Student Name : _____ Student Number: _____

MIDTERM ONE

READ THE DIRECTIONS
ONCE YOU START, MAKE SURE YOUR EXAM HAS 5 PAGES (including the coverpage)

Page 1	14	
Page 2	13	
Page 3	12	
Page 4	11	
Total	**50**	

- Show all work for full credit.
- BOX in your answer to each question.
- Unless otherwise indicated always use EXACT numbers. (i.e $\sqrt{\pi}$ instead of 1.77).
- You may use the fact that $\lim_{x\to 0} \frac{\sin x}{x} = 1$.
- You may use a scientific calculator during this examination; graphing calculators and other electronic devices are not allowed and should be turned off for the duration of the exam.
- If you use trial-and-error, a guess-and-check method, or numerical approximation when an exact method is available, you will not receive full credit.
- You may use one double-sided, hand-written, 8.5 by 11 inch page of notes.
- You have 60 minutes to complete the exam - so, spend on average $\leq$ 15 minutes per page.

1. Evaluate the following limits. If the limit does not exist write 'DNE' and explain why. Showing no work, or just applying a formula will receive zero points.

(a) (3 points) $\lim_{r\to 1} \dfrac{r^2}{(r-1)^3}$

$DNE.$

(b) (3 points) $\lim_{x\to\infty} \dfrac{\sqrt[3]{8x^6+3x^2-7}+x^2}{3x^2-7x}$

$\dfrac{\sqrt[3]{8}+1}{3} = 1$

(c) (4 points) $\lim_{x\to 0} \dfrac{\pi \sin^3(x)}{x(1-\cos(x))}$

π

(d) (4 points) $\lim_{h\to 0} \dfrac{(\frac{\pi}{2}+h)\cos(\frac{\pi}{2}+h) - (\frac{\pi}{2})\cos(\frac{\pi}{2})}{h}$. *Hint: Limit definition of derivative.*

$\dfrac{d}{dx}(x\cos(x))|_{x=\pi/2} = \cos(\pi/2) - (\pi/2)\sin(\pi/2) = -\pi/2.$

2. Using the derivative rules (Power, Product, Quotient, etc...) find the derivatives of the following functions.

   You do not need to simplify your answers!

   (a) (4 points) $f(x) = (\sqrt{x}+1)e^x \cos x$

   $$\frac{1}{2\sqrt{x}}e^x \sin x + (e^x \cos x - \sin x e^x)(\sqrt{x}+1).$$

   (b) (4 points) $g(x) = \dfrac{\sin x + \tan x}{x^e + e^x}$

   $$\frac{(\cos x + \sec^2 x)(x^e + e^x) - (ex^{e-1} + e^x)(\sin x + \tan x)}{(x^e + e^x)^2}.$$

3. (5 points) $f(x) = \dfrac{1}{\sqrt{x+1}}$. Using the **LIMIT DEFINITION OF THE DERIVATIVE** find $f'(1)$.

   $$f'(1) = -\frac{1}{2 \cdot 2^{3/2}}$$

4. Let $F(x) = \begin{cases} xe^x, & x > 1 \\ \frac{e}{2}x^2 + \frac{1}{2}e^x, & x \leq 1 \end{cases}$.

(a) (3 points) Is F continuous at $x = 1$? *You must show your work and explain with a sentence.*
Since the limits $\lim_{x\to 1^+} F(x) = e = \lim_{x\to 1^-} F(x)$ agree we know that F is continuous at $x = 1$.

(b) (4 points) Is F differentiable at $x = 1$? *You must show your work and explain with a sentence.*
Since the derivative from the left ($2e$) is not equal to the derivative from the right ($\frac{3}{2}e$), F is not differentiable at $x = 1$.

5. Matt is grinding a rail on his skateboard with position $p(t) = \frac{1}{3}t^3 - 2t^2 + 3t + 10$.

(a) (2 points) Find Matt's average velocity on the time interval $[0, 1]$.
$\frac{\frac{1}{3}-1+1+10-10}{1-0} = \frac{1}{3}$.

(b) (3 points) Find all times when Matt is neither moving left nor right.
t=3, t=1

6. (2 points) Write the equation of the tangent line to the circle $(x-1)^2+y^2=4$ at $(2,\sqrt{3})$?

$$y-\sqrt{3}=-\frac{2-1}{\sqrt{3}-0}(x-2).$$

7. Let $f(x)=2x^2$ and $g(x)=x^2+1$.

   (a) (2 points) What is the slope and y-intercept of the tangent line to $f(x)$ at $x=a$.

$$m_f=4a, b_f=-2a^2$$

   (b) (2 points) What is the slope and y-intercept of the tangent line to $g(x)$ at $x=b$.

$$m_g=2b, b_g=-b^2+1$$

   (c) (2 points) If a line is simultaneously tangent to $f(x)$ at $x=a$ and $g(x)$ at $x=b$. Write two equations involving only a and b that a and b must satisfy.

$$2a^2=2b$$
$$2a^2=b^2+1$$

(d) (3 points) Find all values of a and b so that the tangent line to $f(x)$ at $x = a$ is simultaneously tangent to $g(x)$ at $x = b$.

$$\left(\frac{1}{\sqrt{2}}, \frac{2}{\sqrt{2}}\right) \qquad \text{and} \qquad \left(-\frac{1}{\sqrt{2}}, -\frac{2}{\sqrt{2}}\right).$$