm S}$. Solve for $\mu_{a[1,0]}$ from (I8):

$$\mu_{a[1,0]} = \frac{\gamma_d S}{\gamma_d + \lambda_b \mu_{b,\rm B}}. \tag{I40}$$

Adding $\mu_{l[1,\text{-}1]}$, derived in (I36), to $\mu_{a[1,0]}$ and using the definition $\mu_{b,\rm S} = \mu_{l[1,\text{-}1]} + \mu_{a[1,0]}$, we get:$\mu_{b,\rm S}$ □

The next lemma shows that, taking the entry rate ν's as given, the four equations characterizing $\mu_{h[0,0]}$, $\mu_{l[0,0]}$, $\mu_{b,\rm S}$, and $\mu_{b,\rm B}$ have a unique positive solution. In doing so, it shows that $S - \mu_{b,\rm S} > 0$, which I use later for the proof of Proposition 10.

Lemma 4. *For any given ν_h and ν_l, a unique positive solution exists for μ's.*

Proof. Make the following change of variables: $q_{\rm BO} = \lambda_r \mu_{l[0,0]}$ and $q_{\rm S} = \lambda_b \mu_{b,\rm S}$, $e_b = \frac{1}{\lambda_b}$, and $e_r = \frac{1}{\lambda_r}$ in (I29)-(I32) and get:

$$\nu_h F_h = \gamma_d \mu_{h[0,0]} + q_{\rm S} \mu_{h[0,0]} \tag{I41}$$

$$\nu_l F_l = \frac{\gamma_u}{\gamma_u + \gamma_d} q_{\rm BO} \left(S - e_b q_{\rm S}\right) + \gamma_u e_r q_{\rm BO} \tag{I42}$$

$$\mu_{b,\rm B} = \mu_{h[0,0]} + \frac{\gamma_u}{q_{\rm S} + \gamma_d} \frac{\mu_{b,\rm B}}{\gamma_u + \gamma_d} \frac{q_{\rm BO} \left(S - e_b q_{\rm S}\right)}{e_b \left(\gamma_u + \gamma_d\right) + \mu_{b,\rm B}} \tag{I43}$$

$$q_{\rm S} = \frac{\gamma_d S}{e_b \gamma_d + \mu_{b,\rm B}} + \frac{q_{\rm BO} \left(S - e_b q_{\rm S}\right)}{e_b \left(\gamma_u + \gamma_d\right) + \mu_{b,\rm B}}. \tag{I44}$$

From (I41), solve for $\mu_{h[0,0]}$ as a function of $q_{\rm S}$ to get:

$$\mu_{h[0,0]} = \frac{\nu_h F_h}{\gamma_d + q_{\rm S}}. \tag{I45}$$

Thus, $\mu_{h[0,0]}$ decreases in $q_{\rm S}$.

Next, I show that $\mu_{b,\rm B}$ decreases in $q_{\rm S}$. From (I42),

$$q_{\rm BO} = \frac{\nu_l F_l}{\gamma_u e_r + \frac{\gamma_u}{\gamma_u + \gamma_d} \left(S - e_b q_{\rm S}\right)}. \tag{I46}$$

Plugging it into (I43) and rearranging gives:

$$\left(\mu_{b,\mathrm{B}} - \mu_{h[0,0]}\right) = \tag{I47}$$
$$\frac{1}{\left(e_b\left(\gamma_u+\gamma_d\right)+\mu_{b,\mathrm{B}}\right)\left(q_\mathrm{s}+\gamma_d\right)}\mu_{b,\mathrm{B}}\frac{1}{\left(e_r\left(\gamma_u+\gamma_d\right)+\left(S-e_bq_\mathrm{s}\right)\right)}\nu_lF_l\left(S-e_bq_\mathrm{s}\right).$$

Rearrange it and get:

$$1=\frac{\mu_{h[0,0]}}{\mu_{b,\mathrm{B}}}+\frac{1}{\left(e_b\left(\gamma_u+\gamma_d\right)+\mu_{b,\mathrm{B}}\right)\left(\gamma_d+q_\mathrm{s}\right)}\frac{1}{\left(e_r\left(\gamma_u+\gamma_d\right)+\left(S-e_bq_\mathrm{s}\right)\right)}\nu_lF_l\left(S-e_bq_\mathrm{s}\right). \tag{I48}$$

Since $\mu_{h[0,0]}$ is an implicit function of q_s, (I48) characterizes $\mu_{b,\mathrm{B}}$ as an implicit function of q_s. Applying the Implicit Function Theorem to (I48), $\mu_{b,\mathrm{B}}$ decreases in q_s.

Next, I simplify (I41)-(I44) into a single equation of one unknown, q_s, and show that q_s is uniquely determined and positive. From (I43),

$$\frac{q_\mathrm{BO}\left(S-e_bq_\mathrm{s}\right)}{e_b\left(\gamma_u+\gamma_d\right)+\mu_{b,\mathrm{B}}}=\left(\mu_{b,\mathrm{B}}-\mu_{h[0,0]}\right)\frac{\left(\gamma_u+\gamma_d\right)\left(q_\mathrm{s}+\gamma_d\right)}{\gamma_u\mu_{b,\mathrm{B}}}. \tag{I49}$$

Plug (I49) into (I44) and multiply both sides by $\mu_{b,\mathrm{B}}$:

$$q_\mathrm{s}\mu_{b,\mathrm{B}}=\frac{\gamma_dS\mu_{b,\mathrm{B}}}{e_b\gamma_d+\mu_{b,\mathrm{B}}}+\left(\mu_{b,\mathrm{B}}-\mu_{h[0,0]}\right)\frac{\left(\gamma_u+\gamma_d\right)\left(q_\mathrm{s}+\gamma_d\right)}{\gamma_u}.$$

Add $q_\mathrm{s}\mu_{h[0,0]}$ to both sides

$$q_\mathrm{s}\mu_{b,\mathrm{B}}+q_\mathrm{s}\mu_{h[0,0]}=\frac{\gamma_dS\mu_{b,\mathrm{B}}}{e_b\gamma_d+\mu_{b,\mathrm{B}}}+\left(\mu_{b,\mathrm{B}}-\mu_{h[0,0]}\right)\frac{\left(\gamma_u+\gamma_d\right)\left(q_\mathrm{s}+\gamma_d\right)}{\gamma_u}+q_\mathrm{s}\mu_{h[0,0]}. \tag{I50}$$

Subtract $q_\mathrm{s}\mu_{b,\mathrm{B}}$ from both sides, simplify, and plug the expression for $\mu_{h[0,0]}$ into the left-hand side of (I50):

$$\frac{\nu_hF_h}{\gamma_d+q_\mathrm{s}}q_\mathrm{s}=\frac{\gamma_dS\mu_{b,\mathrm{B}}}{e_b\gamma_d+\mu_{b,\mathrm{B}}}+\left(\mu_{b,\mathrm{B}}-\mu_{h[0,0]}\right)\frac{\gamma_d}{\gamma_u}\left(\gamma_u+\gamma_d+q_\mathrm{s}\right). \tag{I51}$$

The left-hand side increases in q_s. The first term on the right-hand side, $\frac{\gamma_dS\mu_{b,\mathrm{B}}}{e_b\gamma_d+\mu_{b,\mathrm{B}}}$, increases in $\mu_{b,\mathrm{B}}$ and, hence, decreases in q_s. Consider the second term on the right-hand side. From (I47),

$$\left(\mu_{b,\mathrm{B}}-\mu_{h[0,0]}\right)\left(\gamma_u+\gamma_d+q_\mathrm{s}\right)= \tag{I52}$$
$$\frac{1}{\left(e_b\left(\gamma_u+\gamma_d\right)+\mu_{b,\mathrm{B}}\right)\left(q_\mathrm{s}+\gamma_d\right)}\frac{\mu_{b,\mathrm{B}}\left(\gamma_u+\gamma_d+q_\mathrm{s}\right)}{\left(e_r\left(\gamma_u+\gamma_d\right)+\left(S-e_bq_\mathrm{s}\right)\right)}\nu_lF_l\left(S-e_bq_\mathrm{s}\right).$$

The right-hand side of (I52) decreases in q_s and increases in $\mu_{b,\mathrm{B}}$. Since $\mu_{b,\mathrm{B}}$ decreases in q_s, the right-hand side of (I52), as a result, decreases in q_s. Put together, the left-hand side minus the right-hand side of (I51) (as a function of one unknown, q_s) strictly increases in q_s. At $q_\mathrm{s}=0$, the left-hand side of (I51) is 0, the right-hand side is positive. The left-hand side minus the right-hand side, as a result, is negative. At $q_{\text{S}} = \frac{S}{e_b}$, the left-hand side is $\frac{S}{e_b}\mu_{h[0,0]}$. Consider the right-hand side. From (I47), $\mu_{b,\text{B}} - \mu_{h[0,0]} = 0$. Thus, the right-hand side equals $\frac{\gamma_d S \mu_{b,\text{B}}}{e_b \gamma_d + \mu_{b,\text{B}}}$, which is less than $\frac{S}{e_b}\mu_{h[0,0]}$. Thereby, at $q_{\text{S}} = \frac{S}{e_b}$, the left-hand side minus the right-hand side is positive. Thus, (I51) has a unique solution in $(0, \frac{S}{e_b})$.

The other unknowns, $\mu_{h[0,0]}$, $\mu_{b,\text{B}}$, and q_{BO} are uniquely given as functions of q_{S}. Thus, taking as given ν_h and ν_l in [0, 1], the system of four equations and four unknowns (I29)-(I32) has a unique positive solution. This proof for an existence of a unique positive solution holds for any e_r including $e_r = 0$. Moreover, the solution is such that $q_{\text{S}} < \frac{S}{e_b}$ implying that $\mu_{h[1,0]} = S - e_b q_{\text{S}} > 0$. □

Lemma 5. *The gains from trade from a repo transaction, ω_r, is characterized by:*

$$\left(r + \gamma_d + \gamma_u + q_{\text{LE}}\frac{1}{2} + q_{\text{BO}}\frac{1}{2}\left(1 + \frac{q_{\text{B}}\gamma_u}{(r + \gamma_d + q_{\text{B}} + \gamma_u)(r + \gamma_d + q_{\text{S}})}\right)\right)\omega_r \tag{I53}$$
$$= \frac{q_{\text{B}}}{(r + \gamma_d + \gamma_u + q_{\text{B}})} \cdot$$
$$\cdot \left(x + \frac{r + \gamma_d + \gamma_u + q_{\text{S}}}{r + \gamma_d + q_{\text{S}}}\left[-2y + \frac{r + \gamma_d + q_{\text{B}}}{\left(r + \gamma_d + q_{\text{S}}\frac{1}{2} + q_{\text{B}}\frac{1}{2}\right)}\frac{1}{2}\left(x + q_{\text{BO}}\frac{1}{2}\omega_r\right)\right]\right).$$

Proof. Take the difference between (I17) and (I16):

$$\begin{aligned} rV_{l[1,\text{-}1]} - rV_{l[0,0]} = {} & -fee - \gamma_u\left(V_{l[1,\text{-}1]} - V_{l[0,0]}\right) + q_{\text{B}}\left(V_{l[0,\text{-}1]} - V_{l[1,\text{-}1]} + p_b\right) \\ & + \gamma_d\left(V_{l[0,0]} - V_{l[1,\text{-}1]}\right) - q_{\text{LE}}\left(V_{l[1,\text{-}1]} - V_{l[0,0]}\right). \end{aligned} \tag{I54}$$

Take the difference between (I13) and (I12):

$$\begin{aligned} rV_{h[0,1]l[1,\text{-}1]} - rV_{h[1,0]} = {} & fee + \gamma_d\left(V_{a[1,0]} - V_{h[0,1]l[1,\text{-}1]}\right) - \gamma_u\left(V_{h[0,1]l[1,\text{-}1]} - V_{h[1,0]}\right) \\ & + q_{\text{B}}\left(V_{h[0,1]l[0,\text{-}1]} - V_{h[0,1]l[1,\text{-}1]}\right) - \gamma_d\left(V_{a[1,0]} - V_{h[1,0]}\right) \\ & - q_{\text{BO}}\left(V_{h[0,1]l[1,\text{-}1]} - V_{h[1,0]}\right). \end{aligned} \tag{I55}$$

Then, adding the two differences and simplifying yields:

$$\left(r + \gamma_d + \gamma_u + q_{\text{BO}}\frac{1}{2} + q_{\text{LE}}\frac{1}{2}\right)\omega_r = q_{\text{B}}\left(V_{h[0,1]l[0,\text{-}1]} - V_{h[0,1]l[1,\text{-}1]} + V_{l[0,\text{-}1]} - V_{l[1,\text{-}1]} + p_b\right). \tag{I56}$$

Next, I derive $V_{h[0,1]l[0,\text{-}1]} - V_{h[0,1]l[1,\text{-}1]} + V_{l[0,\text{-}1]} - V_{l[1,\text{-}1]} + p_b$. Take the difference between (I18) and (I17):

$$\begin{aligned} rV_{l[0,\text{-}1]} - rV_{l[1,\text{-}1]} = {} & -(\delta - \eta J - x) - y + \gamma_u\left(V_{a[0,\text{-}1]} - V_{l[0,\text{-}1]}\right) + \gamma_d\left(-c - V_{l[0,\text{-}1]}\right) \\ & - \left(\gamma_u\left(0 - V_{l[1,\text{-}1]}\right) + q_{\text{B}}\left(V_{l[0,\text{-}1]} - V_{l[1,\text{-}1]} + p_b\right) + \gamma_d\left(-V_{l[1,\text{-}1]}\right)\right). \end{aligned}$$

Take the difference between (I14) and (I13):

$$rV_{h[0,1]l[0,\text{-}1]} - rV_{h[0,1]l[1,\text{-}1]} = -\gamma_d \left(V_{h[0,1]l[0,\text{-}1]} - V_{h[0,1]l[1,\text{-}1]}\right) + \gamma_u \left(V_{h[0,1]a[0,\text{-}1]} - V_{h[0,1]l[0,\text{-}1]}\right) \\ - q_{\text{B}} \left(V_{h[0,1]l[0,\text{-}1]} - V_{h[0,1]l[1,\text{-}1]}\right) - \gamma_u \left(V_{h[1,0]} - V_{h[0,1]l[1,\text{-}1]}\right).$$

Add the two differences and add rp_b to both sides:

$$(r + q_{\text{B}}) \left(V_{l[0,\text{-}1]} - V_{l[1,\text{-}1]} + V_{h[0,1]l[0,\text{-}1]} - V_{h[0,1]l[1,\text{-}1]} + p_b\right) \\ = rp_b - (\delta - \eta J - x) - y - \gamma_u \left(-V_{a[0,\text{-}1]} + V_{l[0,\text{-}1]} - V_{l[1,\text{-}1]} - V_{h[0,1]a[0,\text{-}1]} + V_{h[0,1]l[0,\text{-}1]}\right) \\ - \gamma_d \left(c + V_{l[0,\text{-}1]} - V_{l[1,\text{-}1]} + V_{h[0,1]l[0,\text{-}1]} - V_{h[0,1]l[1,\text{-}1]}\right) - \gamma_u \left(V_{h[1,0]} - V_{h[0,1]l[1,\text{-}1]}\right).$$

Add $\gamma_d p_b - \gamma_d p_b$ to the right-hand side and simplify to get:

$$(r + \gamma_d + q_{\text{B}} + \gamma_u) \left(V_{l[0,\text{-}1]} - V_{l[1,\text{-}1]} + V_{h[0,1]l[0,\text{-}1]} - V_{h[0,1]l[1,\text{-}1]} + p_b\right) = \\ rp_b - (\delta - \eta J) + x - y + \gamma_d (p_b - c) + \gamma_u \left(V_{h[0,1]a[0,\text{-}1]} + V_{a[0,\text{-}1]} + p_b - V_{h[1,0]}\right). \tag{I57}$$

Next, I derive $V_{h[0,1]a[0,\text{-}1]} + V_{a[0,\text{-}1]} + p_b - V_{h[1,0]}$. Adding together (I15), (I19), and rp_b, subtracting (I12), and using $p_b - V_{a[1,0]} = \frac{1}{2}\omega_b$ we get

$$(r + \gamma_d + q_{\text{S}}) \left(V_{h[0,1]a[0,\text{-}1]} + V_{a[0,\text{-}1]} + p_b - V_{h[1,0]}\right) = rp_b - (\delta - \eta J) - y + \gamma_d \frac{1}{2}\omega_b - q_{\text{BO}} \frac{1}{2}\omega_r. \tag{I58}$$

Plug (I57) and (I58) back into (I56) and get:

$$\left(r + \gamma_d + \gamma_u + q_{\text{BO}} \frac{1}{2} \left(1 + \frac{q_{\text{B}}\gamma_u}{(r + \gamma_d + q_{\text{B}} + \gamma_u)(r + \gamma_d + q_{\text{S}})}\right) + q_{\text{LE}} \frac{1}{2}\right) \omega_r \\ = \frac{q_{\text{B}}}{(r + \gamma_d + \gamma_u + q_{\text{B}})} \left(x + \frac{r + \gamma_d + \gamma_u + q_{\text{S}}}{r + \gamma_d + q_{\text{S}}} \left(rp_b - (\delta - \eta J) - y + \gamma_d \frac{1}{2}\omega_b\right)\right). \tag{I59}$$

The expression characterizes ω_r but still has p_b and ω_b. Thus, I derive now, $rp_b - (\delta - \eta J) - y + \gamma_d \frac{1}{2}\omega_b$. Re-arrange the bond price equation (I27) and use $(r + \gamma_d)V_{h[0,0]} = q_{\text{S}} \frac{1}{2}\omega_b$ to get

$$rp_b - (\delta - \eta J) + y + \gamma_d \frac{1}{2}\omega_b = \frac{r + \gamma_d + q_{\text{B}}}{\left(r + \gamma_d + q_{\text{S}} \frac{1}{2} + q_{\text{B}} \frac{1}{2}\right) 2} \frac{1}{2} \left(x + q_{\text{BO}} \frac{1}{2}\omega_r\right). \tag{I60}$$

Add (-2y) to both sides:

$$rp_b - (\delta - \eta J) - y + \gamma_d \frac{1}{2}\omega_b = -2y + \frac{r + \gamma_d + q_{\text{B}}}{\left(r + \gamma_d + q_{\text{S}} \frac{1}{2} + q_{\text{B}} \frac{1}{2}\right) 2} \frac{1}{2} \left(x + q_{\text{BO}} \frac{1}{2}\omega_r\right). \tag{I61}$$

Plugging this back into (I59) gives (I53). □

Proof of Proposition 10.a. To simplify derivations, I focus on parameter conditions such that the entry rate of high type investors is given by an the interior solution. I prove the result in three steps. First, I derive the limits of q_{BO}, q_{S}, q_{B}, and q_{LE}. Second, using the limits derived in step 1, I show that $\lim_{\lambda_b \to \infty} \omega_r \to 0$. Finally, in step 3, using the limits of q's and ω_r, I show that the bond price and volume are higher in the presence of short-selling. $\square$

Step 1 In this step, I derive the limits of q_{BO}, q_{S}, q_{B}, and q_{LE}.

Proof. Consider the limit of q_{BO}. From (I42), in any equilibrium with CDS trading (i.e. when $\nu_l > 0$), $0 < \lim_{\lambda_b \to \infty} q_{\text{BO}} < \infty$.

Next, I show that $\lim_{\lambda_b \to \infty} q_{\text{S}} = \infty$. I prove by contradiction. Suppose $\lim_{\lambda_b \to \infty} q_{\text{B}} = \infty$ and $\lim_{\lambda_b \to \infty} q_{\text{S}} < \infty$. Then, $V_{h[0,0]}$, ν_h, and $\mu_{h[0,0]}$ are zero asymptotically. Combine (I43) and (I44),

$$\mu_{b,\text{B}} = \mu_{\text{h}[0,0]} + \frac{\gamma_u}{q_{\text{S}} + \gamma_d} \frac{\mu_{b,\text{B}}}{\gamma_u + \gamma_d} \left(q_{\text{S}} - \frac{\gamma_d S}{e_b \gamma_d + \mu_{b,\text{B}}} \right). \tag{I62}$$

Using $\mu_{h[0,0]} \to 0$ in (I62),

$$\mu_{b,\text{B}} = \frac{\gamma_u}{q_{\text{S}} + \gamma_d} \frac{\mu_{b,\text{B}}}{\gamma_u + \gamma_d} \left(q_{\text{S}} - \frac{\gamma_d S}{e_b \gamma_d + \mu_{b,\text{B}}} \right).$$

Using the conjecture that $q_{\text{S}} < \infty$, simplifying, and rearranging, we get

$$\gamma_u \gamma_d + (q_{\text{S}} + \gamma_d)\, \gamma_d = -\gamma_u \frac{\gamma_d S}{e_b \gamma_d + \mu_{b,\text{B}}}.$$

This is a contradiction as the left-hand size is finite and positive, while the right-hand is negative when $q_{\text{S}} < \infty$. Suppose, instead, $\lim_{\lambda_b \to \infty} q_{\text{S}} < \infty$ and $\lim_{\lambda_b \to \infty} q_{\text{B}} < \infty$. From (I44),

$$q_{\text{S}} = \frac{\gamma_d S}{e_b \gamma_d + \mu_{b,\text{B}}} + \frac{q_{\text{BO}} \left(S - e_b q_{\text{S}} \right)}{e_b \left(\gamma_u + \gamma_d \right) + \mu_{b,\text{B}}}.$$

Setting $e_b = 0$,

$$q_{\text{S}} = \frac{\gamma_d S}{\mu_{b,\text{B}}} + \frac{q_{\text{BO}} S}{\mu_{b,\text{B}}}.$$

Since $\mu_{b,\text{B}} \to 0$ (from the conjecture that $\lim_{\lambda_b \to \infty} q_{\text{B}} < \infty$), $q_{\text{S}} \to \infty$. This a contradiction. Thus, $\lim_{\lambda_b \to \infty} q_{\text{S}} < \infty$ yields contradictions (whether $\lim_{\lambda_b \to \infty} q_{\text{B}} < \infty$ or $\lim_{\lambda_b \to \infty} q_{\text{B}} = \infty$). So it has to be that $q_{\text{S}} \to \infty$.

Next, consider the limits of q_{B} and q_{LE}. For ν_h to be given by an interior solution, q_{B} has to converge to ∞ and, in particular, at a rate such that

$$\frac{q_{\text{S}} \frac{1}{2}}{r + \gamma_d + q_{\text{S}} \frac{1}{2} + q_{\text{B}} \frac{1}{2}} (x + q_{\text{BO}} \frac{1}{2} \omega_r) = (r + \gamma_d) O_h. \tag{I63}$$

Thus, for an interior solution of ν_h, it has to be that $\lim_{\lambda_b \to \infty} q_{\text{B}} \to \infty$. Solving for $\mu_{b,\text{S}}$ from

(I32),

$$\mu_{b,\text{S}} = \frac{S\left(\gamma_d + \frac{q_{\text{B}} q_{\text{BO}}}{q_{\text{B}}+q_{\text{BO}}+\gamma_d+\gamma_u}\right)}{q_{\text{B}} + \gamma_d}.$$

Taking its limit (in particular, using $\lim_{\lambda_b\to\infty} q_{\text{BO}} < \infty$ and $\lim_{\lambda_b\to\infty} q_{\text{B}} = \infty$), we get $\mu_{b,\text{S}} \to 0$. This implies that $\mu_{h[1,0]} \to S$ and $\infty > \lim_{\lambda_b\to\infty} q_{\text{LE}} = \lambda_r S > 0$. □

Step 2

Proof. Now, I derive the limit of ω_r. From (I63),

$$q_{\text{B}} = \frac{-(r+\gamma_d)O_h\left(r+\frac{1}{2}q_{\text{S}}+\gamma_d\right)+\frac{1}{2}q_{\text{S}}\left(x+\frac{1}{2}q_{\text{BO}}\omega_r\right)}{\frac{1}{2}(r+\gamma_d)O_h}.$$

Substitute it into (I53) and subtract the right-hand side of (I53) from the left-hand side. Then, taking the limit as $q_{\text{S}} \to \infty$, we get

$$2y - 2x + (r+\gamma_d)O_h + \left(r+\gamma_d+\gamma_u+\frac{1}{2}q_{\text{LE}}\right)\omega_r = 0.$$

Solving for ω_r:

$$\omega_r = \frac{2x-2y-(r+\gamma_d)O_h}{r+\gamma_d+\gamma_u+\frac{1}{2}q_{\text{LE}}}. \tag{I64}$$

This is positive using Assumption 1 and the above result that $\infty > \lim_{\lambda_b\to\infty} q_{\text{LE}}$.

As an aside, consider the parameter conditions that would ensure an existence of an equilibrium where $\nu_l > 0$. Plug (I64) back into the value function of $l[0,0]$:

$$\begin{aligned} V_{l[0,0]} &= \frac{1}{r+\gamma_u} q_{\text{LE}} \frac{1}{2} \frac{2x-2y-(r+\gamma_d)O_h}{r+\gamma_d+\gamma_u+\frac{1}{2}q_{\text{LE}}} \\ &= \frac{1}{r+\gamma_u}\lambda_r S \frac{1}{2}\frac{2x-2y-(r+\gamma_d)O_h}{r+\gamma_d+\gamma_u+\frac{1}{2}\lambda_r S}. \end{aligned}$$

Thus, as long as

$$\frac{1}{r+\gamma_u}\lambda_r S \frac{1}{2}\frac{2x-2y-(r+\gamma_d)O_h}{r+\gamma_d+\gamma_u+\frac{1}{2}\lambda_r S} \geq O_l,$$

the entry rate of low-valuation investors is positive: $\nu_l > 0$. □

Step 3

Proof. Using the above limits of q's and ω_r, the bond price in the presence of short-selling limits to:

$$\lim_{\lambda_b\to\infty} p_b = \frac{\delta - \eta J + x - y - (r+\gamma_d)O_h}{r} + q_{\text{BO}}\frac{\frac{1}{2}\omega_r}{r}. \tag{I65}$$

Without short-selling it limits to,

$$\lim_{\lambda_b \to \infty} \hat{p}_b = \frac{\delta - \eta J + x - y - (r + \gamma_d) O_h}{r}. \tag{I66}$$

Above, I showed that $q_{\text{BO}} \frac{\frac{1}{2}\omega_r}{r} > 0$. The bond price, as a result, is higher in the presence of short-selling: $\lim_{\lambda_b \to \infty} p_b > \lim_{\lambda_b \to \infty} \hat{p}_b$.

Consider now the effect on bond volume. Setting $e_b = 0$ in (I44),

$$q_{\text{S}} = \frac{\gamma_d S}{\mu_{b,\text{B}}} + \frac{q_{\text{BO}} (S - \mu_{b,\text{S}})}{\mu_{b,\text{B}}}.$$

Multiply both sides by $\mu_{b,\text{B}}$ and use the result that $\mu_{b,\text{S}} \to 0$,

$$q_{\text{S}} \mu_{b,\text{B}} = \gamma_d S + q_{\text{BO}} S. \tag{I67}$$

In the absence of the repo market, the bond volume converges to the first term $\gamma_d S$. From step 1, the second term $q_{\text{BO}} S$ is positive in the presence of short-selling. Short-selling, as a result, increases the trading volume in the bond market. The second term is the flow of repo market matches. They add to the bond volume because they capture the flow of short-sellers who borrow and (short-)sell instantly. □

Proof of Proposition 10.b. I assume the parameter conditions are such that as $\lambda_r \to \infty$, an equilibrium with short selling exists (i.e. where $\nu_l > 0$) and that the entry rate of high-type investors is given by a positive interior solution. The proof of equilibrium existence is tedious but is available upon request. Then, I prove the result in three steps. In step 1, I derive the limits of q_{BO}, q_{LE}, $\mu_{b,\text{B}}$, and ω_r. In step 2, I show $\mu_{b,\text{B}}$ and $\mu_{b,\text{S}}$ increase in ν_l. Finally, in step 3, I use these results in steps 1 and 2 and show that short-selling increases the bond price and volume. □

Step 1

Proof. In this part of the Proposition, I show that $\lim_{\lambda_r \to \infty} q_{\text{BO}} < \infty$, $\lim_{\lambda_r \to \infty} q_{\text{LE}} = \infty$, $\lim_{\lambda_r \to \infty} \mu_{b,\text{B}} < \infty$, $\lim_{\lambda_r \to \infty} \mu_{h[0,0]} < \infty$, $\lim_{\lambda_r \to \infty} \omega_r = 0$ for $\nu_l \geq 0$ and taking ν_l as given.

Consider the limits of q_{BO} and $\mu_{l[0,0]}$. From (I42):

$$q_{\text{BO}} = \frac{\frac{\nu_l F_l}{\gamma_u}}{\frac{1}{\gamma_u + \gamma_d} (S - e_b q_{\text{S}}) + e_r}. \tag{I68}$$

Lemma 4 showed that $S - e_b q_{\text{S}} > 0$. Thus, $\lim_{\lambda_r \to \infty} q_{\text{BO}} < \infty$, and $\lim_{\lambda_r \to \infty} e_r q_{\text{BO}} = \lim_{\lambda_r \to \infty} \mu_{l[0,0]} = 0$.

Next, consider the limits of $\mu_{b,\text{B}}$ and q_{S}. Setting $e_r = 0$ and using $\lim_{\lambda_r \to \infty} e_r q_{\text{BO}} = 0$ in (I41)-(I44), we get:

$$\nu_h F_h = \gamma_d \mu_{h[0,0]} + q_{\text{S}} \mu_{h[0,0]} \tag{I69}$$

$$\nu_l F_l = \frac{\gamma_u}{\gamma_u + \gamma_d} q_{\text{BO}} (S - e_b q_{\text{S}}) \tag{I70}$$

$$\mu_{b,\text{B}}{=}\mu_{h[0,0]} + \frac{\gamma_u}{q_\text{S}+\gamma_d}\frac{\mu_{b,\text{B}}}{\gamma_u+\gamma_d}\frac{q_\text{BO}\left(S-e_b q_\text{S}\right)}{e_b\left(\gamma_u+\gamma_d\right)+\mu_{b,\text{B}}} \tag{I71}$$

$$q_\text{S}{=}\frac{\gamma_d S}{e_b\gamma_d+\mu_{b,\text{B}}} + \frac{q_\text{BO}\left(S-e_b q_\text{S}\right)}{e_b\left(\gamma_u+\gamma_d\right)+\mu_{b,\text{B}}}. \tag{I72}$$

Solving for q_BO and $\mu_{h[0,0]}$ from the first two and substituting them into the other two, we get

$$\mu_{b,\text{B}}{=}\frac{\nu_h F_h}{\gamma_d+q_\text{S}} + \frac{\mu_{b,\text{B}}}{q_\text{S}+\gamma_d}\frac{\nu_l F_l}{e_b\left(\gamma_u+\gamma_d\right)+\mu_{b,\text{B}}}, \tag{I73}$$

$$q_\text{S}{=}\frac{\gamma_d S}{e_b\gamma_d+\mu_{b,\text{B}}} + \frac{\nu_l F_l\left(\gamma_u+\gamma_d\right)}{\gamma_u\left(e_b\left(\gamma_u+\gamma_d\right)+\mu_{b,\text{B}}\right)}. \tag{I74}$$

Solving for q_S from (I73), we get

$$q_\text{S}{=}\frac{\nu_h F_h}{\mu_{b,\text{B}}} + \frac{\nu_l F_l}{e_b\left(\gamma_u+\gamma_d\right)+\mu_{b,\text{B}}} - \gamma_d. \tag{I75}$$

Substituting this into (I74) and simplifying, we get

$$\frac{\nu_h F_h}{\gamma_d} - \mu_{b,\text{B}}\left(1+\frac{S}{\mu_{b,\text{B}}+e_b\gamma_d}+\frac{\nu_l F_l}{\gamma_u\left(\mu_{b,\text{B}}+e_b\left(\gamma_d+\gamma_u\right)\right)}\right) = 0. \tag{I76}$$

For $\mu_{b,\text{B}} = 0$, the left-hand side is positive, while for a large $\mu_{b,\text{B}}$, the left-hand side is negative. The left-hand side strictly decreases in $\mu_{b,\text{B}}$. Thus, it has a unique, positive, and finite solution in $\mu_{b,\text{B}}$. In turn, from (I74), $0 < \lim q_\text{S} < \infty$.

Finally, consider the limits of q_LE and ω_r.

$$\lim_{\lambda_r\to\infty} q_\text{LE} = \lim_{\lambda_r\to\infty}\lambda_r\mu_{h[1,0]} = \lim_{\lambda_r\to\infty}\lambda_r\left(S-\mu_{b,\text{S}}\right) = \infty. \tag{I77}$$

Moreover, $\lim_{\lambda_r\to\infty} q_\text{BO} < \infty$ and $\lim_{\lambda_r\to\infty} q_\text{LE} = \infty$ imply that $\lim_{\lambda_r\to\infty}\omega_r = 0$ from (I53). □

Step 2 In this part of the Proposition, I show that $\mu_{b,\text{B}}$ and $\mu_{b,\text{S}}$ strictly increase with ν_l for $\nu_l \geq 0$.

Proof. Combine the value functions, the definition of ω_r, and that $V_{h[0,0]} = O_h$ to get

$$(r+\gamma_d)O_h = q_\text{S}\frac{x+q_\text{BO}\frac{1}{2}\omega_r}{\left(r+\gamma_d+q_\text{S}\frac{1}{2}+q_\text{B}\frac{1}{2}\right)}. \tag{I78}$$

Then (I42)-(I44), (I53), and (I78) characterize a set of five equations and five unknowns, q_S, $\mu_{b,\text{B}}$, $\mu_{h[0,0]}$, q_BO, and ω_r as implicit functions of ν_l. Set $e_r = 0$ and the limits of q_BO and ω_r in (I42), (I44), and (I78):

$$\nu_l F_l = \frac{\gamma_u}{\gamma_u+\gamma_d}q_\text{BO}\left(S-e_b q_\text{S}\right) \tag{I79}$$

$$q_\text{S}{=}\frac{\gamma_d S}{e_b\gamma_d+\mu_{b,\text{B}}} + \frac{q_\text{BO}\left(S-e_b q_\text{S}\right)}{e_b\left(\gamma_u+\gamma_d\right)+\mu_{b,\text{B}}} \tag{I80}$$

$$(r+\gamma_d)O_h = q_{\mathrm{S}}\frac{x}{\left(r+\gamma_d+q_{\mathrm{S}}\frac{1}{2}+q_{\mathrm{B}}\frac{1}{2}\right)}. \tag{I81}$$

From (I79):

$$q_{\mathrm{BO}}\left(S-e_b q_{\mathrm{S}}\right)=\frac{\nu_l F_l}{\gamma_u}\left(\gamma_u+\gamma_d\right).$$

Substitute it into (I80). Together with (I81), we have two equations that characterize $\mu_{b,\mathrm{B}}$ and q_{S} as implicit functions of ν_l:

$$q_{\mathrm{S}}=\frac{\gamma_d S}{e_b\gamma_d+\mu_{b,\mathrm{B}}}+\frac{\frac{\nu_l F_l}{\gamma_u}\left(\gamma_u+\gamma_d\right)}{e_b\left(\gamma_u+\gamma_d\right)+\mu_{b,\mathrm{B}}} \tag{I82}$$

$$(r+\gamma_d)O_h = q_{\mathrm{S}}\frac{x}{\left(r+\gamma_d+q_{\mathrm{S}}\frac{1}{2}+q_{\mathrm{B}}\frac{1}{2}\right)}. \tag{I83}$$

Substitute the expression for q_{S} from (I82) into (I83) and get:

$$\begin{aligned}(r+\gamma_d)O_h&\left[r+\gamma_d+\left(\frac{\gamma_d S}{e_b\gamma_d+\mu_{b,\mathrm{B}}}+\frac{\frac{\nu_l F_l}{\gamma_u}\left(\gamma_u+\gamma_d\right)}{e_b\left(\gamma_u+\gamma_d\right)+\mu_{b,\mathrm{B}}}\right)\frac{1}{2}+q_{\mathrm{B}}\frac{1}{2}\right]\\&=\left[\frac{\gamma_d S}{e_b\gamma_d+\mu_{b,\mathrm{B}}}+\frac{\frac{\nu_l F_l}{\gamma_u}\left(\gamma_u+\gamma_d\right)}{e_b\left(\gamma_u+\gamma_d\right)+\mu_{b,\mathrm{B}}}\right]x.\end{aligned} \tag{I84}$$

Applying the Implicit Function Theorem,

$$\frac{\partial\mu_{b,\mathrm{B}}}{\partial\nu_l}=\frac{\frac{(\gamma_d+\gamma_u)(r+\gamma_d+\frac{1}{2}\mu_{b,\mathrm{B}}\lambda_b)}{\gamma_d+\gamma_u+\mu_{b,\mathrm{B}}\lambda_b}}{\lambda_b\left(\frac{S\gamma_d}{\gamma_d+\lambda_b\mu_{b,\mathrm{B}}}\frac{r+\gamma_d+\frac{1}{2}\gamma_d+\lambda_b\mu_{b,\mathrm{B}}}{(\gamma_d+\lambda_b\mu_{b,\mathrm{B}})}+\frac{\frac{\nu_l F_l}{\gamma_u}(\gamma_d+\gamma_u)}{\gamma_d+\gamma_u+\lambda_b\mu_{b,\mathrm{B}}}\frac{r+\gamma_d+\frac{1}{2}\gamma_d+\frac{1}{2}\gamma_u+\lambda_b\mu_{b,\mathrm{B}}}{\left(\gamma_d+\gamma_u+\lambda_b\mu_{b,\mathrm{B}}\right)}\right)}.$$

I showed earlier that population masses are positive for any ν_h and ν_l. Thus, the right-hand side is positive, and $\mu_{b,\mathrm{B}}$ strictly increases in ν_l. From (I83), q_{S} strictly increases in $\mu_{b,\mathrm{B}}$ and, hence, in ν_l. Both $\mu_{b,\mathrm{S}}$ and $\mu_{b,\mathrm{B}}$, as a result, increase in ν_l. □

Step 3 In this step, I use the results from steps 1 and 2 and show that short-selling increases the price and the trading volume of the bond.

Proof. As $\lambda_r\to\infty$, using the earlier result that $\lim_{\lambda_r\to\infty}\mu_{b,\mathrm{B}}<\infty$, $\lim_{\lambda_r\to\infty}\omega_r=0$, and $\lim_{\lambda_r\to\infty}q_{\mathrm{BO}}=\lim_{\lambda_r\to\infty}\lambda_r\mu_{l[0,0]}<\infty$, the bond price with short-selling limits to

$$\lim_{\lambda_r\to\infty}p_b=\frac{(\delta-\eta J)+x-y-(r+\gamma_d)O_h}{r}-\frac{(r+2\gamma_d)}{r}\frac{1}{2}\frac{(x-(r+\gamma_d)O_h)}{r+\gamma_d+\lambda_b\mu_{b,\mathrm{B}}\frac{1}{2}} \tag{I85}$$

The bond price without short-selling is

$$p_b=\frac{(\delta-\eta J)+x-y-(r+\gamma_d)O_h}{r}-\frac{(r+2\gamma_d)}{r}\frac{1}{2}\frac{(x-(r+\gamma_d)O_h)}{r+\gamma_d+\lambda_b\mu_{b,\mathrm{B}}\frac{1}{2}} \tag{I86}$$

In the limit then, short-selling affects the bond price only through the second term (the illiquidity discount) and, in particular, through $\mu_{b,\text{B}}$ that is in the denominator of the illiquidity discount term. In step 2, I showed that $\mu_{b,\text{B}}$ and q_{S} are implicit functions of ν_l and that they strictly increase in ν_l for $\nu_l \geq 0$. This implies that $\mu_{b,\text{B}}$ and q_{S} are larger in the presence of short-selling because $\mu_{b,\text{B}}$ and q_{S} evaluated at $\nu_l > 0$ and $\nu_l = 0$ correspond to their levels in the presence and absence of short-selling, respectively. Larger $\mu_{b,\text{B}}$, in turn, implies that $\lim_{\lambda_r \to \infty} d_b < \hat{d}_b$, and, hence, $\lim_{\lambda_r \to \infty} p_b > \hat{p}_b$. The bond volume also increases in the presence of short-selling because the trading volume is the product of $\mu_{b,\text{B}}$ and q_{S}. □

J Short-Selling and CDS

In this section, I present the model with both bond short-selling and CDS trading. The equilibrium variables and conditions are the same as in the previous section with just short-selling except for few changes. Define the meeting intensities in the CDS market as

$$q_{c,\text{B}} \equiv \lambda_c \mu_{l[0,0]}$$

$$q_{c,\text{S}} \equiv \lambda_c \mu_{h[0,0]}$$

I denote the CDS position of an agent on the third position of her portfolio: $[\theta_b, \theta_r, \theta_c]$. If an agent has no CDS position, I continue to denote her position with just $[\theta_b, \theta_r]$ as in the previous section. Then, the inflow-outflow equations characterizing $\mu_{h[0,0]}$ and $\mu_{l[0,0]}$ change as:

$$\nu_h F_h + \gamma_u \mu_{l[0,0,\text{-}1]} = \gamma_d \mu_{h[0,0]} + (q_{\text{S}} + q_{c,\text{B}})\, \mu_{h[0,0]} \tag{J87}$$

$$\nu_l F_l + \gamma_d \mu_{l[1,\text{-}1]} + \gamma_d \mu_{l[0,\text{-}1]} + \gamma_d \mu_{l[0,0,\text{-}1]} = \gamma_u \mu_{l[0,0]} + (q_{\text{LE}} + q_{c,\text{S}})\, \mu_{l[0,0]} \tag{J88}$$

Two additional inflow-outflow equations characterize the masses of investors who have sold CDS ($\mu_{h[0,0,1]}$) and who have bought CDS ($\mu_{l[0,0,\text{-}1]}$), respectively:

$$q_{c,\text{B}} \mu_{h[0,0]} = (\gamma_u + \gamma_d)\, \mu_{h[0,0,1]} \tag{J89}$$

$$q_{c,\text{B}} \mu_{h[0,0]} = (\gamma_u + \gamma_d)\, \mu_{l[0,0,\text{-}1]} \tag{J90}$$

The additional market clearing condition is:

$$\mu_{h[0,0,1]} = \mu_{l[0,0,\text{-}1]}. \tag{J91}$$

Similarly, value functions of $h[0,0]$ and $l[0,0]$ change as

$$rV_{h[0,0]} = \gamma_d \left(0 - V_{h[0,0]}\right) + q_{\text{S}} \left(V_{h[1,0]} - V_{h[0,0]} - p_b\right) + q_{c,\text{B}} \left(V_{h[0,0,1]} - V_{h[0,0]}\right) \tag{J92}$$

$$rV_{l[0,0]} = \gamma_u \left(0 - V_{l[0,0]}\right) + q_{\text{LE}} \left(V_{l[1,\text{-}1]} - V_{l[0,0]}\right) + q_{\text{cs}} \left(V_{l[0,0,\text{-}1]} - V_{l[0,0]}\right) \tag{J93}$$

Two additional value functions are

$$rV_{h[0,0,1]} = p_c - \eta J + x - y + \gamma_d \left(-T_{\text{S}} - V_{h[0,0,1]}\right) \tag{J94}$$

$$rV_{l[0,0,\text{-}1]} = \eta J + x - y - p_c + \gamma_u \left(-T_{\text{B}} - V_{l[0,0,\text{-}1]}\right) \tag{J95}$$

As in the main environment, the CDS spread is given by

$$V_{h[0,0,1]} - V_{h[0,0]} = \frac{1}{2}\left(V_{h[0,0,1]} - V_{h[0,0]} + V_{l[0,0,\text{-}1]} - V_{l[0,0]}\right). \tag{J96}$$

I compare the two environments without CDS and with CDS based on what the entry rates converge to once CDS is introduced. In particular, I use the following algorithm. I, first, let the entry rate of low types to adjust to CDS introduction but keeping the entry rate of high types fixed. Next, I allow the entry rate of high types to adjust to both CDS introduction and the change in the low types' entry rate that CDS introduction induces in the first step. I repeat this process until both entry rates are at equilibrium. Whether low types adjust their entry rate first then high types adjust or vice versa gives the same result. The result is as follows. CDS introduction increases the value of entering for both long and short investors. Short investors, as a result, enter at a higher rate (assuming that their entry rate was given by an interior solution before) until their entry rate converges to the maximum possible: a corner solution. In response to both CDS introduction itself and the resulting increase in low types' entry rate, long investors also enter at a higher equilibrium rate.

K Endogenous Search Efforts

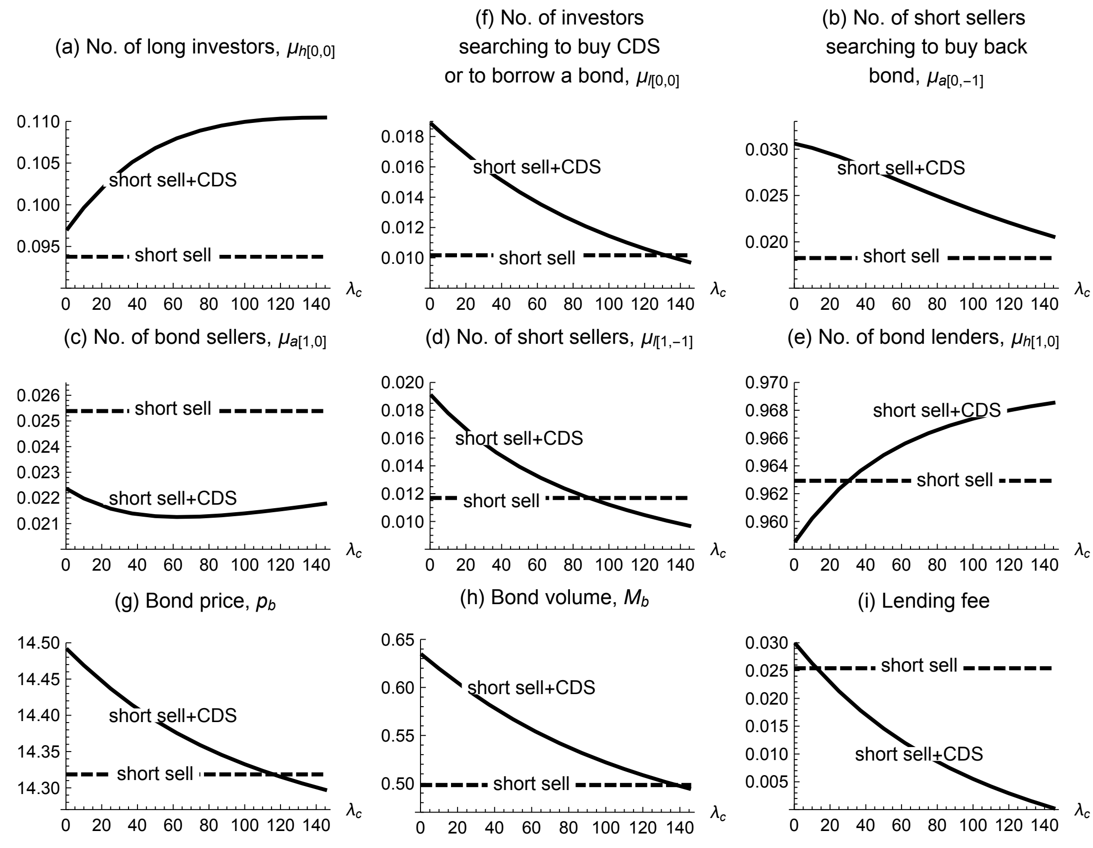
I setup the environment so that all investors who want to rebalance their asset position choose their search effort optimally. In particular, in addition to long investors, bond sellers, $a[1,0]$, and CDS buyers, $l[0,0]$, also choose their search effort. As a result, the total bond volume is $M_b = (\lambda_{b,\text{S}}+\lambda_{b,\text{B}})\mu_{b,\text{S}}\mu_{b,\text{B}}$, while the CDS volume is $M_c = (\lambda_{c,\text{S}}+\lambda_{c,\text{B}})\mu_{c,\text{S}}\mu_{c,\text{B}}$. Then, only for the proof of Proposition 11, I set the search efforts of the short size to zero: $\lambda_{l[0,0]} = 0$ and $\lambda_{a[1,0]} = 0$. I assume throughout that the parameter conditions are such that the entry rate of high type investors is given by an interior solution with and without CDS.

In the paper, I focus on the simpler environment with only the long side choosing search efforts because endogenizing search efforts on both sides of the market complicates derivations significantly. Given the intractability, in Proposition K.1, I only characterize the parameter conditions under which introducing CDS still increases bond market liquidity. Numerically, the results when both sides choose search intensities versus when only one side chooses are analogous.

The HJB equations are derived analogously as in the environment with exogenous

Figure 2: The Marginal Effect of CDS When Investors Already Short-Sell

The figures illustrate the results discussed in Section 4.2. The dashed lines plot the masses of actively searching investors (plots (a)-(e)), the bond price, bond volume, and the repo contract lending fee for an environment in which investors short-sell but do not trade CDS. The solid lines do the same for an environment in which investors both short-sell and trade CDS. They are plotted as functions of CDS market matching efficiency (λ_c). The relevant model is in online Appendix J. The parameter values used to generate the plots are $r = 0.04$, $\alpha = 0.05$, $\sigma_e = 540$, $\rho = 1$, $\delta = 1$, $\eta = 0.0009$, $J = 670$, $F_h = 2.5$, $F_l = 0.18$, $\gamma_d = 0.35$, $\gamma_u = 0.44$, $\lambda_b = 120$, $\lambda_r = 17$, $O_h = 0.4$, and $O_l = 0.925$. I focus on parameter conditions such that short-selling exists in equilibrium both before and after CDS introduction. For example, as $\lambda_c \to 145$, the gains from repo contract goes to zero, so for $\lambda_c > 145$, investors stop short-selling and start using CDS only.



search intensities. As a result, the value functions are characterized by:

$$rV_\tau = -c(\lambda_{b,\tau}, \lambda_{c,\tau}) + ((\delta - \eta J) - x_\tau)\,\theta_b - y|\theta_b| + (p_c - (\eta J + x_\tau))\,\theta_c - y|\theta_c| \quad \text{(K1)}$$
$$+ \sum_{k=1}^{K(\tau)} \gamma(k,\tau) \max_{\tau' \in \mathcal{T}(\tau,k)} \frac{1}{r\alpha}\left(1 - e^{-r\alpha(V_{\tau'} - V_\tau + P(\tau,\tau'))}\right),$$

where $c(\lambda_{b,\tau}, \lambda_{c,\tau})$ is the agent's total search cost. Simplifying (K1) further, for the non-searcher agent types, the value functions are identical to (B3), (B5), and (B6). The value functions of the searcher agents include the cost of their search efforts:

$$rV_{l[0,0]} = \gamma_u(0 - V_{l[0,0]}) - c(0, \lambda_{l[0,0]}) + \frac{M_c}{\mu_{l[0,0]}}\frac{1}{2}\omega_c \quad \text{(K2)}$$

$$rV_{h[0,0]} = \gamma_d(0 - V_{h[0,0]}) - c(\lambda_{b,h[0,0]}, \lambda_{c,h[0,0]}) + \frac{M_b}{\mu_{h[0,0]}}\frac{1}{2}\omega_b + \frac{M_c}{\mu_{h[0,0]}}\frac{1}{2}\omega_c. \quad \text{(K3)}$$

$$rV_{a[1,0]} = (\delta - \eta J) - y - c(\lambda_{a[1,0]}, 0) + \frac{M_b}{\mu_{a[1,0]}}\frac{1}{2}\omega_b \quad \text{(K4)}$$

The volume of matches in the bond and the CDS market are

$$M_b = (\lambda_{b,h[0,0]} + \lambda_{a[1,0]})\mu_{a[1,0]}\mu_{h[0,0]}$$

$$M_c = (\lambda_{c,h[0,0]} + \lambda_{l[0,0]})\mu_{l[0,0]}\mu_{h[0,0]}$$

The first order conditions with respect to the search efforts are:

$$2c_0\lambda_{b,h[0,0]} = \mu_{a[1,0]}\left(-p_b + V_{h[1,0]} - V_{h[0,0]}\right) \quad \text{(K5)}$$

$$2c_0\lambda_{c,h[0,0]} = \mu_{l[0,0]}\left(V_{h[0,1]} - V_{h[0,0]}\right) \quad \text{(K6)}$$

$$2c_0\lambda_{l[0,0]} = \mu_{h[0,0]}\left(V_{l[0,\text{-}1]} - V_{l[0,0]}\right) \quad \text{(K7)}$$

$$2c_0\lambda_{a[1,0]} = \mu_{h[0,0]}\left(p_b - V_{a[1,0]}\right). \quad \text{(K8)}$$

The equilibrium equations are analogous to the baseline environment plus (K5)-(K8) that pin down the optimal search efforts.

Proof of Lemma 1. Combining (B3) and (K3), the reservation value of the buyer is

$$r\left(V_{h[1,0]} - V_{h[0,0]}\right) = (\delta - \eta J + x - y) - \gamma_d\omega_b + c(\lambda_{b,h[0,0]}, \lambda_{c,h[0,0]}) - q_{\text{bs}}\frac{1}{2}\omega_b - q_{\text{cB}}\frac{1}{2}\omega_c.$$

where $q_{\text{bs}} = \frac{M_b}{\mu_{h[0,0]}}$ and $q_{\text{cB}} = \frac{M_c}{\mu_{h[0,0]}}$. Using (K3), we can write it as

$$r\left(V_{h[1,0]} - V_{h[0,0]}\right) = (\delta - \eta J + x - y) - (r + \gamma_d)\,V_{h[0,0]} - \gamma_d\omega_b.$$

Next, consider the seller's reservation value. Combining (B3), (B4), and (K3), we get

$$(r + \gamma_d)\,\omega_b = x + c(\lambda_{b,h[0,0]}, \lambda_{c,h[0,0]}) + c(\lambda_{a[1,0]}, 0) - q_{\text{bs}}\frac{1}{2}\omega_b - q_{\text{bB}}\frac{1}{2}\omega_b - q_{\text{cB}}\frac{1}{2}\omega_c. \quad \text{(K9)}$$

where $q_{b\text{B}} = \frac{M_b}{\mu_{a[1,0]}}$. Using (K3), this becomes

$$(r + \gamma_d)\,\omega_b = x - (r + \gamma_d)\,V_{h[0,0]} + c(\lambda_{a[1,0]}, 0) - q_{b\text{B}}\frac{1}{2}\omega_b. \tag{K10}$$

From (K10),

$$q_{b\text{B}}\frac{1}{2}\omega_b - c(\lambda_{a[1,0]}, 0) = x - (r + \gamma_d)\,\omega_b - (r + \gamma_d)\,V_{h[0,0]}.$$

Plug it into the reservation value of the seller and get:

$$rV_{a[1,0]} = \delta - \eta J + x - y - (r + \gamma_d)\,V_{h[0,0]} - (r + \gamma_d)\,\omega_b.$$

Combining the reservation values of the bond buyer and the seller and using $V_{h[0,0]} = O_h$,

$$rp_b = \delta - \eta J + x - y - (r + \gamma_d)\,O_h - \frac{1}{2}\,(r + 2\gamma_d)\,\omega_b. \tag{K11}$$

From (K10) and the fact $c(\lambda_{a[1,0]}, 0) = 0$ when $\lambda_{a[1,0]} = 0$,

$$\omega_b = \frac{x - (r + \gamma_d)\,O_h}{\left(r + \gamma_d + q_{b\text{B}}\frac{1}{2}\right)}. \tag{K12}$$

Plugging it into (K11), the bond price and the illiquidity discount have the same characterization as in the environment with exogenous search efforts. □

The results so far apply to both the environment in which only the long side chooses their search effort (i.e. $\lambda_{b,h[0,0]}$ and $\lambda_{c,h[0,0]}$ are endogenous, and $\lambda_{a[1,0]} = 0$ and $\lambda_{l[0,0]} = 0$) and the environment in which both sides of the market choose their search effort (i.e. $\lambda_{b,h[0,0]}$, $\lambda_{c,h[0,0]}$, $\lambda_{a[1,0]}$, and $\lambda_{l[0,0]}$ are endogenous). In Proposition 11, only the long side chooses its search efforts. Then, Proposition K.1 allows both sides to choose their search effort and characterizes the parameter conditions under which liquidity spillover effect arises.

Proof of Proposition 11. Consider the set of three equations and three unknowns $\{\mu_{a[1,0]}, \mu_{h[0,0]}, \omega_b\}$:

$$\frac{1}{4c_0}\mu_{h[0,0]}\mu_{a[1,0]}^2\omega_b = \gamma_d(S - \mu_{a[1,0]}) \tag{K13}$$

$$(r + \gamma_d)\omega_b = x - \frac{\mu_{a[1,0]}\left(\mu_{a[1,0]} + 2\mu_{h[0,0]}\right)\omega_b^2}{16c_0} - A \tag{K14}$$

$$(r + \gamma_d)O_h = \frac{\left(\mu_{a[1,0]}\omega_b\right)^2}{16c_0} + A, \tag{K15}$$

where

$$A = \frac{M_c}{\mu_{h[0,0]}}\frac{1}{2}\omega_c - c_0(\lambda_{c,h[0,0]})^2.$$

Eq. (K13) comes from combining the inflow-outflow equations with (K5), (K14) comes from combining (K9) with (K5), and (K15) combines (K3) with (K5)-(K6). With and without CDS, $\{\mu_{a[1,0]}, \mu_{h[0,0]}, \omega_b\}$ is the solution to (K13)–(K15) with $A > 0$ and $A = 0$, respectively.

Applying the Implicit Function Theorem,

$$\frac{\partial \omega_b}{\partial A} = -\frac{4c\left(4c_0\gamma_d + \mu_{a[1,0]}\mu_{h[0,0]}\omega_b\right)}{\mu_{a[1,0]}^2\omega_b\left(2c_0\left(2r+3\gamma_d\right) + \mu_{a[1,0]}\mu_{h[0,0]}\omega_b\right)}$$

$$\frac{\partial \mu_{a[1,0]}}{\partial A} = -\frac{4c\left(8c_0(r+\gamma_d) + \mu_{a[1,0]}\mu_{h[0,0]}\omega_b\right)}{\mu_{a[1,0]}\omega_b^2\left(2c_0\left(2r+3\gamma_d\right) + \mu_{a[1,0]}\mu_{h[0,0]}\omega_b\right)}$$

Thus, ω_b decreases in the presence of the CDS market. This implies that the bond illiquidity discount decreases and the bond price increases. The bond volume is given by: $M_b = \gamma_d(S - \mu_{a[1,0]})$. Since the mass of bond sellers decreases, the bond volume increases.

Since both ω_b and $\mu_{a[1,0]}$ decrease with the introduction of CDS, a direct corollary is that, for a long investor, the marginal benefit of searching in the bond market decreases. A long investor, as a result, lowers its search effort in the bond market. □

Proposition K.1. *Consider the environment where both sides of the market choose their search efforts (i.e.* $\lambda_{b,h[0,0]}$*,* $\lambda_{c,h[0,0]}$*,* $\lambda_{a[1,0]}$*, and* $\lambda_{l[0,0]}$ *are endogenous). Suppose (K19) holds. Then,* $d_b < \hat{d}_b$*, and* $M_b > \hat{M}_b$*.*

Proof. Now the equations equivalent to (K13)–(K15) are

$$\frac{1}{4c_0}(\mu_{a[1,0]} + \mu_{h[0,0]})\mu_{a[1,0]}\mu_{h[0,0]}\omega_b = \gamma_d(S - \mu_{a[1,0]}) \tag{K16}$$

$$(r+\gamma_d)\omega_b = x - \frac{\left(\left(\mu_{a[1,0]} + \mu_{h[0,0]}\right)^2 + 2\mu_{h[0,0]}\mu_{a[1,0]}\right)\omega_b^2}{16c_0} - A \tag{K17}$$

$$(r+\gamma_d)O_h = \frac{\mu_{a[1,0]}\left(\mu_{a[1,0]} + 2\mu_{h[0,0]}\right)\omega_b{}^2}{16c_0} + A, \tag{K18}$$

where

$$A = \frac{M_c}{\mu_{h[0,0]}}\frac{1}{2}\omega_c - c_0(\lambda_{c,h[0,0]})^2.$$

Eq. (K16) comes from combining the inflow-outflow equations with (K5), (K17) comes from combining (K9) with (K5), and (K18) combines (K3) with (K5)-(K6).

Applying the Implicit Function Theorem:

$$\frac{\partial \mu_{a[1,0]}}{\partial A} = -\frac{4c_0\left(8c_0\left(\mu_{a[1,0]} + 2\mu_{h[0,0]}\right)(r+\gamma_d) + \mu_{h[0,0]}\left(\mu_{a[1,0]}^2 + 3\mu_{a[1,0]}\mu_{h[0,0]} + \mu_{h[0,0]}^2\right)\omega_b\right)}{B}$$

$$\frac{\partial \omega_b}{\partial A} = -\frac{4c_0\left(4c_0\left(\mu_{a[1,0]} + \mu_{h[0,0]}\right)\gamma_d + \mu_{h[0,0]}\left(\mu_{a[1,0]}^2 + \mu_{a[1,0]}\mu_{h[0,0]} + \mu_{h[0,0]}^2\right)\omega_b\right)}{B},$$

where

$$B \equiv -16c_0^2(r+\gamma_d)\gamma_d\omega_b + 2c_0\left(\mu_{a[1,0]}^2 + \mu_{a[1,0]}\mu_{h[0,0]} + \mu_{h[0,0]}^2\right)(2r+3\gamma_d)\,\omega_b^2$$
$$+\,\mu_{h[0,0]}\left(\mu_{a[1,0]} + \mu_{h[0,0]}\right)\left(\mu_{a[1,0]}^2 + \mu_{a[1,0]}\mu_{h[0,0]} + \mu_{h[0,0]}^2\right)\omega_b^3$$

Thus, $\frac{\partial\omega_b}{\partial A} < 0$ and $\frac{\partial\mu_{a[1,0]}}{\partial A} < 0$ if

$$B > 0. \tag{K19}$$

Using arguments analogous to the previous proposition, $\frac{\partial\omega_b}{\partial A} < 0$ and $\frac{\partial\mu_{a[1,0]}}{\partial A} < 0$ imply that bond market liquidity and the bond price are higher in the presence of the CDS market. □

L Covered CDS

Covered CDS positions are positions where a bond owner purchases CDS: $[\theta_b, \theta_c] = [1, \text{-}1]$. Lemma 6 shows that such positions do not arise in equilibrium.

Lemma 6. *In the equilibrium of Proposition 2, the mass of agents with covered CDS positions,* $[\theta_b, \theta_c] = [1, \text{-}1]$*, is zero.*

Proof of Lemma 6. This is a corollary from Proposition 2 proof (in particular, the last step). □

The intuition is as follows. Between the high- and average-valuation bond owners, the average type bondholders are the only potential CDS buyers (hence, covered CDS buyers). If an average-type investor buys CDS from a high-valuation investor, the gains from trade is proportional to $x - 2y - (r+\gamma_d)O_h$. But by Assumption 1 this is negative. That is, when high and average types enter a CDS contract, the holding cost both sides incur together with the entry cost outweigh the total hedging benefit. Thus, bondholders do not buy CDS, and only low-valuation investors buy CDS.

Changing the environment so that covered CDS positions do arise would only change the benchmark environment. Whether bondholders buy CDS or not in the benchmark, the marginal effect of naked CDS positions relative to the benchmark is likely to be the same.