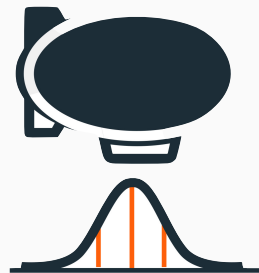


LONGITUDINAL MODELING AND MISSING DATA HANDLING IN **BLIMP**

Craig K. Enders, UCLA



WWW.APPLIEDMISSINGDATA.COM/BLIMP

BLIMP 3.0

Blimp 3 offers powerful latent variable modeling and imputation for incomplete data sets with up to three levels. Blimp's unique Bayesian computational architecture allows easy specification of complex analyses that are difficult or impossible to fit in other software packages.

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[User's Guide](#)



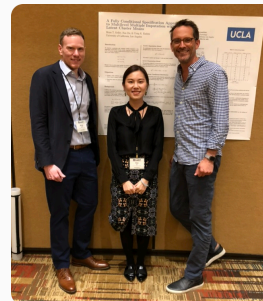
Workshops and Training

Enders, C. K. (2023, March). Longitudinal modeling and missing data handling in Blimp. Workshop presented at the Advanced Techniques for Longitudinal Data Analysis in Social Science (ATLASS) conference. Bielefeld, Germany.

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ACKNOWLEDGEMENTS

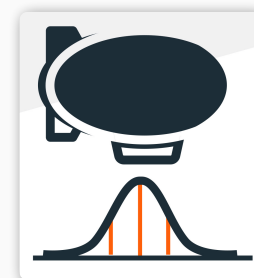
Brian Keller & Han Du



WORKSHOP OUTLINE

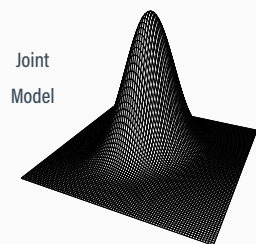
- Linear growth
- Nonlinear change
- Incomplete predictors
- Interaction effects
- Heterogeneous within-cluster variation
- Structural modeling
- Missing not at random processes
- Nonnormal data

THE **BLIMP** MODELING FRAMEWORK



MODELING FRAMEWORKS

Multivariate modeling

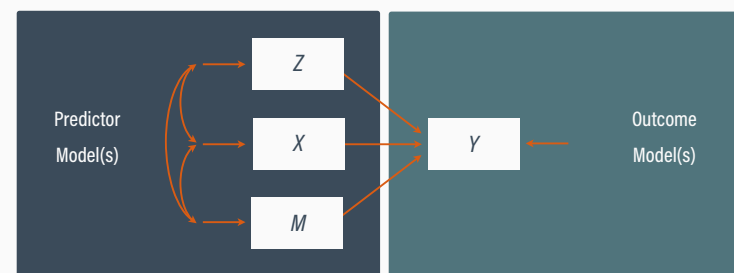


- Classic methods often assume a multivariate distribution (e.g., normal)
- e.g., FIML and joint model multiple imputation

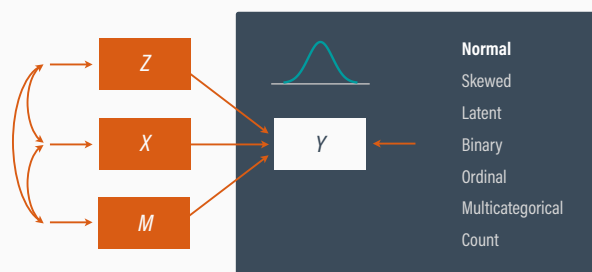
MODELING FRAMEWORKS

Multivariate modeling

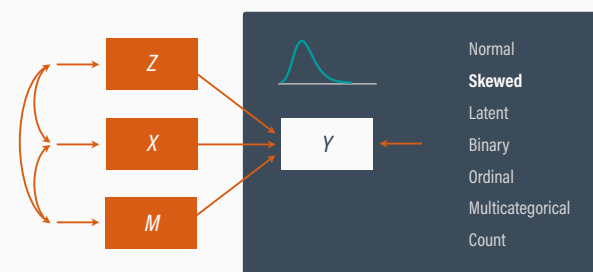
Factored regression specification (Blimp)



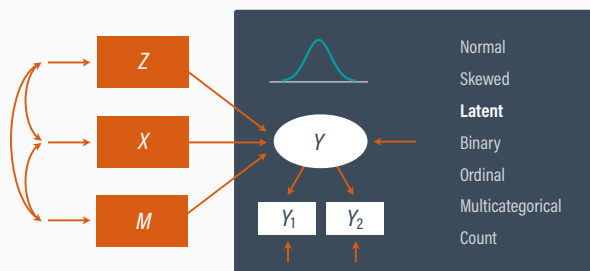
OUTCOME VARIABLE TYPES



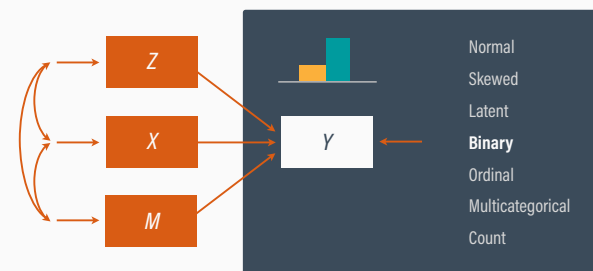
OUTCOME VARIABLE TYPES



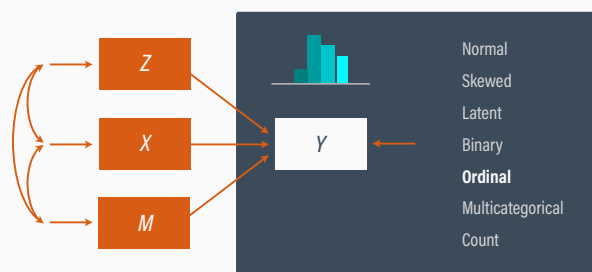
OUTCOME VARIABLE TYPES



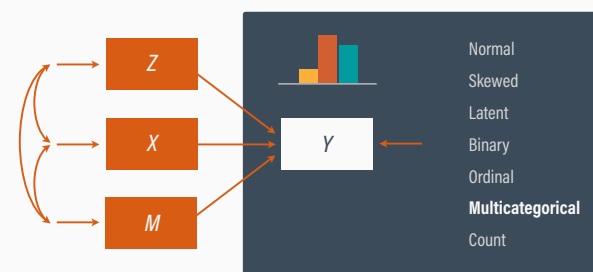
OUTCOME VARIABLE TYPES



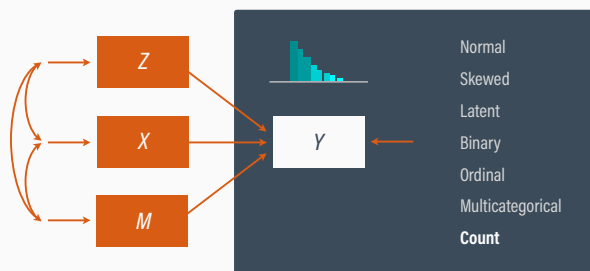
OUTCOME VARIABLE TYPES



OUTCOME VARIABLE TYPES

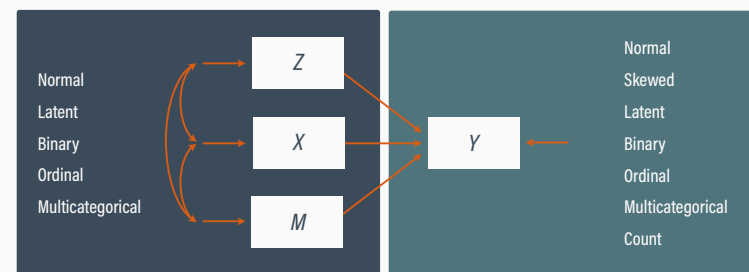


OUTCOME VARIABLE TYPES



PREDICTOR VARIABLE TYPES

Multivariate Distribution = Univariate Outcome Model \times Multivariate Predictor Model

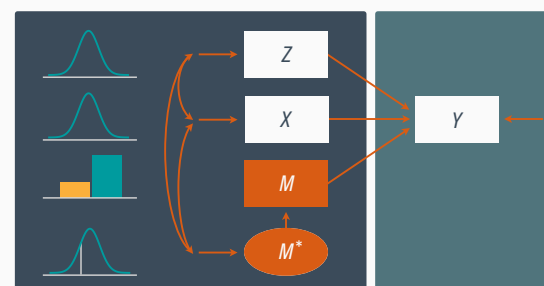


LATENT RESPONSE FORMULATION



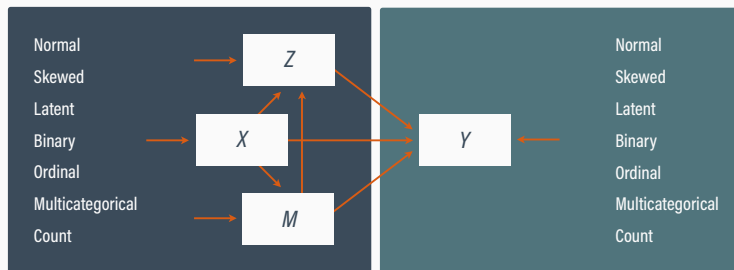
CATEGORICAL PREDICTOR

Multivariate Distribution = Univariate Outcome Model \times Multivariate Predictor Model



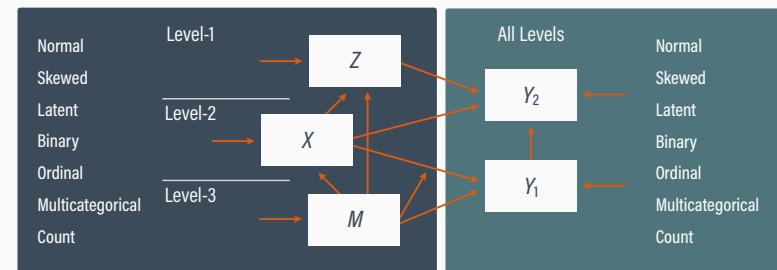
SEQUENTIAL PREDICTOR TYPES

Multivariate Distribution = Univariate Outcome Model \times Univariate Predictor Models



MULTILEVEL MODELING

Multivariate Distribution = Univariate Outcome Model \times Multivariate or Univariate Predictor Models



MCMC ESTIMATION

Estimate regression models

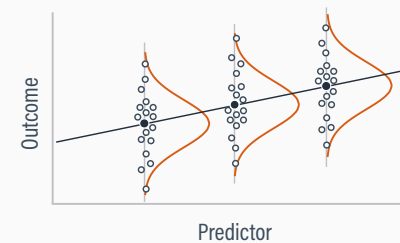


Impute missing values

- » Do for $t = 1$ to T iterations
 - » Estimate focal model parameters, conditional on filled-in data
 - » Estimate predictor model parameters, conditional on filled-in data
 - » Impute outcome scores, conditional on the focal model parameters
 - » Impute predictors, conditional on the focal and predictor model parameters
- » Repeat

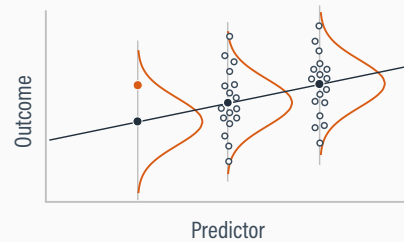
MISSING DATA IMPUTATION

- Imputations are sampled at random from distributions of plausible values
- One or more sets of model parameters define the center and spread



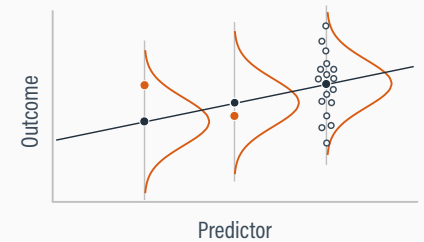
MISSING DATA IMPUTATION

- Imputations are sampled at random from distributions of plausible values
- One or more sets of model parameters define the center and spread



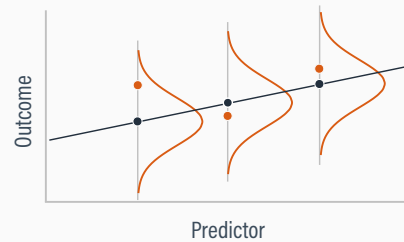
MISSING DATA IMPUTATION

- Imputations are sampled at random from distributions of plausible values
- One or more sets of model parameters define the center and spread



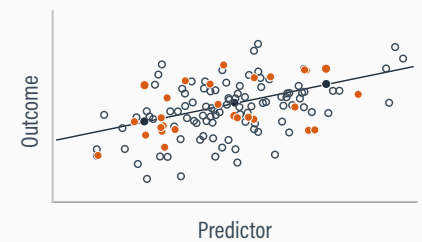
MISSING DATA IMPUTATION

- Imputations are sampled at random from distributions of plausible values
- One or more sets of model parameters define the center and spread



MISSING DATA IMPUTATION

- Imputations are sampled at random from distributions of plausible values
- One or more sets of model parameters define the center and spread



MISSING DATA MECHANISMS

PARTITIONING THE DATA

Complete			=	Observed			+	Missing			Indicators		
Y ₁	Y ₂	Y ₃		Y ₁	Y ₂	Y ₃		Y ₁	Y ₂	Y ₃	M ₁	M ₂	M ₃
4	4	3		4	4	3					0	0	0
3	3	5		3	NA	5			3		0	1	0
7	1	6		7	1	6					0	0	0
2	1	6		NA	1	6		2			1	0	0
5	9	3	=	5	9	3	+				0	0	0
3	2	2		3	NA	NA			2	2	0	1	1
1	6	7		1	6	7					0	0	0
9	4	9		9	4	9					0	0	0
2	5	6		2	NA	6			5		0	1	0

RUBIN'S MISSINGNESS MECHANISMS

- Missing data mechanisms describe different ways in which the pattern of 0s and 1s in \mathbf{M} relate to the observed or missing data
- Missingness may be independent of the data, or it could relate to the observed or missing parts (or both)
- Mechanisms are essentially missingness models

MISSING COMPLETELY AT RANDOM

- The probability of missing values is completely unrelated to the data

$$f(\mathbf{M} = 1 | \mathbf{Y}_{\text{obs}}, \mathbf{Y}_{\text{mis}}, \phi) = f(\mathbf{M} = 1 | \phi)$$

- Purely haphazard missingness

Missingness Conditioning on the Data

M			Y _{obs}			Y _{mis}		
M ₁	M ₂	M ₃	Y ₁	Y ₂	Y ₃	Y ₁	Y ₂	Y ₃
0	0	0	4	4	3			
0	1	0	3	NA	5		3	
0	0	0	7	1	6			
1	0	0	NA	1	6	2		
0	0	0	5	9	3			
0	1	1	3	NA	NA		2	2
0	0	0	1	6	7			
0	0	0	9	4	9			
0	1	0	2	NA	6		5	

(CONDITIONALLY) MISSING AT RANDOM

- The probability of missing values is unrelated to the missing (latent) data

$$f(\mathbf{M} = 1 | \mathbf{Y}_{\text{obs}}, \mathbf{Y}_{\text{mis}}, \phi) = f(\mathbf{M} = 1 | \mathbf{Y}_{\text{obs}}, \phi)$$

- Missingness is haphazard after conditioning on observed data

Missingness Conditioning on the Data

M			Y _{obs}			Y _{mis}		
M ₁	M ₂	M ₃	Y ₁	Y ₂	Y ₃	Y ₁	Y ₂	Y ₃
0	0	0	4	4	3			
0	1	0	3	NA	5		3	
0	0	0	7	1	6			
1	0	0	NA	1	6	2		
0	0	0	5	9	3			
0	1	1	3	NA	NA	2	2	
0	0	0	1	6	7			
0	0	0	9	4	9			
0	1	0	2	NA	6		5	

MISSING NOT AT RANDOM

- The probability of missing values is related to the missing (latent) data

$$f(\mathbf{M} = 1 | \mathbf{Y}_{\text{obs}}, \mathbf{Y}_{\text{mis}}, \phi) \text{ or } f(\mathbf{M} = 1 | \mathbf{Y}_{\text{mis}}, \phi)$$

- The observed data may or may not additionally determine missingness

Missingness Conditioning on the Data

M			Y _{obs}			Y _{mis}		
M ₁	M ₂	M ₃	Y ₁	Y ₂	Y ₃	Y ₁	Y ₂	Y ₃
0	0	0	4	4	3			
0	1	0	3	NA	5		3	
0	0	0	7	1	6			
1	0	0	NA	1	6	2		
0	0	0	5	9	3			
0	1	1	3	NA	NA	2	2	
0	0	0	1	6	7			
0	0	0	9	4	9			
0	1	0	2	NA	6		5	

CLINICAL TRIAL DATA

■ Predictors
■ Outcome

Variable	Definition	Missing %	Scale
<i>PERSON</i>	Person-level (level-2) identifier	0	Integer index
<i>WAVE</i>	Data collection wave	0	Integer index (1 to 4)
<i>MONTH</i>	Months relative to 4-week follow-up	0	Integer index (-1 to 2)
<i>BREATHCO</i>	Breath CO reading	23.94	Numeric (0 to 46)
<i>DRINKS</i>	Number of drinks per drinking day	15.00	Numeric (0 to 34)
<i>CIGS</i>	Number of cigarettes per smoking day	14.85	Numeric (0 to 50)
<i>FEMALE</i>	Gender dummy code	0	0 = Male, 1 = Female
<i>CONDITION</i>	Treatment condition	0	0 = Single-medication, 1 = Dual-medication
<i>NICDEP</i>	Nicotine dependence dummy code	6.67	0 = Very Low, to Low 1 = Moderate to Very High
<i>QUIT16</i>	Quit smoking at 4-month follow-up	33.94	0 = Did not quit, 1 = Quit smoking
<i>DROPOUT</i>	Dropout indicator	13.33	0 = In study 1 = Dropped out

LONGITUDINAL PROCESSES

- The conditional MAR process assumes that missingness at occasion t is explained by the observed values of other variables (covariates and repeated measurements)
- Two plausible MNAR processes: one's underlying growth trajectory is responsible for missing data, or the unseen value of the outcome at a particular occasion predicts nonresponse

MULTILEVEL GROWTH MODELS

MOTIVATING EXAMPLE

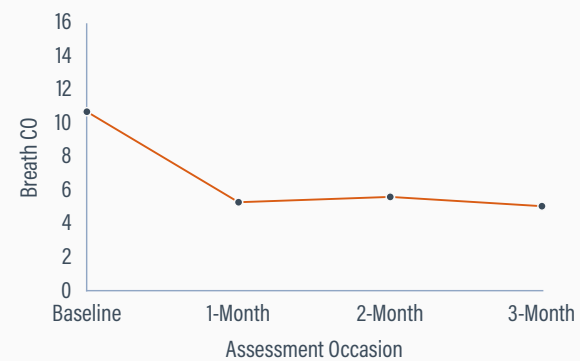
- Smoking cessation clinical trial with four repeated measurements nested in participants
- Two treatment arms: varenicline vs. varenicline + naltrexone
- Cigarette⁴ smoking (breath CO) assessed at baseline, 4-week, 8-week, and 12-week follow-ups

CLINICAL TRIAL DATA

■ Predictors
■ Outcome

Variable	Definition	Missing %	Scale
<i>PERSON</i>	Person-level (level-2) identifier	0	Integer index
<i>WAVE</i>	Data collection wave	0	Integer index (1 to 4)
<i>MONTH</i>	Months relative to 4-week follow-up	0	Integer index (-1 to 2)
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<i>DROPOUT</i>	Dropout indicator	13.33	0 = In study 1 = Dropped out

OBSERVED MEANS

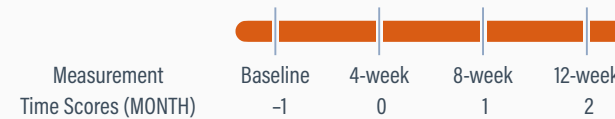


GROWTH MODELING OVERVIEW

- Multilevel growth models express outcome changes as a function of a temporal predictor (e.g., change per month)
- The basic model features an average change trajectory (fixed effects) and person-specific trajectories (random effects)
- Person-specific trajectories can be outcomes and predictors

TIME SCORES

- The growth model requires a meaningful zero value for the temporal predictor (measurement occasion, passage of time)
- Substantive concerns should dictate centering, I use months since the 4-week follow-up assessment



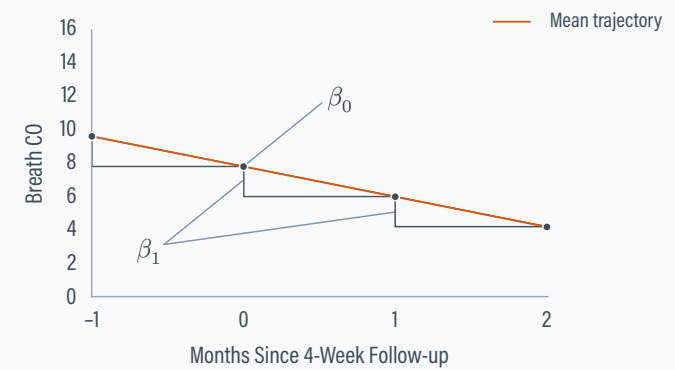
LINEAR GROWTH MODEL

- Individual intercepts and slopes are normally distributed latent variables, β_0 and β_1 define the mean trajectory, and time-specific residuals are normal with constant variation

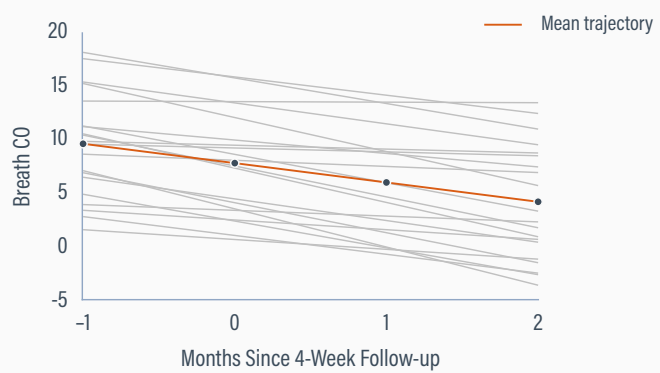
$$BREATHCO_{ti} = \beta_{0i} + \beta_{1i}(MONTH_{ti}) + \varepsilon_{ti}$$

$$\begin{pmatrix} \beta_{0i} \\ \beta_{1i} \end{pmatrix} \sim N \left(\begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix}, \Sigma_{\beta_i} \right) \quad \varepsilon_{ti} \sim N(0, \sigma_{\varepsilon}^2)$$

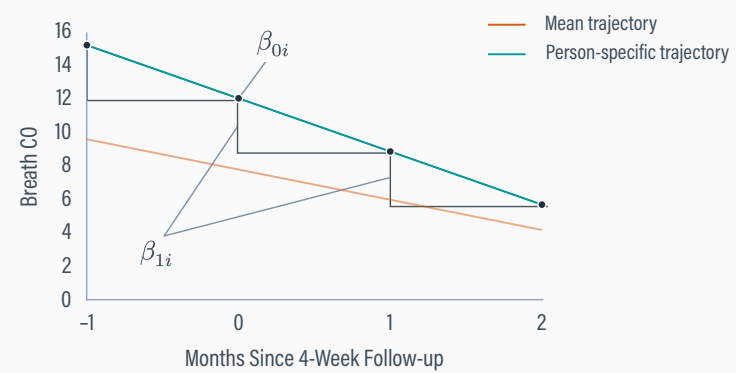
MEAN INTERCEPT AND SLOPE



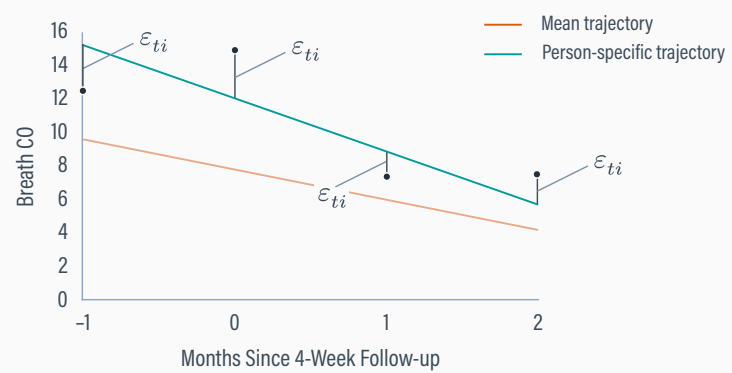
PERSON-SPECIFIC TRAJECTORIES



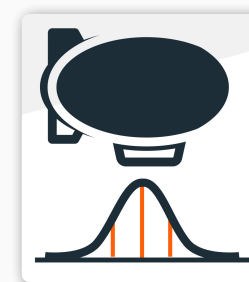
PERSON-SPECIFIC INTERCEPT AND SLOPE



TIME-SPECIFIC RESIDUALS



LINEAR GROWTH
MODELS IN **BLIMP**



CLINICAL TRIAL DATA

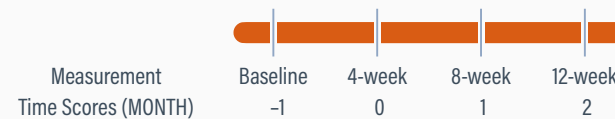
■ Predictors
■ Outcome

Variable	Definition	Missing %	Scale
<i>PERSON</i>	Person-level (level-2) identifier	0	Integer index
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<i>DROPOUT</i>	Dropout indicator	13.33	0 = In study 1 = Dropped out

ANALYSIS MODEL

- Linear growth model with random intercepts and slopes (person-specific intercepts and growth rates)

$$BREATHCO_{ti} = \beta_{0i} + \beta_{1i}(MONTH_{ti}) + \varepsilon_{ti}$$



BLIMP SCRIPT

```
DATA: clinicaltrial.dat;  
VARIABLES: person wave month breathCO drinks cigs female condition nicdep quit16 dropout;  
CLUSTERID: person;  
MISSING: 999;  
FIXED: month;  
MODEL:  
breathCO ~ month | month;  
BURN: 5000;  
ITERATIONS: 10000;  
SEED: 90291;
```

DATA AND VARIABLES

```
DATA: clinicaltrial.dat;  
VARIABLES: person wave month breathCO drinks cigs female condition nicdep quit16 dropout;  
CLUSTERID: person;  
MISSING: 999;  
FIXED: month;  
MODEL:  
breathCO ~ month | month;  
BURN: 5000;  
ITERATIONS: 10000;  
SEED: 90291;
```

MODEL DETAILS

DATA: clinicaltrial.dat;

VARIABLES: person wave month breathCO drinks cigs female condition nicdep quit16 dropout;

CLUSTERID: person;

MISSING: 999;

FIXED: month;

MODEL:

breathCO ~ month | month;

BURN: 5000;

ITERATIONS: 10000;

SEED: 90291;

COMPUTATIONAL DETAILS

DATA: clinicaltrial.dat;

VARIABLES: person wave month breathCO drinks cigs female condition nicdep quit16 dropout;

CLUSTERID: person;

MISSING: 999;

FIXED: month;

MODEL:

breathCO ~ month | month;

BURN: 5000;

ITERATIONS: 10000;

SEED: 90291;

DATA AND VARIABLES

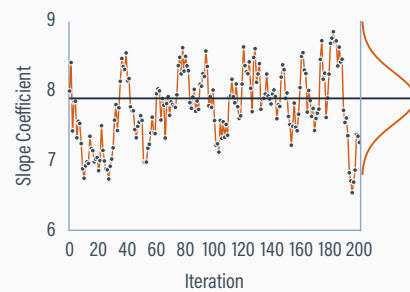
DATA: clinicaltrial.dat; # ascii text data
VARIABLES: person wave month breathCO drinks cigs female
condition nicdep quit16 dropout; # variable order
CLUSTERID: person; # level-2 identifier variable
MISSING: 999; # missing value code

MODEL DETAILS

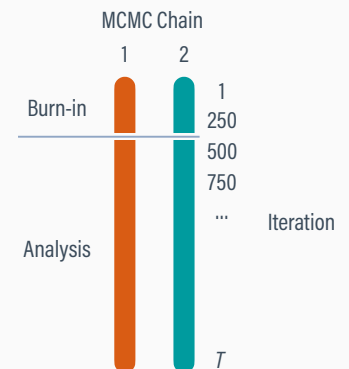
FIXED: month; # complete predictors
MODEL: # regression equations
breathCO ~ month | month; # DV ~ predictors | random predictors

MCMC ESTIMATION

- MCMC uses computer simulation to “draw” or “sample” parameters from a distribution of plausible values
- Estimates continually vary across iterations in a random pattern

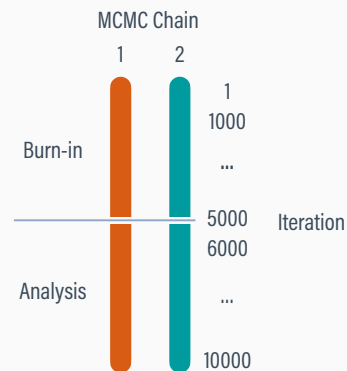


MCMC ALGORITHM



COMPUTATIONAL DETAILS

BURN: 5000; # burn-in iterations
ITERATIONS: 10000; # analysis iterations
SEED: 90291; # random number seed



MISSING DATA IMPUTATION

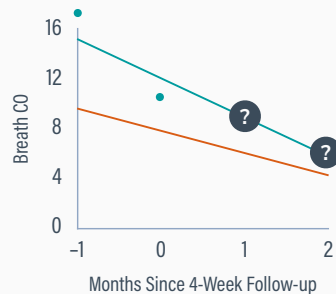
- Missing values are imputed at each MCMC iteration

$$BREATHCO_{ti(mis)} \sim N(\beta_{0i} + \beta_{1i}(MONTH_{ti}), \sigma_{\epsilon}^2)$$

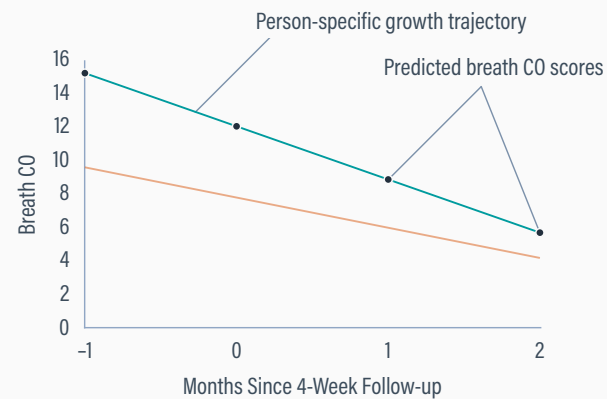
- The model parameters and random effects define normal distributions of imputations around person-specific trajectories

ILLUSTRATION

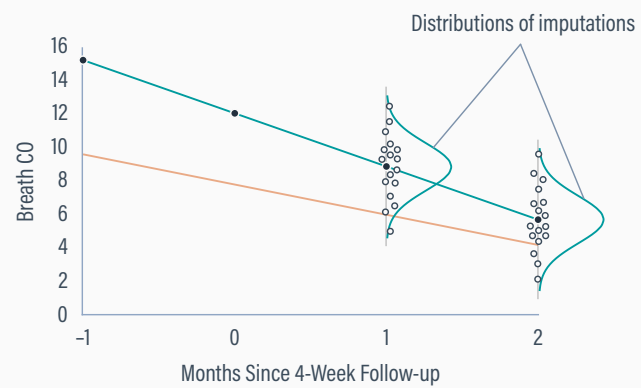
- A participant provides baseline and one-month follow-up scores then drops out
- Missing scores are imputed by sampling from a distribution of plausible values



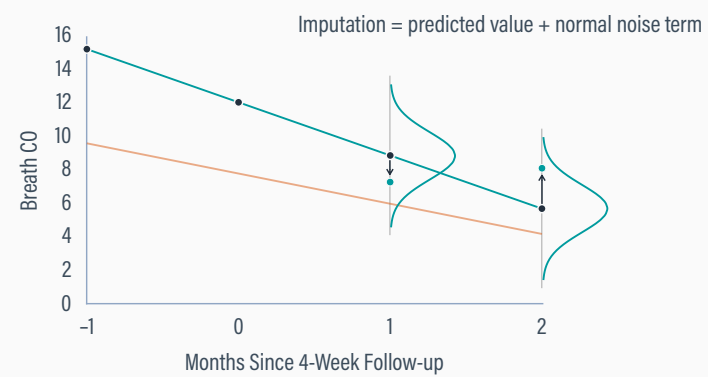
IMPUTATION AT 8- AND 12-WEEK FOLLOW-UP



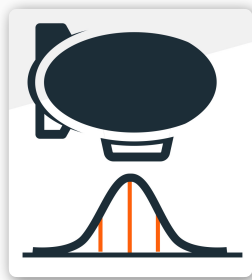
IMPUTATION AT 8- AND 12-WEEK FOLLOW-UP



IMPUTATION AT 8- AND 12-WEEK FOLLOW-UP



UNDERSTANDING BLIMP OUTPUT



MCMC CONVERGENCE

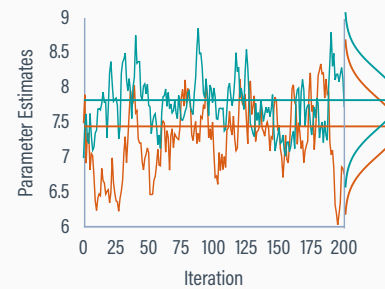
- MCMC converges when posterior distributions are stationary
- Parameter estimates oscillate around a stable mean, and variation doesn't change with additional iterations
- Set burn-in cycles > number of iterations needed to converge

POTENTIAL SCALE REDUCTION

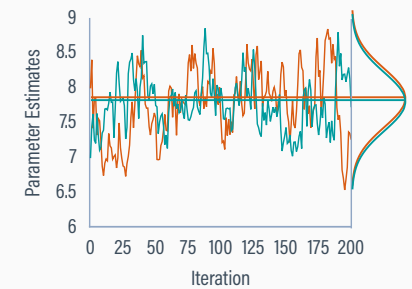
- The potential scale reduction (PSR) factor compares parameter distributions from two MCMC processes
- MCMC converges when two chains give estimates with same means and spread
- PSRs for all parameters should be < 1.05

PSR GRAPHIC

MCMC has not converged (PSR > 1.05)



MCMC has converged (PSR < 1.05)



PSR DIAGNOSTIC OUTPUT

BURN-IN POTENTIAL SCALE REDUCTION (PSR) OUTPUT:

NOTE: Split chain PSR is being used. This splits each chain's iterations to create twice as many chains.

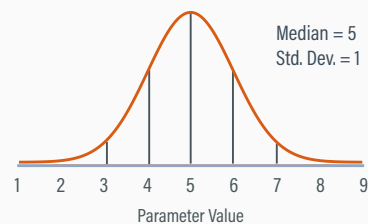
Comparing iterations across 2 chains	Highest PSR	Parameter #
126 to 250	1.186	10
251 to 500	1.090	3
376 to 750	1.037	3
501 to 1000	1.033	10
...
2001 to 4000	1.019	10
2126 to 4250	1.010	10
2251 to 4500	1.010	10
2376 to 4750	1.005	10
>> Worst PSR < 1.05	2501 to 5000	1.010 2

BAYESIAN POSTERIOR SUMMARIES

- Probability distributions are tools for expressing our knowledge about a parameter in a Bayesian analysis
- The posterior distribution describes plausible parameter values that are consistent with the data (no repeated sampling)
- We use summary statistics to describe parameter distributions

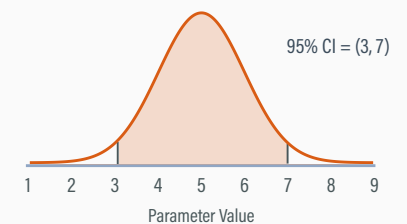
MEDIAN AND STANDARD DEVIATION

- The posterior median and standard deviation quantify the most likely parameter value and uncertainty
- Analogous to a point estimate and standard error, sans repeated sampling



95% CREDIBLE INTERVALS

- The 95% credible interval gives limits spanning 95% of the parameter's range
- Akin to a confidence interval, but references a range of highly plausible parameter values



POINT ESTIMATES

OUTCOME MODEL ESTIMATES:

Summaries based on 10000 iterations using 2 chains.

Outcome Variable: **breathCO**

Parameters	Median	StdDev	2.5%	97.5%	PSR	N_Eff
Variances:						
L2 : Var(Intercept)	19.826	3.219	14.342	27.084	1.002	2214.658
L2 : Cov(month,Intercept)	-1.155	1.162	-3.526	1.070	1.001	861.249
L2 : Var(month)	2.274	0.863	0.815	4.218	1.011	290.118
Residual Var.	18.381	1.746	15.316	22.168	1.003	910.242
Coefficients:						
Intercept	7.837	0.409	7.052	8.645	1.003	1020.162
month	-1.800	0.220	-2.235	-1.367	1.001	2631.023
Standardized Coefficients:						
month	-0.301	0.035	-0.369	-0.230	1.000	2595.367
Proportion Variance Explained						
by Coefficients	0.091	0.021	0.053	0.136	1.000	2596.186
by Level-2 Random Intercepts	0.431	0.046	0.340	0.520	1.001	1764.658
by Level-2 Random Slopes	0.064	0.024	0.023	0.116	1.010	322.944
by Level-1 Residual Variation	0.411	0.044	0.331	0.504	1.006	750.430



Parameter Values

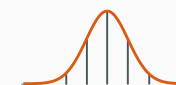
MEASURES OF UNCERTAINTY

OUTCOME MODEL ESTIMATES:

Summaries based on 10000 iterations using 2 chains.

Outcome Variable: **breathCO**

Parameters	Median	StdDev	2.5%	97.5%	PSR	N_Eff
Variances:						
L2 : Var(Intercept)	19.826	3.219	14.342	27.084	1.002	2214.658
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Parameter Values

95% CREDIBLE INTERVALS

OUTCOME MODEL ESTIMATES:

Summaries based on 10000 iterations using 2 chains.

Outcome Variable: **breathCO**

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by Level-1 Residual Variation	0.411	0.044	0.331	0.504	1.006	750.430



Parameter Values

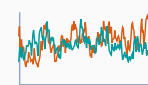
DIAGNOSTICS

OUTCOME MODEL ESTIMATES:

Summaries based on 10000 iterations using 2 chains.

Outcome Variable: **breathCO**

Parameters	Median	StdDev	2.5%	97.5%	PSR	N_Eff
Variances:						
L2 : Var(Intercept)	19.826	3.219	14.342	27.084	1.002	2214.658
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Iteration

COEFFICIENTS

OUTCOME MODEL ESTIMATES:

Summaries based on 10000 iterations using 2 chains.

Outcome Variable: **breathCO**

Parameters	Median	StdDev	2.5%	97.5%	PSR	N_Eff
Variances:						
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by Level-2 Random Slopes	0.064	0.024	0.023	0.116	1.010	322.944
by Level-1 Residual Variation	0.411	0.044	0.331	0.504	1.006	750.430

STANDARDIZED COEFFICIENTS

OUTCOME MODEL ESTIMATES:

Summaries based on 10000 iterations using 2 chains.

Outcome Variable: **breathCO**

Parameters	Median	StdDev	2.5%	97.5%	PSR	N_Eff
Variances:						
L2 : Var(Intercept)	19.826	3.219	14.342	27.084	1.002	2214.658
L2 : Cov(month,Intercept)	-1.155	1.162	-3.526	1.070	1.001	861.249
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by Level-1 Residual Variation	0.411	0.044	0.331	0.504	1.006	750.430

VARIANCES AND COVARIANCES

OUTCOME MODEL ESTIMATES:

Summaries based on 10000 iterations using 2 chains.

Outcome Variable: **breathCO**

Parameters	Median	StdDev	2.5%	97.5%	PSR	N_Eff
Variances:						
L2 : Var(Intercept)	19.826	3.219	14.342	27.084	1.002	2214.658
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EFFECT SIZE ESTIMATES

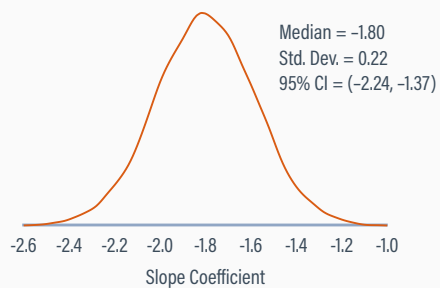
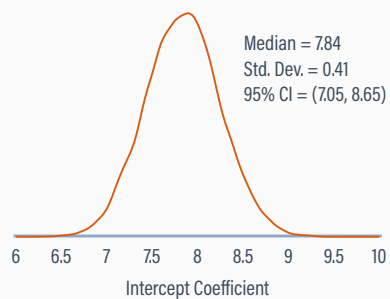
OUTCOME MODEL ESTIMATES:

Summaries based on 10000 iterations using 2 chains.

Outcome Variable: **breathCO**

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Variances:						
L2 : Var(Intercept)	19.826	3.219	14.342	27.084	1.002	2214.658
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by Level-1 Residual Variation	0.411	0.044	0.331	0.504	1.006	750.430

POSTERIOR DISTRIBUTIONS



COEFFICIENT OUTPUT

OUTCOME MODEL ESTIMATES:

Summaries based on 10000 iterations using 2 chains.

Outcome Variable: **breathCO**

Parameters	Median	StdDev	2.5%	97.5%	PSR	N_Eff

Variances:						
...						

Coefficients:

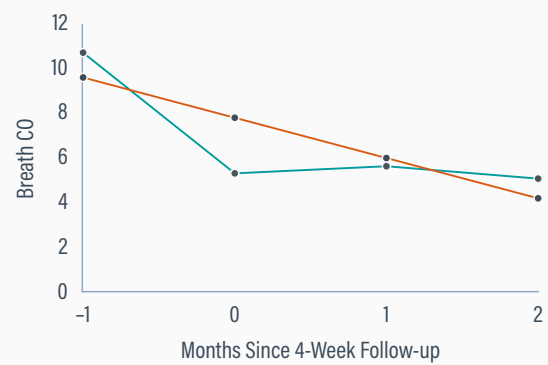
Intercept	7.837	0.409	7.052	8.645	1.003	1020.162
month	-1.800	0.220	-2.235	-1.367	1.001	2631.023

Standardized Coefficients:

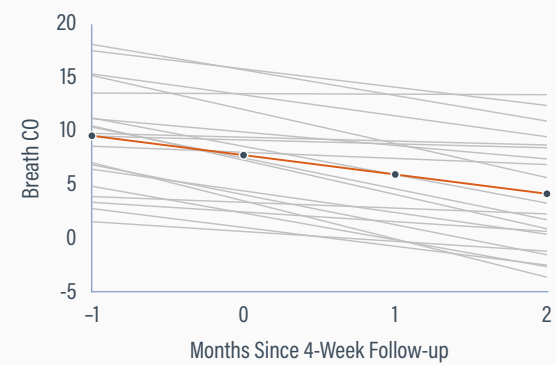
...

Average monthly change rate
Model-predicted mean at MONTH = 0

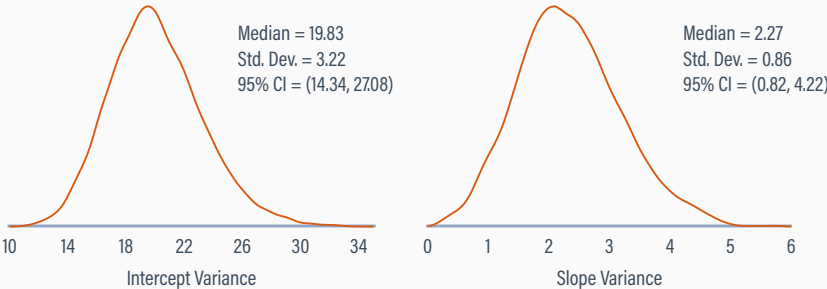
OBSERVED VS. PREDICTED MEANS



INDIVIDUAL TRAJECTORIES



POSTERIOR DISTRIBUTIONS



VARIANCE COMPONENT OUTPUT

OUTCOME MODEL ESTIMATES:

Summaries based on 10000 iterations using 2 chains.

Outcome Variable: **breathCO**

Parameters	Median	StdDev	2.5%	97.5%	PSR	N_Eff
Variances:						
L2 : Var(Intercept)	19.826	3.219	14.342	27.084	1.002	2214.658
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Residual Var.	18.381	1.746	15.316	22.168	1.003	910.242
Coefficients:						
...						

EFFECT SIZE OUTPUT

OUTCOME MODEL ESTIMATES:

Summaries based on 10000 iterations using 2 chains.

Outcome Variable: **breathCO**

Parameters	Median	StdDev	2.5%	97.5%	PSR	N_Eff
------------	--------	--------	------	-------	-----	-------

Variances:

...

Proportion	Variance Explained					
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by Level-2 Random Intercepts	0.431	0.046	0.340	0.520	1.001	1764.658
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» Rights & Sterba (2019)

HOW MANY ITERATIONS?

- MCMC estimates are autocorrelated across iterations
- The effective number of MCMC samples estimates the number of independent estimates after removing autocorrelation
- Gelman et al. (2014, p. 267) recommend at least 100 independent MCMC samples per parameter

EFFECTIVE NUMBER OF MCMC SAMPLES

OUTCOME MODEL ESTIMATES:

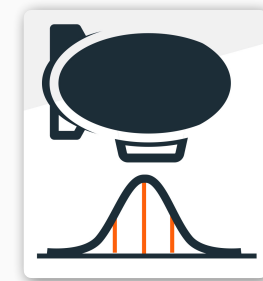
Summaries based on 10000 iterations using 2 chains.

Outcome Variable: **breathCO**

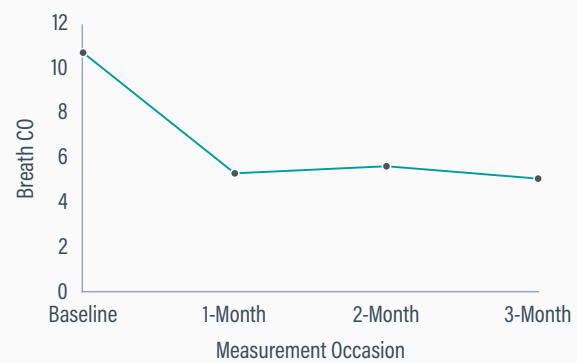
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Variances:						
L2 : Var(Intercept)	19.826	3.219	14.342	27.084	1.002	2214.658
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by Level-1 Residual Variation	0.411	0.044	0.331	0.504	1.006	750.430

>> Effective number of MCMC samples
for every parameter is > 100

NONLINEAR GROWTH MODELS IN **BLIMP**



OBSERVED MEANS



NONLINEAR MODELING OPTIONS

- Transform the time metric (e.g., square root of MONTHS)
- Piecewise growth model (multiple epochs of linear trends)
- Polynomial growth trajectories

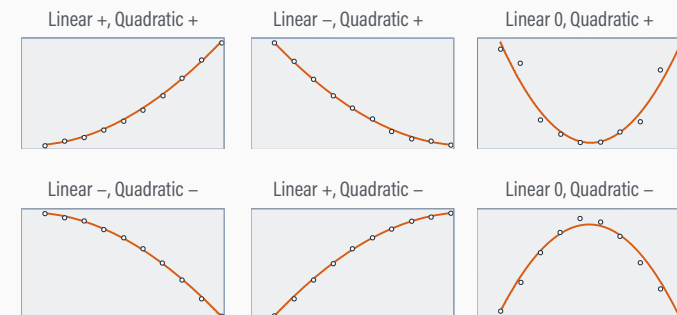
POLYNOMIAL REGRESSION

- A quadratic function uses a squared term as a predictor

$$Y = \beta_0 + \beta_1 X + \beta_2 X^2 + \varepsilon$$

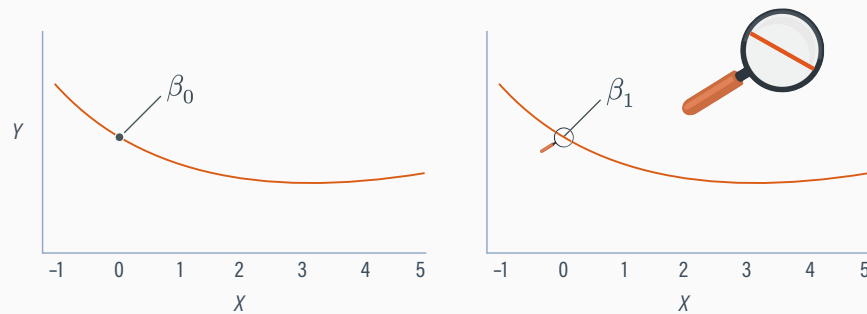
- β_0 and β_1 are the expected value and instantaneous linear change when $X = 0$, and β_2 captures curvature

POLYNOMIAL COMBINATIONS



LOWER-ORDER TERMS AND SCALING

Establishing a meaningful zero value is important!



ANALYSIS MODEL

- Growth model with a quadratic fixed effect but no quadratic random effect (common curvature across persons)

$$BREATHCO_{ti} = \beta_{0i} + \beta_{1i}(MONTH_{ti}) + \beta_2(MONTH_{ti}^2) + \varepsilon_{ti}$$



BLIMP SCRIPT

```
DATA: clinicaltrial.dat;  
VARIABLES: person wave month breathCO drinks cigs condition gender nicdep quit16 dropout;  
CLUSTERID: person;  
MISSING: 999;  
FIXED: month;  
MODEL:  
breathCO ~ month (month^2) | month;      << Quadratic fixed effect  
BURN: 5000;  
ITERATIONS: 10000;  
SEED: 90291;
```

ANALYSIS MODEL

- Growth model with a quadratic fixed effect and a quadratic random effect (person-specific curvature)

$$BREATHCO_{ti} = \beta_{0i} + \beta_{1i}(MONTH_{ti}) + \beta_{2i}(MONTH_{ti}^2) + \varepsilon_{ti}$$



BLIMP SCRIPT

```
DATA: clinicaltrial.dat;  
VARIABLES: person wave month breathCO drinks cigs condition gender nicdep quit16 dropout;  
CLUSTERID: person;  
MISSING: 999;  
FIXED: month;  
MODEL:  
breathCO ~ month (month^2) | month (month^2);    << Quadratic fixed and random effects  
BURN: 5000;  
ITERATIONS: 10000;  
SEED: 90291;
```

COEFFICIENT OUTPUT

OUTCOME MODEL ESTIMATES:

Summaries based on 10000 iterations using 2 chains.

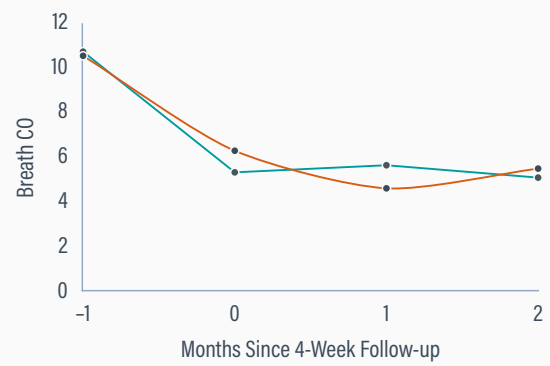
Outcome Variable: **breathCO**

Parameters	Median	StdDev	2.5%	97.5%	PSR	N_Eff
...						
Variances:						
...						
Coefficients:						
Intercept	6.308	0.448	5.433	7.200	1.008	613.994
month	-2.961	0.342	-3.620	-2.276	1.003	1324.276
(month^2)	1.281	0.200	0.886	1.673	1.002	2075.754

Standardized Coefficients:
...

- Curvature (change in linear trend over time)
- Instantaneous linear change at MONTH = 0
- Model-predicted mean at MONTH = 0

OBSERVED VS. PREDICTED MEANS



VARIANCE COMPONENT OUTPUT

OUTCOME MODEL ESTIMATES:

Summaries based on 10000 iterations using 2 chains.

Outcome Variable: **breathCO**

Parameters	Median	StdDev	2.5%	97.5%	PSR	N_Eff
------------	--------	--------	------	-------	-----	-------

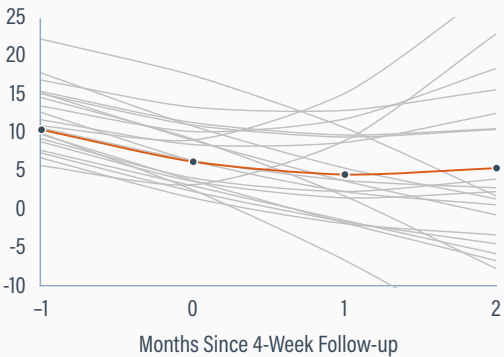
Variances:						
L2 : Var(Intercept)	20.849	3.718	14.824	29.308	1.005	536.477
L2 : Cov(month,Intercept)	2.370	1.984	-1.178	6.574	1.002	855.422
L2 : Var(month)	11.426	2.184	7.727	16.181	1.005	1119.236
L2 : Cov(month^2,Intercept)	-1.244	1.169	-3.843	0.733	1.004	286.116
L2 : Cov(month^2,month)	-4.975	1.165	-7.661	-3.055	1.011	408.096
L2 : Var(month^2)	2.387	0.707	1.280	4.055	1.017	237.353
Residual Var.	10.823	1.183	8.779	13.436	1.005	965.422

Coefficients:

...

- Curvature variance
- Variance of instantaneous linear change rates at MONTH = 0
- Variance of intercepts at MONTH = 0

SPAGHETTI PLOT



EFFECT SIZE OUTPUT

OUTCOME MODEL ESTIMATES:

Summaries based on 10000 iterations using 2 chains.

Outcome Variable: **breathCO**

Parameters	Median	StdDev	2.5%	97.5%	PSR	N_Eff

Variances:						
...						
Proportion Variance Explained						
by Coefficients	0.119	0.025	0.072	0.171	1.003	1345.826
by Level-2 Random Intercepts	0.463	0.043	0.382	0.548	1.001	1702.503
by Level-2 Random Slopes	0.166	0.029	0.114	0.227	1.005	823.026
by Level-1 Residual Variation	0.248	0.031	0.193	0.315	1.004	1007.734

BAYESIAN WALD TEST

- Asparouhov and Muthén (2021) proposed a Bayesian Wald test that mimics familiar likelihood-based Wald tests

$$T = (\theta - \theta_0)' \Sigma_{\theta}^{-1} (\theta - \theta_0)$$

- T is the sum of squared standardized differences (chi-square metric) between the posterior means and null hypothesis

BLIMP SCRIPT

DATA: clinicaltrial.dat;

VARIABLES: person wave month breathCO drinks cigs condition gender nicdep quit16 dropout;

CLUSTERID: person;

MISSING: 999;

FIXED: month;

MODEL:

breathCO ~ month@beta1 (month^2)@beta2 | month (month^2);

TEST: beta2 = 0;

<< Single-parameter test

BURN: 5000;

ITERATIONS: 10000;

SEED: 90291;

MODEL FIT OUTPUT

MODEL FIT:

INFORMATION CRITERIA

Marginal Likelihood	
DIC2	4076.173
WAIC	4138.998

...

WALD TESTS (Asparouhov & Muthén, 2021)

Test #1

Full:

[1] breathCO ~ Intercept month@beta1 (month^2)@beta2 | Intercept month

Restricted:

[1] breathCO ~ Intercept month@beta1 (month^2)@beta2 | Intercept month

Constraints in Restricted:

[1] beta2 = 0

Wald Statistic (Chi-Square)	41.007
Number of Parameters Tested (df)	1
Probability	0.000

MODEL COMPARISON

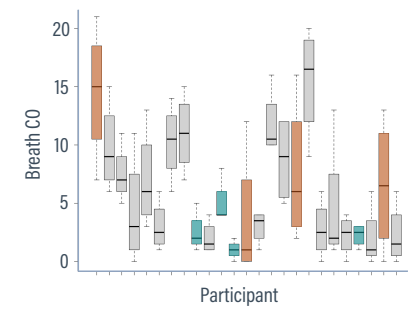
Criterion	Linear	Quadratic Fixed	Quadratic Random
Marginal DIC	4196.55	4138.49	4076.17
Marginal WAIC	4258.86	4201.08	4139.00
R ² Fixed effects	.09	.12	.12
R ² Intercepts	.43	.45	.46
R ² Slopes	.06	.08	.17
R ² Residual	.41	.35	.24

MODELING HETEROGENEOUS VARIATION IN **BLIMP**



VARIANCE HETEROGENEITY

- Participants exhibit substantial differences in their within-person variation of breath CO
- The standard growth model assumes constant within-cluster variation



MODELING WITHIN-CLUSTER VARIATION

- Cluster-specific residual variances that do not depend on other variables (Kasim & Raudenbush, 1998)
- Location and scale models where within-cluster variation is modeled as a function of level-1 and level-2 predictors
- Cluster-specific residual variation in location and scale models can be an outcome and a level-2 predictor

HETEROGENOUS VARIANCE SCRIPT

```
DATA: clinicaltrial.dat;  
VARIABLES: person wave month breathCO drinks cigs condition gender nicdep quit16 dropout;  
CLUSTERID: person;  
MISSING: 999;  
FIXED: month;  
MODEL:  
breathCO ~ month (month^2) | month (month^2);  
BURN: 5000;  
ITERATIONS: 10000;  
SEED: 90291;  
OPTIONS: hev;
```

>> Kasim & Raudenbush (1998)

ANALYSIS MODEL

- Location and scale models introduce a regression with the logarithm of the within-cluster variation as the outcome

$$BREATHCO_{ti} = \beta_{0i} + \beta_{1i}(MONTH_{ti}) + \beta_{2i}(MONTH_{ti}^2) + \varepsilon_{ti}$$
$$\sigma_{\varepsilon_{ti}}^2 = \exp(\gamma_0 + \epsilon_i)$$

- Level-1 and level-2 variables can predict within-cluster variation, and variation can predict distal outcomes

LOCATION AND SCALE MODEL SCRIPT

DATA: clinicaltrial.dat;

VARIABLES: person wave month breathCO drinks cigs female condition nicdep quit16 dropout;

CLUSTERID: person;

LATENT: person = logvar; << Define level-2 latent variable called logvar

MISSING: 999;

FIXED: month;

MODEL:

breathCO ~ month (month^2) | month (month^2);

var(breathCO) ~ 1@logvar; << Log of within-cluster variation equal to the latent variable

BURN: 10000;

ITERATIONS: 20000;

SEED: 90291;

COEFFICIENT OUTPUT

OUTCOME MODEL ESTIMATES:

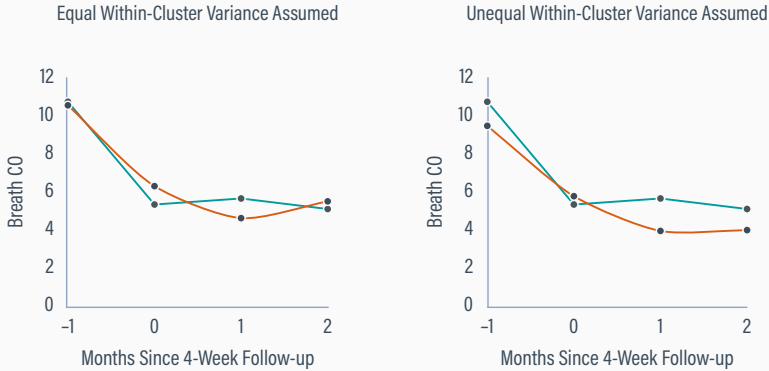
Summaries based on 20000 iterations using 2 chains.

Outcome Variable: **breathCO**

Parameters	Median	StdDev	2.5%	97.5%	PSR	N_Eff

Variances:						
...						
Coefficients:						
Intercept	5.780	0.453	4.944	6.729	1.002	98.688
month	-2.758	0.286	-3.309	-2.168	1.009	240.943
(month^2)	0.934	0.191	0.535	1.309	1.003	185.840
...						

OBSERVED VS. PREDICTED MEANS



VARIANCE COMPONENT OUTPUT

OUTCOME MODEL ESTIMATES:

Summaries based on 10000 iterations using 2 chains.

Outcome Variable: **breathCO**

Parameters	Median	StdDev	2.5%	97.5%	PSR	N_Eff
Variances:						
L2 : Var(Intercept)	16.105	3.550	10.065	23.972	1.008	250.216
L2 : Cov(month,Intercept)	1.205	1.431	-1.362	4.226	1.003	509.765
L2 : Var(month)	5.148	1.527	2.767	8.746	1.011	209.675
L2 : Cov(month^2,Intercept)	-2.449	1.033	-4.653	-0.554	1.007	173.358
L2 : Cov(month^2,month)	-2.625	0.861	-4.639	-1.251	1.023	168.084
L2 : Var(month^2)	1.612	0.557	0.673	2.875	1.038	97.502
Mean Residual Var.	6.739	1.013	4.854	8.880	1.021	133.344
Q25% Residual Var.	2.195	0.509	1.410	3.389	1.011	135.047
Q50% Residual Var.	6.702	1.185	4.676	9.308	1.014	189.349
Q75% Residual Var.	20.614	3.840	14.321	29.243	1.012	236.991

...

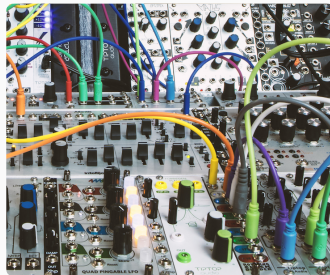
— = exp(quantiles of within-cluster variance on log metric)
 — = exp(mean within-cluster variance on log metric)

INCOMPLETE PREDICTOR VARIABLES IN **BLIMP**



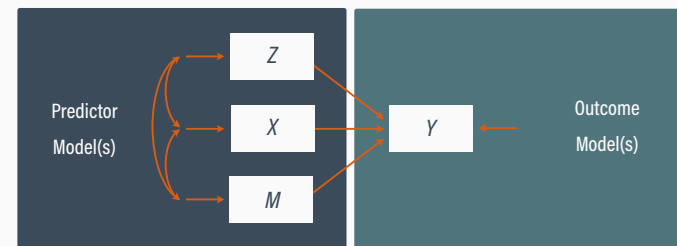
FACTORED REGRESSION MODELING

- Incomplete predictors require a model
- Factored regression uses a modular specification where a sequence of models replaces a multivariate function



FACTORED REGRESSION OVERVIEW

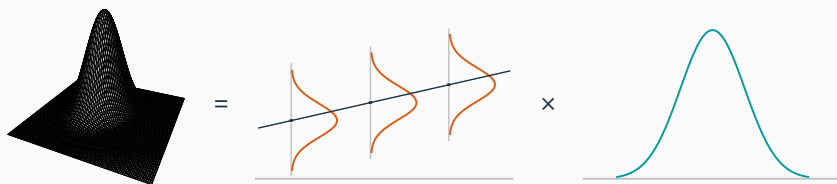
- Factored regression specifications invoke separate models for incomplete predictor variables and outcomes



BIVARIATE FACTORED REGRESSION

Bivariate Distribution = Univariate Outcome Model \times Univariate Predictor Model

$$f(Y, X) = f(Y|X) \times f(X)$$



MISSING PREDICTOR VARIABLES

- The factored regression specification dictates the distribution of missing predictor scores

$$f(X|Y) \propto f(Y|X) \times f(X) = \text{[Three orange normal curves with a regression line]} \times \text{[Teal normal curve]}$$

- The distribution of X given Y is the product of the two model-implied distributions (normal curves)

SIMPLE REGRESSION ILLUSTRATION

- The factored regression invokes two linear regressions

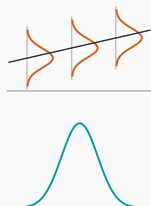
Factorization

$$f(Y, X) = f(Y|X) \times f(X)$$

Fitted models

$$Y = \beta_0 + \beta_1 X + \varepsilon$$

$$X = \gamma_0 + \epsilon$$



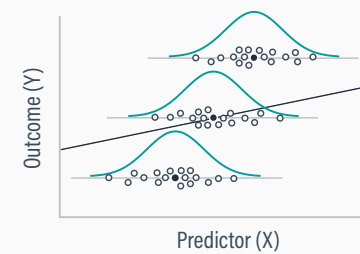
PREDICTOR IMPUTATIONS

- Two sets of model parameters define the center and spread of the imputations

$$X_{mis} \sim N(\hat{X}, \sigma_{X|Y}^2)$$

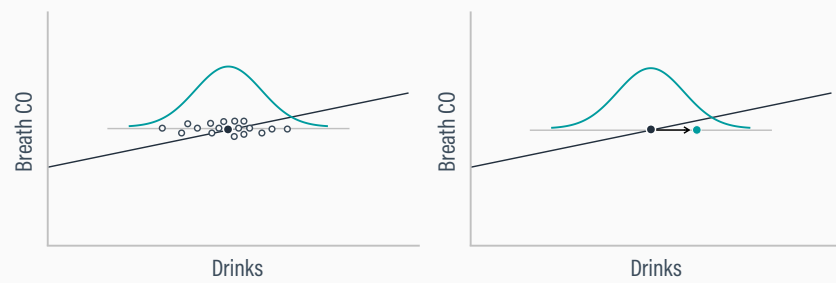
$$\sigma_{X|Y}^2 = \left(\frac{1}{\sigma_\epsilon^2} + \frac{\beta_1^2}{\sigma_\epsilon^2} \right)^{-1}$$

$$\hat{X} = \sigma_{X|Y}^2 \times \left(\frac{\gamma_0}{\sigma_\epsilon^2} + \frac{\beta_1(Y - \beta_0)}{\sigma_\epsilon^2} \right)$$



IMPUTATION EXAMPLE

Imputation = predicted value + random normal noise

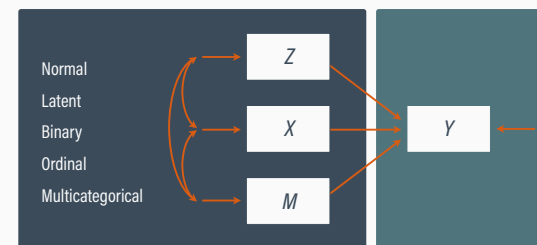


BLIMP DEFAULT SPECIFICATION

Multivariate Distribution = Univariate Outcome Model \times Multivariate Predictor Model

$$f(Y, M, X, Z) = f(Y|M, X, Z) \times f(M, X, Z)$$

MODEL:
 $y \sim z \times m;$



CLINICAL TRIAL DATA

■ Predictors
■ Outcome

Variable	Definition	Missing %	Scale
<i>PERSON</i>	Person-level (level-2) identifier	0	Integer index
<i>WAVE</i>	Data collection wave	0	Integer index (1 to 4)
<i>MONTH</i>	Months relative to 4-week follow-up	0	Integer index (-1 to 2)
<i>BREATHCO</i>	Breath CO reading	23.94	Numeric (0 to 46)
<i>DRINKS</i>	Number of drinks per drinking day	15.00	Numeric (0 to 34)
<i>CIGS</i>	Number of cigarettes per smoking day	14.85	Numeric (0 to 50)
<i>FEMALE</i>	Gender dummy code	0	0 = Male, 1 = Female
<i>CONDITION</i>	Treatment condition	0	0 = Single-medication, 1 = Dual-medication
<i>NICDEP</i>	Nicotine dependence dummy code	6.67	0 = Very Low, to Low 1 = Moderate to Very High
<i>QUIT16</i>	Quit smoking at 4-month follow-up	33.94	0 = Did not quit, 1 = Quit smoking
<i>DROPOUT</i>	Dropout indicator	13.33	0 = In study 1 = Dropped out

ILLUSTRATIVE LINEAR MODEL

- Two incomplete predictors: DRINKS is a time-varying random covariate, and NICDEP is a binary level-2 predictor

$$\begin{aligned}
 BREATHCO_{ti} = & \beta_{0i} + \beta_{1i}(MONTH_{ti}) + \beta_{2i}(DRINKS_{ti} - \mu_{i(D)}) \\
 & + \beta_3(NICDEP_i) + \varepsilon_{ti}
 \end{aligned}$$

- DRINKS is centered at its latent group means

DISAGGREGATING LEVEL-1 PREDICTORS

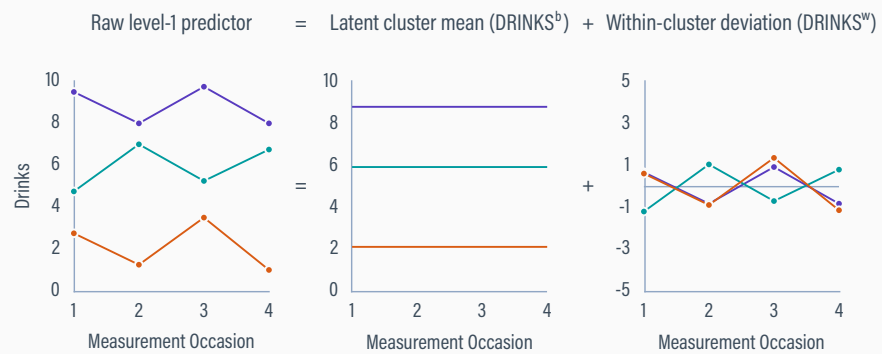
- Blimp disaggregates predictors with nonzero ICCs into within-cluster and between-cluster components

Within-cluster deviation of a score
around the latent group mean

$$DRINKS_{ti} = \underbrace{(DRINKS_{ti} - \mu_{i(D)})}_{\text{Within-cluster deviation}} + \underbrace{(\mu_{i(D)} - \mu_{(D)})}_{\text{Between-cluster deviation}} = DRINKS_{ti}^w + DRINKS_i^b$$

Between-cluster deviation of a latent
group mean around the grand mean

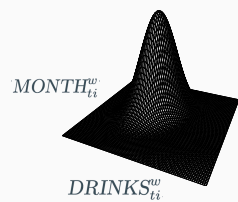
PARTITION FOR THREE PARTICIPANTS



PREDICTOR DISTRIBUTIONS

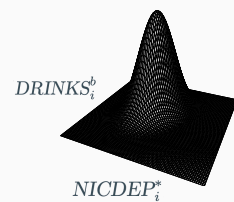
Level-1

$$\begin{pmatrix} MONTH_{ti}^w \\ DRINKS_{ti}^w \end{pmatrix} \sim N \left(\begin{pmatrix} \mu_{(M)} \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_M^2 & \sigma_{MD}^w \\ \sigma_{DM}^w & \sigma_D^2 \end{pmatrix} \right)$$



Level-2

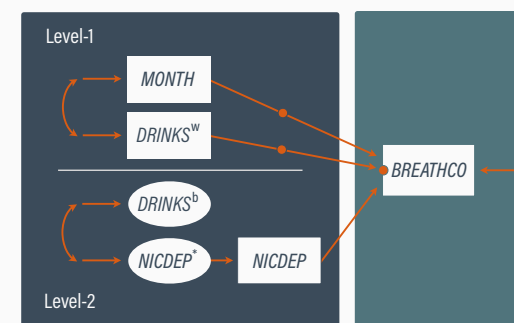
$$\begin{pmatrix} DRINKS_i^b \\ NICDEP_i^* \end{pmatrix} \sim N \left(\begin{pmatrix} \mu_{(D)} \\ \mu_{(N^*)} \end{pmatrix}, \begin{pmatrix} \sigma_D^2 & \sigma_{DN^*}^b \\ \sigma_{N^*D}^b & 1 \end{pmatrix} \right)$$



PATH DIAGRAM

Joint Distribution = Univariate Outcome Model × Multivariate Predictor Model

ORDINAL: nicdep;
CENTER:
 groupmean = drinks;
MODEL:
 breathCO ~ month drinks nicdep
 | month drinks;



DISTRIBUTION OF IMPUTATIONS

- The distribution of the DRINKS imputations depends on the focal model and a secondary model linking it to the predictors

$$\text{var}(DRINKS_{ti(mis)} | BREATHCO_{ti}, MONTH_{ti}, NICDEP_i) = \left(\frac{1}{\sigma_\epsilon^2} + \frac{\beta_{2i}^2}{\sigma_\epsilon^2} \right)^{-1}$$

- Random coefficients induce heteroscedastic variation that depends on the magnitude of a person's random slope

LATENT CONTEXTUAL EFFECTS

- Linear growth model with an incomplete predictor at each level and latent group means as an additional level-2 covariate

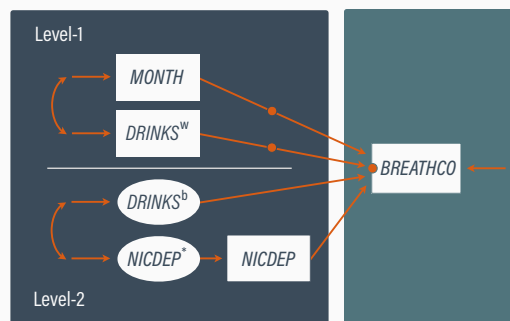
$$BREATHCO_{ti} = \beta_{0i} + \beta_{1i}(MONTH_{ti}) + \beta_{2i}(DRINKS_i^w) + \beta_3(DRINKS_i^b) + \beta_4(NICDEP_i) + \epsilon_{ti}$$

- The influence of drinking is partitioned into a within- and between-person regression slopes

PATH DIAGRAM

Joint Distribution = Univariate Outcome Model × Multivariate Predictor Model

ORDINAL: nicdep;
CENTER:
 groupmean = drinks;
 grandmean = drinks.mean;
MODEL:
 breathCO ~ month drinks nicdep
 drinks.mean | month drinks;



CLINICAL TRIAL DATA

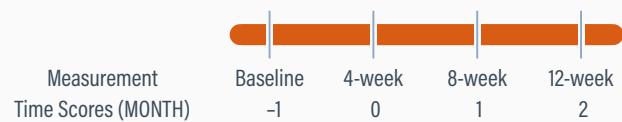
■ Predictors
■ Outcome

Variable	Definition	Missing %	Scale
PERSON	Person-level (level-2) identifier	0	Integer index
WAVE	Data collection wave	0	Integer index (1 to 4)
MONTH	Months relative to 4-week follow-up	0	Integer index (-1 to 2)
BREATHCO	Breath CO reading	23.94	Numeric (0 to 46)
DRINKS	Number of drinks per drinking day	15.00	Numeric (0 to 34)
CIGS	Number of cigarettes per smoking day	14.85	Numeric (0 to 50)
FEMALE	Gender dummy code	0	0 = Male, 1 = Female
CONDITION	Treatment condition	0	0 = Single-medication, 1 = Dual-medication
NICDEP	Nicotine dependence dummy code	6.67	0 = Very Low, to Low 1 = Moderate to Very High
QUIT16	Quit smoking at 4-month follow-up	33.94	0 = Did not quit, 1 = Quit smoking
DROPOUT	Dropout indicator	13.33	0 = In study 1 = Dropped out

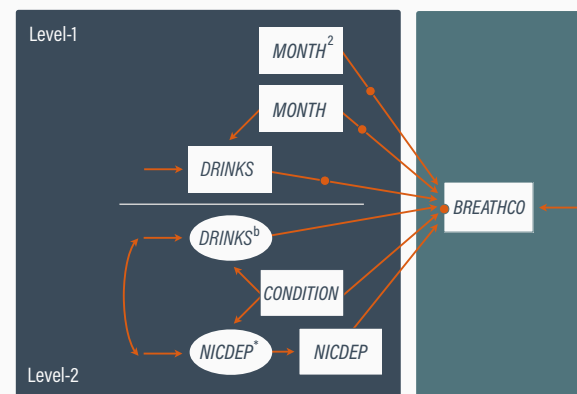
ANALYSIS MODEL

- Quadratic growth model with a random time-varying covariate, latent group means, and time-invariant predictors

$$BREATHCO_{ti} = \beta_{0i} + \beta_{1i}(MONTH_{ti}) + \beta_{2i}(MONTH_{ti}^2) + \beta_{3i}(DRINKS_{ti}^w) + \beta_4(DRINKS_i^b - \mu_{(DRINKS)}) + \beta_5(CONDITION_i) + \beta_6(NICDEP_i - \mu_{(NICDEP)}) + \varepsilon_{ti}$$



PATH DIAGRAM



BLIMP SCRIPT

```
DATA: clinicaltrial.dat;  
VARIABLES: person wave month breathCO drinks cigs female condition nicdep quit16 dropout;  
CLUSTERID: person;  
ORDINAL: condition nicdep;  
MISSING: 999;  
FIXED: month condition;  
CENTER: groupmean = drinks, grandmean = drinks.mean nicdep;  
MODEL:  
breathCO ~ month (month^2) drinks drinks.mean condition nicdep | month (month^2) drinks;  
BURN: 10000;  
ITERATIONS: 20000;  
SEED: 90291;
```

PSR DIAGNOSTIC OUTPUT

BURN-IN POTENTIAL SCALE REDUCTION (PSR) OUTPUT:

NOTE: Split chain PSR is being used. This splits each chain's iterations to create twice as many chains.

Comparing iterations across 2 chains	Highest PSR	Parameter #
251 to 500	1.299	7
501 to 1000	1.103	34
751 to 1500	1.107	15
1001 to 2000	1.038	27
...
4001 to 8000	1.103	4
4251 to 8500	1.055	10
4501 to 9000	1.038	10
4751 to 9500	1.025	10
>> Worst PSR < 1.05	5001 to 10000	1.016 10

COEFFICIENT OUTPUT

OUTCOME MODEL ESTIMATES:

Summaries based on 20000 iterations using 2 chains.

Outcome Variable: **breathCO**

Parameters	Median	StdDev	2.5%	97.5%	PSR	N_Eff
------------	--------	--------	------	-------	-----	-------

Variances:
...

Coefficients:						
Intercept	5.682	0.591	4.504	6.810	1.003	1125.433
month	-2.456	0.372	-3.191	-1.736	1.002	1598.430
drinks	0.312	0.133	0.065	0.582	1.009	554.416
drinks.mean[person]	0.319	0.278	-0.210	0.887	1.013	446.532
condition	1.345	0.740	-0.066	2.815	1.002	1100.831
nicdep	2.741	0.767	1.207	4.220	1.001	1095.901
(month^2)	1.076	0.209	0.663	1.486	1.001	2296.637

Standardized Coefficients:
...

VARIANCE COMPONENT OUTPUT

OUTCOME MODEL ESTIMATES:

Summaries based on 20000 iterations using 2 chains.

Outcome Variable: **breathCO**

Parameters	Median	StdDev	2.5%	97.5%	PSR	N_Eff
------------	--------	--------	------	-------	-----	-------

Variances:						
L2 : Var(Intercept)	18.483	3.575	12.495	26.628	1.001	1203.014
L2 : Cov(month,Intercept)	5.756	2.161	2.058	10.481	1.002	1234.223
L2 : Var(month)	10.041	2.251	6.337	15.134	1.000	1197.078
L2 : Cov(drinks,Intercept)	1.410	0.697	0.182	2.965	1.005	452.609
L2 : Cov(drinks,month)	-0.066	0.532	-1.004	1.098	1.013	273.380
L2 : Var(drinks)	0.441	0.248	0.108	1.073	1.030	162.376
L2 : Cov(month^2,Intercept)	-2.763	1.214	-5.548	-0.762	1.000	801.204
L2 : Cov(month^2,month)	-4.263	1.124	-6.811	-2.435	1.002	709.560
L2 : Cov(month^2,drinks)	0.013	0.263	-0.574	0.477	1.014	251.194
L2 : Var(month^2)	2.058	0.657	1.035	3.597	1.003	445.892
Residual Var.	10.196	1.178	8.134	12.723	1.004	1370.317

Coefficients:
...

EFFECT SIZE OUTPUT

OUTCOME MODEL ESTIMATES:

Summaries based on 20000 iterations using 2 chains.

Outcome Variable: **breathCO**

Parameters	Median	StdDev	2.5%	97.5%	PSR	N_Eff
------------	--------	--------	------	-------	-----	-------

Variances:
...

Proportion Variance Explained						
by Coefficients	0.192	0.036	0.126	0.266	1.001	1456.191
by Level-2 Random Intercepts	0.352	0.048	0.260	0.446	1.010	447.559
by Level-2 Random Slopes	0.235	0.045	0.162	0.337	1.013	319.565
by Level-1 Residual Variation	0.215	0.029	0.163	0.278	1.003	1453.998

INTERACTION
EFFECTS IN **BLIMP**

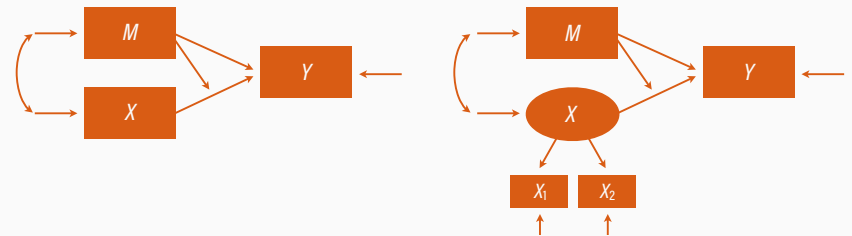


INCOMPLETE PRODUCT TERMS

- Factored regression specifications readily accommodate incomplete interactive and other nonlinear effects
- Product terms appear as deterministic functions of lower-order terms in the focal regression (growth) model
- The models and distributions of the predictors are unchanged

INTERACTION EFFECTS

Multivariate Distribution = Univariate Outcome Model \times Predictor Models



PREDICTOR IMPUTATIONS

- The product term induces heteroscedastic variation, such that the spread of the X imputations depends on the value of M

$$X_{mis} \sim \left(\hat{X}, \sigma_{X|Y,M}^2 \right)$$

$$\sigma_{X|Y,M}^2 = \left(\frac{1}{\sigma_\epsilon^2} + \frac{(\beta_1 + \beta_3 M)^2}{\sigma_\epsilon^2} \right)^{-1}$$

$$\hat{X} = \sigma_{X|Y,M}^2 \times \left(\frac{\gamma_0 + \gamma_1 M}{\sigma_\epsilon^2} + \frac{(\beta_1 + \beta_3 M)(Y - \beta_0 - \beta_2 M)}{\sigma_\epsilon^2} \right)$$


ANALYSIS MODEL

- Quadratic growth model with a random time-varying covariate, time-invariant predictors, and within-cluster and cross-level interaction effects


$$BREATHCO_{ti} = \beta_{0i} + \beta_{1i}(MONTH_{ti}) + \beta_{2i}(MONTH_{ti}^2) + \beta_{3i}(DRINKS_{ti}^w)$$

$$+ \beta_4(DRINKS_i^b - \mu_{(DRINKS)}) + \beta_5(CONDITION_i) + \beta_6(NICDEP_i - \mu_{(NICDEP)})$$

$$+ \beta_7(MONTH_{ti})(DRINKS_{ti}^w) + \beta_8(MONTH_{ti})(CONDITION_i) + \epsilon_{ti}$$

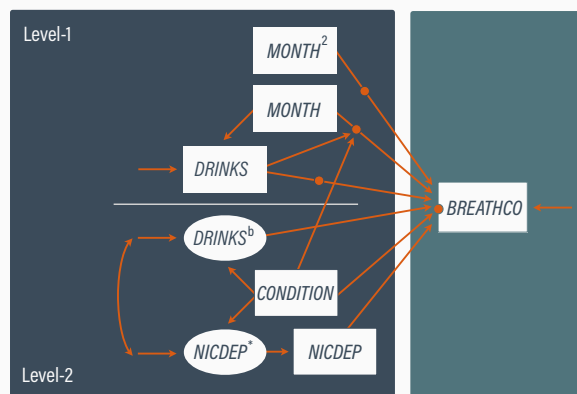


Within-person interaction



Cross-level interaction

PATH DIAGRAM



BLIMP SCRIPT

DATA: clinicaltrial.dat;
VARIABLES: person wave month breathCO drinks cigs female condition nicdep quit16 dropout;
CLUSTERID: person;
ORDINAL: condition nicdep;
MISSING: 999;
FIXED: month condition;
CENTER: groupmean = drinks, grandmean = drinks.mean nicdep;
MODEL:
 breathCO ~ month (month^2) drinks drinks.mean condition nicdep
 month*drinks month*condition | month (month^2) drinks;
BURN: 10000;
ITERATIONS: 20000;
SEED: 90291;

« Product terms

PSR DIAGNOSTIC OUTPUT

BURN-IN POTENTIAL SCALE REDUCTION (PSR) OUTPUT:

NOTE: Split chain PSR is being used. This splits each chain's iterations to create twice as many chains.

Comparing iterations across 2 chains	Highest PSR	Parameter #
251 to 500	1.518	9
501 to 1000	1.203	6
751 to 1500	1.118	12
1001 to 2000	1.074	4
...
4001 to 8000	1.047	4
4251 to 8500	1.041	5
4501 to 9000	1.044	15
4751 to 9500	1.043	4
5001 to 10000	1.034	9

>> Worst PSR < 1.05

COEFFICIENT OUTPUT

OUTCOME MODEL ESTIMATES:

Summaries based on 20000 iterations using 2 chains.

Outcome Variable: **breathCO**

Parameters	Median	StdDev	2.5%	97.5%	PSR	N_Eff
------------	--------	--------	------	-------	-----	-------

Variances:

...

Coefficients:

Intercept	5.545	0.558	4.436	6.619	1.001	1134.035
month	-2.210	0.419	-3.033	-1.382	1.000	1638.356
drinks	0.368	0.131	0.118	0.623	1.000	722.465
drinks.mean[person]	0.222	0.252	-0.257	0.749	1.008	508.331
condition	1.309	0.729	-0.113	2.754	1.004	1147.243
nicdep	2.682	0.763	1.193	4.213	1.002	1087.752
(month*2)	0.886	0.216	0.465	1.314	1.001	2295.107
month*drinks	-0.263	0.099	-0.453	-0.065	1.001	1020.609
month*condition	-0.129	0.416	-0.937	0.704	1.001	1861.082

Standardized Coefficients:

...

Cross-level interaction
Within-person interaction

BLIMP SCRIPT: CONDITIONAL EFFECTS

MODEL:

```
breathCO ~ month (month^2) drinks drinks.mean condition nicdep  
month*drinks month*condition | month (month^2) drinks;
```

SIMPLE: month | drinks;

SIMPLE: month | condition;

« Monthly change by levels of the moderators

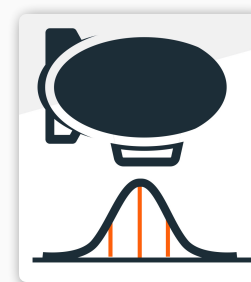
CONDITIONAL EFFECT OUTPUT

Conditional Effects	Median	StdDev	2.5%	97.5%	PSR	N_Eff
...						
month drinks @ +1 SD						
Intercept	6.695	0.746	5.209	8.132	1.000	869.797
Slope	-2.141	0.430	-2.973	-1.280	1.000	1402.021
month drinks @ 0						
Intercept	5.545	0.558	4.436	6.619	1.001	1134.035
Slope	-1.322	0.298	-1.907	-0.733	1.000	1904.117
month drinks @ -1 SD						
Intercept	4.387	0.635	3.158	5.641	1.001	1077.089
Slope	-0.511	0.428	-1.332	0.338	1.001	1190.200
...						

CONDITIONAL EFFECT OUTPUT

Conditional Effects	Median	StdDev	2.5%	97.5%	PSR	N_Eff
...						
month condition @ 0						
Intercept	5.545	0.558	4.436	6.619	1.001	1134.035
Slope	-1.322	0.298	-1.907	-0.733	1.000	1904.117
month condition @ 1						
Intercept	6.859	0.625	5.649	8.102	1.002	834.610
Slope	-1.448	0.333	-2.087	-0.782	1.001	1482.718



STRUCTURAL
MODELS IN **BLIMP**



STRUCTURAL MODELS

- Level-1 variables can predictor other level-1 variables
- Level-2 variables and the random effects (or latent group means) of level-1 variables can predict level-2 variables
- Outcome variables can be any combination of normal, skewed continuous, latent, binary, ordinal, multicategorical, and count

CLINICAL TRIAL DATA

 Predictors
 Outcomes

Variable	Definition	Missing %	Scale
<i>PERSON</i>	Person-level (level-2) identifier	0	Integer index
<i>WAVE</i>	Data collection wave	0	Integer index (1 to 4)
<i>MONTH</i>	Months relative to 4-week follow-up	0	Integer index (-1 to 2)
<i>BREATHCO</i>	Breath CO reading	23.94	Numeric (0 to 46)
<i>DRINKS</i>	Number of drinks per drinking day	15.00	Numeric (0 to 34)
<i>CIGS</i>	Number of cigarettes per smoking day	14.85	Numeric (0 to 50)
<i>FEMALE</i>	Gender dummy code	0	0 = Male, 1 = Female
<i>CONDITION</i>	Treatment condition	0	0 = Single-medication, 1 = Dual-medication
<i>NICDEP</i>	Nicotine dependence dummy code	6.67	0 = Very Low, to Low 1 = Moderate to Very High
<i>QUIT16</i>	Quit smoking at 4-month follow-up	33.94	0 = Did not quit, 1 = Quit smoking
<i>DROPOUT</i>	Dropout indicator	13.33	0 = In study 1 = Dropped out

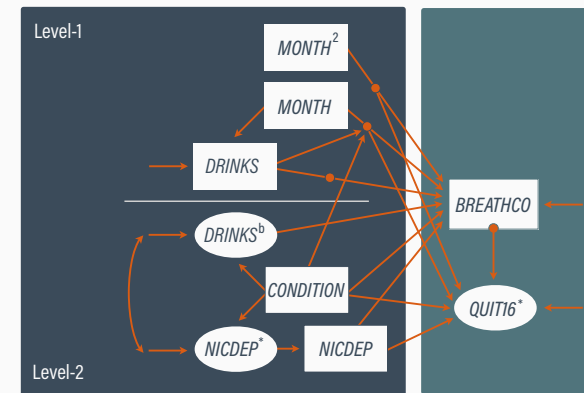
ANALYSIS MODEL

- Quadratic growth model with trajectories and time-invariant covariates predicting quit status at the 4-month follow-up

$$\begin{aligned}
 BREATHCO_{ti} = & \beta_{0i} + \beta_{1i}(MONTH_{ti}) + \beta_{2i}(MONTH_{ti}^2) + \beta_{3i}(DRINKS_{ti}^w) \\
 & + \beta_4(DRINKS_{ti}^b - \mu_{(DRINKS)}) + \beta_5(CONDITION_i) + \beta_6(NICDEP_i - \mu_{(NICDEP)}) \\
 & + \beta_7(MONTH_{ti})(DRINKS_{ti}^w) + \beta_8(MONTH_{ti})(CONDITION_i) + \varepsilon_{ti}
 \end{aligned}$$

$$\text{logit}(QUIT16_i) = \gamma_0 + \gamma_1(\beta_{0i}) + \gamma_2(\beta_{1i}) + \gamma_3(\beta_{2i}) + \gamma_4(CONDITION_i) + \gamma_5(NICDEP_i)$$

PATH DIAGRAM



BLIMP SCRIPT

DATA: clinicaltrial.dat;

VARIABLES: person wave month breathCO drinks cigs female condition nicdep quit16 dropout;

CLUSTERID: person;

ORDINAL: condition nicdep quit16;

MISSING: 999;

RANDEFFECT:

raniceps = breathCO | 1 [person];

ranslopes = breathCO | month [person];

rancurves = breathCO | month^2 [person];

« Define random effects as latent variables

BLIMP SCRIPT, CONTINUED

FIXED: month condition;

CENTER: groupmean = drinks, grandmean = drinks.mean nicdep;

MODEL:

breathCO ~ month (month^2) drinks drinks.mean condition nicdep

month*drinks month*condition | month (month^2) drinks;

logit(quit16) ~ raniceps ranslopes rancurves condition nicdep; « Logistic outcome model

BURN: 60000;

ITERATIONS: 150000;

SEED: 90291;

PSR DIAGNOSTIC OUTPUT

BURN-IN POTENTIAL SCALE REDUCTION (PSR) OUTPUT:

NOTE: Split chain PSR is being used. This splits each chain's iterations to create twice as many chains.

Comparing iterations across 2 chains	Highest PSR	Parameter #
1501 to 3000	1.362	10
3001 to 6000	1.158	6
4501 to 9000	1.145	30
6001 to 12000	1.169	34
...
24001 to 48000	1.076	6
25501 to 51000	1.038	36
27001 to 54000	1.046	36
28501 to 57000	1.038	36
30001 to 60000	1.045	6

>> Worst PSR < 1.05

COEFFICIENT OUTPUT

OUTCOME MODEL ESTIMATES:

Summaries based on 150000 iterations using 2 chains.

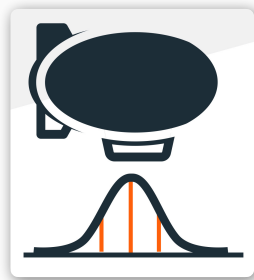
Outcome Variable: `logit(quit16)`

Parameters	Median	StdDev	2.5%	97.5%	PSR	N_Eff
Coefficients:						
Intercept	0.001	0.764	-1.608	1.441	1.000	1952.803
raniceps	-0.921	0.789	-3.541	-0.421	1.005	109.498
ranslopes	-1.519	0.995	-3.152	0.898	1.013	148.055
rancurves	-3.079	2.298	-6.462	2.756	1.013	98.556
condition	-0.241	1.084	-2.409	1.913	1.000	2414.594
nicdep	-2.077	1.145	-4.677	-0.152	1.001	2022.065
Odds Ratios:						
Intercept	1.001	1.509	0.200	4.223	1.000	2774.290
raniceps	0.398	0.162	0.029	0.657	1.004	121.467
ranslopes	0.219	0.998	0.043	2.456	1.007	279.311
rancurves	0.046	33.830	0.002	15.733	1.001	4426.390
condition	0.785	3.300	0.090	6.774	1.001	3848.116
nicdep	0.125	0.430	0.009	0.859	1.000	8129.363

Proportion Variance Explained

...

MISSING NOT AT RANDOM MODELS IN **BLIMP**



MISSING NOT AT RANDOM MODELS

- A missing not at random (MNAR) process occurs when the unseen score values carry information about missingness
- Selection and pattern mixture modeling are the two primary frameworks, both are available for longitudinal analyses
- Selection models feature an additional regression with a binary missing data or dropout indicator as the dependent variable

LONGITUDINAL MNAR PROCESSES

- Random coefficient-dependent missingness occurs when one's underlying growth trajectory is responsible for missing data
- Outcome-dependent missingness occurs when the unseen value of the dependent variable at a particular occasion predicts nonresponse

MISSING DATA INDICATORS

- A missing data indicator codes each instance of the outcome variable as 0 = observed and 1 = missing
- If most participants permanently attrit, then the binary dropout indicator is coded as 0 = observed at occasion t , 1 = dropped out at occasion t , and NA = dropped out prior to occasion t

DROPOUT INDICATOR CODING

Completer pattern	Obs 0	Obs 0	Obs 0	Obs 0
Dropout pattern 1	Obs 0	Obs 0	Obs 0	Mis 1
Dropout pattern 2	Obs 0	Obs 0	Mis 1	Mis NA
Dropout pattern 3	Obs 0	Mis 1	Mis NA	Mis NA



CLINICAL TRIAL DATA

■ Predictors
■ Outcomes

Variable	Definition	Missing %	Scale
<i>PERSON</i>	Person-level (level-2) identifier	0	Integer index
<i>WAVE</i>	Data collection wave	0	Integer index (1 to 4)
<i>MONTH</i>	Months relative to 4-week follow-up	0	Integer index (-1 to 2)
<i>BREATHCO</i>	Breath CO reading	23.94	Numeric (0 to 46)
<i>DRINKS</i>	Number of drinks per drinking day	15.00	Numeric (0 to 34)
<i>CIGS</i>	Number of cigarettes per smoking day	14.85	Numeric (0 to 50)
<i>FEMALE</i>	Gender dummy code	0	0 = Male, 1 = Female
<i>CONDITION</i>	Treatment condition	0	0 = Single-medication, 1 = Dual-medication
<i>NICDEP</i>	Nicotine dependence dummy code	6.67	0 = Very Low, to Low 1 = Moderate to Very High
<i>QUIT16</i>	Quit smoking at 4-month follow-up	33.94	0 = Did not quit, 1 = Quit smoking
<i>DROPOUT</i>	Dropout indicator	13.33	0 = In study 1 = Dropped out

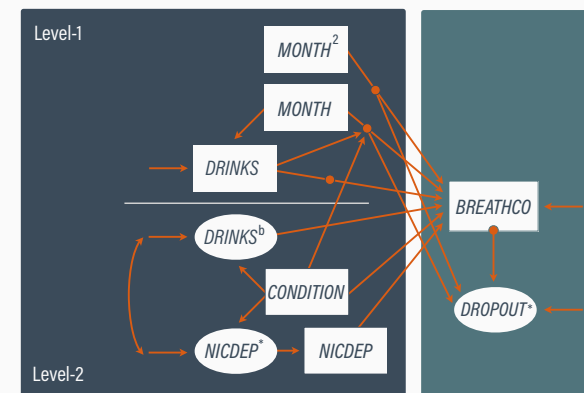
WU-CARROLL MODEL

- Growth model with occasion-specific dummy codes and random coefficients predicting the dropout indicator

$$BREATHCO_{ti} = \beta_{0i} + \beta_{1i}(MONTH_{ti}) + \beta_{2i}(MONTH_{ti}^2) + \beta_{3i}(DRINKS_{ti}^w) + \beta_{4i}(DRINKS_{ti}^b - \mu_{(DRINKS)}) + \beta_{5i}(CONDITION_i) + \beta_{6i}(NICDEP_i - \mu_{(NICDEP)}) + \beta_{7i}(MONTH_{ti})(DRINKS_{ti}^w) + \beta_{8i}(MONTH_{ti})(CONDITION_i) + \varepsilon_{ti}$$

$$DROPOUT_{ti}^* = \gamma_0 + \gamma_1(T_{2i}) + \gamma_2(T_{3i}) + \gamma_3(T_{4i}) + \gamma_4(\beta_{0i}) + \gamma_5(\beta_{1i}) + \gamma_6(\beta_{2i}) + \epsilon_{ti}$$

PATH DIAGRAM



BLIMP SCRIPT

```
DATA: clinicaltrial.dat;  
VARIABLES: person wave month breathCO drinks cigs female condition nicdep quit16 dropout;  
CLUSTERID: person;  
ORDINAL: condition nicdep dropout;  
MISSING: 999;  
RANDEFFECT:  
raniceps = breathCO | 1 [person];  
ranslopes = breathCO | month [person];  
rancurves = breathCO | month^2 [person]
```

BLIMP SCRIPT, CONTINUED

```
MODEL:  
breathCO ~ month (month^2) drinks drinks.mean condition nicdep  
month*drinks month*condition | month (month^2) drinks;  
dropout ~ 1@-3 (month == 0) (month == 1) (month == 2) raniceps ranslopes rancurves | 1@0;  
BURN: 50000;  
ITERATIONS: 100000;  
SEED: 90291;
```

PSR DIAGNOSTIC OUTPUT

BURN-IN POTENTIAL SCALE REDUCTION (PSR) OUTPUT:

NOTE: Split chain PSR is being used. This splits each chain's iterations to create twice as many chains.

Comparing iterations across 2 chains	Highest PSR	Parameter #
1251 to 2500	1.477	35
2501 to 5000	1.514	37
3751 to 7500	1.290	43
5001 to 10000	1.357	36
...
20001 to 40000	1.063	42
21251 to 42500	1.023	32
22501 to 45000	1.023	6
23751 to 47500	1.031	6
>> Worst PSR < 1.05	25001 to 50000	1.045 6

COEFFICIENT OUTPUT

OUTCOME MODEL ESTIMATES:

Summaries based on 100000 iterations using 2 chains.

Outcome Variable: **breathCO**

Parameters	Median	StdDev	2.5%	97.5%	PSR	N_Eff
------------	--------	--------	------	-------	-----	-------

Variances:

...

Coefficients:

Intercept	5.089	0.667	3.778	6.385	1.002	305.714
month	-2.456	0.496	-3.448	-1.500	1.001	295.011
drinks	0.360	0.143	0.091	0.650	1.003	393.015
drinks.mean[person]	0.260	0.247	-0.205	0.765	1.001	443.548
condition	1.541	0.712	0.205	2.995	1.001	772.555
nicdep	2.919	0.775	1.378	4.395	1.002	714.591
month^2	0.959	0.255	0.461	1.466	1.003	306.829
month*drinks	-0.280	0.098	-0.473	-0.089	1.001	768.577
month*condition	-0.142	0.410	-0.943	0.660	1.002	2068.035

Standardized Coefficients:

...

COEFFICIENT OUTPUT

OUTCOME MODEL ESTIMATES:

Summaries based on 100000 iterations using 2 chains.

...

Outcome Variable: **dropout**

Parameters	Median	StdDev	2.5%	97.5%	PSR	N_Eff
Variances:						
Residual Var.	1.000	0.000	1.000	1.000	nan	nan
Coefficients:						
Intercept	-3.000	0.000	-3.000	-3.000	nan	nan
ranccepts	-0.062	0.068	-0.200	0.075	1.007	313.659
ranslopes	-0.112	0.610	-1.196	1.193	1.013	170.750
rancurves	-0.281	1.446	-2.831	2.895	1.017	173.331
(month == 0)	2.191	0.149	1.858	2.445	1.003	727.572
(month == 1)	1.378	0.207	0.938	1.748	1.001	3130.294
(month == 2)	1.396	0.208	0.966	1.782	1.000	5385.965
Thresholds:						
Tau 1	0.000	0.000	0.000	0.000	nan	nan
...						

DIGGLE-KENWARD MODEL

- Growth model with occasion-specific dummy codes and concurrent and lagged outcome scores predicting dropout

$$\begin{aligned} BREATHC O_{ti} = & \beta_{0i} + \beta_{1i}(MONTH_{ti}) + \beta_{2i}(MONTH_{ti}^2) + \beta_{3i}(DRINKS_{ti}^w) \\ & + \beta_{4i}(DRINKS_{ti}^b - \mu_{(DRINKS)}) + \beta_{5i}(CONDITION_i) + \beta_{6i}(NICDEP_i - \mu_{(NICDEP)}) \\ & + \beta_{7i}(MONTH_{ti})(DRINKS_{ti}^w) + \beta_{8i}(MONTH_{ti})(CONDITION_i) + \varepsilon_{ti} \end{aligned}$$

$$\begin{aligned} DROPOUT_{ti}^* = & \gamma_0 + \gamma_1(T_{2i}) + \gamma_2(T_{3i}) + \gamma_3(T_{4i}) \\ & + \gamma_4(BREATHC O_{ti}) + \gamma_5(BRTHCOLAG_{ti}) + \epsilon_{ti} \end{aligned}$$

BLIMP SCRIPT

```
DATA: clinicaltrial.dat;  
VARIABLES: person wave month breathCO drinks cigs female condition nicdep quit16 dropout;  
CLUSTERID: person;  
ORDINAL: condition nicdep dropout;  
MISSING: 999;  
TRANSFORM:  
brthCOLag = lag1(breathCO, month, person);
```

« Compute lagged outcome

BLIMP SCRIPT, CONTINUED

```
FIXED: month condition;  
CENTER: groupmean = drinks, grandmean = drinks.mean nicdep;  
MODEL:  
breathCO ~ month (month^2) drinks drinks.mean condition nicdep  
month*drinks month*condition | month (month^2) drinks;  
brthCOLag ~ breathCO month drinks condition nicdep | 1@0;  
dropout ~ 1@-3 (month == 0) (month == 1) (month == 2) breathCO brthCOLag | 1@0;  
BURN: 20000;  
ITERATIONS: 10000;  
SEED: 90291;
```

PSR DIAGNOSTIC OUTPUT

BURN-IN POTENTIAL SCALE REDUCTION (PSR) OUTPUT:

NOTE: Split chain PSR is being used. This splits each chain's iterations to create twice as many chains.

Comparing iterations across 2 chains	Highest PSR	Parameter #
501 to 1000	1.507	9
1001 to 2000	1.113	6
1501 to 3000	1.149	6
2001 to 4000	1.108	6
...
8001 to 16000	1.035	6
8501 to 17000	1.009	6
9001 to 18000	1.013	41
9501 to 19000	1.016	41
>> Worst PSR < 1.05	10001 to 20000	1.010 4

COEFFICIENT OUTPUT

OUTCOME MODEL ESTIMATES:

Summaries based on 10000 iterations using 2 chains.

Outcome Variable: **breathCO**

Parameters	Median	StdDev	2.5%	97.5%	PSR	N_Eff
------------	--------	--------	------	-------	-----	-------

Variances:

...

Coefficients:

Intercept	5.769	0.601	4.662	6.953	1.005	468.534
month	-2.159	0.425	-2.993	-1.314	1.004	525.492
drinks	0.263	0.129	0.018	0.518	1.006	300.078
drinks.mean[person]	0.408	0.293	-0.127	1.004	1.010	222.795
condition	1.548	0.756	0.061	2.980	1.004	493.066
nicdep	2.954	0.771	1.466	4.505	1.002	586.808
(month*2)	0.895	0.216	0.475	1.316	1.002	588.550
month*drinks	-0.184	0.097	-0.379	0.002	1.013	394.952
month*condition	-0.173	0.409	-1.001	0.602	1.001	1299.504

Standardized Coefficients:

...

COEFFICIENT OUTPUT

OUTCOME MODEL ESTIMATES:

Summaries based on 10000 iterations using 2 chains.

...

Outcome Variable: dropout

Parameters	Median	StdDev	2.5%	97.5%	PSR	N_Eff
Variances:						
Residual Var.	1.000	0.000	1.000	1.000	nan	nan
Coefficients:						
Intercept	-3.000	0.000	-3.000	-3.000	nan	nan
breathCO	0.047	0.029	-0.014	0.100	1.008	178.917
brthCOlag	-0.029	0.023	-0.075	0.015	1.007	218.258
(month == 0)	2.292	0.165	1.967	2.618	1.001	962.839
(month == 1)	1.324	0.209	0.903	1.722	1.002	654.210
(month == 2)	1.294	0.224	0.834	1.715	1.004	441.393

Thresholds:						
Tau 1	0.000	0.000	0.000	0.000	nan	nan

Standardized Coefficients:						
...						

MODEL COMPARISON

Parameter	MAR		MNAR-WC		MNAR-DK	
	Mdn	SD	Mdn	SD	Mdn	SD
Intercept	5.55	0.56	5.09	0.67	5.77	0.60
MONTH	-2.21*	0.42	-2.46*	0.50	-2.16*	0.43
MONTH ²	0.89*	0.22	0.96*	0.26	0.90*	0.22
DRINKS ^w	0.37*	0.13	0.36*	0.14	0.26*	0.13
DRINKS ^b	0.22	0.25	0.26	0.25	0.41	0.29
CONDITION	1.31	0.73	1.54*	0.71	1.55*	0.76
NICDEP	2.68*	0.76	2.92*	0.78	2.95*	0.77
TIME × DRINKS ^w	-0.26*	0.10	-0.28*	0.10	-0.18	0.10
TIME × CONDITION	-0.13	0.42	-0.14	0.41	-0.17	0.41
R ² Fixed effects	.21	.04	.24	.04	.20	.04
R ² Intercepts	.34	.05	.33	.05	.40	.05
R ² Slopes	.25	.05	.24	.04	.20	.04

* = significant at p < .05

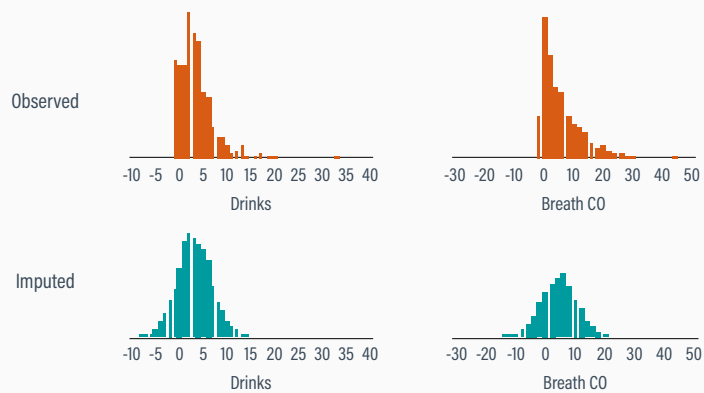
MODELING NONNORMAL DATA IN **BLIMP**



NONNORMAL DATA

- Generating normally distributed imputations for skewed variables usually produces implausible out-of-range values
- Means and coefficients are often unaffected, but other estimates can be distorted
- Conduct sensitivity analyses by generating imputations from nonnormal distributions and/or applying transformations

OBSERVED VS. IMPUTED DATA



YEO-JOHNSON TRANSFORMATION

- The Yeo-Johnson normal distribution defines a transformed (latent) variable X^* with a particular mean and variance
- The distribution invokes a shape parameter λ that links the nonnormal X scores to a normalized (latent) variable
- Incomplete variables are modeled on the normalized metric, and the MCMC algorithm iteratively estimates λ

TRANSFORMATION FUNCTION

- The Yeo-Johnson power function subsumes a range of common transformations used in practice (e.g., logarithmic, inverse, Box-Cox)

$$X_i^* = \begin{cases} ((X_i + 1)^\lambda - 1)/\lambda & \text{if } X_i \geq 0 \text{ and } \lambda \neq 0 \\ \ln(X_i + 1) & \text{if } X_i \geq 0 \text{ and } \lambda = 0 \\ -((-X_i + 1)^{2-\lambda} - 1)/(2 - \lambda) & \text{if } X_i < 0 \text{ and } \lambda \neq 2 \\ -\ln(-X_i + 1) & \text{if } X_i < 0 \text{ and } \lambda = 2 \end{cases}$$

SEQUENTIAL SPECIFICATION

- Modeling skewed predictors requires a so-called sequential specification that factorizes the multivariate predictor distribution into a sequence of univariate regressions
- Conceptually, a path model is used to link predictors
- Each incomplete predictor is an outcome in its own regression

NONNORMAL PREDICTORS

- The nonnormal predictor appears as a normalized (latent) variable in its own regression model
- The nonnormal version of the variable appears as a predictor in the focal regression (growth) model
- An inverse transformation converts the normalized latent imputes to nonnormal imputes on the original metric

PATH DIAGRAM

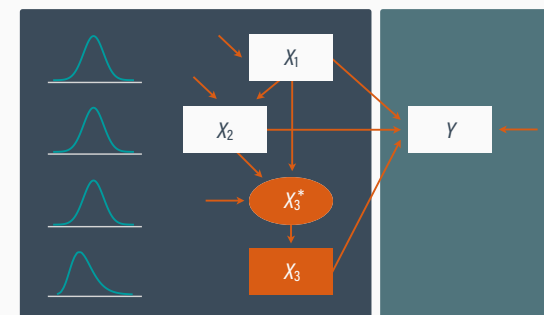
Joint Distribution = Univariate Outcome Model \times Univariate Predictor Models

MODEL:

$x_2 \sim x_1;$

$\gg y_{jt}(x_3) \sim x_2 x_1;$

$y \sim x_1 x_2 x_3;$



ANALYSIS MODEL

Model With Normalized Predictor as Outcome

$$DRINKS_{ti}^* = \gamma_{0i} + \gamma_1(MONTH_{ti}) + \gamma_2(NICDEP_i) + \gamma_3(CONDITION_i) + \epsilon_{ti}$$

Focal Model with Skewed Predictor

$$\begin{aligned} BREATHCO_{ti} = & \beta_{0i} + \beta_{1i}(MONTH_{ti}) + \beta_{2i}(MONTH_{ti}^2) + \beta_{3i}(DRINKS_{ti}^w) \\ & + \beta_4(DRINKS_i^b - \mu_{(DRINKS)}) + \beta_5(CONDITION_i) + \beta_6(NICDEP_i - \mu_{(NICDEP)}) \\ & + \beta_7(MONTH_{ti})(DRINKS_{ti}^w) + \beta_8(MONTH_{ti})(CONDITION_i) + \epsilon_{ti} \end{aligned}$$

BLIMP MODEL SCRIPT

- Centering the skewed variable at its median value of 4 in the normalized model improves MCMC convergence

MODEL:

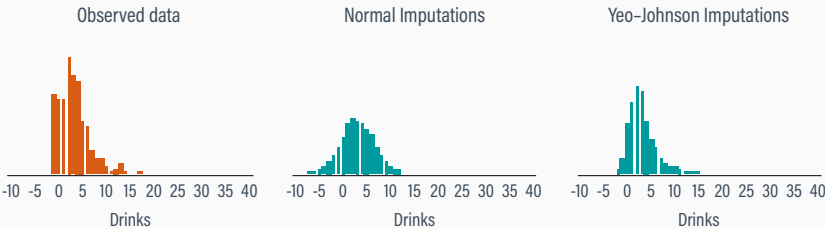
focal.model:

```
breathCO ~ month (month^2) drinks drinks.mean condition nicdep  
          month*drinks month*condition | month (month^2) drinks;
```

predictor.model:

```
yjt(drinks - 4) ~ time nicdep condition;
```

OBSERVED VS. IMPUTED DATA



COEFFICIENT OUTPUT

OUTCOME MODEL ESTIMATES:

Summaries based on 20000 iterations using 2 chains.

Outcome Variable: **breathCO**

Parameters	Median	StdDev	2.5%	97.5%	PSR	N_Eff
Variances:						
...						
Coefficients:						
Intercept	5.535	0.568	4.441	6.680	1.002	964.847
month	-2.236	0.418	-3.054	-1.403	1.001	1940.700
drinks	0.343	0.134	0.086	0.616	1.003	615.314
drinks.mean[person]	0.248	0.318	-0.345	0.906	1.011	415.224
condition	1.403	0.718	-0.021	2.824	1.001	1198.627
nicdep	2.787	0.751	1.299	4.242	1.002	1124.305
(month*2)	0.917	0.212	0.503	1.331	1.001	2519.477
month*drinks	-0.240	0.095	-0.428	-0.054	1.001	838.642
month*condition	-0.154	0.408	-0.955	0.642	1.000	2773.145

Standardized Coefficients:

...

COEFFICIENT OUTPUT

OUTCOME MODEL ESTIMATES:

Summaries based on 20000 iterations using 2 chains.

...

Outcome Variable: `yjt(drinks - 4)`

Parameters	Median	StdDev	2.5%	97.5%	PSR	N_Eff

Variances:						
L2 : Var(Intercept)	2.978	0.537	2.080	4.184	1.001	3091.170
Residual Var.	5.380	0.383	4.691	6.198	1.000	4958.543

Coefficients:						
Intercept	0.044	0.246	-0.430	0.537	1.000	3103.794
month	-0.909	0.090	-1.084	-0.730	1.001	5346.106
nicdep	-0.372	0.354	-1.057	0.320	1.002	2400.282
condition	-0.454	0.338	-1.105	0.227	1.001	3199.748

Transformation:						
Yeo-Johnson (lambda)	0.628	0.025	0.577	0.678	1.003	1921.032

Standardized Coefficients:

...

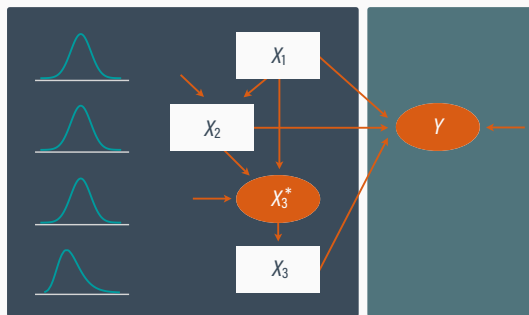
NONNORMAL OUTCOMES

- A nonnormal outcome appears as a normalized (latent) variable in the focal regression (growth) model
- Normalization changes the dependent variable's metric, so estimates will differ due to rescaling
- Blimp can save original and normalized imputes for inspection

NORMALIZED OUTCOME MODEL

Joint Distribution = Univariate Outcome Model \times Univariate Predictor Models

MODEL:
 $x_2 \sim x_1$;
 $y|t(x_3) \sim x_2 x_1$;
>> $y|t(y) \sim x_1 x_2 x_3$;



ANALYSIS MODEL

Model With Normalized Predictor as Outcome

$$DRINKS_{ti}^* = \gamma_{0i} + \gamma_1(MONTH_{ti}) + \gamma_2(NICDEP_i) + \gamma_3(CONDITION_i) + \epsilon_{ti}$$

Focal Model with Skewed Predictor and Normalized Outcome

$$\begin{aligned} BREATHCO_{ti}^* = & \beta_{0i} + \beta_{1i}(MONTH_{ti}) + \beta_{2i}(MONTH_{ti}^2) + \beta_{3i}(DRINKS_{ti}^w) \\ & + \beta_4(DRINKS_i^b - \mu_{(DRINKS)}) + \beta_5(CONDITION_i) + \beta_6(NICDEP_i - \mu_{(NICDEP)}) \\ & + \beta_7(MONTH_{ti})(DRINKS_{ti}^w) + \beta_8(MONTH_{ti})(CONDITION_i) + \epsilon_{ti} \end{aligned}$$

BLIMP MODEL SCRIPT

- Centering the skewed variables at their median (4 and 8) in the normalized models improves MCMC convergence

MODEL:

focal.model:

```
yjt(breathCO - 8) ~ month (month^2) drinks drinks.mean condition nicdep  
month*drinks month*condition | month (month^2) drinks;
```

predictor.model:

```
yjt(drinks - 4) ~ time nicdep condition;
```

COEFFICIENT OUTPUT

OUTCOME MODEL ESTIMATES:

Summaries based on 20000 iterations using 2 chains.

Outcome Variable: `yjt(breathCO - 8)`

Parameters	Median	StdDev	2.5%	97.5%	PSR	N_Eff
------------	--------	--------	------	-------	-----	-------

Variances:

...

Coefficients:

Intercept	-5.274	0.670	-6.614	-3.985	1.003	710.678
month	-2.936	0.407	-3.735	-2.138	1.001	1161.525
drinks	0.159	0.119	-0.069	0.399	1.001	595.354
drinks.mean[person]	0.456	0.310	-0.159	1.072	1.003	345.554
condition	1.499	0.691	0.141	2.860	1.003	1067.435
nicdep	2.847	0.715	1.414	4.244	1.001	1176.606
(month^2)	1.213	0.223	0.778	1.646	1.001	1433.258
month*drinks	-0.158	0.088	-0.335	0.011	1.004	1375.647
month*condition	-0.194	0.383	-0.938	0.568	1.001	1832.944

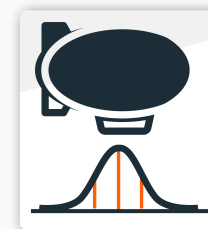
Standardized Coefficients:

...

MODEL COMPARISON

Parameter	Normal Imputations		Y-J Predictor		Y-J Predictor & Outcome	
	<i>Mdn</i>	<i>SD</i>	<i>Mdn</i>	<i>SD</i>	<i>Mdn</i>	<i>SD</i>
Intercept	5.55	0.56	5.54	0.57	-5.27	0.67
<i>MONTH</i>	-2.21*	0.42	-2.24*	0.42	-2.94*	0.41
<i>MONTH</i> ²	0.89*	0.22	0.92*	0.21	1.21*	0.22
<i>DRINKS</i> ^{ac}	0.37*	0.13	0.34*	0.13	0.16*	0.12
<i>DRINKS</i> ^b	0.22	0.25	0.25	0.32	0.46	0.31
<i>CONDITION</i>	1.31	0.73	1.40	0.72	1.50*	0.69
<i>NICDEP</i>	2.68*	0.76	2.79*	0.75	2.85*	0.72
<i>TIME</i> × <i>DRINKS</i> ^{ac}	-0.26*	0.10	-0.24*	0.10	-0.16*	0.09
<i>TIME</i> × <i>CONDITION</i>	-0.13	0.42	-0.15	0.41	-0.19	0.38
R ² Fixed effects	.21	.04	.21	.04	.23	.04
R ² Intercepts	.34	.05	.34	.05	.39	.04
R ² Slopes	.25	.05	.24	.05	.18	.04

* = significant at $p < .05$



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