Outline

Stuff For Your Note Sheet

1. Formula for Parametrizing a circle $y(t) = A\sin(\omega t - \theta) + B$: $x(t) = A\cos(\omega t - \theta) + C$
2. Trig Identities
   (a) Double and Half Angle Stuff
   (b) Wierd Sum Rules
   (c) Derivatives of arctrig functions
   (d) Law of Sines
   (e) Law of Cosines
   (f) Reference Triangles
3. Definitions of Continuity and Limit
4. Limit Definition of Derivative
5. Parametric First and Second Derivative Definitions
6. DUFADIP-U (*Diagram, Unknowns/Knows, Formula, Algebra, Differentiate, Isolate, Plug In*)
7. PLARHDTSLAW-C (*Picture, Label, Assess (Constraint/Formula), Relationship, Domain, Hone Down, Take Derivative, Set Equal to Zero, Local Max or Min?, Ascertain Globalness, Write a Sentence, Check you answered the question*)
8. Remember you must have an indeterminant quotient to apply L'höpital.
9. Remember how to prove a point is a global max or min when optimizing
   (a) If your domain is closed, i.e $[a, b]$, then confirm your critical point is a local min or local max via second derivative test or perhaps by checking the first derivative at nearby points. Then test all critical points and endpoints and make sure you say something about the domain being closed.
   (b) If your domain is not closed, i.e $(0, \infty)$ then you will most likely have only one critical point. Prove it is a local max or min and then state that since there is only one critical point you know it is a global max or min (Make sure you understand why).

1 Limits

1.1 Limits from MT1

Most of these you can use L'Hospital for. But always remember to first check if the limit is of the form $\frac{\text{constant}}{0}$ in which case you need to look at left and right limits. Also, some limits like $\lim_{x\to\infty} \frac{3x^2-9}{\sqrt{x^4+7x+3}}$ are easier to use our techniques from MT1 then to use L'Hospital.

1.2 L'Hospital

$$\lim_{t\to 0} \frac{t\tan(t)}{1-\cos t}$$

$$\lim_{x\to\infty} x\sin(\frac{\pi}{x})$$

$$\lim_{x\to\infty} (1-\frac{\pi}{x})^{x/\pi}.$$

1.3 Continuity

Find c so that the function $F(x) = \begin{cases} ce^{x^2}, & x \geq \sqrt{2} \\ x^2, & x < \sqrt{2} \end{cases}$ is continuous. Explain.

1.4 Limit Definition of Derivative

Calculate the derivatives of the following functions using the limit definition:

$$f(x) = \frac{1}{1+x^2}$$

$$g(t) = \frac{t}{3t+5}$$

1.5 Geometric Limits

Let P be the point on the graph of $y = e^{-x^2/7}$ with x-coordinate $a \neq 0$. The normal line to the graph through P will have y-intercept b. (a) Find $\lim_{a\to 0} b$. $(-3/2)$ (b) Find $\lim_{a\to\infty} b$. -5/2.

2 Explicit Derivatives

2.1 Drawing graph of f' versus f

To practice, just draw something call it f then draw f'

2.2 Chain Rule, Product Rule, Quotient Rule

2.3 Trig and ArcTrig

2.4 Periodic Derivatives

$$f(x) = e^{2x} - x^n + 30x^{n-1} + \frac{1}{x}$$

$$\text{Find } f^{(n)}(x)$$

3 Implicit Derivatives

3.1 Logarithmic Differentiation

$$y = \ln(x)^{\ln(x)}$$

$$y = (\sqrt{x})^{e^{\cos(\ln(x))}}$$

3.2 Strange Curves

Consider the curve given by the following equation

$$x + x^2 - 2y - \sin(x + y) = 0.$$

(a) Find $\frac{dy}{dx}$. (b) Check that $(-3, 3)$ is on the curve. Compute the equation of the tangent line at this point. (c) Estimate b so that $(b, 2.8)$ lies on the curve.

4 Parametrics

Let $(x(t), y(t)) = (e^{-t} + t, te^{-(t/2)})$. (a) Find $\frac{dy}{dx}$. (b) Find the coordinates of the point at which the tangent line to the curve is horizontal. (c) Find the coordinates at which the tangent line is vertical. (d) Find $\frac{d^2y}{dx^2}(0)$.

Let C be the curve given by the parametric equations $(x(t), y(t)) = (2t^2 - t, t^2 + t + 1)$. (a) Find all points P on C for which the tangent line to C at P passes through the point $(1, 0)$. (b) Suppose at P the particle leaves C and continues at its current speed towards $(1, 0)$. How long does it take to reach $(0, 1)$.

5 Tangent Line Approximation

5.1 Single Variable

Find the linear approximation of the function $y = 2\ln(1 + x) + e^x + 5x^2$ at the point $x = 0$ and use it to approximate the solution to the equation

$$1.01 = 2\ln(1 + x) + e^x + 5x^2.$$

5.2 Implicit

Estimate using tangent line approximation y_0 for the point $(-\pi, y_0)$ using the basepoint (-3,3) on the curve: $x + x^2 - 2y - \sin(x + y) = 0$.

5.3 Over or Underestimate?

To check just take second derivative or use a graph. $f'' < 0$ *then over,* $f'' > 0$ *then under.* f'' *is evaluated at basepoint*

6 Related Rates

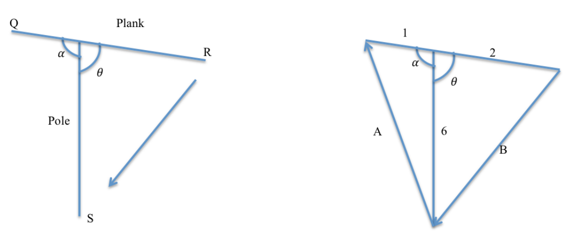
6.1 With Angles

A UFO flies horizontally at a constant speed at an altitude of 15 km and pases directly over a tracking telescope on the ground. When the angle of elevation is $\pi/3$, this angle is decreasing at a rate of .1 rad/min. How fast is the UFO flying?

6.2 Law of Sines

6.3 Law of Cosines

A 3 meter plank is balanced on a 6 meter pole so that 1 meter lies on the left and 2 meters on the right side of the pole.



Suppose the point R is moving downward so that the distance between it and point S is shrinking at 3 m/s. When R is a distance of $4\sqrt{2}$ meters from S the law of cosines tells us that $\theta = \frac{\pi}{3}$ and that the distance from Q to S is $\sqrt{31}$. At this point in time find the rate the distance between Q and S is changing. That is, in the diagram on the right find $\frac{dA}{dt}$ for the initial conditions stated above. *The Law of Cosines is that* $\underline{B^2 = C^2 + D^2 - 2CD\cos\theta}$ *with* θ *the angle opposite side* B.

6.4 Pythagorean

A right triangle is undergoing a transformation so that on leg is increasing at 3 cm/sec and the other leg is decreasing at 2 cm/sec. How fast is the ratio $\frac{\text{perimeter}}{area}$ changing?

6.5 Similar Triangles

A balloon is rising at a rate of 8 ft/sec at a horizontal distance of 12 feet from a 20ft tall light post. At what rate is the shadow of the balloon moving along the ground when the balloon is 5ft above the ground?

6.6 Volumes or other Geometric Formulas

Matt is baking a cylindrical cake. Both the radius and the height of the cake are expanding. The height is increasing at a rate of 2 m/s and the surface area is increasing at a rate of 36π m^2/s. When the height is 2 m and the radius is 4 m at what rate is the volume of the cake changing?

6.7 Curves

A marble is rolling along the curve

$$xe^{2\sin\theta} = 1$$

If the x coordinate of the marble is changing at a rate of 2 $\frac{cm}{sec}$ at the point $(x, \theta) = (1, \pi)$, then at what rate is the θ coordinate changing?

7 Curve Sketching

1. Max and Min
2. Points of inflection
3. Increasing and decreasing on intervals
4. Vertical and Horizontal Asymptotes
5. Drawing Graphs from This Data

$$y = e^x(x-1)^2$$

$$y = \frac{x^3}{x^2-1}$$

Do (a-d) with the two functions just listed

8 Optimization

8.1 Perimeter Stuff

A farmer has 136 meters of fencing. She wants to make one or two rectangular enclosures. One will be a square and the other will have its long side twice as long as its short side. What should she do to minimize the total area of the enclosures? What should she do to maximize the area?

8.2 Area

A rectangular poster is to have an area of 2700 cm^2 with 3-cm margins at the bottom and sides and a 5-cm margin at the top. What dimensions will vibe the largest printed area? Justify your answer is a maximum.

8.3 Volume

A mass of clay of volume $(4/3)\pi$ in^3 is formed into two spheres. How should the clay by divided to make the total surface are of the spheres a maximum? How to make it a minimum?

8.4 With Curves

Find the equation of the line passing through the point $(3, 2)$ which cuts off the triangle of least area from the first quadrant.

Find the point on the ellipse $x^2 + y^2 - xy = 1$ with the largest y coordinate.

8.5 Random Stuff That is Hard to Anticipate

Gordo wants two positive real numbers such that their product is 100 and the sum of the first number with the square of the second number is as small as possible. What should Gordo choose?