CHAPTER 6 DYNAMICS OF FLUID FLOW

▸ 6.1 INTRODUCTION

In the previous chapter, we studied the velocity and acceleration at a point in a fluid flow, without taking into consideration the forces causing the flow. This chapter includes the study of forces causing fluid flow. Thus dynamics of fluid flow is the study of fluid motion with the forces causing flow. The dynamic behaviour of the fluid flow is analysed by the Newton's second law of motion, which relates the acceleration with the forces. The fluid is assumed to be incompressible and non-viscous.

▸ 6.2 EQUATIONS OF MOTION

According to Newton's second law of motion, the net force F_x acting on a fluid element in the direction of x is equal to mass m of the fluid element multiplied by the acceleration a_x in the x-direction. Thus mathematically,

$$F_x = m.a_x \quad \text{...(6.1)}$$

In the fluid flow, the following forces are present :

(*i*) F_g, gravity force.
(*ii*) F_p, the pressure force.
(*iii*) F_v, force due to viscosity.
(*iv*) F_t, force due to turbulence.
(*v*) F_c, force due to compressibility.

Thus in equation (6.1), the net force

$$F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x + (F_c)_x.$$

(*i*) If the force due to compressibility, F_c is negligible, the resulting net force

$$F_x = (F_g)_x + (F_p)_x + (F_v)_x + (F_t)_x$$

and equation of motions are called **Reynold's equations of motion.**

(*ii*) For flow, where (F_t) is negligible, the resulting equations of motion are known as **Navier-Stokes Equation.**

(*iii*) If the flow is assumed to be ideal, viscous force (F_v) is zero and equation of motions are known as **Euler's equation of motion.**

6.3 EULER'S EQUATION OF MOTION

This is equation of motion in which the forces due to gravity and pressure are taken into consideration. This is derived by considering the motion of a fluid element along a stream-line as :

Consider a stream-line in which flow is taking place in s-direction as shown in Fig. 6.1. Consider a cylindrical element of cross-section dA and length ds. The forces acting on the cylindrical element are:

1. Pressure force pdA in the direction of flow.
2. Pressure force $\left(p + \frac{\partial p}{\partial s} ds\right) dA$ opposite to the direction of flow.
3. Weight of element $\rho g dA ds$.

Let θ is the angle between the direction of flow and the line of action of the weight of element.

The resultant force on the fluid element in the direction of s must be equal to the mass of fluid element × acceleration in the direction s.

$$\therefore \quad pdA - \left(p + \frac{\partial p}{\partial s} ds\right) dA - \rho g dA ds \cos\theta$$

$$= \rho dA ds \times a_s \qquad \ldots(6.2)$$

where a_s is the acceleration in the direction of s.

Now $\quad a_s = \frac{dv}{dt}$, where v is a function of s and t.

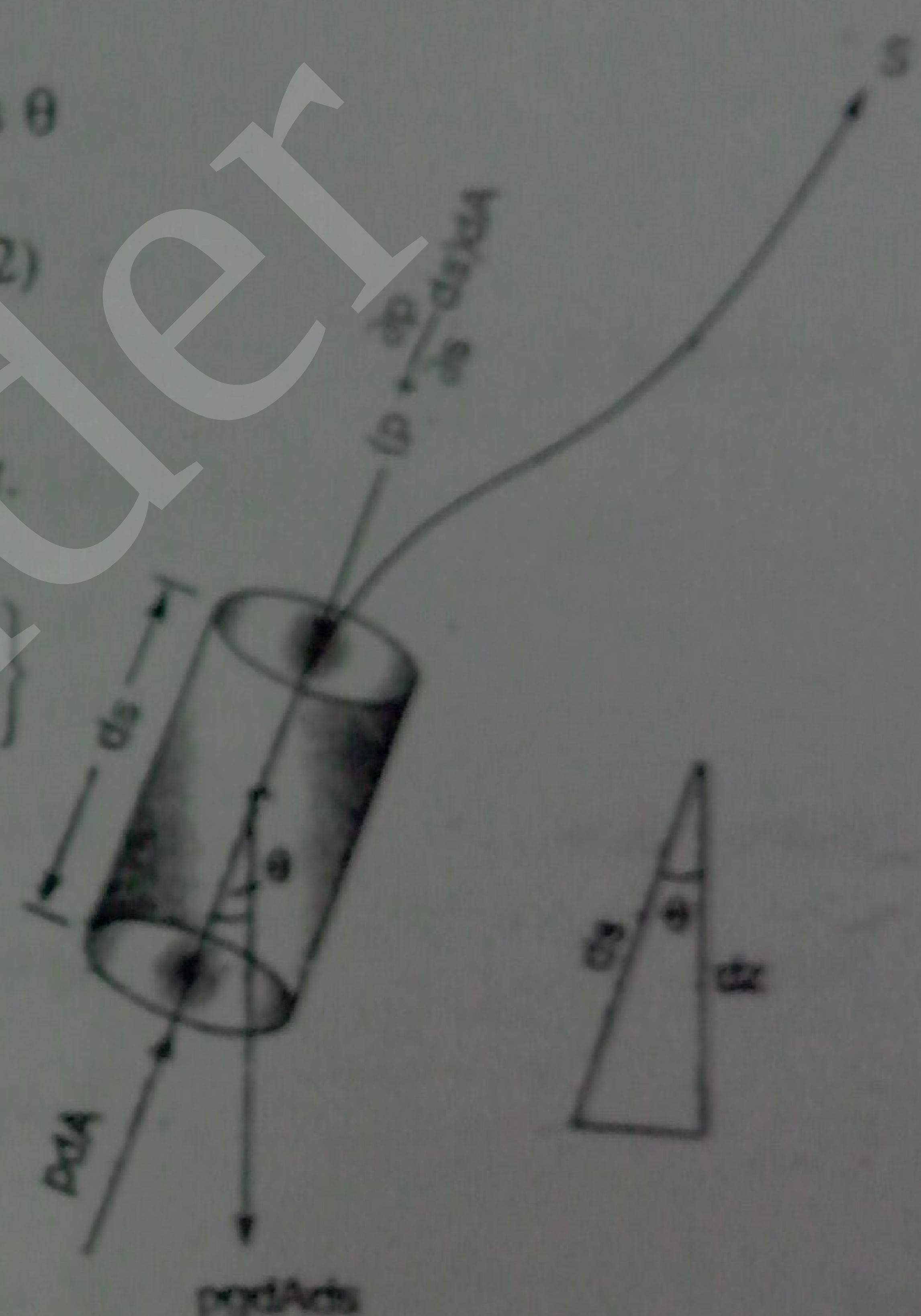
$$= \frac{\partial v}{\partial s}\frac{ds}{dt} + \frac{\partial v}{\partial t} = \frac{v\partial v}{\partial s} + \frac{\partial v}{\partial t} \quad \left\{\because \frac{ds}{dt} = v\right\}$$

If the flow is steady, $\frac{\partial v}{\partial t} = 0$

$$\therefore \quad a_s = \frac{v\partial v}{\partial s}$$

Substituting the value of a_s in equation (6.2) and simplifying the equation, we get

$$-\frac{\partial p}{\partial s} ds dA - \rho g\, dA ds \cos\theta = \rho dA ds \times \frac{v\partial v}{\partial s}$$



(a) (b)

Fig. 6.1 *Forces on a fluid element.*

Dividing by $\rho ds dA$, $-\frac{\partial p}{\rho \partial s} - g\cos\theta = \frac{v\partial v}{\partial s}$

or $$\frac{\partial p}{\rho \partial s} + g\cos\theta + v\frac{\partial v}{\partial s} = 0$$

But from Fig. 6.1 (b), we have $\cos\theta = \frac{dz}{ds}$

$$\therefore \quad \frac{1}{\rho}\frac{dp}{ds} + g\frac{dz}{ds} + \frac{vdv}{ds} = 0 \quad \text{or} \quad \frac{dp}{\rho} + gdz + vdv = 0$$

or $$\frac{dp}{\rho} + gdz + vdv = 0 \qquad \ldots(6.3)$$

Equation (6.3) is known as Euler's equation of motion.

▶ 6.4 BERNOULLI'S EQUATION FROM EULER'S EQUATION

Bernoulli's equation is obtained by integrating the Euler's equation of motion (6.3) as

$$\int \frac{dp}{\rho} + \int g dz + \int v dv = \text{constant}$$

If flow is incompressible, ρ is constant and

$$\therefore \qquad \frac{p}{\rho} + gz + \frac{v^2}{2} = \text{constant}$$

or

$$\frac{p}{\rho g} + z + \frac{v^2}{2g} = \text{constant}$$

or

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{constant} \qquad \ldots(6.4)$$

Equation (6.4) is a Bernoulli's equation in which

$\frac{p}{\rho g}$ = pressure energy per unit weight of fluid or pressure head.

$v^2/2g$ = kinetic energy per unit weight or kinetic head.

z = potential energy per unit weight or potential head.

▶ 6.5 ASSUMPTIONS

The following are the assumptions made in the derivation of Bernoulli's equation :

(*i*) The fluid is ideal, *i.e.*, viscosity is zero (*ii*) The flow is steady

(*iii*) The flow is incompressible (*iv*) The flow is irrotational.

Problem 6.1 *Water is flowing through a pipe of 5 cm diameter under a pressure of 29.43 N/cm² (gauge) and with mean velocity of 2.0 m/s. Find the total head or total energy per unit weight of the water at a cross-section, which is 5 m above the datum line.*

Solution. Given :

Diameter of pipe = 5 cm = 0.5 m

Pressure, $p = 29.43 \text{ N/cm}^2 = 29.43 \times 10^4 \text{ N/m}^2$

Velocity, $v = 2.0$ m/s

Datum head, $z = 5$ m

Total head = pressure head + kinetic head + datum head

Pressure head $= \frac{p}{\rho g} = \frac{29.43 \times 10^4}{1000 \times 9.81} = 30 \text{ m}$ $\left\{\rho \text{ for water} = 1000 \frac{\text{kg}}{\text{m}^3}\right\}$

Kinetic head $= \frac{v^2}{2g} = \frac{2 \times 2}{2 \times 9.81} = 0.204 \text{ m}$

$\therefore$ Total head $= \frac{p}{\rho g} + \frac{v^2}{2g} + z = 30 + 0.204 + 5 =$ **35.204 m. Ans.**

Problem 6.2 *A pipe, through which water is flowing, is having diameters, 20 cm and 10 cm at the cross-sections 1 and 2 respectively. The velocity of water at section 1 is given 4.0 m/s. Find the velocity head at sections 1 and 2 and also rate of discharge.*

Solution. Given :

$$D_1 = 20 \text{ cm} = 0.2 \text{ m}$$

∴ Area,
$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (.2)^2 = 0.0314 \text{ m}^2$$

$$V_1 = 4.0 \text{ m/s}$$

$$D_2 = 0.1 \text{ m}$$

∴
$$A_2 = \frac{\pi}{4} (.1)^2 = .00785 \text{ m}^2$$

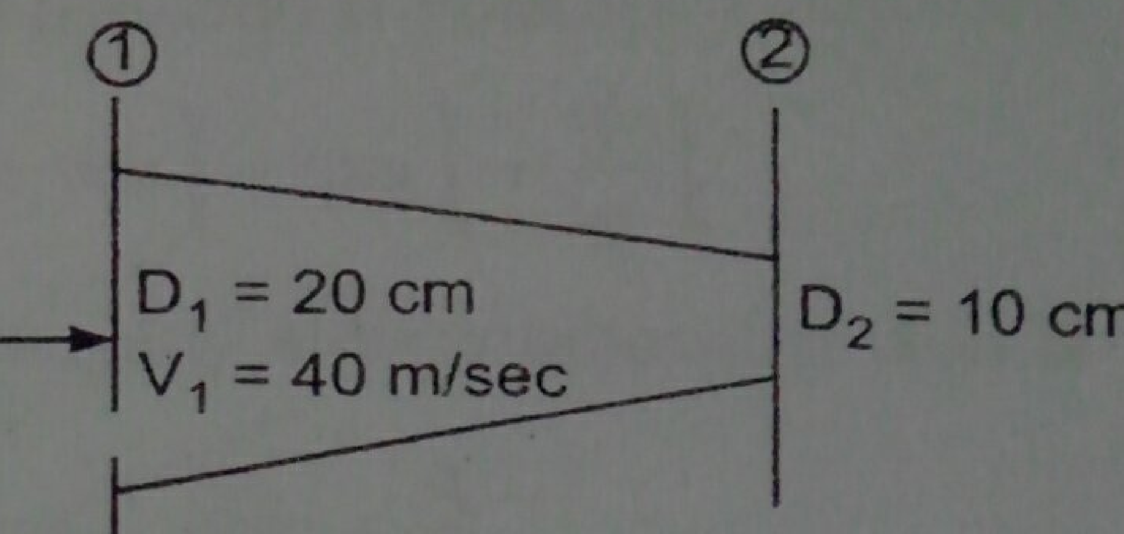


Fig. 6.2

(*i*) Velocity head at section 1

$$= \frac{V_1^2}{2g} = \frac{4.0 \times 4.0}{2 \times 9.81} = \mathbf{0.815 \text{ m. Ans.}}$$

(*ii*) Velocity head at section 2 = $V_2^2/2g$

To find V_2, apply continuity equation at 1 and 2

∴
$$A_1V_1 = A_2V_2 \quad \text{or} \quad V_2 = \frac{A_1V_1}{A_2} = \frac{.0314}{.00785} \times 4.0 = 16.0 \text{ m/s}$$

∴ Velocity head at section 2 $= \frac{V_2^2}{2g} = \frac{16.0 \times 16.0}{2 \times 9.81} = \mathbf{83.047 \text{ m. Ans.}}$

(*iii*) Rate of discharge $= A_1V_1$ or A_2V_2

$= 0.0314 \times 4.0 = 0.1256 \text{ m}^3\text{/s}$

$=$ **125.6 litres/s. Ans.** $\{\because 1 \text{ m}^3 = 1000 \text{ litres}\}$

Problem 6.3 *State Bernoulli's theorem for steady flow of an incompressible fluid. Derive an expression for Bernoulli's equation from first principle and state the assumptions made for such a derivation.*

Solution. Statement of Bernoulli's Theorem. It states that in a steady, ideal flow of an incompressible fluid, the total energy at any point of the fluid is constant. The total energy consists of pressure energy, kinetic energy and potential energy or datum energy. These energies per unit weight of the fluid are :

Pressure energy $= \frac{p}{\rho g}$

Kinetic energy $= \frac{v^2}{2g}$

Datum energy $= z$

Thus mathematically, Bernoulli's theorem is written as

$$\frac{p}{\rho g} + \frac{v^2}{2g} + z = \text{Constant.}$$

Derivation of Bernoulli's theorem. For derivation of Bernoulli's theorem, Articles 6.3 and 6.4 should be written.

Assumptions are given in Article 6.5.

Problem 6.4 *The water is flowing through a pipe having diameters 20 cm and 10 cm at sections 1 and 2 respectively. The rate of flow through pipe is 35 litres/s. The section 1 is 6 m above datum and section 2 is 4 m above datum. If the pressure at section 1 is 39.24 N/cm², find the intensity of pressure at section 2.*

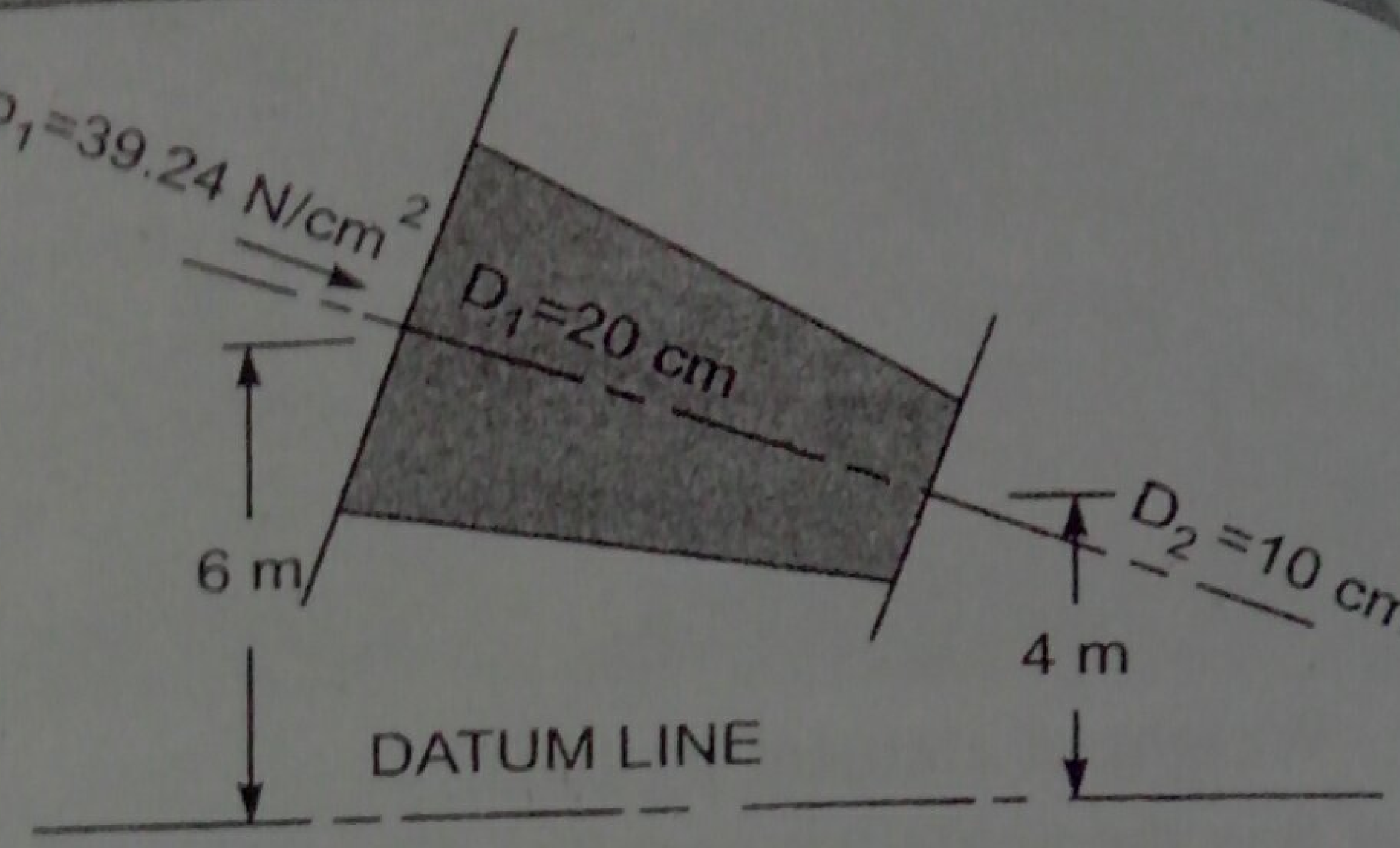


Fig. 6.3

Solution. Given :

At section 1,
$$D_1 = 20 \text{ cm} = 0.2 \text{ m}$$
$$A_1 = \frac{\pi}{4}(.2)^2 = .0314 \text{ m}^2$$
$$p_1 = 39.24 \text{ N/cm}^2 = 39.24 \times 10^4 \text{ N/m}^2$$
$$z_1 = 6.0 \text{ m}$$

At section 2,
$$D_2 = 0.10 \text{ m}$$
$$A_2 = \frac{\pi}{4}(0.1)^2 = .00785 \text{ m}^2$$
$$z_2 = 4 \text{ m}$$
$$p_2 = ?$$

Rate of flow,
$$Q = 35 \text{ lit/s} = \frac{35}{1000} = .035 \text{ m}^3\text{/s}$$

Now
$$Q = A_1V_1 = A_2V_2$$

$\therefore$
$$V_1 = \frac{Q}{A_1} = \frac{.035}{.0314} = \mathbf{1.114 \text{ m/s}}$$

and
$$V_2 = \frac{Q}{A_2} = \frac{.035}{.00785} = \mathbf{4.456 \text{ m/s}}$$

Applying Bernoulli's equation at sections 1 and 2, we get
$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

or
$$\frac{39.24 \times 10^4}{1000 \times 9.81} + \frac{(1.114)^2}{2 \times 9.81} + 6.0 = \frac{p_2}{1000 \times 9.81} + \frac{(4.456)^2}{2 \times 9.81} + 4.0$$

or
$$40 + 0.063 + 6.0 = \frac{p_2}{9810} + 1.012 + 4.0$$

or
$$46.063 = \frac{p_2}{9810} + 5.012$$

$\therefore$
$$\frac{p_2}{9810} = 46.063 - 5.012 = 41.051$$

$\therefore$
$$p_2 = 41.051 \times 9810 \text{ N/m}^2$$
$$= \frac{41.051 \times 9810}{10^4} \text{ N/cm}^2 = \mathbf{40.27 \text{ N/cm}^2. \text{ Ans.}}$$

Problem 6.5 *Water is flowing through a pipe having diameter 300 mm and 200 mm at the bottom and upper end respectively. The intensity of pressure at the bottom end is 24.525 N/cm² and the pressure at the upper end is 9.81 N/cm². Determine the difference in datum head if the rate of flow through pipe is 40 lit/s.*

Solution. Given :

Section 1, $D_1 = 300 \text{ mm} = 0.3 \text{ m}$

$p_1 = 24.525 \text{ N/cm}^2 = 24.525 \times 10^4 \text{ N/m}^2$

Section 2, $D_2 = 200 \text{ mm} = 0.2 \text{ m}$

$p_2 = 9.81 \text{ N/cm}^2 = 9.81 \times 10^4 \text{ N/m}^2$

Rate of flow $= 40$ lit/s

or $$Q = \frac{40}{1000} = 0.04 \text{ m}^3/\text{s}$$

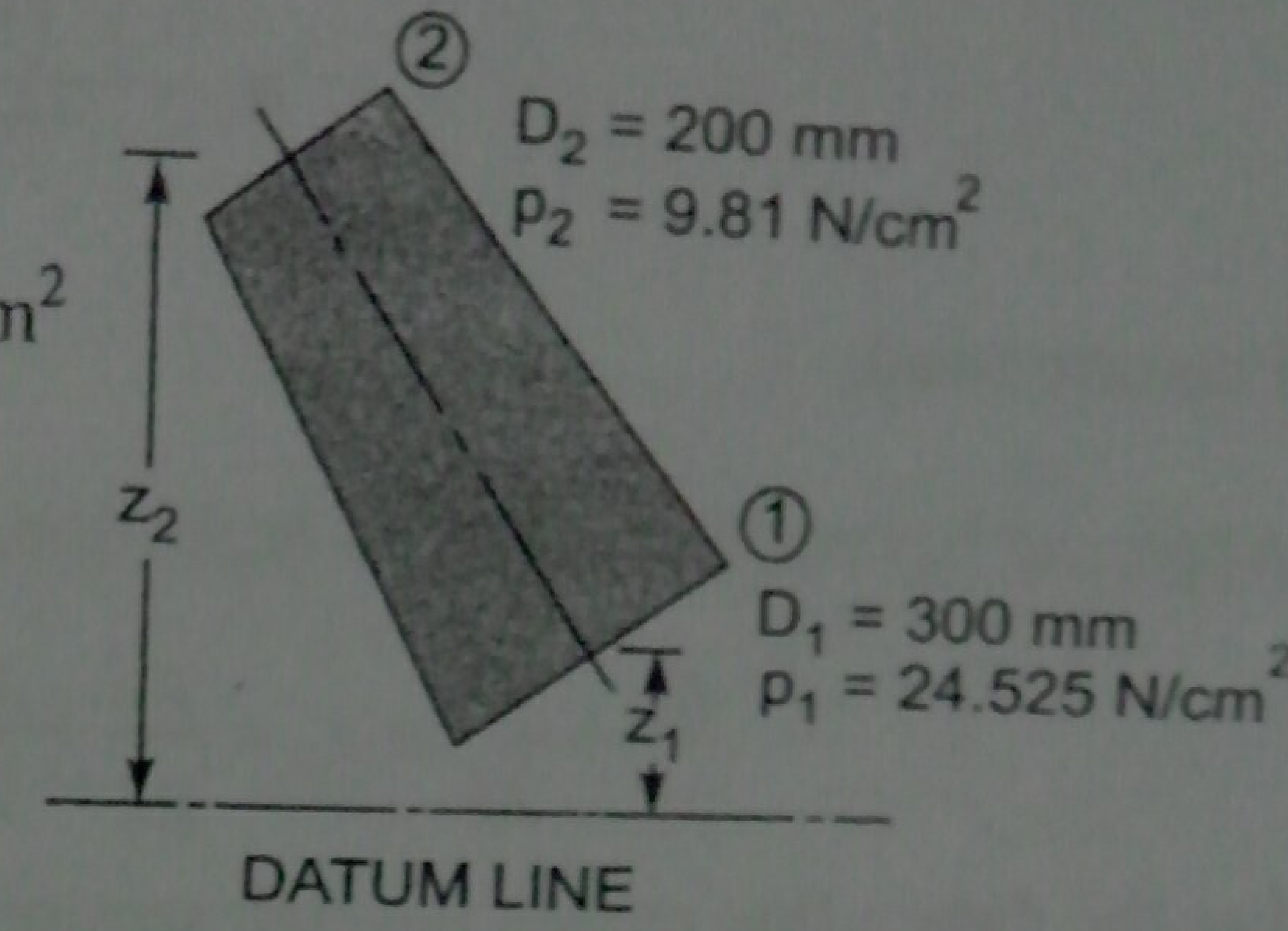


Fig. 6.4

Now $$A_1V_1 = A_2V_2 = \text{rate of flow} = 0.04$$

$\therefore$ $$V_1 = \frac{.04}{A_1} = \frac{.04}{\frac{\pi}{4}D_1^2} = \frac{0.04}{\frac{\pi}{4}(0.3)^2} = 0.5658 \text{ m/s}$$

$$\simeq 0.566 \text{ m/s}$$

$$V_2 = \frac{.04}{A_2} = \frac{.04}{\frac{\pi}{4}(D_2)^2} = \frac{0.04}{\frac{\pi}{4}(0.2)^2} = 1.274 \text{ m/s}$$

Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

or $$\frac{24.525 \times 10^4}{1000 \times 9.81} + \frac{.566 \times .566}{2 \times 9.81} + z_1 = \frac{9.81 \times 10^4}{1000 \times 9.81} + \frac{(1.274)^2}{2 \times 9.81} + z_2$$

or $$25 + .32 + z_1 = 10 + 1.623 + z_2$$

or $$25.32 + z_1 = 11.623 + z_2$$

$\therefore$ $$z_2 - z_1 = 25.32 - 11.623 = 13.697 = 13.70 \text{ m}$$

$\therefore$ Difference in datum head $= z_2 - z_1 =$ **13.70 m. Ans.**

Problem 6.6 *The water is flowing through a taper pipe of length 100 m having diameters 600 mm at the upper end and 300 mm at the lower end, at the rate of 50 litres/s. The pipe has a slope of 1 in 30. Find the pressure at the lower end if the pressure at the higher level is 19.62 N/cm².*

Solution. Given :

Length of pipe, $L = 100$ m

Dia. at the upper end, $D_1 = 600 \text{ mm} = 0.6 \text{ m}$

$\therefore$ Area, $$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} \times (.6)^2$$

$$= 0.2827 \text{ m}^2$$

p_1 = pressure at upper end

$= 19.62 \text{ N/cm}^2$

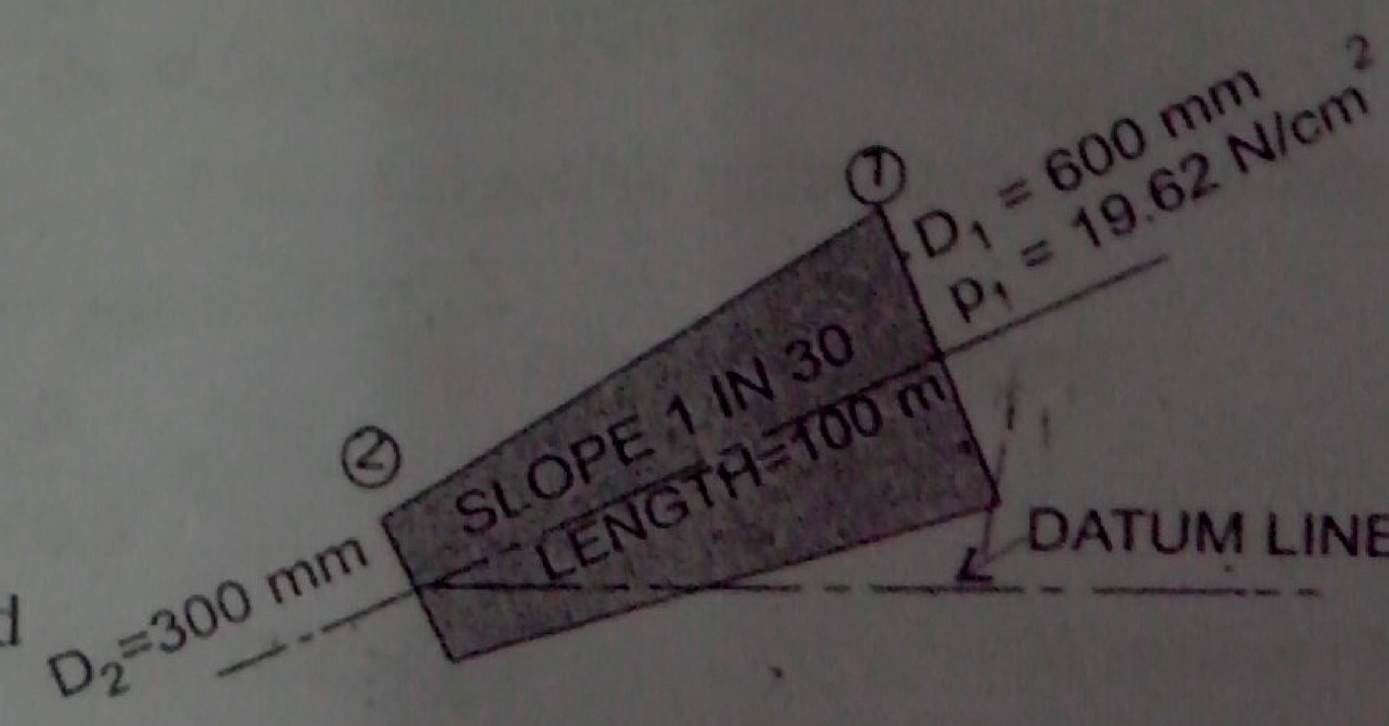


Fig. 6.5

$$= 19.62 \times 10^4 \text{ N/m}^2$$

Dia. at lower end, $D_2 = 300 \text{ mm} = 0.3 \text{ m}$

$\therefore$ Area, $A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} (.3)^2 = 0.07068 \text{ m}$

$$Q = \text{rate of flow} = 50 \text{ litres/s} = \frac{50}{1000} = 0.05 \text{ m}^3\text{/s}$$

Let the datum line passes through the centre of the lower end.

Then $z_2 = 0$

As slope is 1 in 30 means $z_1 = \frac{1}{30} \times 100 = \frac{10}{3} \text{ m}$

Also we know $Q = A_1V_1 = A_2V_2$

$\therefore$ $V_1 = \frac{Q}{A} = \frac{0.05}{.2827} = 0.1768 \text{ m/sec} = 0.177 \text{ m/s}$

and $V_2 = \frac{Q}{A_2} = \frac{0.5}{.07068} = 0.7074 \text{ m/sec} = 0.707 \text{ m/s}$

Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

or $$\frac{19.62 \times 10^4}{1000 \times 9.81} + \frac{.177^2}{2 \times 9.81} + \frac{10}{3} = \frac{p_2}{\rho g} + \frac{.707^2}{2 \times 9.81} + 0$$

or $$20 + 0.001596 + 3.334 = \frac{p_2}{\rho g} + 0.0254$$

or $$23.335 - 0.0254 = \frac{p_2}{1000 \times 9.81}$$

or $p_2 = 23.3 \times 9810 \text{ N/m}^2 = 228573 \text{ N/m}^2 =$ **22.857 N/cm². Ans.**

6.6 BERNOULLI'S EQUATION FOR REAL FLUID

The Bernoulli's equation was derived on the assumption that fluid is inviscid (non-viscous) and therefore frictionless. But all the real fluids are viscous and hence offer resistance to flow. Thus there are always some losses in fluid flows and hence in the application of Bernoulli's equation, these losses have to be taken into consideration. Thus the Bernoulli's equation for real fluids between points 1 and 2 is given as

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_L \quad \text{...(6.5)}$$

where h_L is loss of energy between points 1 and 2.

Problem 6.7 *A pipe of diameter 400 mm carries water at a velocity of 25 m/s. The pressures at the points A and B are given as 29.43 N/cm² and 22.563 N/cm² respectively while the datum head at A and B are 28 m and 30 m. Find the loss of head between A and B.*

Solution. Given :

Dia. of pipe, $D = 400 \text{ mm} = 0.4 \text{ m}$

Velocity, $V = 25 \text{ m/s}$

At point *A*, $p_A = 29.43 \text{ N/cm}^2 = 29.43 \times 10^4 \text{ N/m}^2$

$z_A = 28 \text{ m}$

$v_A = v = 25 \text{ m/s}$

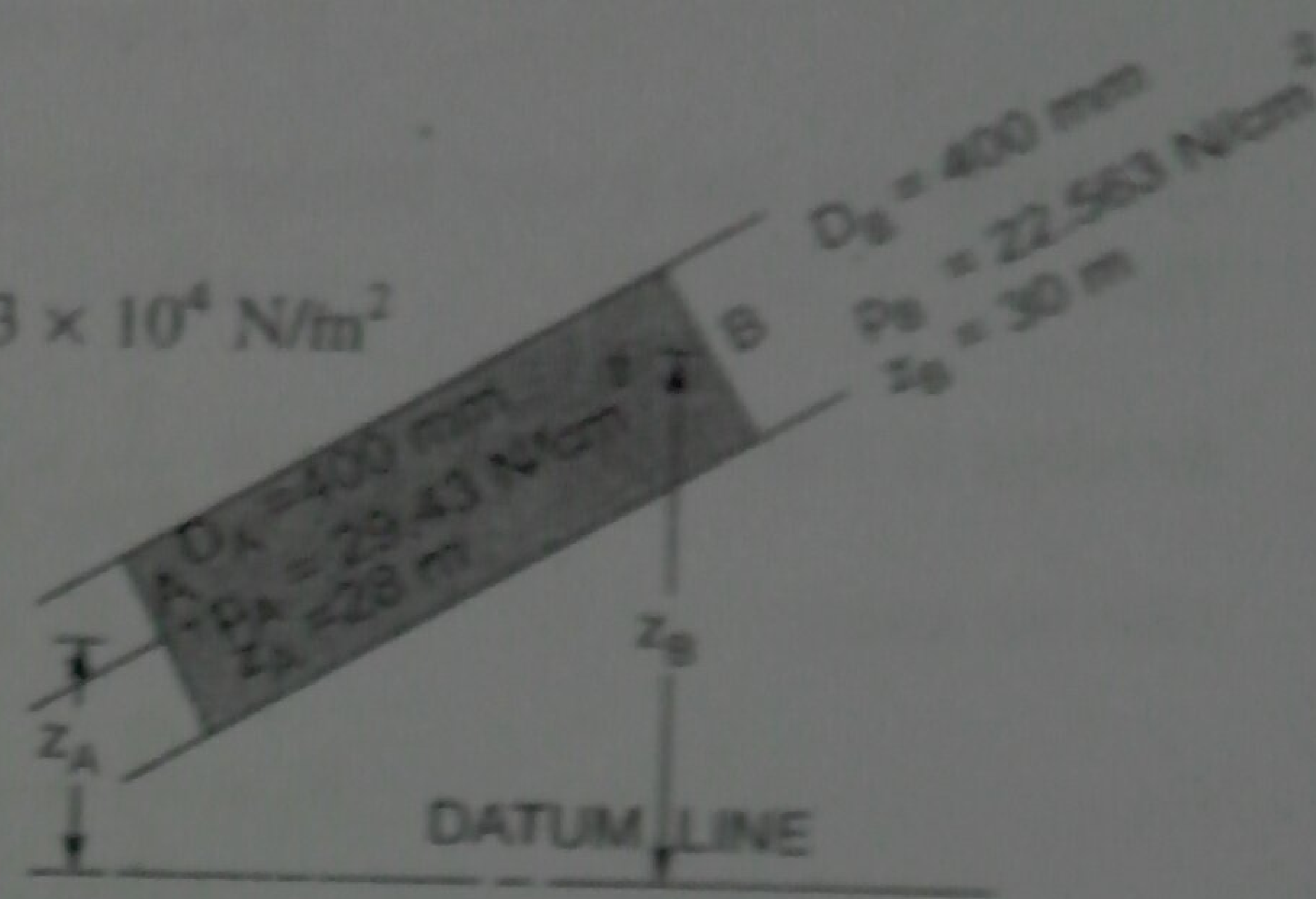


Fig. 6.6

∴ Total energy at *A*, $$E_A = \frac{p_A}{\rho g} + \frac{v_A^2}{2g} + z_A$$

$$= \frac{29.43 \times 10^4}{1000 \times 9.81} + \frac{25^2}{2 \times 9.81} + 28$$

$$= 30 + 31.85 + 28 = 89.85 \text{ m}$$

At point *B*, $p_B = 22.563 \text{ N/cm}^2 = 22.563 \times 10^4 \text{ N/m}^2$

$z_B = 30 \text{ m}$

$v_B = v = v_A = 25 \text{ m/s}$

∴ Total energy at *B*, $$E_B = \frac{p_B}{\rho g} + \frac{v_B^2}{2g} + z_B$$

$$= \frac{22.563 \times 10^4}{1000 \times 9.81} + \frac{25^2}{2 \times 9.81} + 30 = 23 + 31.85 + 30 = 84.85 \text{ m}$$

∴ Loss of energy $= E_A - E_B = 89.85 - 84.85 = 5.0$ **m. Ans.**

Problem 6.8 *A conical tube of length 2.0 m is fixed vertically with its smaller end upwards. The velocity of flow at the smaller end is 5 m/s while at the lower end it is 2 m/s. The pressure head at the smaller end is 2.5 m of liquid. The loss of head in the tube is* $\frac{0.35(v_1 - v_2)^2}{2g}$, *where* v_1 *is the velocity at the smaller end and* v_2 *at the lower end respectively. Determine the pressure head at the lower end. Flow takes place in the downward direction.*

Solution. Let the smaller end is represented by (1) and lower end by (2)

Given :

Length of tube, $L = 2.0 \text{ m}$

$v_1 = 5 \text{ m/s}$

$p_1/\rho g = 2.5 \text{ m of liquid}$

$v_2 = 2 \text{ m/s}$

Loss of head $= h_L = \frac{0.35(v_1 - v_2)^2}{2g}$

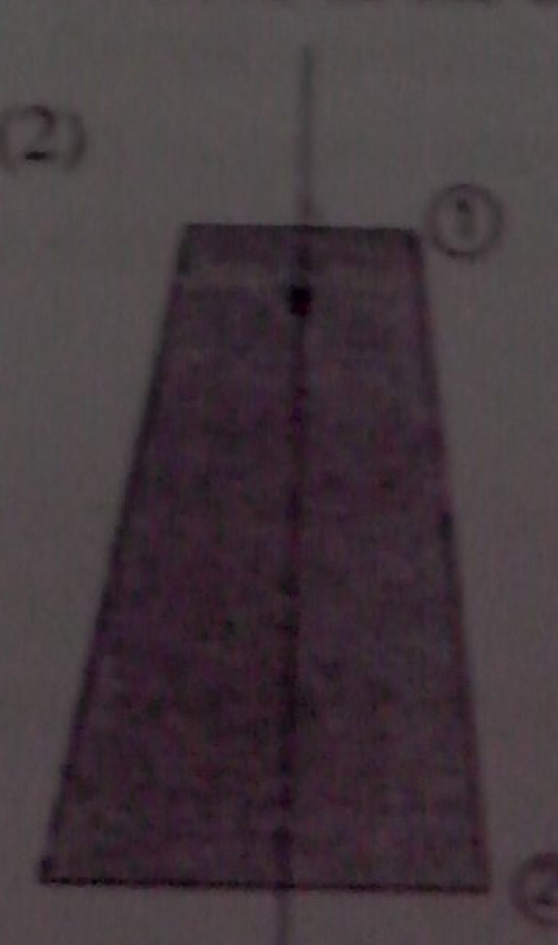


Fig. 6.7

$$= \frac{0.35\,[5-2]^2}{2g} = \frac{0.35 \times 9}{2 \times 9.81} = 0.16 \text{ m}$$

Pressure head, $\frac{p_2}{\rho g} = ?$

Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2 + h_L$$

Let the datum line passes through section (2). Then $z_2 = 0$, $z_1 = 2.0$

$$\therefore \quad 2.5 + \frac{5^2}{2 \times 9.81} + 2.0 = \frac{p_2}{\rho g} + \frac{2^2}{2 \times 9.81} + 0 + 0.16$$

$$2.5 + 1.27 + 2.0 = \frac{p_2}{\rho g} + 0.203 + .16$$

or $$\frac{p_2}{\rho g} = (2.5 + 1.27 + 2.0) - (.203 + .16)$$

$$= 5.77 - .363 = \mathbf{5.407 \text{ m of fluid. Ans.}}$$

Problem 6.9 *A pipeline carrying oil of specific gravity 0.87, changes in diameter from 200 mm diameter at a position A to 500 mm diameter at a position B which is 4 metres at a higher level. If the pressures at A and B are 9.81 N/cm² and 5.886 N/cm² respectively and the discharge is 200 litres/s determine the loss of head and direction of flow.*

Solution. Discharge, $Q = 200 \text{ lit/s} = 0.2 \text{ m}^3/\text{s}$

Sp. gr. of oil $= 0.87$

$\therefore$ ρ for oil $= .87 \times 1000 = 870 \frac{\text{kg}}{\text{m}^3}$

Given : At section *A*, $D_A = 200 \text{ mm} = 0.2 \text{ m}$

Area, $A_A = \frac{\pi}{4}(D_A)^2 = \frac{\pi}{4}(.2)^2$

$= 0.0314 \text{ m}^2$

$p_A = 9.81 \text{ N/cm}^2$

$= 9.81 \times 10^4 \text{ N/m}^2$

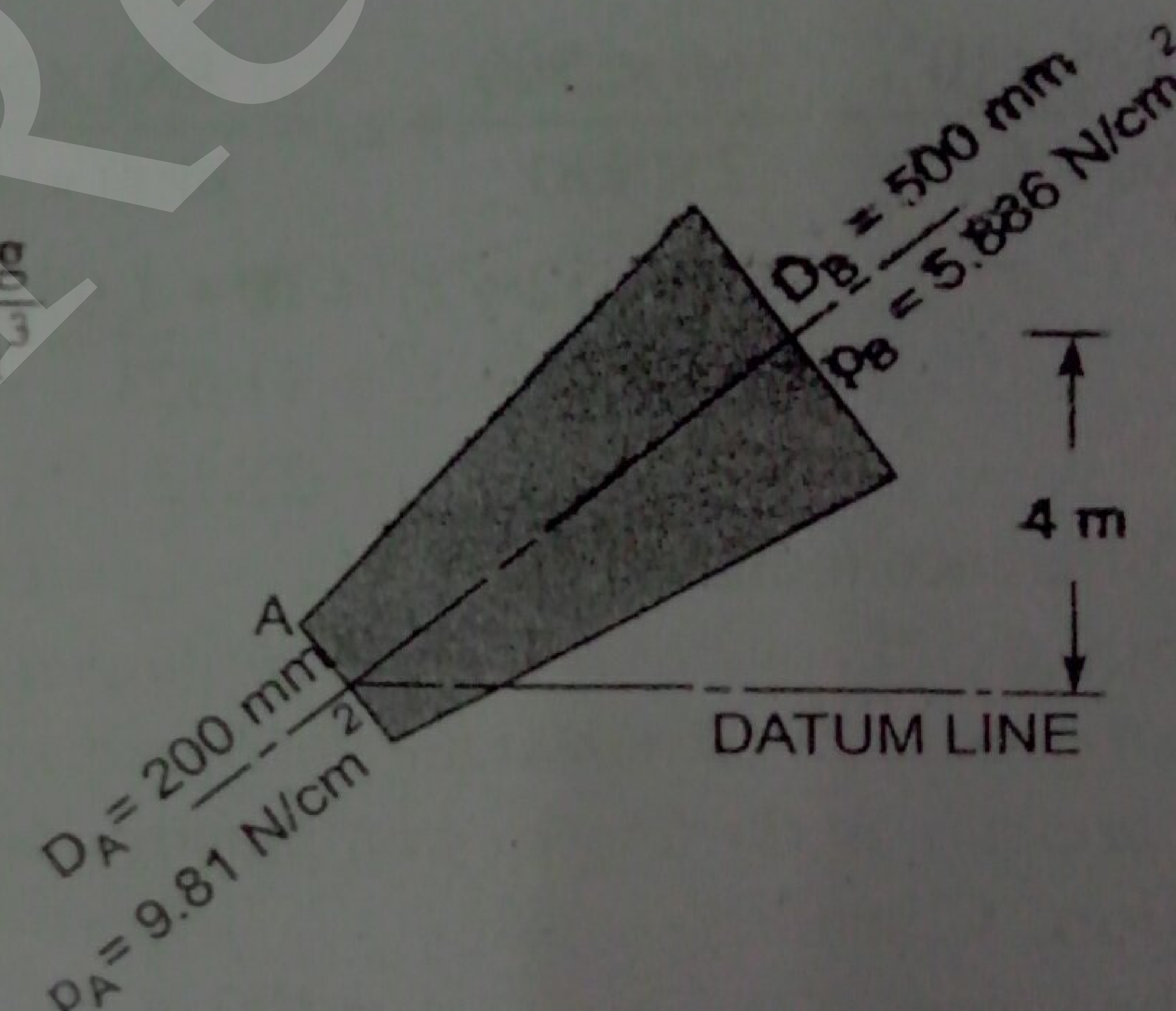


Fig. 6.8

If datum line is passing through *A*, then

$Z_A = 0$

$$V_A = \frac{Q}{A_A} = \frac{0.2}{0.0314} = 6.369 \text{ m/s}$$

At section *B*, $D_B = 500 \text{ mm} = 0.50 \text{ m}$

Area, $A_B = \frac{\pi}{4} D_B^2 = \frac{\pi}{4}(.5)^2 = 0.1963 \text{ m}^2$

$p_B = 5.886 \text{ N/cm}^2 = 5.886 \times 10^4 \text{ N/m}^2$

$$Z_B = 4.0 \text{ m}$$

$$V_B = \frac{Q}{\text{Area}} = \frac{0.2}{.1963} = 1.018 \text{ m/s}$$

Total energy at A

$$= E_A = \frac{p_A}{\rho g} + \frac{V_A^2}{2g} + Z_A$$

$$= \frac{9.81 \times 10^4}{870 \times 9.81} + \frac{(6.369)^2}{2 \times 9.81} + 0 = 11.49 + 2.067 = 13.557 \text{ m}$$

Total energy at B

$$= E_B = \frac{p_B}{\rho g} + \frac{V_B^2}{2g} + Z_B$$

$$= \frac{5.886 \times 10^4}{870 \times 9.81} + \frac{(1.018)^2}{2 \times 9.81} + 4.0 = 6.896 + 0.052 + 4.0 = 10.948 \text{ m}$$

(*i*) **Direction of flow.** As E_A is more than E_B and hence flow is taking place from A to B. **Ans.**

(*ii*) Loss of head $= h_L = E_A - E_B = 13.557 - 10.948 =$ **2.609 m. Ans.**

6.7 PRACTICAL APPLICATIONS OF BERNOULLI'S EQUATION

Bernoulli's equation is applied in all problems of incompressible fluid flow where energy considerations are involved. But we shall consider its application to the following measuring devices :

1. Venturimeter.
2. Orifice meter.
3. Pitot-tube.

6.7.1 Venturimeter. A venturimeter is a device used for measuring the rate of a flow of a fluid flowing through a pipe. It consists of three parts :

(*i*) A short converging part, (*ii*) Throat, and (*iii*) Diverging part. It is based on the Principle of Bernoulli's equation.

Expression for rate of flow through venturimeter

Consider a venturimeter fitted in a horizontal pipe through which a fluid is flowing (say water), as shown in Fig. 6.9.

Let d_1 = diameter at inlet or at section (1),

p_1 = pressure at section (1)

v_1 = velocity of fluid at section (1),

a = area at section (1) $= \frac{\pi}{4} d_1^2$

and d_2, p_2, v_2, a_2 are corresponding values at section (2).

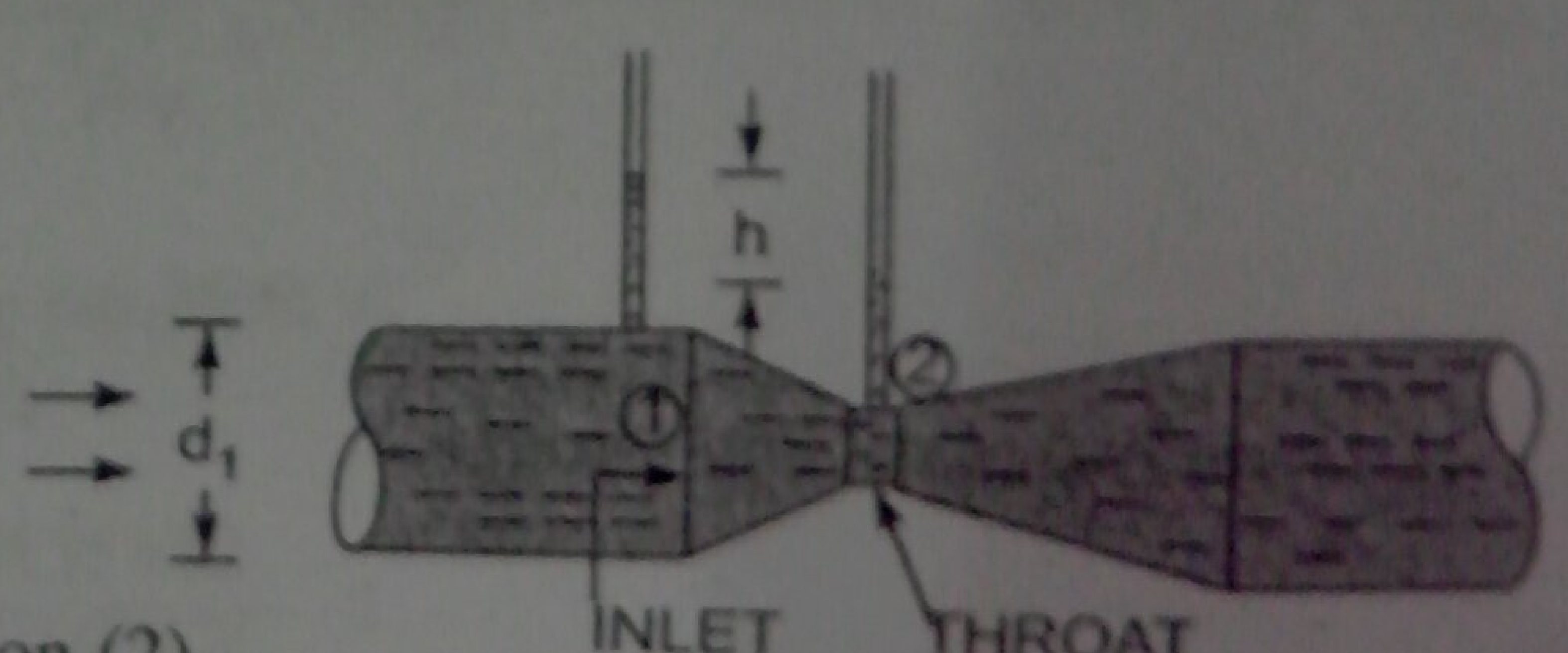


Fig. 6.9 *Venturimeter.*

Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

As pipe is horizontal, hence $z_1 = z_2$

$$\therefore \quad \frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} \quad \text{or} \quad \frac{p_1 - p_2}{\rho g} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

But $\frac{p_1 - p_2}{\rho g}$ is the difference of pressure heads at sections 1 and 2 and it is equal to h or $\frac{p_1 - p_2}{\rho g} = h$

Substituting this value of $\frac{p_1 - p_2}{\rho g}$ in the above equation, we get

$$h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \quad \text{...(6.6)}$$

Now applying continuity equation at sections 1 and 2

$$a_1 v_1 = a_2 v_2 \quad \text{or} \quad v_1 = \frac{a_2 v_2}{a_1}$$

Substituting this value of v_1 in equation (6.6)

$$h = \frac{v_2^2}{2g} - \frac{\left(\frac{a_2 v_2}{a_1}\right)^2}{2g} = \frac{v_2^2}{2g}\left[1 - \frac{a_2^2}{a_1^2}\right] = \frac{v_2^2}{2g}\left[\frac{a_1^2 - a_2^2}{a_1^2}\right]$$

or
$$v_2^2 = 2gh \frac{a_1^2}{a_1^2 - a_2^2}$$

$\therefore$
$$v_2 = \sqrt{2gh \frac{a_1^2}{a_1^2 - a_2^2}} = \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gh}$$

$\therefore$ **Discharge,**
$$Q = a_2 v_2$$
$$= a_2 \frac{a_1}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} \quad \text{...(6.7)}$$

Equation (6.7) gives the discharge under ideal conditions and is called, theoretical discharge. Actual discharge will be less than theoretical discharge.

$\therefore$
$$Q_{act} = C_d \times \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh} \quad \text{...(6.8)}$$

where C_d = Co-efficient of venturimeter and its value is less than 1.

Value of 'h' given by differential U-tube manometer

Case I. Let the differential manometer contains a liquid which is heavier than the liquid flowing through the pipe. Let

S_h = Sp. gravity of the heavier liquid

S_o = Sp. gravity of the liquid flowing through pipe

x = Difference of the heavier liquid column in U-tube

Then
$$h = x\left[\frac{S_h}{S_o} - 1\right] \quad \text{...(6.9)}$$

Case II. If the differential manometer contains a liquid which is lighter than the liquid flowing through the pipe, the value of h is given by

$$h = x\left[1 - \frac{S_l}{S_o}\right] \quad \text{...(6.10)}$$

where S_l = Sp. gr. of lighter liquid in U-tube

S_o = Sp. gr. of fluid flowing through pipe

x = Difference of the lighter liquid columns in U-tube.

Case III. Inclined Venturimeter with Differential U-tube manometer. The above two cases are given for a horizontal venturimeter. This case is related to inclined venturimeter having differential U-tube manometer. Let the differential manometer contains heavier liquid then h is given as

$$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = x\left[\frac{S_h}{S_o} - 1\right] \quad \text{...(6.11)}$$

Case IV. Similarly, for inclined venturimeter in which differential manometer contains a liquid which is lighter than the liquid flowing through the pipe, the value of h is given as

$$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = x\left[1 - \frac{S_l}{S_o}\right] \quad \text{...(6.12)}$$

Problem 6.10 *A horizontal venturimeter with inlet and throat diameters 30 cm and 15 cm respectively is used to measure the flow of water. The reading of differential manometer connected to the inlet and the throat is 20 cm of mercury. Determine the rate of flow. Take $C_d = 0.98$.*

Solution. Given :

Dia. at inlet, $d_1 = 30$ cm

∴ Area at inlet, $a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4}(30)^2 = 706.85 \text{ cm}^2$

Dia. at throat, $d_2 = 15$ cm

∴ $a_2 = \frac{\pi}{4} \times 15^2 = 176.7 \text{ cm}^2$

$C_d = 0.98$

Reading of differential manometer = x = 20 cm of mercury.

∴ Difference of pressure head is given by (6.9)

or
$$h = x\left[\frac{S_h}{S_o} - 1\right]$$

where S_h = Sp. gravity of mercury = 13.6, S_o = Sp. gravity of water = 1

$$= 20\left[\frac{13.6}{1} - 1\right] = 20 \times 12.6 \text{ cm} = 252.0 \text{ cm of water.}$$

The discharge through venturimeter is given by eqn. (6.8)

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{706.85 \times 176.7}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 9.81 \times 252}$$

$$= \frac{86067593.36}{\sqrt{499636.9 - 31222.9}} = \frac{86067593.36}{684.4}$$

$$= 125756 \text{ cm}^3/\text{s} = \frac{125756}{1000} \text{ lit/s} = \textbf{125.756 lit/s. Ans.}$$

Problem 6.11 *An oil of sp. gr. 0.8 is flowing through a venturimeter having inlet diameter 20 cm and throat diameter 10 cm. The oil-mercury differential manometer shows a reading of 25 cm. Calculate the discharge of oil through the horizontal venturimeter. Take $C_d = 0.98$.*

Solution. Given :

Sp. gr. of oil, $S_o = 0.8$

Sp. gr. of mercury, $S_h = 13.6$

Reading of differential manometer, $x = 25$ cm

$\therefore$ Difference of pressure head, $h = x\left[\frac{S_h}{S_o} - 1\right]$

$$= 25\left[\frac{13.6}{0.8} - 1\right] \text{ cm of oil} = 25\,[17 - 1] = 400 \text{ cm of oil.}$$

Dia. at inlet, $d_1 = 20$ cm

$\therefore$ $a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} \times 20^2 = 314.16 \text{ cm}^2$

$d_2 = 10$ cm

$\therefore$ $a_2 = \frac{\pi}{4} \times 10^2 = 78.54 \text{ cm}^2$

$C_d = 0.98$

$\therefore$ The discharge Q is given by equation (6.8)

or $$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times 400}$$

$$= \frac{21421375.68}{\sqrt{98696 - 6168}} = \frac{21421375.68}{304} \text{ cm}^3/\text{s}$$

$$= 70465 \text{ cm}^3/\text{s} = \textbf{70.465 litres/s. Ans.}$$

Problem 6.12 *A horizontal venturimeter with inlet diameter 20 cm and throat diameter 10 cm is used to measure the flow of oil of sp. gr. 0.8. The discharge of oil through venturimeter is 60 litres/s. Find the reading of the oil-mercury differential manometer. Take $C_d = 0.98$.*

Solution. Given : $d_1 = 20$ cm

$\therefore$ $a_1 = \frac{\pi}{4} 20^2 = 314.16 \text{ cm}^2$

$d_2 = 10$ cm

$$\therefore \quad a_2 = \frac{\pi}{4} \times 10^2 = 78.54 \text{ cm}^2$$

$$C_d = 0.98$$

$$Q = 60 \text{ litres/s} = 60 \times 1000 \text{ cm}^3\text{/s}$$

Using the equation (6.8), $Q = C_d \dfrac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$

or $$60 \times 1000 = 9.81 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times h} = \frac{1071068.78\sqrt{h}}{304}$$

or $$\sqrt{h} = \frac{304 \times 60000}{1071068.78} = 17.029$$

$$\therefore \quad h = (17.029)^2 = 289.98 \text{ cm of oil}$$

But $$h = x\left[\frac{S_h}{S_o} - 1\right]$$

where S_h = Sp. gr. of mercury = 13.6

S_o = Sp. gr. of oil = 0.8

x = Reading of manometer

$$\therefore \quad 289.98 = x\left[\frac{13.6}{0.8} - 1\right] = 16x$$

$$\therefore \quad x = \frac{289.98}{16} = 18.12 \text{ cm.}$$

∴ **Reading of oil-mercury differential manometer = 18.12 cm. Ans.**

Problem 6.13 *A horizontal venturimeter with inlet diameter 20 cm and throat diameter 10 cm is used to measure the flow of water. The pressure at inlet is 17.658 N/cm² and the vacuum pressure at the throat is 30 cm of mercury. Find the discharge of water through venturimeter. Take $C_d = 0.98$.*

Solution. Given :

Dia. at inlet, $d_1 = 20$ cm

$$\therefore \quad a_1 = \frac{\pi}{4} \times (20)^2 = 314.16 \text{ cm}^2$$

Dia. at throat, $d_2 = 10$ cm

$$\therefore \quad a_2 = \frac{\pi}{4} \times 10^2 = 78.74 \text{ cm}^2$$

$$p_1 = 17.658 \text{ N/cm}^2 = 17.658 \times 10^4 \text{ N/m}^2$$

ρ for water $= 1000 \dfrac{\text{kg}}{\text{m}^3}$ and $\therefore \dfrac{p_1}{\rho g} = \dfrac{17.658 \times 10^4}{9.81 \times 1000} = 18$ m of water

$$\frac{p_2}{\rho g} = -30 \text{ cm of mercury}$$

$$= -0.30 \text{ m of mercury} = -0.30 \times 13.6 = -4.08 \text{ m of water}$$

∴ Differential head $= h = \frac{p_1}{\rho g} - \frac{p_2}{\rho g} = 18 - (-4.08)$

$= 18 + 4.08 = 22.08$ m of water $= 2208$ cm of water

The discharge Q is given by equation (6.8)

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= 0.98 \times \frac{314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.74)^2}} \times \sqrt{2 \times 981 \times 2208}$$

$$= \frac{50328837.21}{304} \times 165555 \text{ cm}^3/\text{s} = \mathbf{165.555 \text{ lit/s. Ans.}}$$

Problem 6.14 *The inlet and throat diameters of a horizontal venturimeter are 30 cm and 10 cm respectively. The liquid flowing through the meter is water. The pressure intensity at inlet is 13.734 N/cm² while the vacuum pressure head at the throat is 37 cm of mercury. Find the rate of flow. Assume that 4% of the differential head is lost between the inlet and throat. Find also the value of C_d for the venturimeter.*

Solution. Given :

Dia. at inlet, $d_1 = 30$ cm

∴ $a_1 = \frac{\pi}{4}(30)^2 = 706.85 \text{ cm}^2$

Dia. at throat, $d_2 = 10$ cm

∴ $a_2 = \frac{\pi}{4}(10)^2 = 78.54 \text{ cm}^2$

Pressure, $p_1 = 13.734 \text{ N/cm}^2 = 13.734 \times 10^4 \text{ N/m}^2$

∴ Pressure head, $\frac{p_1}{\rho g} = \frac{13.734 \times 10^4}{1000 \times 9.81} = 14$ m of water

$\frac{p_2}{\rho g} = -37$ cm of mercury

$= \frac{-37 \times 13.6}{100}$ m of water $= -5.032$ m of water

Differential head, $h = p_1/\rho g - p_2/\rho g$

$= 14.0 - (-5.032) = 14.0 + 5.032$

$= 19.032$ m of water $= 1903.2$ cm

Head lost, $h_f = 4\%$ of $h = \frac{4}{100} \times 19.032 = 0.7613$ m

∴ $C_d = \sqrt{\frac{h - h_f}{h}} = \sqrt{\frac{19.032 - .7613}{19.032}} = 0.98$

$\therefore$ Discharge $= C_d \dfrac{a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$

$$= \frac{0.98 \times 706.85 \times 78.54 \times \sqrt{2 \times 981 \times 1903.2}}{\sqrt{(706.85)^2 - (78.54)^2}}$$

$$= \frac{105132247.8}{\sqrt{499636.9 - 6168}} = 149692.8 \text{ cm}^3/\text{s} = \mathbf{0.14969\ m^3/s.\ Ans.}$$

PROBLEMS ON INCLINED VENTURIMETER

Problem 6.15 *A 30 cm × 15 cm venturimeter is inserted in a vertical pipe carrying water, flowing in the upward direction. A differential mercury manometer connected to the inlet and throat gives a reading of 20 cm. Find the discharge. Take $C_d = 0.98$.*

Solution. Given :

Dia. at inlet, $d_1 = 30$ cm

$\therefore$ $a_1 = \dfrac{\pi}{4}(30)^2 = 706.85 \text{ cm}^2$

Dia. at throat, $d_2 = 15$ cm

$\therefore$ $a_2 = \dfrac{\pi}{4}(15)^2 = 176.7 \text{ cm}^2$

$$h = x\left[\frac{S_h}{S_o} - 1\right] = 20\left[\frac{13.6}{1.0} - 1.0\right] = 20 \times 12.6 = 252.0 \text{ cm of water}$$

$C_d = 0.98$

Discharge, $Q = C_d \dfrac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$

$$= 0.98 \times \frac{706.85 \times 176.7}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 981 \times 252}$$

$$= \frac{86067593.36}{\sqrt{499636.3 - 31222.9}} = \frac{86067593.36}{684.4}$$

$= 125756 \text{ cm}^3/\text{s} = \mathbf{125.756\ lit/s.\ Ans.}$

Problem 6.16 *A 20 cm × 10 cm venturimeter is inserted in a vertical pipe carrying oil of sp. gr. 0.8, the flow of oil is in upward direction. The difference of levels between the throat and inlet section is 50 cm. The oil mercury differential manometer gives a reading of 30 cm of mercury. Find the discharge of oil. Neglect losses.*

Solution. Dia. at inlet, $d_1 = 20$ cm

$\therefore$ $a_1 = \dfrac{\pi}{4}(20)^2 = 314.16 \text{ cm}^2$

Dia. at throat, $d_2 = 10$ cm

$\therefore \quad a_2 = \frac{\pi}{4}(10)^2 = 78.54 \text{ cm}^2$

Sp. gr. of oil, $S_o = 0.8$

Sp. gr. of mercury, $S_g = 13.6$

Differential manometer reading, $x = 30$ cm

$$\therefore \quad h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = x\left[\frac{S_g}{S_o} - 1\right]$$

$$= 30\left[\frac{13.6}{0.8} - 1\right] = 30\,[17 - 1] = 30 \times 16 = 480 \text{ cm of oil}$$

$$C_d = 1.0$$

The discharge,

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

$$= \frac{1.0 \times 314.16 \times 78.54}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times 480} \text{ cm}^3/\text{s}$$

$$= \frac{23932630.7}{304} = 78725.75 \text{ cm}^3/\text{s} = 78.725 \text{ litres/s. Ans.}$$

Problem 6.17 *In a vertical pipe conveying oil of specific gravity 0.8, two pressure gauges have been installed at A and B where the diameters are 16 cm and 8 cm respectively. A is 2 metres above B. The pressure gauge readings have shown that the pressure at B is greater than at A by 0.981 N/cm². Neglecting all losses, calculate the flow rate. If the gauges at A and B are replaced by tubes filled with the same liquid and connected to a U-tube containing mercury, calculate the difference of level of mercury in the two limbs of the U-tube.*

Solution. Given :

Sp. gr. of oil, $S_o = 0.8$

$\therefore$ Density, $\rho = 0.8 \times 1000 = 800 \frac{\text{kg}}{\text{m}^3}$

Dia. at A, $D_A = 16 \text{ cm} = 0.16 \text{ m}$

$\therefore$ Area at A, $A_1 = \frac{\pi}{4}(.16)^2 = 0.0201 \text{ m}^2$

Dia. at B, $D_B = 8 \text{ cm} = 0.08 \text{ m}$

$\therefore$ Area at B, $A_2 = \frac{\pi}{4}(.08)^2 = 0.005026 \text{ m}^2$

(*i*) Difference of pressures, $p_B - p_A = 0.981 \text{ N/cm}^2$

$$= 0.981 \times 10^4 \text{ N/m}^2 = \frac{9810 \text{N}}{\text{m}^2}$$

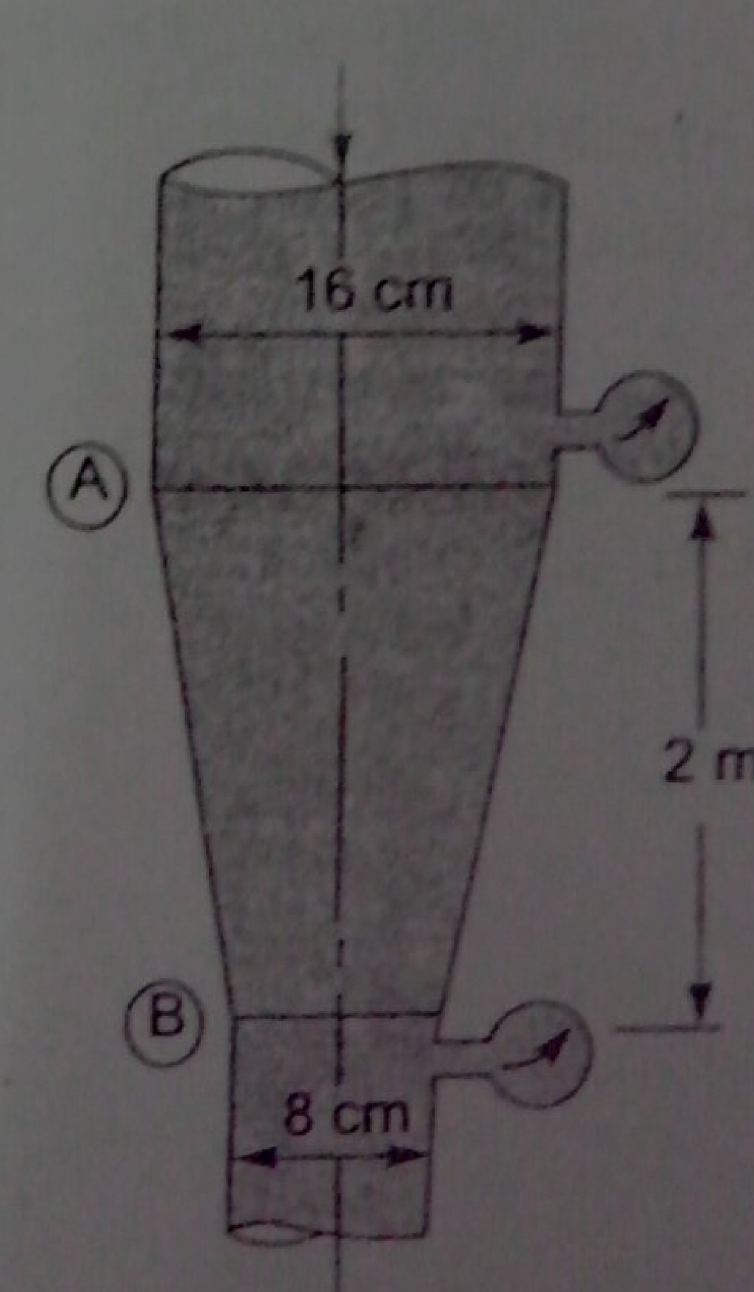


Fig. 6.9 (*a*)

Difference of pressure head

$$\therefore \quad \frac{p_B - p_A}{\rho g} = \frac{9810}{800 \times 9.81} = 1.25 \qquad (\because \rho = 800 \text{ kg/m}^3)$$

Applying Bernoulli's theorem at A and B and taking the reference line passing through section B, we get

$$\frac{p_A}{\rho g}+\frac{V_A^2}{2g}+Z_A=\frac{p_B}{\rho g}+\frac{V_B^2}{2g}+Z_B$$

or
$$\frac{p_A}{\rho g}-\frac{p_B}{p_g}-Z_A-Z_B=\frac{V_B^2}{2g}-\frac{V_A^2}{2g}$$

or
$$\left(\frac{p_A-p_B}{\rho g}\right)+2.0-0.0=\frac{V_B^2}{2g}-\frac{V_A^2}{2g}$$

or
$$-1.25+2.0=\frac{V_B^2}{2g}-\frac{V_A^2}{2g}\qquad\left(\because \frac{p_B-p_A}{\rho g}=1.25\right)$$

$$0.75=\frac{V_B^2}{2g}-\frac{V_A^2}{2g}\qquad\ldots(i)$$

Now applying continuity equation at A and B, we get

$$V_A\times A_1=V_B\times A_2$$

or
$$V_B=\frac{V_A\times A_1}{A_2}=\frac{V_A\times\frac{\pi}{4}(.16)^2}{\frac{\pi}{4}(.08)^2}=4V_A$$

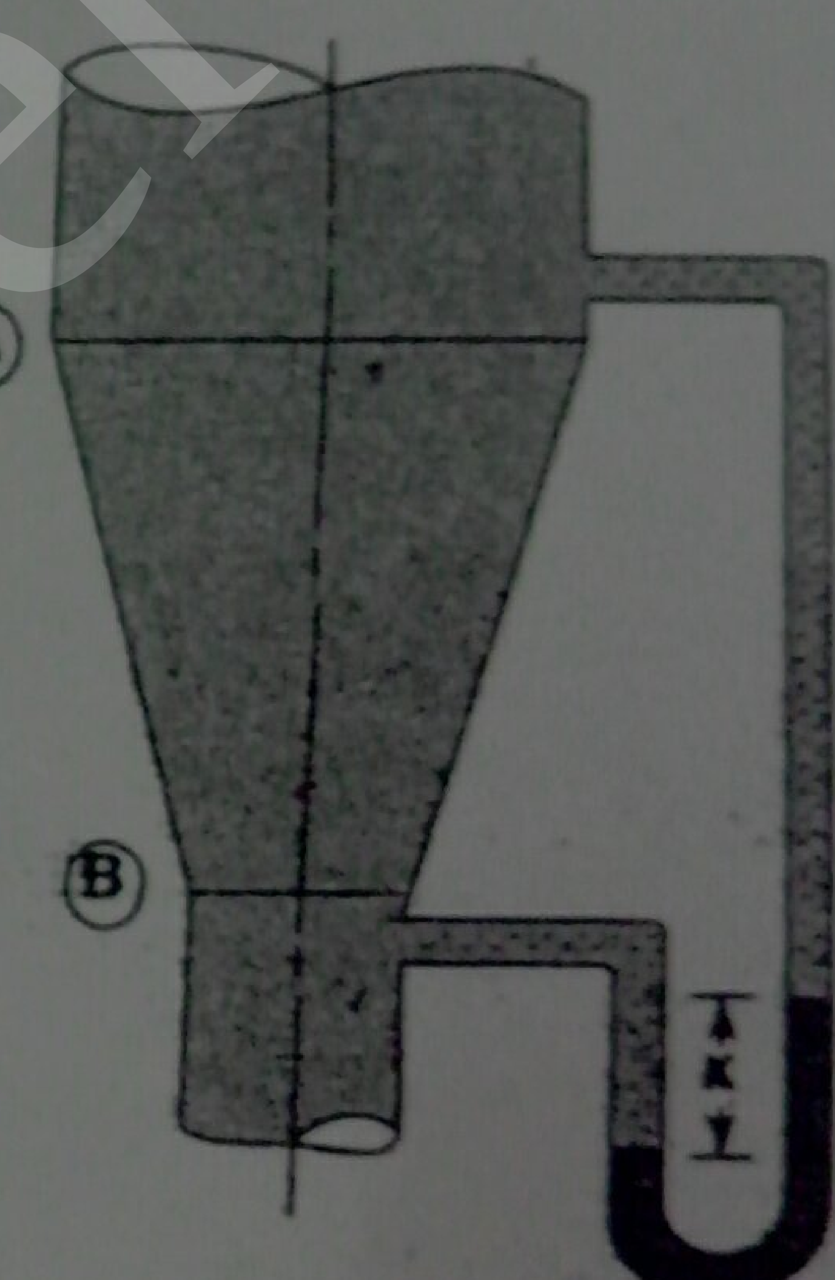


Fig. 6.9 (*b*)

Substituting the value of V_B in equation (*i*), we get

$$0.75=\frac{16V_A^2}{2g}-\frac{V_A^2}{2g}=\frac{15V_A^2}{2g}$$

$\therefore$
$$V_A=\sqrt{\frac{0.75\times2\times9.81}{15}}=\mathbf{0.99\ m/s}$$

$\therefore$ Rate of flow,
$$Q=V_A\times A_1$$
$$=0.99\times0.0201=\mathbf{0.01989\ m^3/s.\ Ans.}$$

(*ii*) Difference of level of mercury in the U-tube.

Let h = Difference of mercury level.

Then
$$h=x\left(\frac{S_g}{S_o}-1\right)$$

where
$$h=\left(\frac{p_A}{\rho g}+Z_A\right)-\left(\frac{p_B}{\rho g}+Z_B\right)=\frac{p_A-p_B}{\rho g}+Z_A-Z_B$$
$$=-1.25+2.0-0$$
$$=0.75\qquad\left(\because \frac{p_B-p_A}{\rho g}=1.25\right)$$

$\therefore$
$$0.75=x\left[\frac{13.6}{0.8}-1\right]=x\times16$$

$\therefore$
$$x=\frac{0.75}{16}=0.04687\text{ m}=\mathbf{4.687\ cm.\ Ans.}$$

Problem 6.18 *Find the discharge of water flowing through a pipe 30 cm diameter placed in an inclined position where a venturimeter is inserted, having a throat diameter of 15 cm. The difference of pressure between the main and throat is measured by a liquid of sp. gr. 0.6 in an inverted U-tube which gives a reading of 30 cm. The loss of head between the main and throat is 0.2 times the kinetic head of the pipe.*

Solution. Dia. at inlet, $d_1 = 30$ cm

$\therefore$ $a_1 = \frac{\pi}{4}(30)^2 = 706.85 \text{ cm}^2$

Dia. at throat, $d_2 = 15$ cm

$\therefore$ $a_2 = \frac{\pi}{4}(15)^2 = 176.7 \text{ cm}^2$

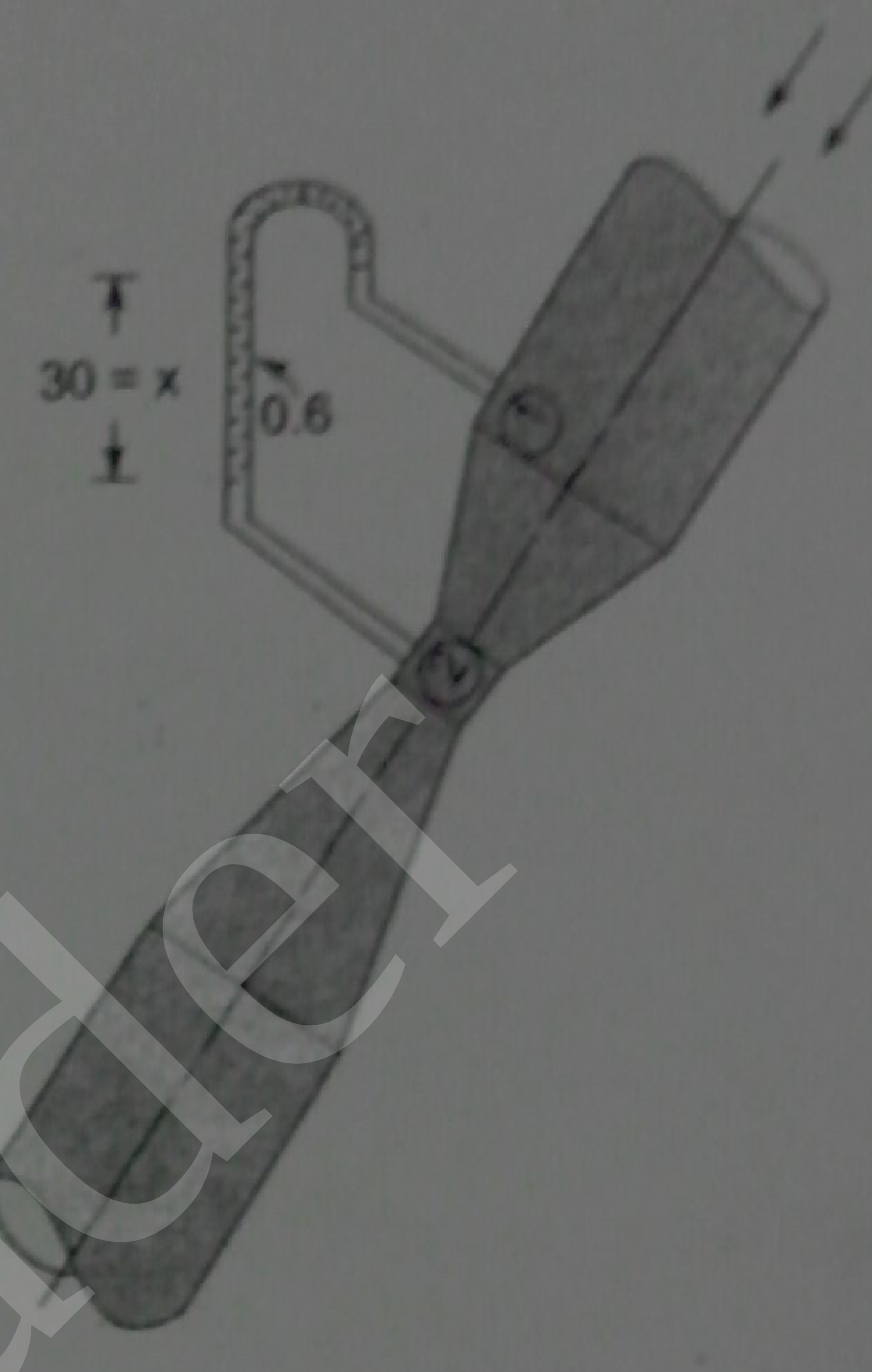


Fig. 6.10

Reading of differential manometer, $x = 30$ cm

Difference of pressure head, h is given by

$$\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = h$$

Also $$h = x\left[1 - \frac{S_l}{S_o}\right]$$

where $S_l = 0.6$ and $S_o = 1.0$

$$= 30\left[1 - \frac{0.6}{1.0}\right] = 30 \times .4 = 12.0 \text{ cm of water}$$

Loss of head, $h_L = 0.2 \times$ kinetic head of pipe $= 0.2 \times \frac{v_1^2}{2g}$

Now applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + z_2 + \frac{v_2^2}{2g} + h_L$$

or $$\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) + \frac{v_1^2}{2g} - \frac{v_2^2}{2g} = h_L$$

But $$\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = h = 12.0 \text{ cm of water}$$

and $$h_L = 0.2 \times v_1^2/2g$$

$\therefore$ $$12.0 + \frac{v_1^2}{2g} - \frac{v_2^2}{2g} = 0.2 \times \frac{v_1^2}{2g}$$

$\therefore$ $$12.0 + 0.8\frac{v_1^2}{2g} - \frac{v_2^2}{2g} = 0 \qquad \text{...(1)}$$

Applying continuity equation at sections (1) and (2), we get

$$a_1v_1 = a_2v_2$$

$\therefore$
$$v_1 = \frac{a_2}{a_1} v_2 = \frac{\frac{\pi}{4}(15)^2 v_2}{\frac{\pi}{4}(30)^2} = \frac{v_2}{4}$$

Substituting this value of v_1 in equation (1), we get

$$12.0 + \frac{0.8}{2g}\left(\frac{v_2}{4}\right)^2 - \frac{v_2^2}{2g} = 0 \text{ or } 12.0 + \frac{v_2^2}{2g}\left[\frac{0.8}{16} - 1\right] = 0$$

or
$$\frac{v_2^2}{2g}[.05 - 1] = -12.0 \text{ or } \frac{0.95\, v_2^2}{2g} = 12.0$$

$\therefore$
$$v_2 = \sqrt{\frac{2 \times 981 \times 12.0}{0.95}} = 157.4 \text{ cm/s}$$

$\therefore$ Discharge $= a_2v_2$

$= 176.7 \times 157.4 \text{ cm}^3\text{/s} = 27800 \text{ cm}^3\text{/s} =$ **27.8 litres/s. Ans.**

Problem 6.19 *A 30 cm × 15 cm venturimeter is provided in a vertical pipe line carrying oil of specific gravity 0.9, the flow being upwards. The difference in elevation of the throat section and entrance section of the venturimeter is 30 cm. The differential U-tube mercury manometer shows a gauge deflection of 25 cm. Calculate :*

(i) the discharge of oil, and

(ii) the pressure difference between the entrance section and the throat section. Take the co-efficient of discharge as 0.98 and specific gravity of mercury as 13.6.

Solution. Given :

Dia. at inlet, $d_1 = 30$ cm

$\therefore$ Area, $a_1 = \frac{\pi}{4}(30)^2 = 706.85 \text{ cm}^2$

Dia. at throat, $d_2 = 15$ cm

$\therefore$ Area, $a_2 = \frac{\pi}{4}(15)^2 = 176.7 \text{ cm}^2$

Let section (1) represents inlet and section (2) represents throat. Then $z_2 - z_1 = 30$ cm

Sp. gr. of oil, $S_o = 0.9$

Sp. gr. of mercury, $S_g = 13.6$

Reading of diff. manometer, $x = 25$ cm

The differential head, h is given by

$$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right)$$

$$= x\left[\frac{S_g}{S_o} - 1\right] = 25\left[\frac{13.6}{0.9} - 1\right] = 352.77 \text{ cm of oil}$$

(*i*) The discharge, Q of oil $= C_d \dfrac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$

$$= \frac{0.98 \times 706.85 \times 176.7}{\sqrt{(706.85)^2 - (176.7)^2}} = \sqrt{2 \times 981 \times 352.77}$$

$$= \frac{101832219.9}{684.4} = 148790.5 \text{ cm}^3/\text{s}$$

$= \textbf{148.79 litres/s. Ans.}$

(*ii*) Pressure difference between entrance and throat section

$$h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = 352.77$$

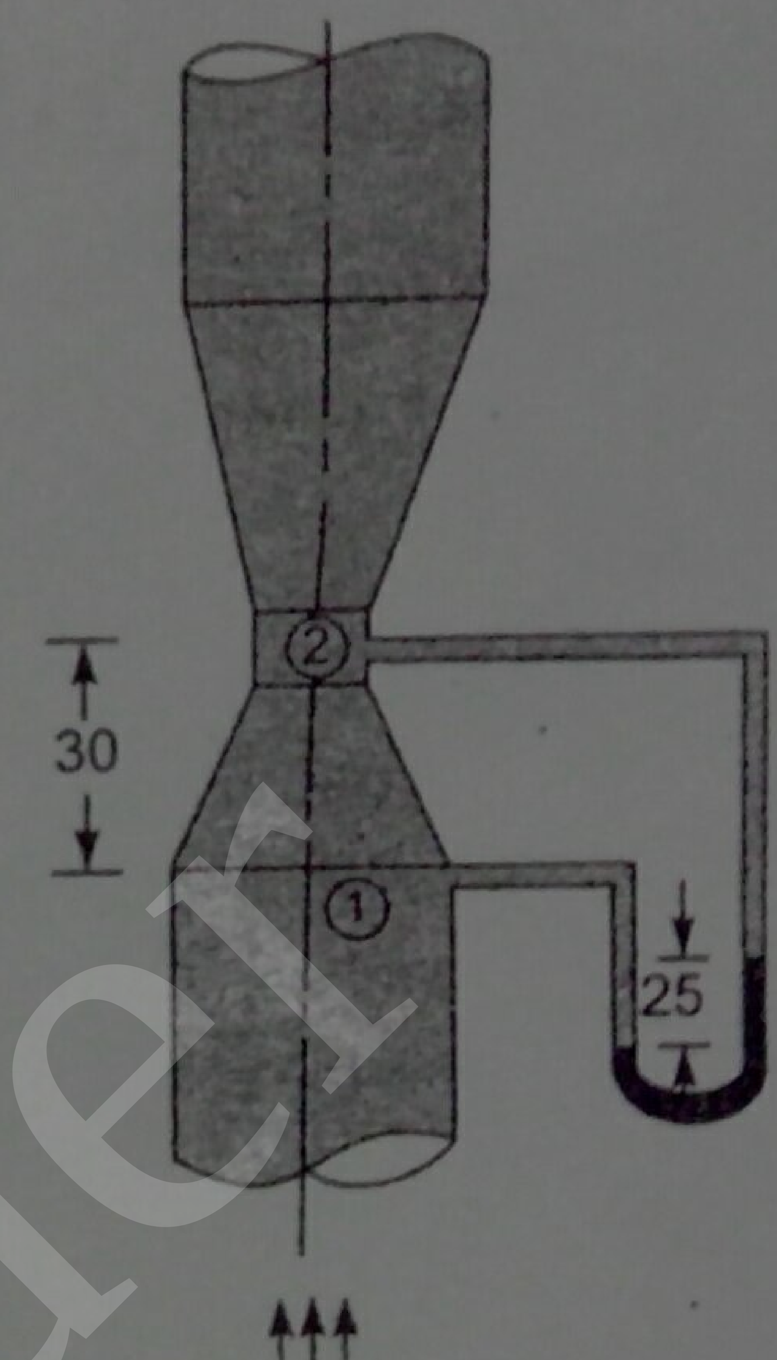


Fig. 6.11

or $$\left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) + z_1 - z_2 = 352.77$$

But $$z_2 - z_1 = 30 \text{ cm}$$

$\therefore$ $$\left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) - 30 = 352.77$$

$\therefore$ $$\frac{p_1}{\rho g} - \frac{p_2}{\rho g} = 352.77 + 30 = 382.77 \text{ cm of oil} = \textbf{3.8277 m of oil. Ans.}$$

or $$(p_1 - p_2) = 3.8277 \times \rho g$$

But density of oil $= \text{Sp. gr. of oil} \times 1000 \text{ kg/m}^3$

$= 0.9 \times 1000 = 900 \text{ kg/cm}^3$

$\therefore$ $$(p_1 - p_2) = 3.8277 \times 900 \times 9.81 \frac{\text{N}}{\text{m}^2}$$

$$= \frac{33795}{10^4} \text{ N/cm}^2 = \textbf{3.3795 N/cm}^2\textbf{. Ans.}$$

Problem 6.20 *Crude oil of specific gravity 0.85 flows upwards at a volume rate of flow of 60 litre per second through a vertical venturimeter with an inlet diameter of 200 mm and a throat diameter of 100 mm. The co-efficient of discharge of the venturimeter is 0.98. The vertical distance between the pressure tappings is 300 mm.*

(i) If two pressure gauges are connected at the tappings such that they are positioned at the levels of their corresponding tapping points, determine the difference of readings in N/cm² of the two pressure gauges.

(ii) If a mercury differential manometer is connected, in place of pressure gauges, to the tappings such that the connecting tube upto mercury are filled with oil, determine the difference in the level of the mercury column.

Solution. Given :

Specific gravity of oil, $S_o = 0.85$

$\therefore$ Density, $\rho = 0.85 \times 1000 = 850$ kg/m^3

Discharge, $Q = 60$ litre/s

$$= \frac{60}{1000} = 0.06 \text{ m}^3\text{/s}$$

Inlet dia, $d_1 = 200$ mm $= 0.2$ m

$\therefore$ Area, $a_1 = \frac{\pi}{4}(.2)^2 = 0.0314 \text{ m}^2$

Throat dia., $d_2 = 100$ mm $= 0.1$ m

$\therefore$ Area, $a_2 = \frac{\pi}{4}(0.1)^2 = 0.00785 \text{ m}^2$

Value of C_d $= 0.98$

Let section (1) represents inlet and section (2) represents throat. Then

$$z_2 - z_1 = 300 \text{ mm} = 0.3 \text{ m}$$

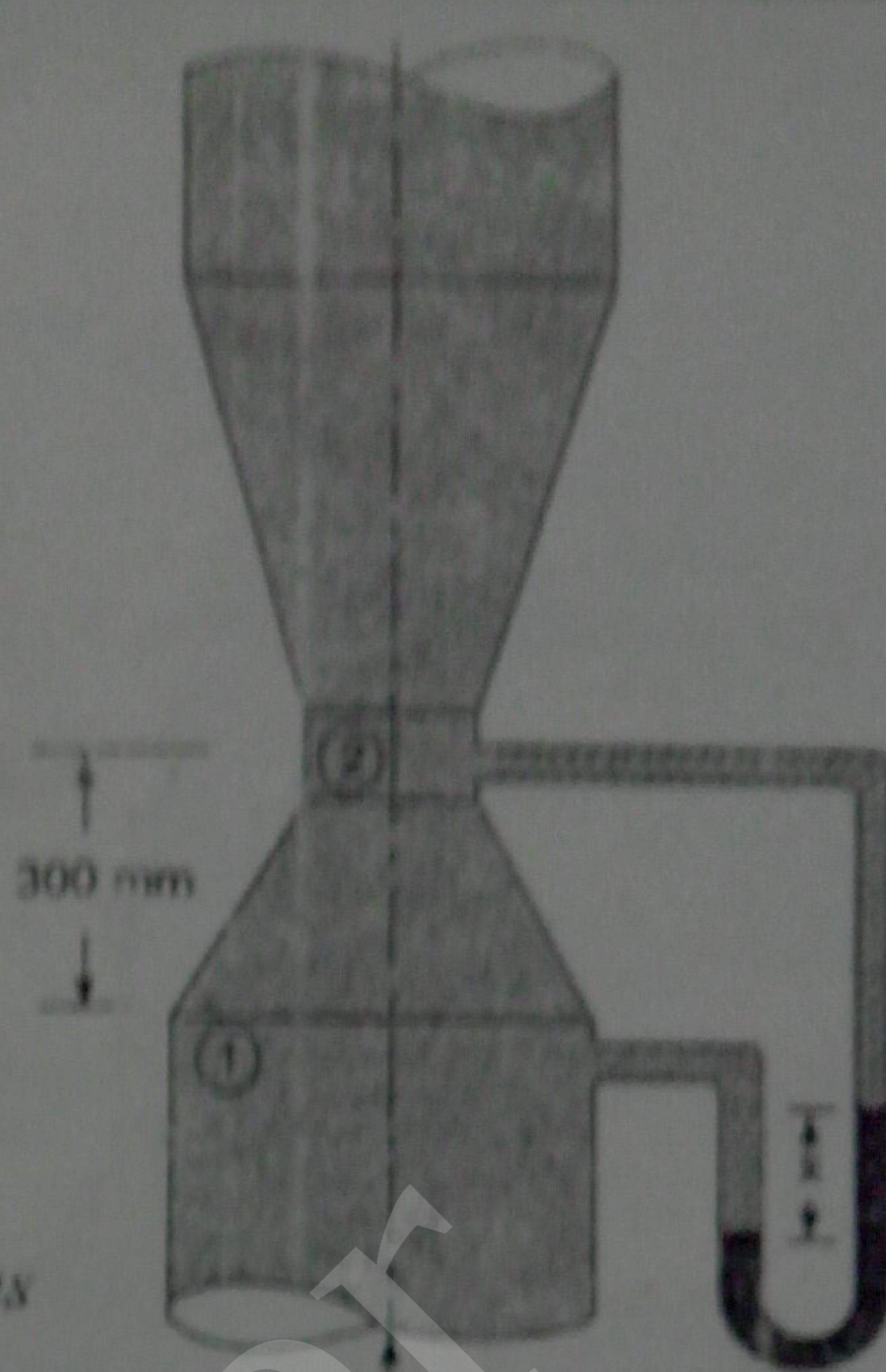


Fig. 6.11 (*d*)

(i) Difference of readings in N/cm^2 of the two pressure gauges

The discharge Q is given by,

$$Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$$

or
$$0.06 = \frac{0.98 \times 0.0314 \times 0.00785}{\sqrt{0.0314^2 - 0.00785^2}} \times \sqrt{2 \times 9.81 \times h}$$

$$= \frac{0.98 \times 0.00024649}{0.0304} \times 4.429\sqrt{h}$$

$\therefore$
$$\sqrt{h} = \frac{0.06 \times 0.0304}{0.98 \times 0.00024649 \times 4.429} = 1.705$$

$\therefore$
$$h = 1.705^2 = 2.908 \text{ m}$$

But for a vertical venturimeter, $h = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right)$

$\therefore$
$$2.908 = \left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = \left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) + z_1 - z_2$$

$$\frac{p_1 - p_2}{\rho g} = 2.908 + z_2 - z_1 = 2.908 + 0.3 \qquad (\because \; z_2 - z_1 = 0.3 \text{ m})$$

$$= 3.208 \text{ m of oil}$$

$\therefore$
$$p_1 - p_2 = \rho g \times 3.208$$

$$= 850 \times 9.81 \times 3.208 \text{ N/m}^2 = \frac{850 \times 9.81 \times 3.208}{10^4} \text{ N/cm}^2$$

$$= \mathbf{2.675 \text{ N/cm}^2}. \textbf{ Ans.}$$

(ii) Difference in the levels of mercury columns (i.e., x)

The value of h is given by, $h = x\left[\frac{S_g}{S_o} - 1\right]$

$$\therefore \quad 2.908 = x\left[\frac{13.6}{0.85} - 1\right] = x\,[16 - 1] = 15\,x$$

$$\therefore \quad x = \frac{2.908}{15} = 0.1938 \text{ m} = \mathbf{19.38 \text{ cm of oil. Ans.}}$$

Problem 6.21 *In a 100 mm diameter horizontal pipe a venturimeter of 0.5 contraction ratio has been fixed. The head of water on the metre when there is no flow is 3 m (gauge). Find the rate of flow for which the throat pressure will be 2 metres of water absolute. The co-efficient of discharge is 0.97. Take atmospheric pressure head = 10.3 m of water.*

Solution. Given :

Dia. of pipe, $d_1 = 100 \text{ mm} = 10 \text{ cm}$

$\therefore$ Area, $a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4}(10)^2 = 78.54 \text{ cm}^2$

Dia. at throat, $d_2 = 0.5 \times d_1 = 0.5 \times 10 = 5 \text{ cm}$

$\therefore$ Area, $a_2 = \frac{\pi}{4}(5)^2 = 19.635 \text{ cm}^2$

Head of water for no flow $= \frac{p_1}{\rho g} = 3 \text{ m (gauge)} = 3 + 10.3 = 13.3 \text{ m (abs.)}$

Throat pressure head $= \frac{p_2}{\rho g} = 2 \text{ m of water absolute.}$

$\therefore$ **Difference of pressure head,** $h = \frac{p_1}{\rho g} - \frac{p_2}{\rho g} = 13.3 - 2.0 = 11.3 \text{ m} = 1130 \text{ cm}$

$\therefore$ Rate of flow, Q is given by $Q = C_d \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \times \sqrt{2gh}$

$$= 0.97 \times \frac{78.54 \times 19.635}{\sqrt{(78.54)^2 - (19.635)^2}} \times \sqrt{2 \times 981 \times 1130}$$

$$= \frac{2227318.17}{76} = 29306.8 \text{ cm}^3/\text{s} = \mathbf{29.306 \text{ litres/s. Ans.}}$$

6.7.2 Orifice Meter or Orifice Plate. It is a device used for measuring the rate of flow of a fluid through a pipe. It is a cheaper device as compared to venturimeter. It also works on the same principle as that of venturimeter. It consists of a flat circular plate which has a circular sharp edged hole called orifice, which is concentric with the pipe. The orifice diameter is kept generally 0.5 times the diameter of the pipe, though it may vary from 0.4 to 0.8 times the pipe diameter.

A differential manometer is connected at section (1), which is at a distance of about 1.5 to 2.0 times the pipe diameter upstream from the orifice plate, and at section (2), which is at a distance of about half the diameter of the orifice on the downstream side from the orifice plate.

Let p_1 = pressure at section (1),
v_1 = velocity at section (1),
a_1 = area of pipe at section (1), and

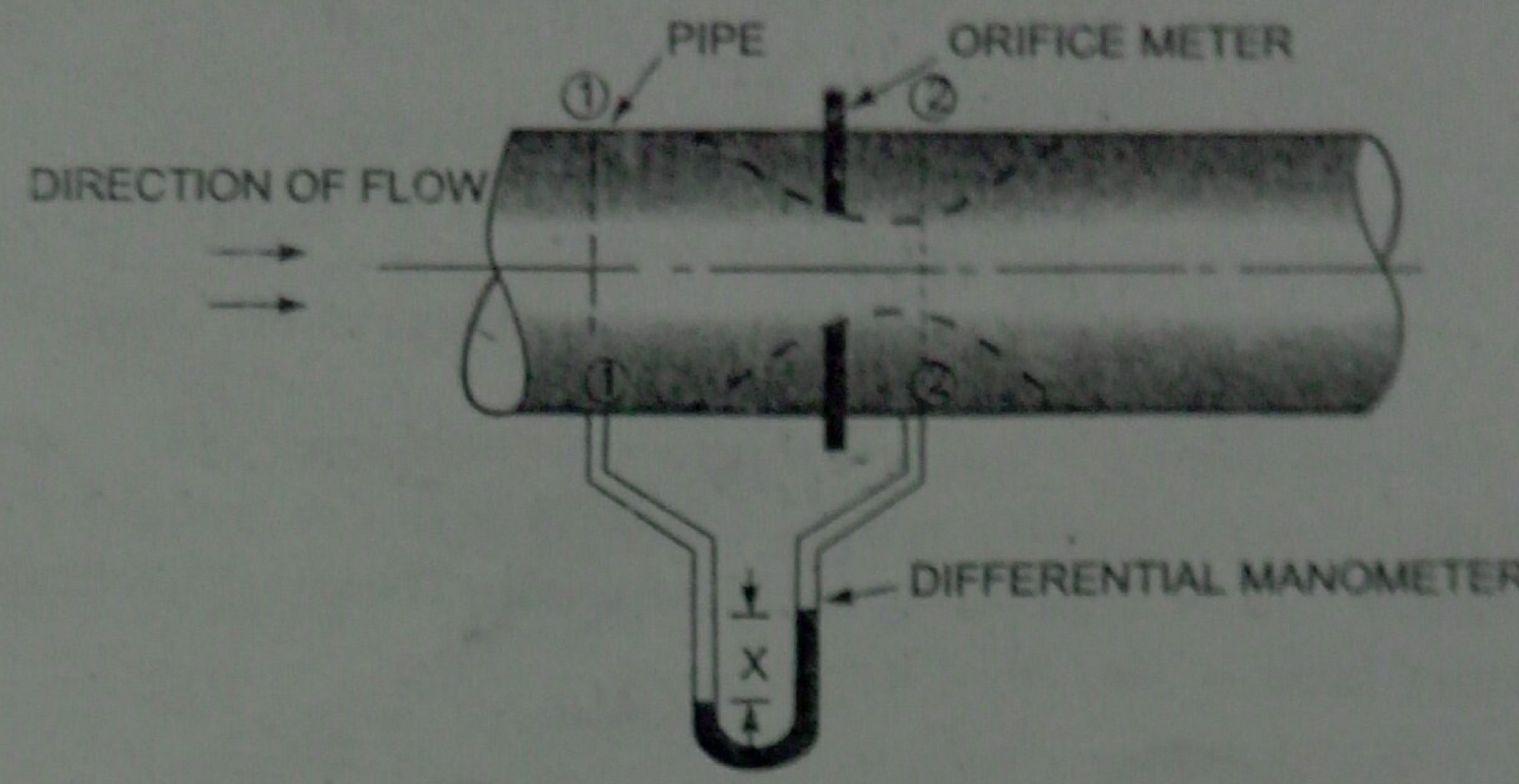


Fig. 6.12. *Orifice meter.*

p_2, v_2, a_2 are corresponding values at section (2). Applying Bernoulli's equation at sections (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

or
$$\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

But
$$\left(\frac{p_1}{\rho g} + z_1\right) - \left(\frac{p_2}{\rho g} + z_2\right) = h = \text{Differential head}$$

$\therefore$
$$h = \frac{v_2^2}{2g} - \frac{v_1^2}{2g} \quad \text{or} \quad 2gh = v_2^2 - v_1^2$$

or
$$v_2 = \sqrt{2gh + v_1^2} \qquad \text{...(i)}$$

Now section (2) is at the vena-contracta and a_2 represents the area at the vena-contracta. If a_0 is the area of orifice then, we have

$$C_c = \frac{a_2}{a_0}$$

where C_c = Co-efficient of contraction

$\therefore$
$$a_2 = a_0 \times C_c \qquad \text{...(ii)}$$

By continuity equation, we have

$$a_1 v_1 = a_2 v_2 \quad \text{or} \quad v_1 = \frac{a_2}{a_1} v_2 = \frac{a_0 C_c}{a_1} v_2 \qquad \text{...(iii)}$$

Substituting the value of v_1 in equation (i), we get

$$v_2 = \sqrt{2gh + \frac{a_0^2 C_c^2 v_2^2}{a_1^2}}$$

or

$$v_2^2 = 2gh + \left(\frac{a_0}{a_1}\right)^2 C_c^2 v_2^2 \text{ or } v_2^2\left[1-\left(\frac{a_0}{a_1}\right)^2 C_c^2\right] = 2gh$$

$\therefore$

$$v_2 = \frac{\sqrt{2gh}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

$\therefore$ The discharge $Q = v_2 \times a_2 = v_2 \times a_0 C_c$ $\{\because a_2 = a_0 C_c \text{ from } (ii)\}$

$$= \frac{a_0 C_c \sqrt{2gh}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 C_c^2}} \quad \ldots(iv)$$

The above expression is simplified by using

$$C_d = C_c \frac{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

$\therefore$

$$C_c = C_d \frac{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 C_c^2}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}}$$

Substituting this value of C_c in equation (iv), we get

$$Q = a_0 \times C_d \frac{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 C_c^2}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}} \times \frac{\sqrt{2gh}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2 C_c^2}}$$

$$= \frac{C_d a_0 \sqrt{2gh}}{\sqrt{1-\left(\frac{a_0}{a_1}\right)^2}} = \frac{C_d a_0 a_1 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}. \quad \ldots(6.13)$$

$a_0 C_c = a_2$

where C_d = Co-efficient of discharge for orifice meter.

The co-efficient of discharge for orifice meter is much smaller than that for a venturimeter.

Problem 6.22 *An orifice meter with orifice diameter 10 cm is inserted in a pipe of 20 cm diameter. The pressure gauges fitted upstream and downstream of the orifice meter gives readings of 19.62 N/cm² and 9.81 N/cm² respectively. Co-efficient of discharge for the orifice meter is given as 0.6. Find the discharge of water through pipe.*

Solution. Given :

Dia. of orifice, $d_0 = 10$ cm

$\therefore$ Area, $a_0 = \frac{\pi}{4}(10)^2 = 78.54\ \text{cm}^2$

Dia. of pipe, $d_1 = 20$ cm

$\therefore$ Area, $a_1 = \frac{\pi}{4}(20)^2 = 314.16\ \text{cm}^2$

$p_1 = 19.62\ \text{N/cm}^2 = 19.62 \times 10^4\ \text{N/m}^2$

$\therefore$ $$\frac{p_1}{\rho g} = \frac{19.62 \times 10^4}{1000 \times 9.81} = 20 \text{ m of water}$$

Similarly $$\frac{p_2}{\rho g} = \frac{9.81 \times 10^4}{1000 \times 9.81} = 10 \text{ m of water}$$

$\therefore$ $$h = \frac{p_1}{\rho g} - \frac{p_2}{\rho g} = 20.0 - 10.0 = 10 \text{ m of water} = 1000 \text{ cm of water}$$

$C_d = 0.6$

The discharge, Q is given by equation (6.13)

$$Q = C_d \frac{a_0 a_1}{\sqrt{a_1^2 - a_0^2}} \times \sqrt{2gh}$$

$$= 0.6 \times \frac{78.54 \times 314.16}{\sqrt{(314.16)^2 - (78.54)^2}} \times \sqrt{2 \times 981 \times 1000}$$

$$= \frac{20736838.09}{304} = 68213.28 \text{ cm}^3\text{/s} = 68.21 \text{ litres/s. Ans.}$$

Problem 6.23 *An orifice meter with orifice diameter 15 cm is inserted in a pipe of 30 cm diameter. The pressure difference measured by a mercury oil differential manometer on the two sides of the orifice meter gives a reading of 50 cm of mercury. Find the rate of flow of oil of sp. gr. 0.9 when the co-efficient of discharge of the orifice meter = 0.64.*

Solution. Given :

Dia. of orifice, $d_0 = 15$ cm

$\therefore$ Area, $a_0 = \frac{\pi}{4}(15)^2 = 176.7\ \text{cm}^2$

Dia. of pipe, $d_1 = 30$ cm

$\therefore$ Area, $a_1 = \frac{\pi}{4}(30)^2 = 706.85\ \text{cm}^2$

Sp. gr. of oil, $S_o = 0.9$

Reading of diff. manometer, $x = 50$ cm of mercury

$\therefore$ Differential head $$h = x\left[\frac{S_g}{S_o} - 1\right] = 50\left[\frac{13.6}{0.9} - 1\right] \text{ cm of oil}$$

$= 50 \times 14.11 = 705.5$ cm of oil

$C_d = 0.64$

∴ The rate of the flow, Q is given by equation (6.13)

$$Q = C_d \cdot \frac{a_0 a_1}{\sqrt{a_1^2 - a_0^2}} \times \sqrt{2gh}$$

$$= 0.64 \times \frac{176.7 \times 706.85}{\sqrt{(706.85)^2 - (176.7)^2}} \times \sqrt{2 \times 981 \times 705.5}$$

$$= \frac{94046317.78}{684.4} = 137414.25 \text{ cm}^3/\text{s} = 137.414 \text{ litres/s. Ans.}$$

6.7.3 Pitot-tube. It is a device used for measuring the velocity of flow at any point in a pipe or a channel. It is based on the principle that if the velocity of flow at a point becomes zero, the pressure there is increased due to the conversion of the kinetic energy into pressure energy. In its simplest form, the pitot-tube consists of a glass tube, bent at right angles as shown in Fig. 6.13.

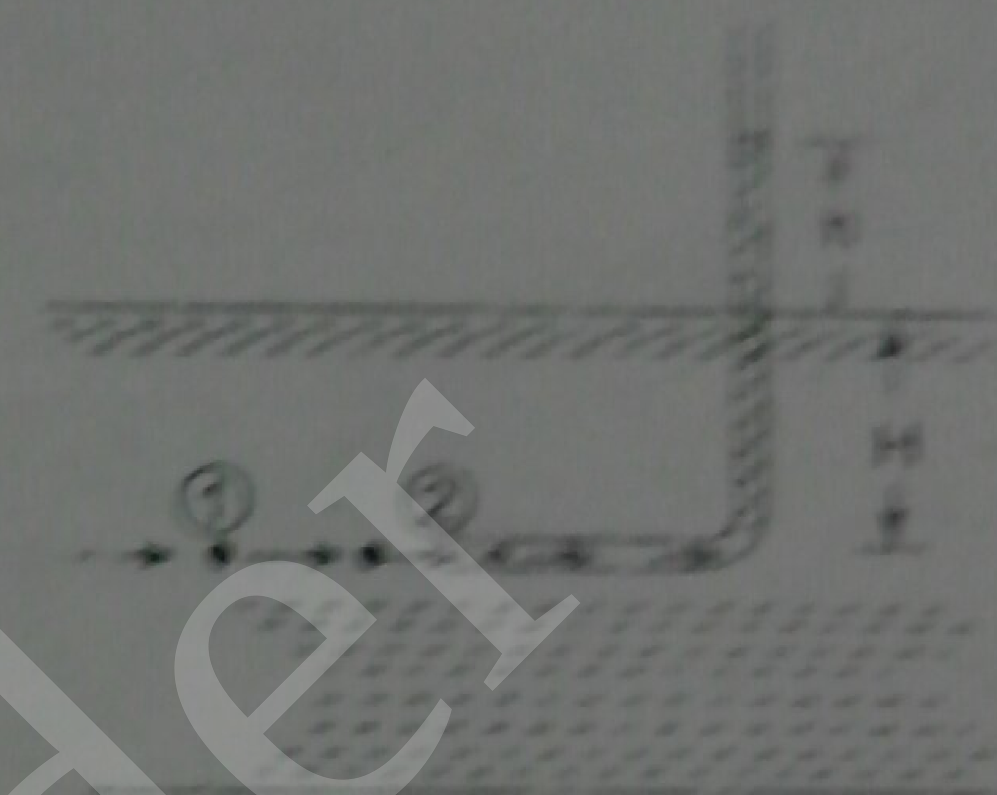


Fig. 6.13 Pitot-tube.

The lower end, which is bent through 90° is directed in the up-stream direction as shown in Fig. 6.13. The liquid rises up in the tube due to the conversion of kinetic energy into pressure energy. The velocity is determined by measuring the rise of liquid in the tube.

Consider two points (1) and (2) at the same level in such a way that point (2) is just at the inlet of the pitot-tube and point (1) is far away from the tube.

Let p_1 = intensity of pressure at point (1)

v_1 = velocity of flow at (1)

p_2 = pressure at point (2)

v_2 = velocity at point (2), which is zero

H = depth of tube in the liquid

h = rise of liquid in the tube above the free surface.

Applying Bernoulli's equation at points (1) and (2), we get

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

But $z_1 = z_2$ as points (1) and (2) are on the same line and $v_2 = 0$.

$$\frac{p_1}{\rho g} = \text{pressure head at (1)} = H$$

$$\frac{p_2}{\rho g} = \text{pressure head at (2)} = (h + H)$$

Substituting these values, we get

$$\therefore \quad H + \frac{v_1^2}{2g} = (h + H) \quad \therefore \quad h = \frac{v_1^2}{2g} \quad \text{or} \quad v_1 = \sqrt{2gh}$$

This is theoretical velocity. Actual velocity is given by

$$(v_1)_{act} = C_v \sqrt{2gh}$$

where C_v = Co-efficient of pitot-tube

∴ Velocity at any point $\quad v = C_v \sqrt{2gh}$ ...(6.14)

Velocity of flow in a pipe by pitot-tube. For finding the velocity at any point in a pipe by pitot-tube, the following arrangements are adopted :

1. Pitot-tube along with a vertical piezometer tube as shown in Fig. 6.14.
2. Pitot-tube connected with piezometer tube as shown in Fig. 6.15.
3. Pitot-tube and vertical piezometer tube connected with a differential *U*-tube manometer as shown in Fig. 6.16.

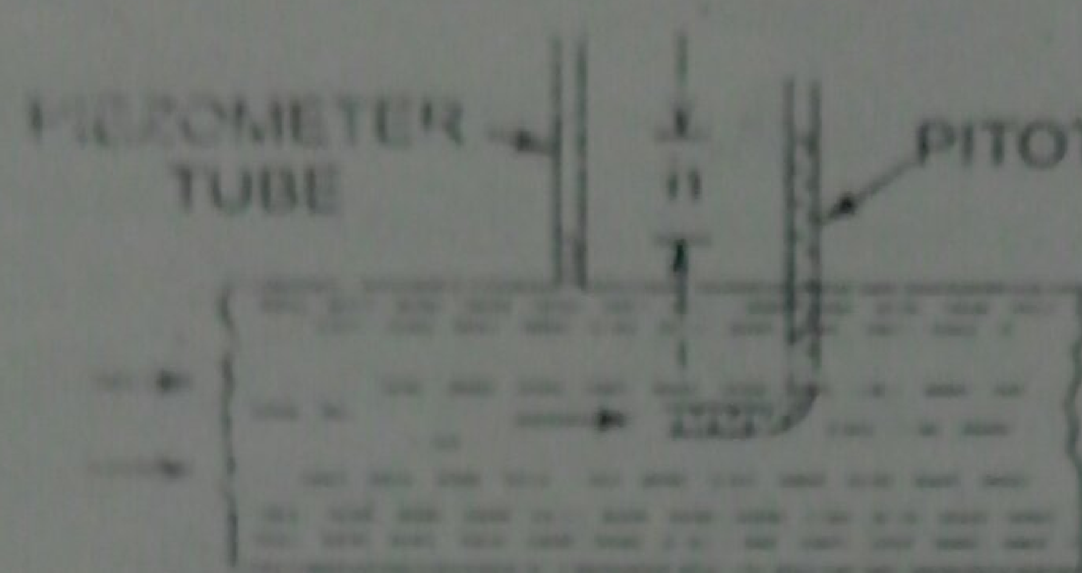


Fig. 6.14

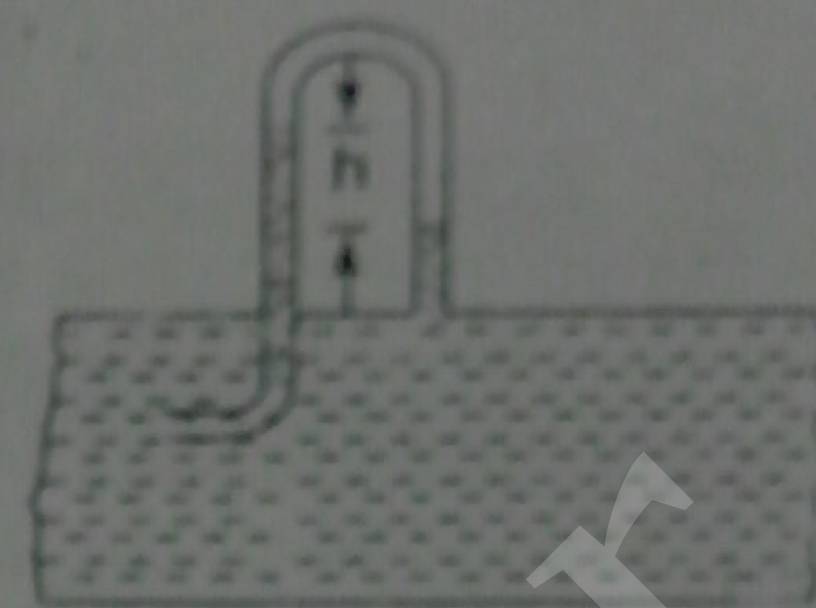


Fig. 6.15

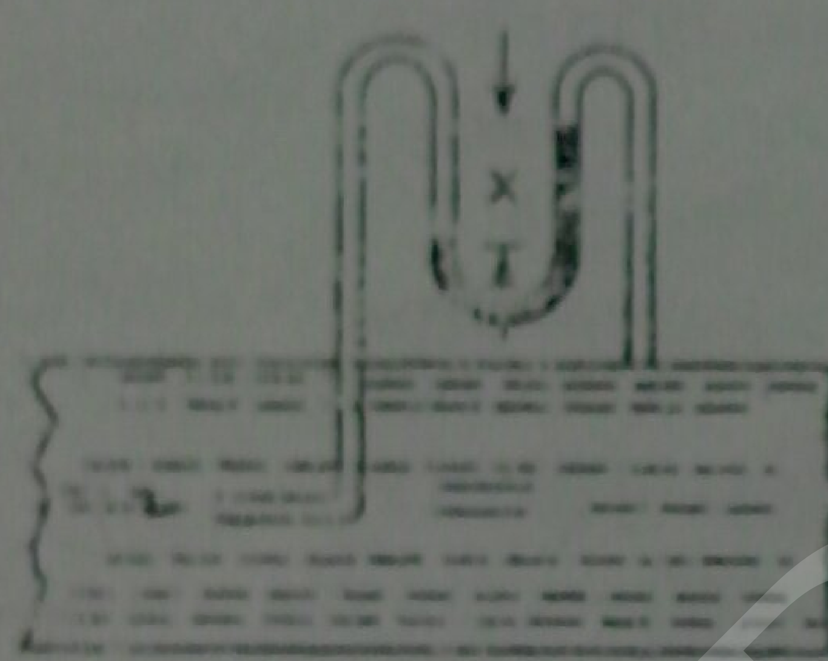


Fig. 6.16

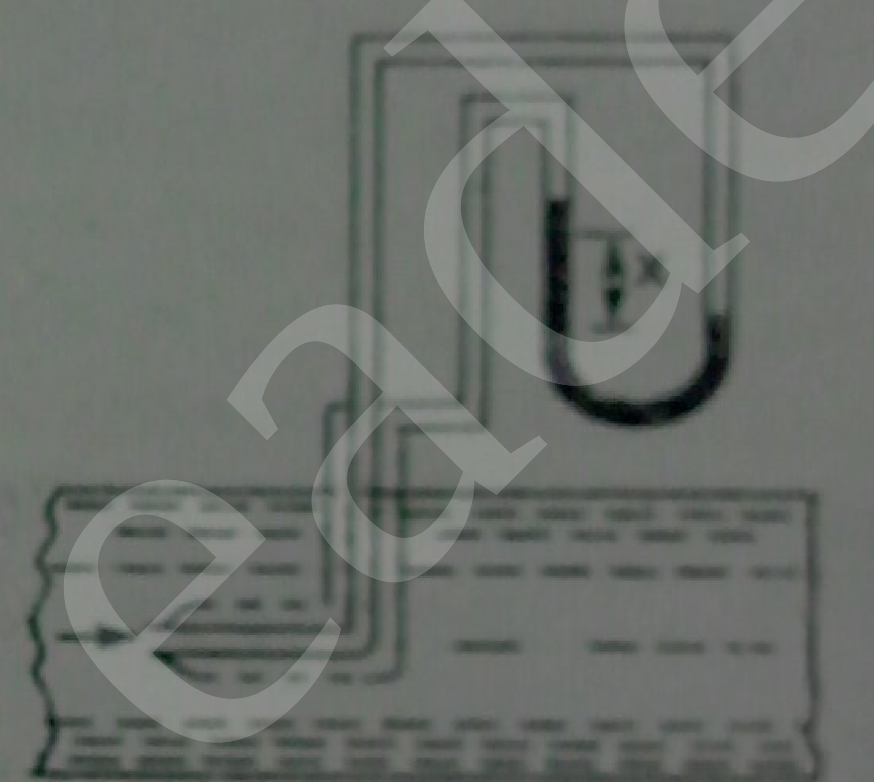


Fig. 6.17

4. Pitot-static tube, which consists of two circular concentric tubes one inside the other with some annular space in between as shown in Fig. 6.17. The outlet of these two tubes are connected to the differential manometer where the difference of pressure head '*h*' is measured by knowing the difference of the levels of the manometer liquid say *x*. Then $h = x\left[\dfrac{S_g}{S_o} - 1\right]$.

Problem 6.24. *A pitot-static tube placed in the centre of a 300 mm pipe line has one orifice pointing upstream and other perpendicular to it. The mean velocity in the pipe is 0.80 of the central velocity. Find the discharge through the pipe if the pressure difference between the two orifices is 60 mm of water. Take the co-efficient of pitot tube as $C_v = 0.98$.*

Solution. Given :

Dia. of pipe, $\quad d = 300 \text{ mm} = 0.30 \text{ m}$

Diff. of pressure head, $\quad h = 60 \text{ mm of water} = .06 \text{ m of water}$

$C_v = 0.98$

Mean velocity, $\quad \overline{V} = 0.80 \times \text{Central velocity}$

Central velocity is given by equation (6.14)

$$= C_v \sqrt{2gh} = 0.98 \times \sqrt{2 \times 9.81 \times .06} = 1.063 \text{ m/s}$$

... of pipe at section (1) and
v_2, p_2, A_2 = corresponding ...

$\therefore \quad \overline{V} = 0.80 \times 1.063 = 0.8504$ m/s

Discharge, $\quad Q$ = Area of pipe $\times \overline{V}$

$$= \frac{\pi}{4} d^2 \times \overline{V} = \frac{\pi}{4} (.30)^2 \times 0.8504 = \mathbf{0.06\ m^3/s.\ Ans.}$$

Problem 6.25 *Find the velocity of the flow of an oil through a pipe, when the difference of mercury level in a differential U-tube manometer connected to the two tappings of the pitot-tube is 100 mm. Take co-efficient of pitot-tube 0.98 and sp. gr. of oil = 0.8.*

Solution. Given :

Diff. of mercury level, $\quad x = 100$ mm $= 0.1$ m

Sp. gr. of oil, $\quad S_o = 0.8$

Sp. gr. of mercury, $\quad S_g = 13.6$

$C_v = 0.98$

Diff. of pressure head, $\quad h = x\left[\frac{S_g}{S_o} - 1\right] = .1\left[\frac{13.6}{0.8} - 1\right] = 1.6$ m of oil

$\therefore$ Velocity of flow $\quad = C_v \sqrt{2gh} = 0.98\sqrt{2 \times 9.81 \times 1.6} = \mathbf{5.49\ m/s.\ Ans.}$

Problem 6.26 *A pitot-static tube is used to measure the velocity of water in a pipe. The stagnation pressure head is 6 m and static pressure head is 5 m. Calculate the velocity of flow assuming the co-efficient of tube equal to 0.98.*

Solution. Given :

Stagnation pressure head, $\quad h_s = 6$ m

Static pressure head, $\quad h_t = 5$ m

$\therefore \quad h = 6 - 5 = 1$ m

Velocity of flow, $\quad V = C_v \sqrt{2gh} = 0.98\sqrt{2 \times 9.81 \times 1} = \mathbf{4.34\ m/s.\ Ans.}$

Problem 6.27 *A sub-marine moves horizontally in sea and has its axis 15 m below the surface of water. A pitot-tube properly placed just in front of the sub-marine and along its axis is connected to the two limbs of a U-tube containing mercury. The difference of mercury level is found to be 170 mm. Find the speed of the sub-marine knowing that the sp. gr. of mercury is 13.6 and that of sea water is 1.026 with respect of fresh water.*

Solution. Given :

Diff. of mercury level, $\quad x = 170$ mm $= 0.17$ m

Sp. gr. of mercury, $\quad S_g = 13.6$

Sp. gr. of sea-water, $\quad S_o = 1.026$

$\therefore \quad h = x\left[\frac{S_g}{S_o} - 1\right] = 0.17\left[\frac{13.6}{1.026} - 1\right] = 2.0834$ m

$\therefore \quad V = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 2.0834} = 6.393$ m/s

$$= \frac{6.393 \times 60 \times 60}{1000} \text{ km/hr} = \mathbf{23.01\ km/hr.\ Ans.}$$

Problem 6.28 *A pitot-tube is inserted in a pipe of 300 mm diameter. The static pressure in pipe is 100 mm of mercury (vacuum). The stagnation pressure at the centre of the pipe, recorded by the* *pitot-tube is 0.981 N/cm². Calculate the rate of flow of water through pipe, if the mean velocity of flow is 0.85 times the central velocity. Take $C_v = 0.98$.*

Solution. Given :

Dia. of pipe, $d = 300 \text{ mm} = 0.30 \text{ m}$

∴ Area, $a = \frac{\pi}{4} d^2 = \frac{\pi}{4} (.3)^2 = 0.07068 \text{ m}^2$

Static pressure head $= 100$ mm of mercury (vacuum)

$$= -\frac{100}{1000} \times 13.6 = -1.36 \text{ m of water}$$

Stagnation pressure $= .981 \text{ N/cm}^2 = .981 \times 10^4 \text{ N/m}^2$

∴ Stagnation pressure head $= \frac{.981 \times 10^4}{\rho g} = \frac{.981 \times 10^4}{1000 \times 9.81} = 1 \text{ m}$

∴ h = Stagnation pressure head – Static pressure head

$$= 1.0 - (-1.36) = 1.0 + 1.36 = 2.36 \text{ m of water}$$

∴ Velocity at centre $= C_v \sqrt{2gh}$

$$= 0.98 \times \sqrt{2 \times 9.81 \times 2.36} = 6.668 \text{ m/s}$$

Mean velocity, $\overline{V} = 0.85 \times 6.668 = 5.6678 \text{ m/s}$

∴ Rate of flow of water $= \overline{V} \times$ area of pipe

$$= 5.6678 \times 0.07068 \text{ m}^3/\text{s} = 0.4006 \text{ m}^3/\text{s}.$$ **Ans.**

▶ 6.8 THE MOMENTUM EQUATION

It is based on the law of conservation of momentum or on the momentum principle, which states that the net force acting on a fluid mass is equal to the change in momentum of flow per unit time in that direction. The force acting on a fluid mass '*m*' is given by the Newton's second law of motion,

$$F = m \times a$$

where a is the acceleration acting in the same direction as force F.

But $$a = \frac{dv}{dt}$$

∴ $$F = m \frac{dv}{dt}$$

$$= \frac{d(mv)}{dt} \quad \{m \text{ is constant and can be taken inside the differential}\}$$

∴ $$F = \frac{d(mv)}{dt} \qquad \text{...(6.15)}$$

Equation (6.15) is known as the momentum principle.

Equation (6.15) can be written as $F.dt = d(mv)$ ...(6.16)

which is known as the *impulse-momentum equation* and states that the impulse of a force F acting on a fluid of mass m in a short interval of time dt is equal to the change of momentum $d(mv)$ in the direction of force.

Force exerted by a flowing fluid on a pipe bend

The impulse-momentum equation (6.16) is used to determine the resultant force exerted by a flowing fluid on a pipe bend.

Consider two sections (1) and (2), as shown in Fig. 6.18.

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Let v_1 = velocity of flow at section (1),

p_1 = pressure intensity at section (1),

A_1 = area of cross-section of pipe at section (1) and

v_2, p_2, A_2 = corresponding values of velocity, pressure and area at section (2).

Let F_x and F_y be the components of the forces exerted by the flowing fluid on the bend in x-and y-directions respectively. Then the force exerted by the bend on the fluid in the directions of x and y will be equal to F_x and F_y but in the opposite directions. Hence component of the force exerted by bend on the fluid in the x-direction $= -F_x$ and in the direction of $y = -F_y$. The other external forces acting on the fluid are p_1A_1 and p_2A_2 on the sections (1) and (2) respectively. Then momentum equation in x-direction is given by

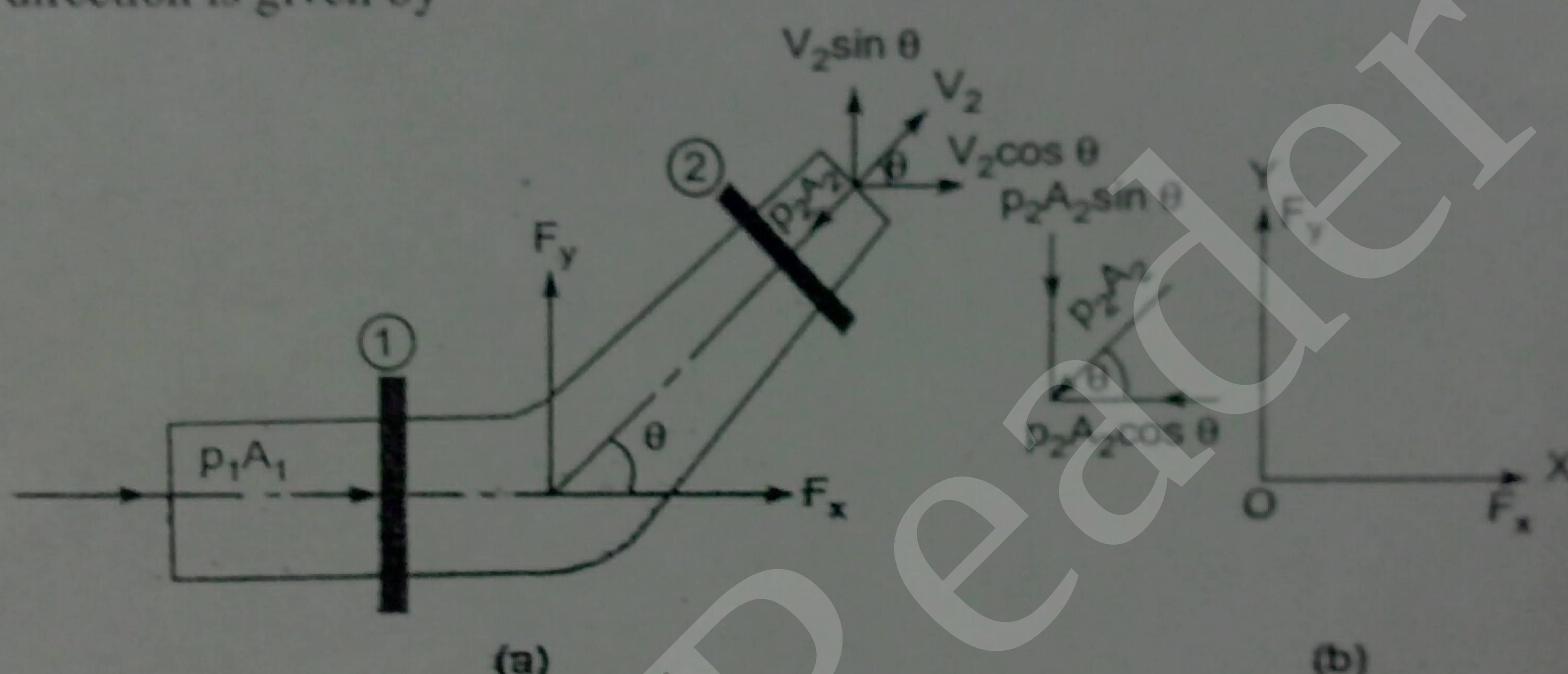

Fig. 6.18 *Forces on bend.*

Net force acting on fluid in the direction of x = Rate of change of momentum in x-direction

$\therefore \quad p_1A_1 - p_2A_2 \cos\theta - F_x$ = (Mass per sec) (change of velocity)

$= \rho Q$ (Final velocity in the direction of x – Initial velocity in the direction of x)

$$= \rho Q (V_2 \cos\theta - V_1) \quad ...(6.17)$$

$$\therefore \quad F_x = \rho Q (V_1 - V_2 \cos\theta) + p_1A_1 - p_2A_2 \cos\theta \quad ...(6.18)$$

Similarly the momentum equation in y-direction gives

$$0 - p_2A_2 \sin\theta - F_y = \rho Q (V_2 \sin\theta - 0) \quad ...(6.19)$$

$$\therefore \quad F_y = \rho Q (-V_2 \sin\theta) - p_2A_2 \sin\theta \quad ...(6.20)$$

Now the resultant force (F_R) acting on the bend

$$= \sqrt{F_x^2 + F_y^2} \quad ...(6.21)$$

And the angle made by the resultant force with horizontal direction is given by

$$\tan\theta = \frac{F_y}{F_x} \quad ...(6.22)$$

Problem 6.29 *A 45° reducing bend is connected in a pipe line, the diameters at the inlet and outlet of the bend being 600 mm and 300 mm respectively. Find the force exerted by water on the bend if the intensity of pressure at inlet to bend is 8.829 N/cm² and rate of flow of water is 600 litres/s.*