CHAPTER

14 Operational Amplifiers

14.1 INTRODUCTION

An operational amplifier or op-amp is a very high gain differential amplifier with high input impedance and low output impedance. Typical uses of the operational amplifier are to provide voltage amplitude changes (amplitude and polarity), oscillators, filter circuits, and many types of instrumentation circuits. An op-amp contains a number of differential amplifier stages to achieve a very high voltage gain.

Figure 14.1 shows a basic op-amp with two inputs and one output as would result using a differential amplifier input stage. Recall from Chapter 12 that each input results in either the same or an opposite polarity (or phase) output, depending on whether the signal is applied to the plus (+) or the minus (−) input.

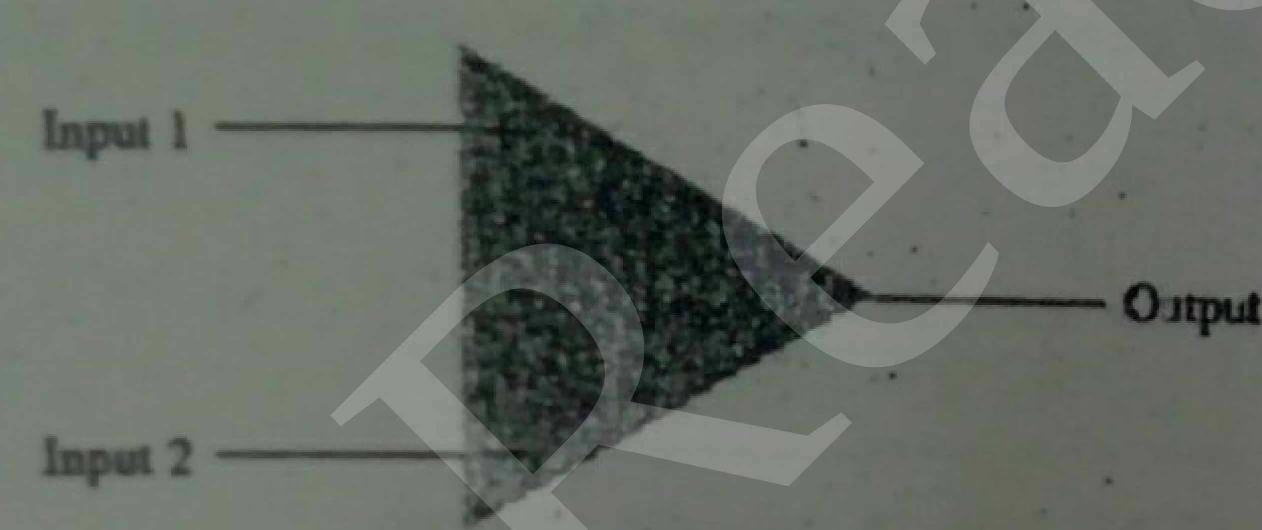

Figure 14.1 Basic op-amp

Single-Ended Input

Single-ended input operation results when the input signal is connected to one input with the other input connected to ground. Figure 14.2 shows the signals connected for

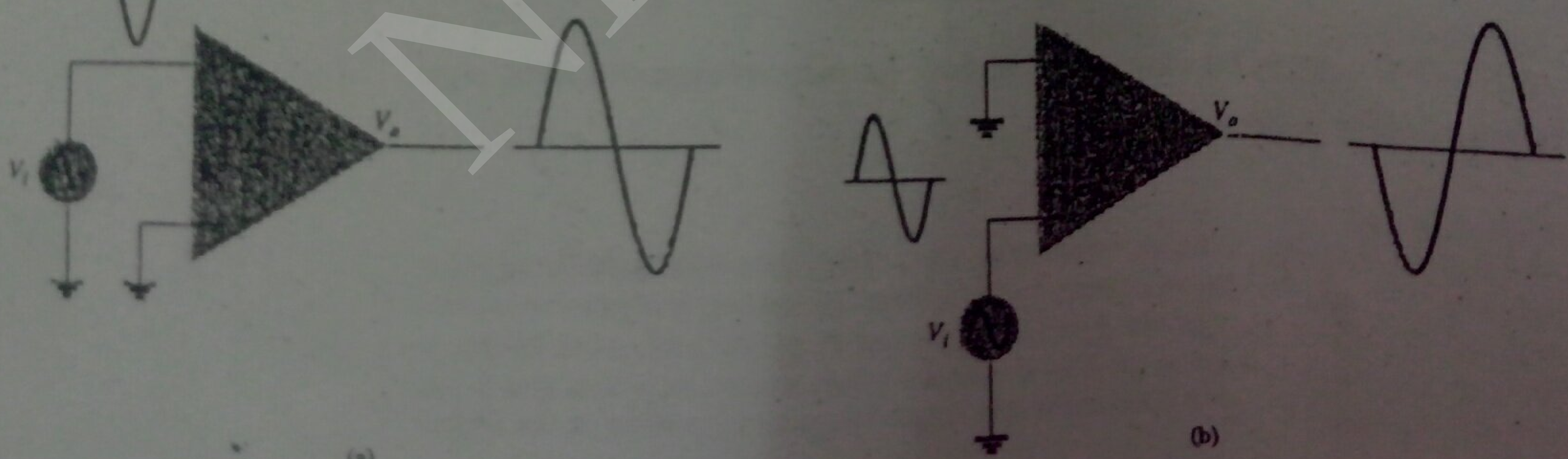

Figure 14.2 Single-ended operation.

this operation. In Fig. 14.2a the input is applied to the plus input (with minus input at ground), which results in an output having the same polarity as the applied input signal. Figure 14.2b shows an input signal applied to the minus input, the output then being opposite in phase to the applied signal.

Double-Ended (Differential) Input

In addition to using only one input, it is possible to apply signals at each input—this being a double-ended operation. Figure 14.3a shows an input, V_d, applied between the two input terminals (recall that neither input is at ground), with the resulting amplified output in phase with that applied between the plus and minus inputs. Figure 14.3b shows the same action resulting when two separate signals are applied to the inputs, the difference signal being $V_{i_1} - V_{i_2}$.

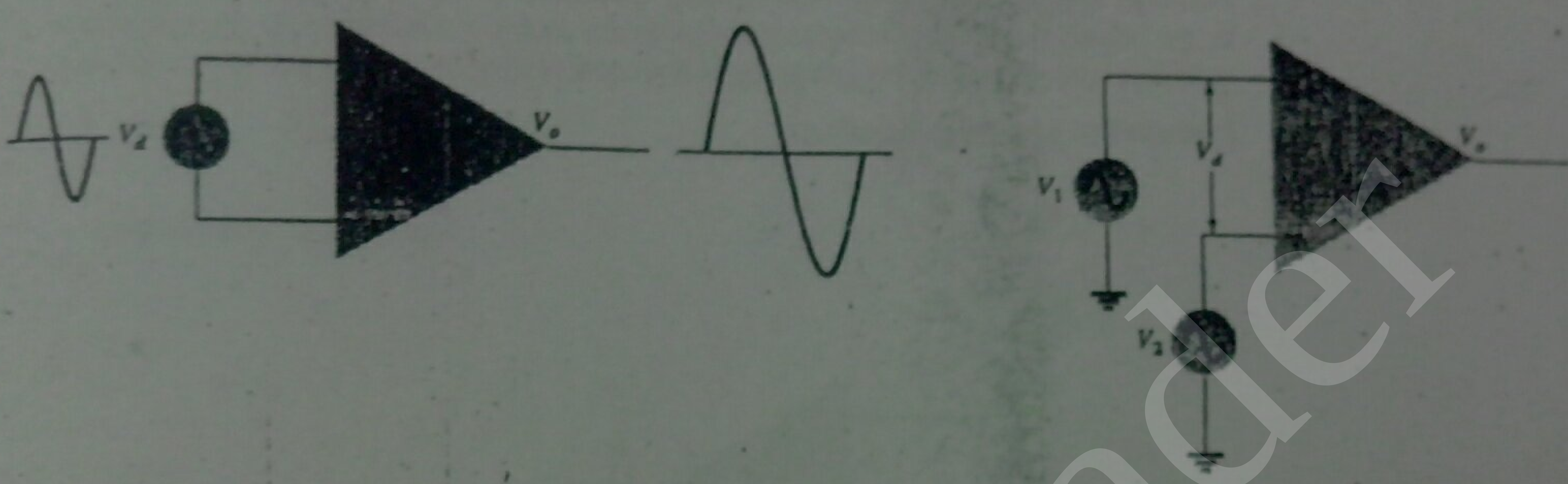

Figure 14.3 Double-ended (differential) operation.

Double-Ended Output

While the operation discussed so far had a single output, the op-amp can also be operated with opposite outputs, as shown in Fig. 14.4. An input applied to either input will result in outputs from both output terminals, these outputs always being opposite in polarity. Figure 14.5 shows a single-ended input with a double-ended output. As shown, the signal applied to the plus input results in two amplified outputs of opposite polarity. Figure 14.6 shows the same operation with a single output

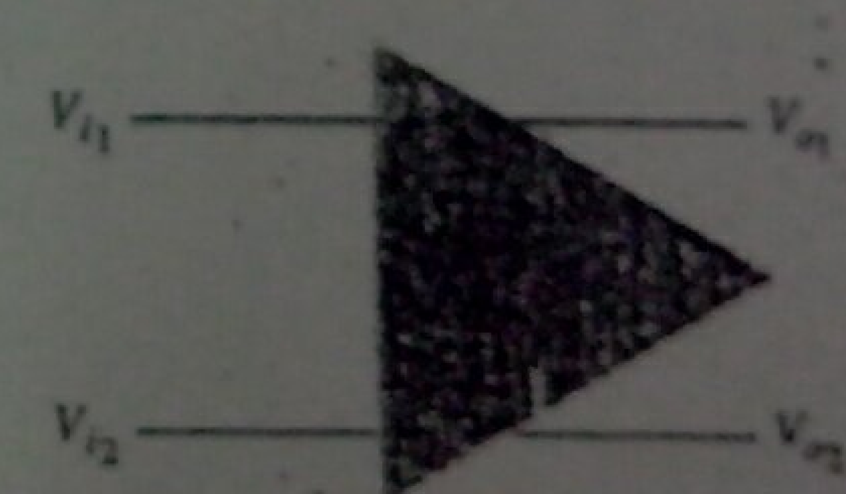

Figure 14.4 Double-ended output.

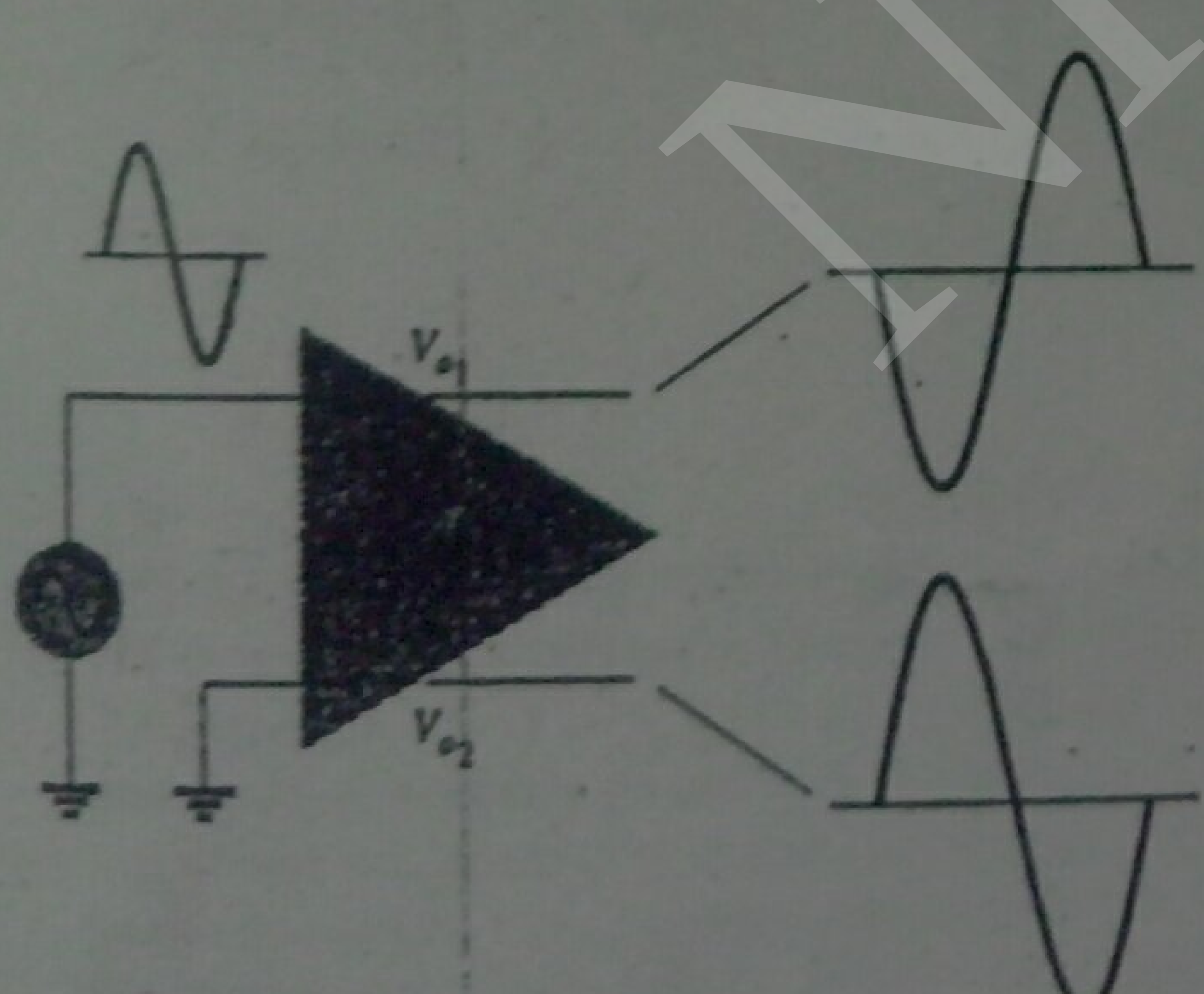

Figure 14.5 Double-ended output with single-ended input.

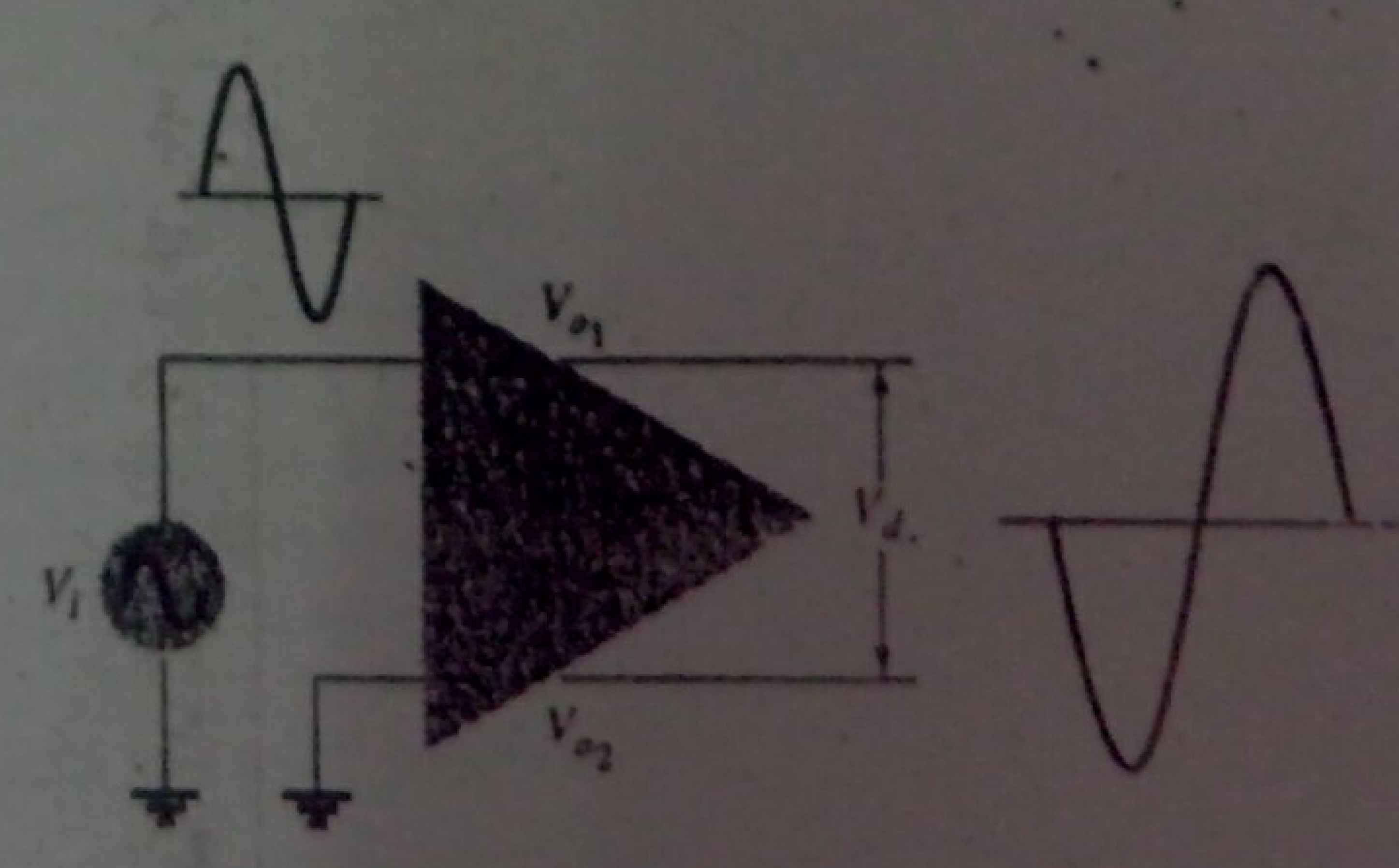

Figure 14.6 Double-ended output

measured between output terminals (not with respect to ground). This difference output signal is $V_{o_1} - V_{o_2}$. The difference output is also referred to as a *floating signal* since neither output terminal is the ground (reference) terminal. Notice that the difference output is twice as large as either V_{o_1} or V_{o_2} since they are of opposite polarity and subtracting them results in twice their amplitude [i.e., 10 V − (−10 V) = 20 V]. Figure 14.7 shows differential input, differential output operation. The input is applied between the two input terminals and the output taken from between the two output terminals. This is fully differential operation.

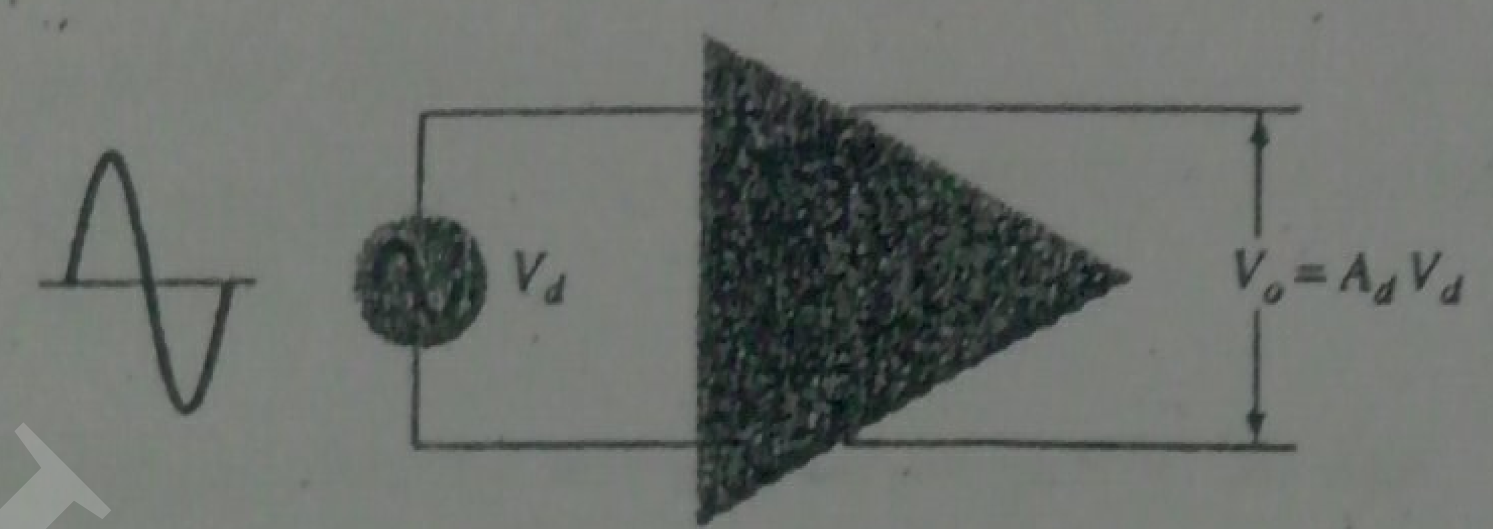

Figure 14.7 Differential-input, differential-output operation.

Common-Mode Operation

When the same input signals are applied to both inputs, common-mode operation results, as shown in Fig. 14.8. Ideally, the two inputs are equally amplified and since they result in opposite polarity signals at the output, these signals cancel, resulting in 0 V output. Practically, a small output signal will result.

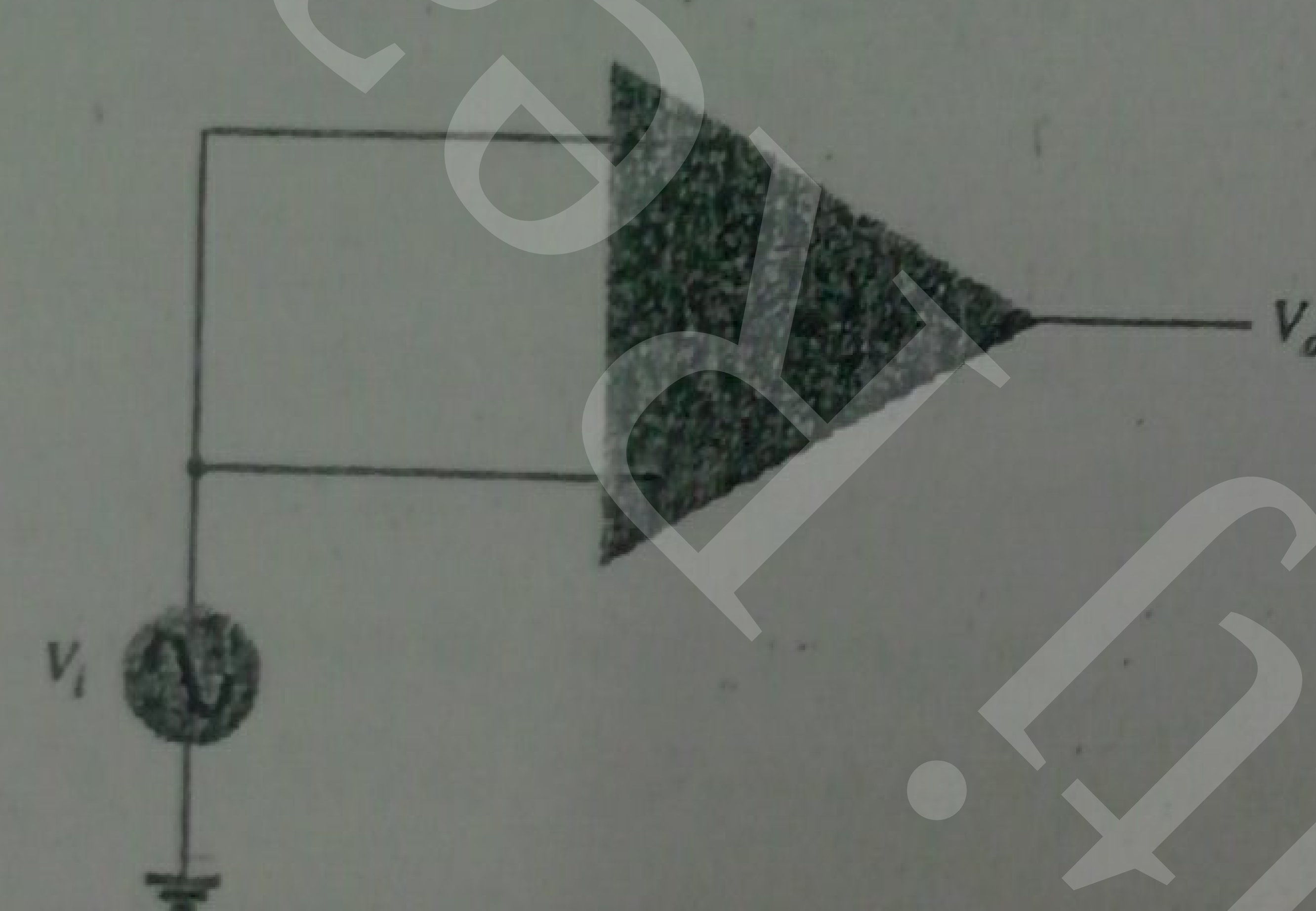

Figure 14.8 Common-mode operation.

Common-Mode Rejection

A significant feature of a differential connection is that the signals which are opposite at the inputs are highly amplified, while those which are common to the two inputs are only slightly amplified—the overall operation being to amplify the difference signal while rejecting the common signal at the two inputs. Since noise (any unwanted input signal) is generally common to both inputs, the differential connection tends to provide attenuation of this unwanted input while providing an amplified output of the difference signal applied to the inputs. This operating feature, referred to as common-mode rejection, is discussed more fully in the next section.

14.2 DIFFERENTIAL AND COMMON-MODE OPERATION

One of the more important features of a differential circuit connection, as provided in an op-amp, is the circuit's ability to greatly amplify signals that are opposite at the two inputs, while only slightly amplifying signals that are common to both inputs. An op-amp provides an output component that is due to the amplification of the difference of the signals applied to the plus and minus inputs and a component due to the signals common to both inputs. Since amplification of the opposite input signals is much greater than that of the common input signals, the circuit provides a common-mode rejection as described by a numerical value called the common-mode rejection ratio (CMRR).

Differential Inputs

When separate inputs are applied to the op-amp, the resulting difference signal is the difference between the two inputs.

$$\boxed{V_d = V_{i_1} - V_{i_2}} \qquad (14.1)$$

Common Inputs

When both input signals are the same, a common signal element due to the two inputs can be defined as the average of the sum of the two signals,

$$\boxed{V_c = \tfrac{1}{2}(V_{i_1} + V_{i_2})} \qquad (14.2)$$

Output Voltage

Since any signals applied to an op-amp in general have both in-phase and out-of-phase components, the resulting output can be expressed as

$$V_o = A_d V_d + A_c V_c \qquad (14.3)$$

where V_d = difference voltage given by Eq. (14.1)
V_c = common voltage given by Eq. (14.2)
A_d = differential gain of the amplifier
A_c = common-mode gain of the amplifier

Opposite Polarity Inputs

If opposite polarity inputs applied to an op-amp are ideally opposite signals, $V_{i_1} = -V_{i_2} = V_s$, the resulting difference voltage is

Eq. (14.1): $V_d = V_{i_1} - V_{i_2} = V_s - (-V_s) = 2V_s$

while the resulting common voltage is

Eq. (14.2): $V_c = \tfrac{1}{2}(V_{i_1} + V_{i_2}) = \tfrac{1}{2}[V_s + (-V_s)] = 0$

so that the resulting output voltage is

Eq. (14.3): $V_o = A_d V_d + A_c V_c = A_d(2V_s) + 0 = 2A_d V_s$

This shows that when the inputs are an ideal opposite signal (no common element), the output is the differential gain times twice the input signal applied to one of the inputs.

Same Polarity Inputs

If the same polarity inputs are applied to an op-amp, $V_{i_1} = V_{i_2} = V_s$, the resulting difference voltage is

Eq. (14.1): $V_d = V_{i_1} - V_{i_2} = V_s - V_s = 0$

while the resulting common voltage is

$$\text{Eq. (14.2):}\quad V_c = \tfrac{1}{2}(V_{i_1} + V_{i_2}) = \tfrac{1}{2}(V_s + V_s) = V_s$$

so that the resulting output voltage is

$$\text{Eq. (14.3):}\quad V_o = A_d V_d + A_c V_c = A_d(0) + A_c V_s = A_c V_s$$

This shows that when the inputs are ideal in-phase signals (no difference signal), the output is the common-mode gain times the input signal, V_s, which shows that only common-mode operation occurs.

Common-Mode Rejection

... above provide the relationships that can be used to measure A_d and A_c ...its.

1. *To measure A_d:* Set $V_{i_1} = -V_{i_2} = V_s = 0.5$ V, so that

$$\text{Eq. (14.1):}\quad V_d = (V_{i_1} - V_{i_2}) = (0.5\text{ V} - (-0.5\text{ V})) = 1\text{ V}$$

and

$$\text{Eq. (14.2):}\quad V_c = \tfrac{1}{2}(V_{i_1} + V_{i_2}) = \tfrac{1}{2}[0.5\text{ V} + (-0.5\text{ V})] = 0\text{ V}$$

Under these conditions the output voltage is

$$\text{Eq. (14.3):}\quad V_o = A_d V_d + A_c V_c = A_d(1\text{ V}) + A_c(0) = A_d$$

Thus, setting the input voltages $V_{i_1} = -V_{i_2} = 0.5$ V results in an output voltage numerically equal to the value of A_d.

2. *To measure A_c:* Set $V_{i_1} = V_{i_2} = V_s = 1$ V, so that

$$\text{Eq. (14.1):}\quad V_d = (V_{i_1} - V_{i_2}) = (1\text{ V} - 1\text{ V}) = 0\text{ V}$$

and

$$\text{Eq. (14.2):}\quad V_c = \tfrac{1}{2}(V_{i_1} + V_{i_2}) = \tfrac{1}{2}(1\text{ V} + 1\text{ V}) = 1\text{ V}$$

Under these conditions the output voltage is

$$\text{Eq. (14.3):}\quad V_o = A_d V_d + A_c V_c = A_d(0\text{ V}) + A_c(1\text{ V}) = A_c$$

Thus, setting the input voltages $V_{i_1} = V_{i_2} = 1$ V results in an output voltage numerically equal to the value of A_c.

Common-Mode Rejection Ratio

Having obtained A_d and A_c (as in the measurement procedure discussed above), we can now calculate a value for the common-mode rejection ratio (CMRR) which is defined by the following equation:

$$\boxed{\text{CMRR} = \frac{A_d}{A_c}} \qquad (14.4)$$

The value of CMRR can also be expressed in logarithmic terms as

$$\boxed{\text{CMRR (log)} = 20\log_{10}\frac{A_d}{A_c}} \qquad (dB) \qquad (14.5)$$

Calculate the CMRR for the circuit measurements shown in Fig. 14.9.

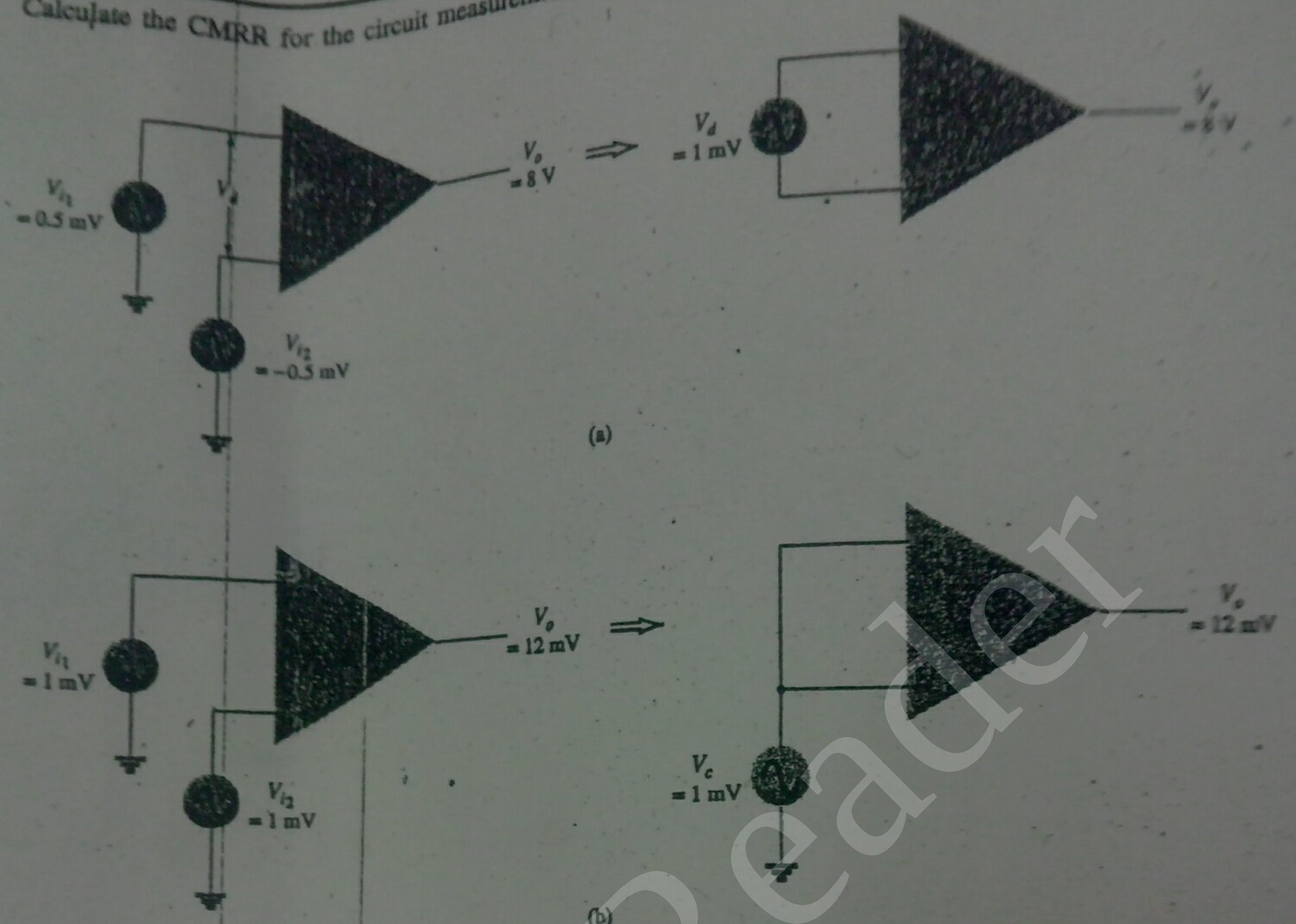

Figure 14.9 Differential and common-mode operation: (a) differential-mode; (b) common-mode.

Solution

From the measurement shown in Fig. 14.9a, using the procedure in step 1 above, we obtain

$$A_d = \frac{V_o}{V_d} = \frac{8\text{ V}}{1\text{ mV}} = 8000$$

From the measurement shown in Fig. 14.9b, using the procedure in step 2 above, gives us

$$A_c = \frac{V_o}{V_c} = \frac{12\text{ mV}}{1\text{ mV}} = 12$$

Using Eq. (14.4), the value of CMRR is

$$\text{CMRR} = \frac{A_d}{A_c} = \frac{8000}{12} = \mathbf{666.7}$$

which can also be expressed as

$$\text{CMRR} = 20\log_{10}\frac{A_d}{A_c} = 20\log_{10}666.7 = \mathbf{56.48\text{ dB}}$$

It should be clear that the desired operation will have A_d very large with A_c very small. That is, the signal components of opposite polarity will appear greatly amplified at the output, whereas the signal components that are in phase will mostly cancel out so that the common-mode gain, A_c, is very small. Ideally, the value of the CMRR is infinite. Practically, the larger the value of CMRR, the better the circuit operation.

We can express the output voltage in terms of the value of CMRR as follows.

$$\text{Eq. (14.3):}\quad V_o = A_dV_d + A_cV_c = A_dV_d\left(1 + \frac{A_cV_c}{A_dV_d}\right) = A_dV_d\left(1 + \frac{A_cV_c}{A_dV_d}\right)$$

Using Eq. (14.4), we can write the above as

$$\boxed{V_o = A_dV_d\left(1 + \frac{1}{\text{CMRR}}\frac{V_c}{V_d}\right)} \tag{14.6}$$

Even when both V_d and V_c components of signal are present, Eq. (14.6) shows that for large values of CMRR the output voltage will be due mostly to the difference signal, with the common-mode component greatly reduced or rejected. Some practical examples should help clarify this idea.

EXAMPLE 14.2

Determine the output voltage of an op-amp for input voltages of $V_{i_1} = 150\ \mu\text{V}$, $V_{i_2} = 140\ \mu\text{V}$. The amplifier has a differential gain of $A_d = 4000$ and the value of CMRR is:

(a) 100.

(b) 10^5.

Solution

$$\text{Eq. (14.1):}\quad V_d = V_{i_1} - V_{i_2} = (150 - 140)\ \mu\text{V} = 10\ \mu\text{V}$$

$$\text{Eq. (14.2):}\quad V_c = \frac{1}{2}(V_{i_1} + V_{i_2}) = \frac{150\ \mu\text{V} + 140\mu\text{V}}{2} = 145\ \mu\text{V}$$

$$\text{(a) Eq. (14.6):}\quad V_o = A_dV_d\left(1 + \frac{1}{\text{CMRR}}\frac{V_c}{V_d}\right) = (4000)(10\ \mu\text{V})\left(1 + \frac{1}{100}\frac{145\ \mu\text{V}}{10\ \mu\text{V}}\right)$$

$$= 40\ \text{mV}(1.145) = \mathbf{45.8\ mV}$$

$$\text{(b)}\quad V_o = (4000)(10\ \mu\text{V})\left(1 + \frac{1}{10^5}\frac{145\ \mu\text{V}}{10\ \mu\text{V}}\right) = 40\ \text{mV}(1.000145) = \mathbf{40.006\ mV}$$

Example 14.2 shows that the larger the value of CMRR, the closer the output voltage is to the difference input times the difference gain with the common-mode signal being rejected.

14.3 OP-AMP BASICS

An operational amplifier is a very high gain amplifier having very high input impedance (typically a few megohms) and low output impedance (less than 100 Ω). The basic circuit is made using a difference amplifier having two inputs (plus and minus) and at least one output. Figure 14.10 shows a basic op-amp unit. As discussed earlier,

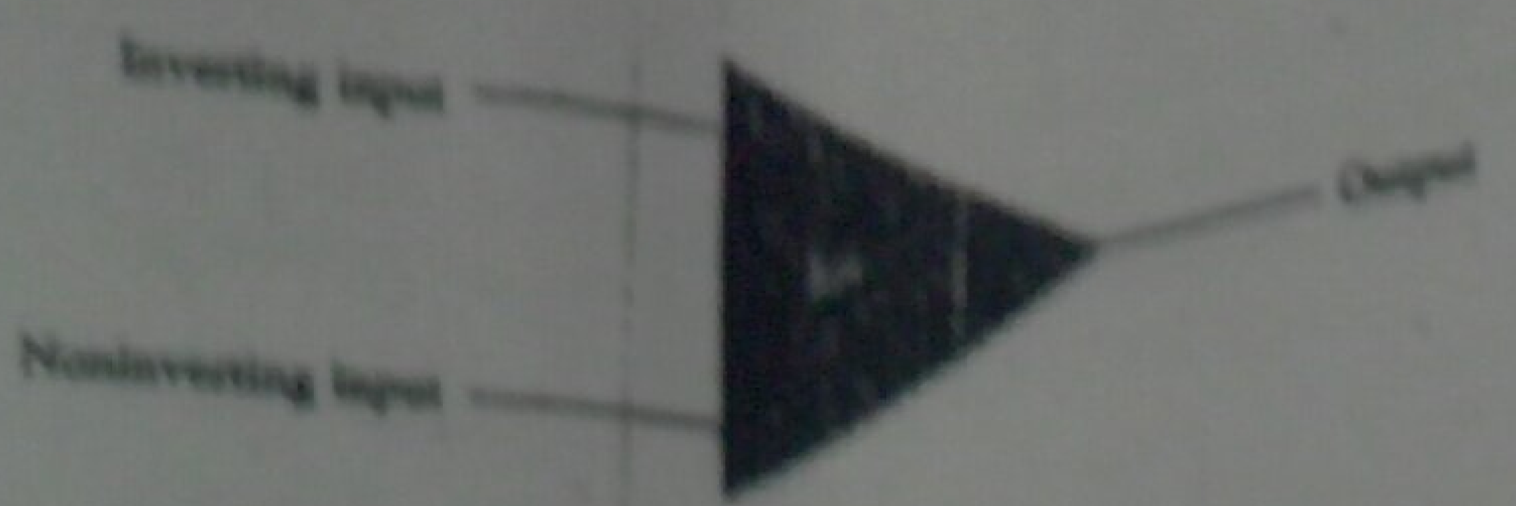

Figure 14.10 Basic op-amp.

the plus (+) input produces an output that is in phase with the signal applied, while an input to the minus (−) input results in an opposite polarity output. The ac equivalent circuit of the op-amp is shown in Fig. 14.11a. As shown, the input signal applied between input terminals sees an input impedance, R_i, typically very high. The output voltage is shown to be the amplifier gain times the input signal taken through an output impedance, R_o, which is typically very low. An ideal op-amp circuit, as shown in Fig. 14.11b, would have infinite input impedance, zero output impedance, and an infinite voltage gain.

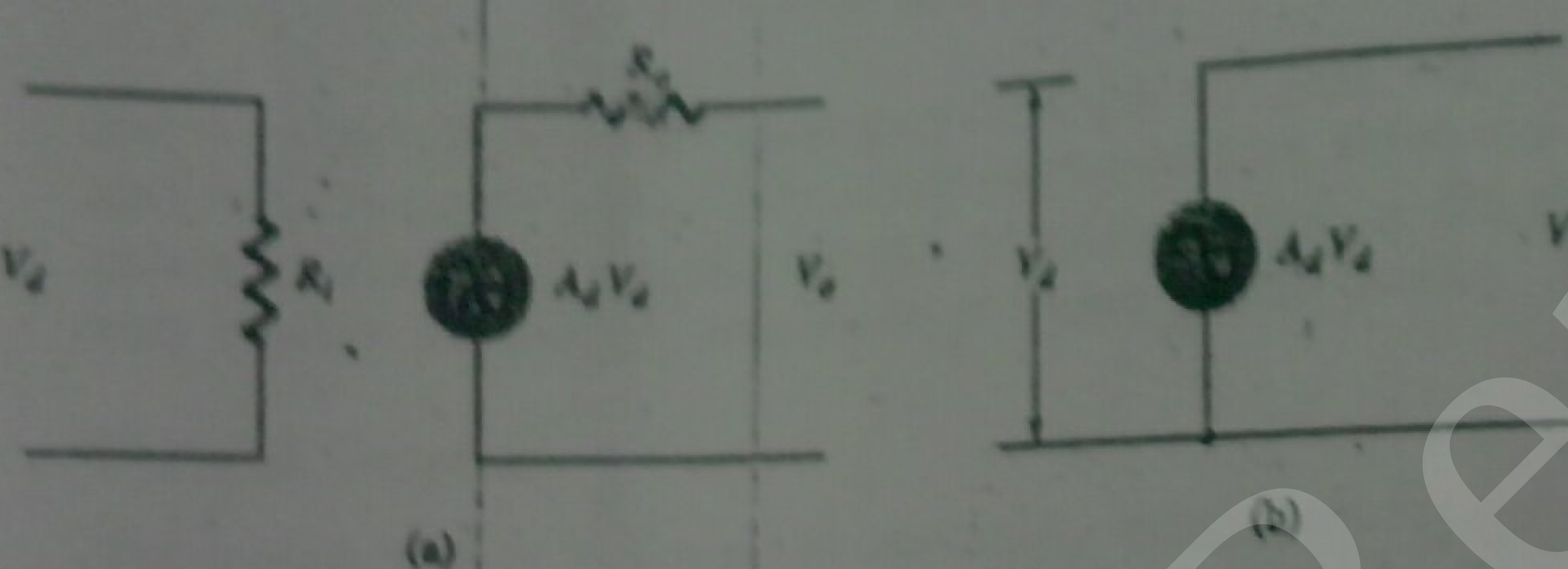

Figure 14.11 Ac equivalent of op-amp circuit: (a) practical; (b) ideal.

The basic circuit connection using an op-amp is shown in Fig. 14.12. As shown, the circuit provides operation as a constant-gain multiplier or scale changer. An input signal, V_1, is applied through resistor R_1 to the minus input. The output is then connected back to the same minus input through resistor R_f. The plus input is connected to ground. Since the signal V_1 is essentially applied to the minus input, the resulting output is opposite in phase to the input signal. Figure 14.13a shows the op-amp replaced by its ac equivalent circuit. If we use the ideal op-amp equivalent circuit, replacing R_i by an infinite resistance and R_o by zero resistance, the ac equivalent circuit is that shown in Fig. 14.13b. The circuit is then redrawn, as shown in Fig. 14.13c, from which circuit analysis is carried out.

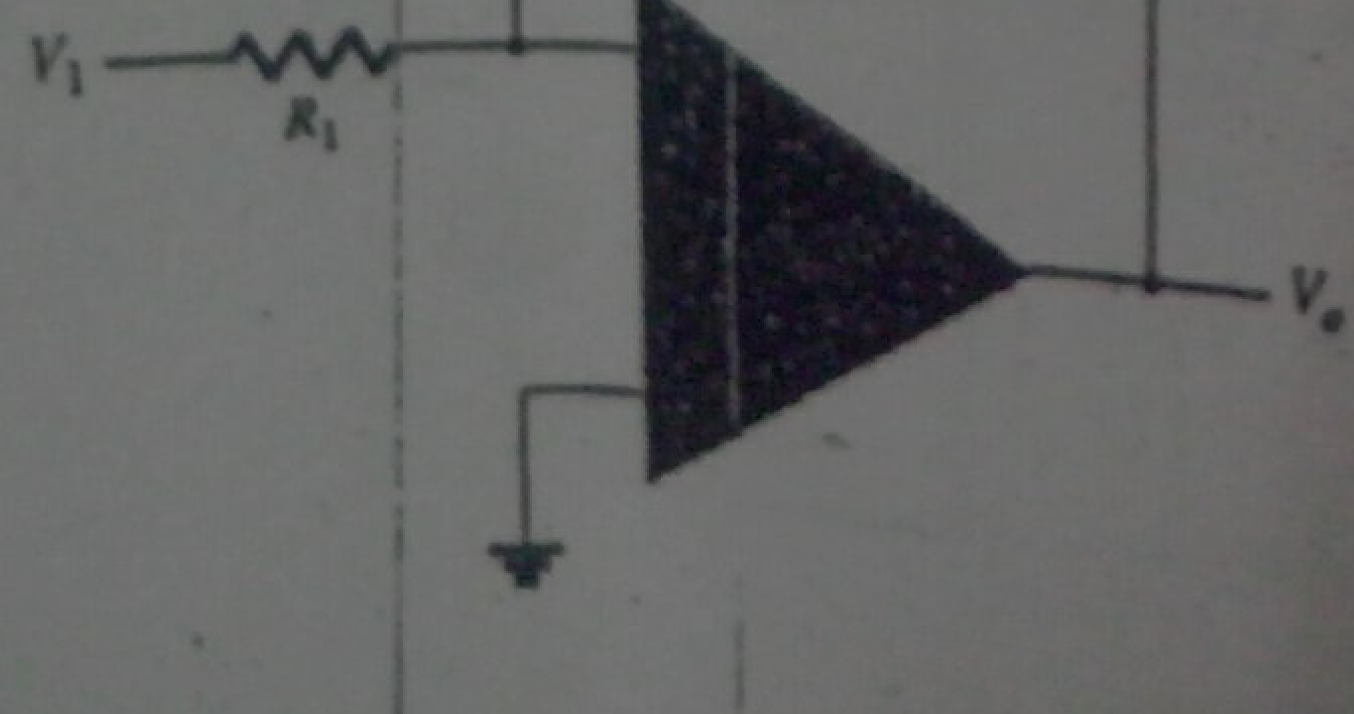

Figure 14.12 Basic op-amp connection.

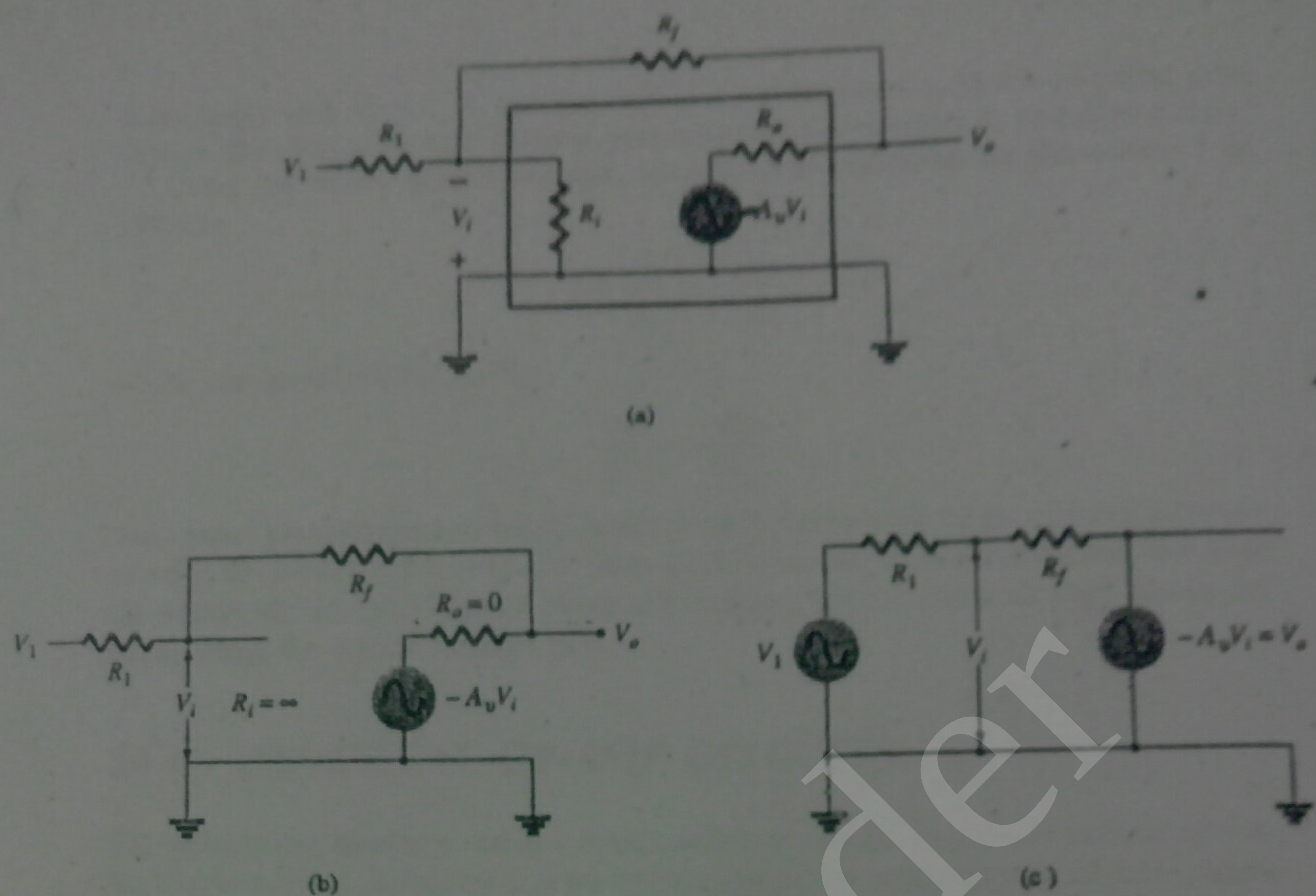

Figure 14.13 Operation of op-amp as constant-gain multiplier: (a) op-amp ac equivalent circuit; (b) ideal op-amp equivalent circuit; (c) redrawn equivalent circuit.

Using superposition we can solve for the voltage V_1 in terms of the components due to each of the sources. For source V_1 only ($-A_vV_i$ set to zero),

$$V_{i_1} = \frac{R_f}{R_1 + R_f} V_1$$

For source $-A_vV_i$ only (V_1 set to zero),

$$V_{i_2} = \frac{R_1}{R_1 + R_f}(-A_vV_i)$$

The total voltage V_i is then

$$V_i = V_{i_1} + V_{i_2} = \frac{R_f}{R_1 + R_f} V_1 + \frac{R_1}{R_1 + R_f}(-A_vV_i)$$

which can be solved for V_i as

$$V_i = \frac{R_f}{R_f + (1 + A_v)R_1} V_1 \tag{14.7}$$

If $A_v \gg 1$ and $A_vR_1 \gg R_f$, as is usually true, then

$$V_i = \frac{R_f}{A_vR_1} V_1$$

Solving for V_o/V_i, we get

$$\frac{V_o}{V_i} = \frac{-A_vV_i}{V_i} = \frac{-A_v}{V_i}\frac{R_fV_1}{A_vR_1} = -\frac{R_f}{R_1}\frac{V_1}{V_i}$$

so that

$$\frac{V_o}{V_1} = -\frac{R_f}{R_1} \qquad (14.8)$$

The result, in Eq. (14.8), shows that the ratio of overall output to input voltage is dependent only on the values of resistors R_1 and R_f—provided that A_v is very large.

Unity Gain

If $R_f = R_1$, the gain is

$$\text{voltage gain} = -\frac{R_f}{R_1} = -1$$

so that the circuit provides a unity voltage gain with 180° phase inversion. If R_f is exactly R_1, the voltage gain is exactly 1.

Constant Magnitude Gain

If R_f is some multiple of R_1, the overall amplifier gain is a constant. For example, if $R_f = 10R_1$, then

$$\text{voltage gain} = -\frac{R_f}{R_1} = -10$$

and the circuit provides a voltage gain of exactly 10 along with an 180° phase inversion from the input signal. If we select precise resistor values for R_f and R_1, we can obtain a wide range of gains, the gain being as accurate as the resistors used and is only slightly affected by temperature and other circuit factors.

Virtual Ground

The output voltage is limited by the supply voltage of, typically, a few volts. As stated before, voltage gains are very high. If, for example, $V_o = -10$ V and $A_v = 20{,}000$, the input voltage would then be

$$V_i = \frac{-V_o}{A_v} = \frac{10\text{ V}}{20{,}000} = 0.5\text{ mV}$$

If the circuit has an overall gain (V_o/V_1) of, say, 1, the value of V_1 would then be 10 V. Compared to all other input and output voltages, the value of V_i, is then small and may be considered 0 V.

Note that although $V_i \approx 0$ V, it is not exactly 0 V. (The output voltage is a few volts, due to the very small input V_i times a very large gain A_v.) The fact that $V_i \approx 0$ V leads to the concept that at the amplifier input there exists a virtual short circuit or virtual ground.

The concept of a virtual short implies that although the voltage is nearly 0 V, there is no current through the amplifier input to ground. Figure 14.14 depicts the virtual ground concept. The heavy line is used to indicate that we may consider that a

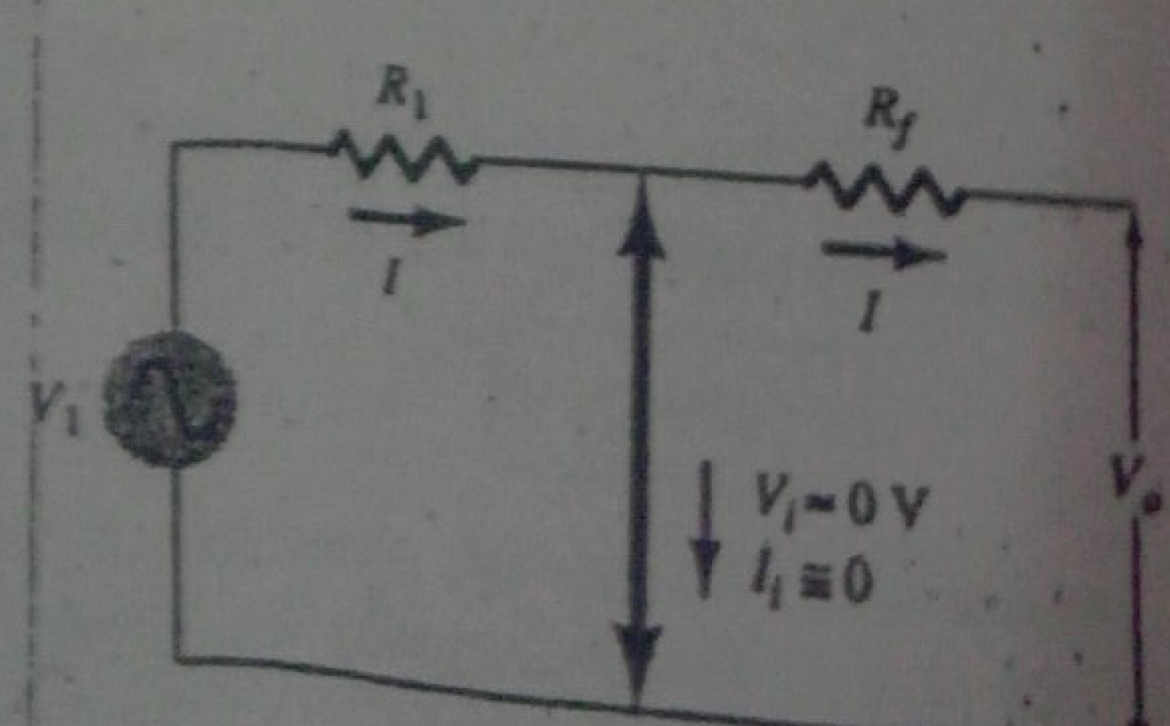

Figure 14.14 Virtual ground in an op-amp.

short exists with $V_i = 0$ V, but that this is a virtual short so that no current goes through the short to ground. Current goes only through resistors R_1 and R_f as shown.

Using the virtual ground concept, we can write equations for the current I as follows:

$$I = \frac{V_1}{R_1} = -\frac{V_o}{R_f}$$

which can be solved for V_o/V_1:

$$\frac{V_o}{V_1} = -\frac{R_f}{R_1}$$

The virtual ground concept, which depends on A_v being very large, allowed a simple solution to determine the overall voltage gain. It should be understood that although the circuit of Fig. 14.14 is not physically correct, it does allow an easy means for determining the overall voltage gain.

14.4 PRACTICAL OP-AMP CIRCUITS

The op-amp can be connected in a large number of circuits to provide various operating characteristics. In this section we cover a few of the most common of these circuit connections.

Inverting Amplifier

The most widely used constant-gain amplifier circuit is the inverting amplifier, shown in Fig. 14.15. The output is obtained by multiplying the input by a fixed or constant gain, set by the input resistor (R_1) and feedback resistor (R_f)—this output also being inverted from the input. Using Eq. (14.8) we can write

$$V_o = -\frac{R_f}{R_1}V_1$$

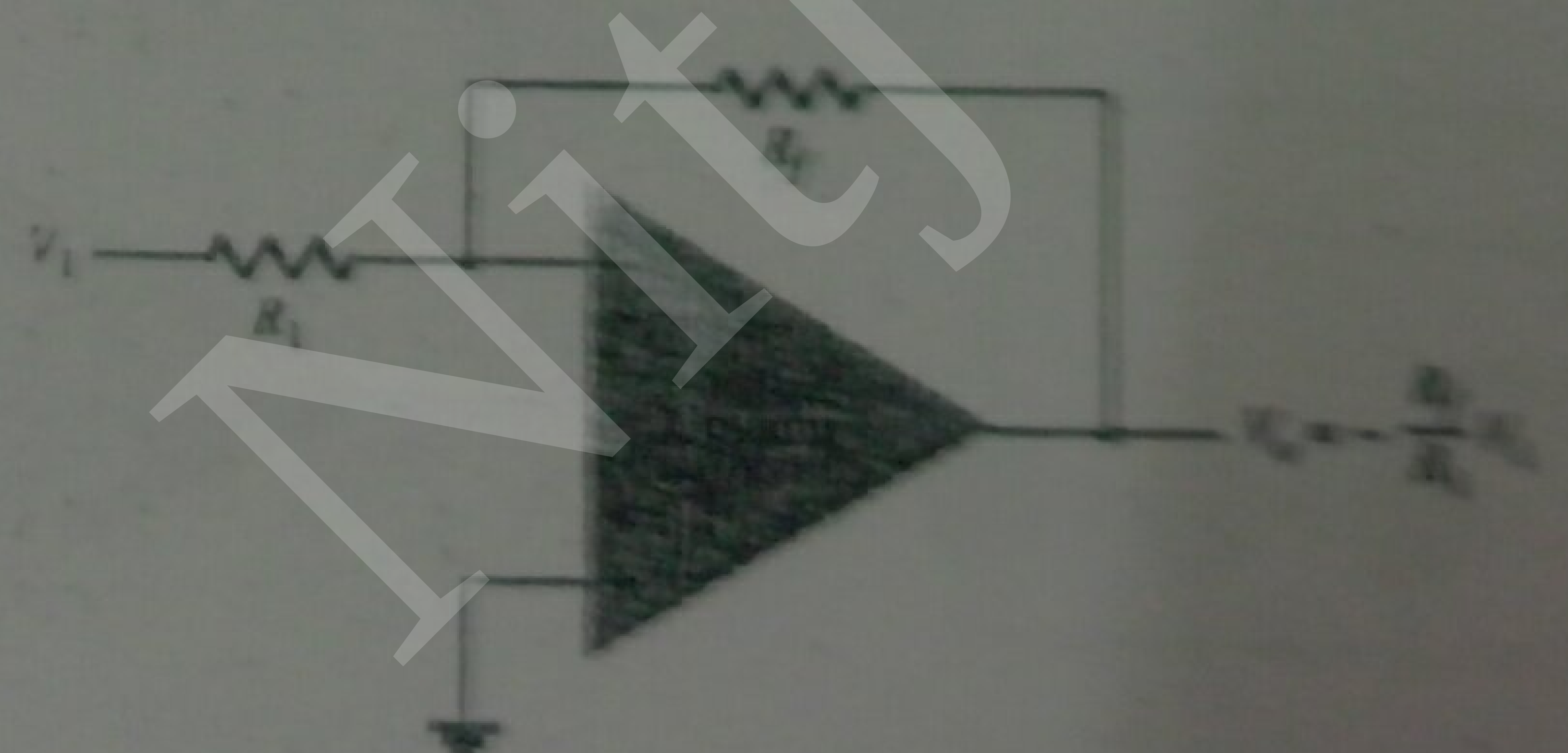

Figure 14.15 Inverting constant-gain multiplier.

EXAMPLE 14.3

If the circuit of Fig. 14.15 has $R_1 = 100$ kΩ and $R_f = 500$ kΩ, what output voltage results for an input of $V_1 = 2$ V?

Solution

$$\text{Eq. (14.8):}\quad V_o = -\frac{R_f}{R_1}V_1 = -\frac{500\text{ k}\Omega}{100\text{ k}\Omega}(2\text{ V}) = -10\text{ V}$$

Noninverting Amplifier

The connection of Fig. 14.16a shows an op-amp circuit that works as a noninverting amplifier or constant-gain multiplier. It should be noted that the inverting amplifier connection is more widely used because it has better frequency stability (discussed later). To determine the voltage gain of the circuit, we can use the equivalent representation shown in Fig. 14.16b. Note that the voltage across R_1 is V_1 since $V_i \approx 0$ V. This must be equal to the output voltage, through a voltage divider of R_1 and R_f, so that

$$V_1 = \frac{R_1}{R_1 + R_f} V_o$$

which results in

$$\boxed{\frac{V_o}{V_1} = \frac{R_1 + R_f}{R_1} = 1 + \frac{R_f}{R_1}} \tag{14.9}$$

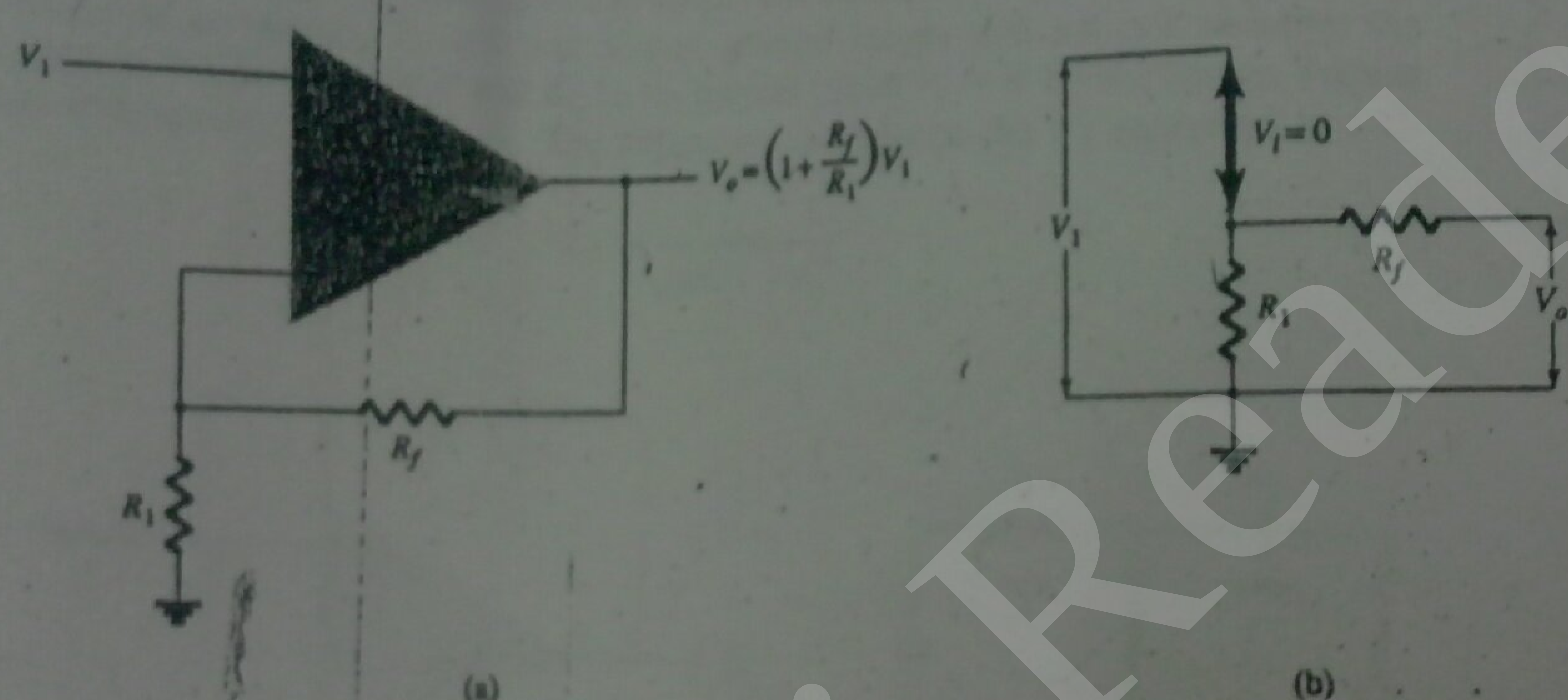

Figure 14.16 Noninverting constant-gain multiplier.

EXAMPLE 14.4

Calculate the output voltage of a noninverting amplifier (as in Fig. 14.16) for values of $V_1 = 2$ V, $R_f = 500$ kΩ, and $R_1 = 100$ kΩ.

Solution

Eq. (14.9): $V_o = \left(1 + \frac{R_f}{R_1}\right) V_1 = \left(1 + \frac{500 \text{ k}\Omega}{100 \text{ k}\Omega}\right)(2 \text{ V}) = 6(2 \text{ V}) = +12 \text{ V}$

Unity Follower

The unity-follower circuit, shown in Fig. 14.17a, provides a gain of unity (1) with no polarity or phase reversal. From the equivalent circuit (see Fig. 14.17b) it is clear that

$$\boxed{V_o = V_1} \tag{14.10}$$

and that the output is the same polarity and magnitude as the input. The circuit operates like an emitter- or source-follower circuit except that the gain is exactly unity.

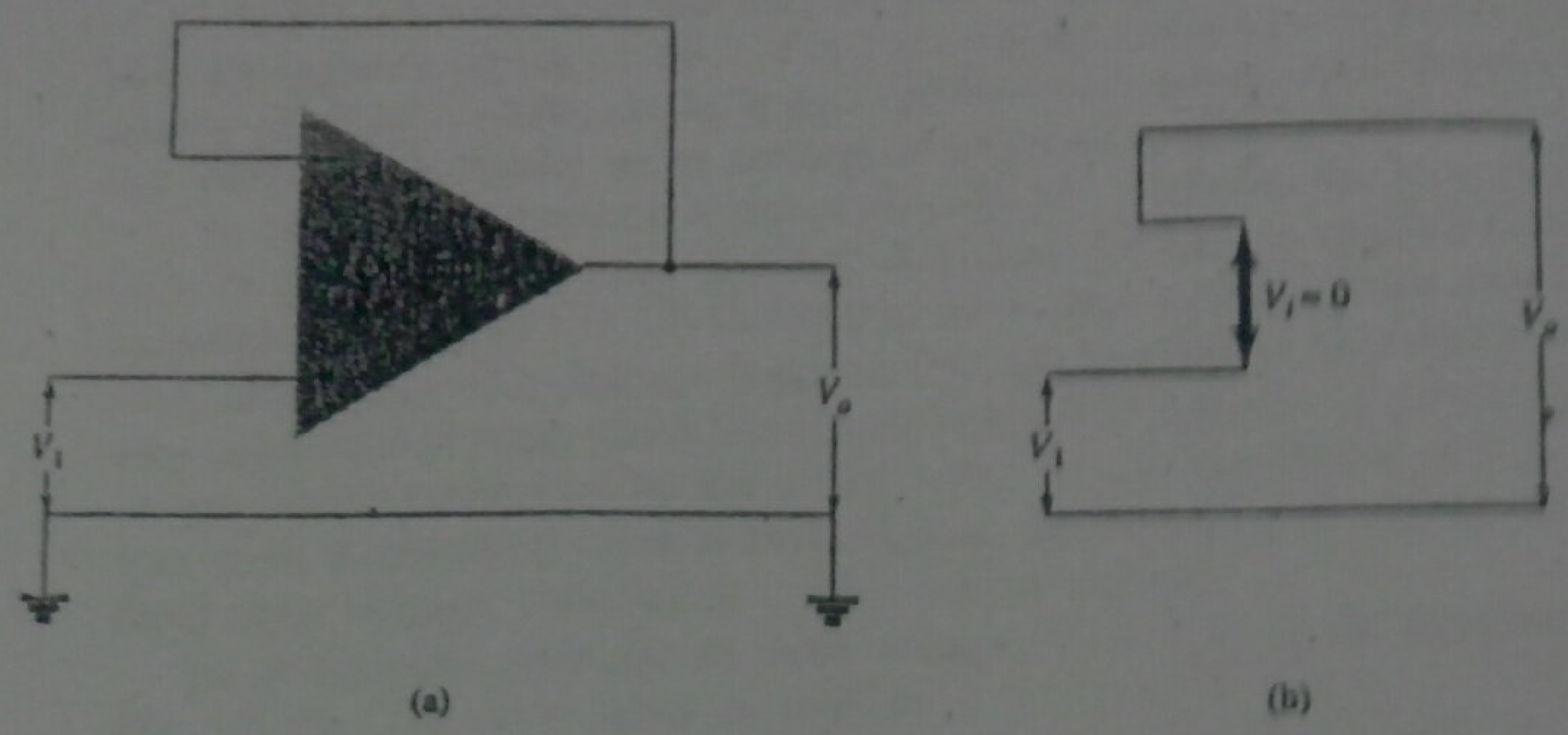

Figure 14.17 (a) Unity follower; (b) virtual-ground equivalent circuit.

Summing Amplifier

Probably the most used of the op-amp circuits is the summing amplifier circuit shown in Fig. 14.18a. The circuit shows a three-input summing amplifier circuit which provides a means of algebraically summing (adding) three voltages, each multiplied by a constant-gain factor. Using the equivalent representation shown in Fig. 14.18b, the output voltage can be expressed in terms of the inputs as

$$V_o = -\left(\frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3\right) \tag{14.11}$$

In other words, each input adds a voltage to the output multiplied by its separate constant-gain multiplier. If more inputs are used, they each add an additional component to the output.

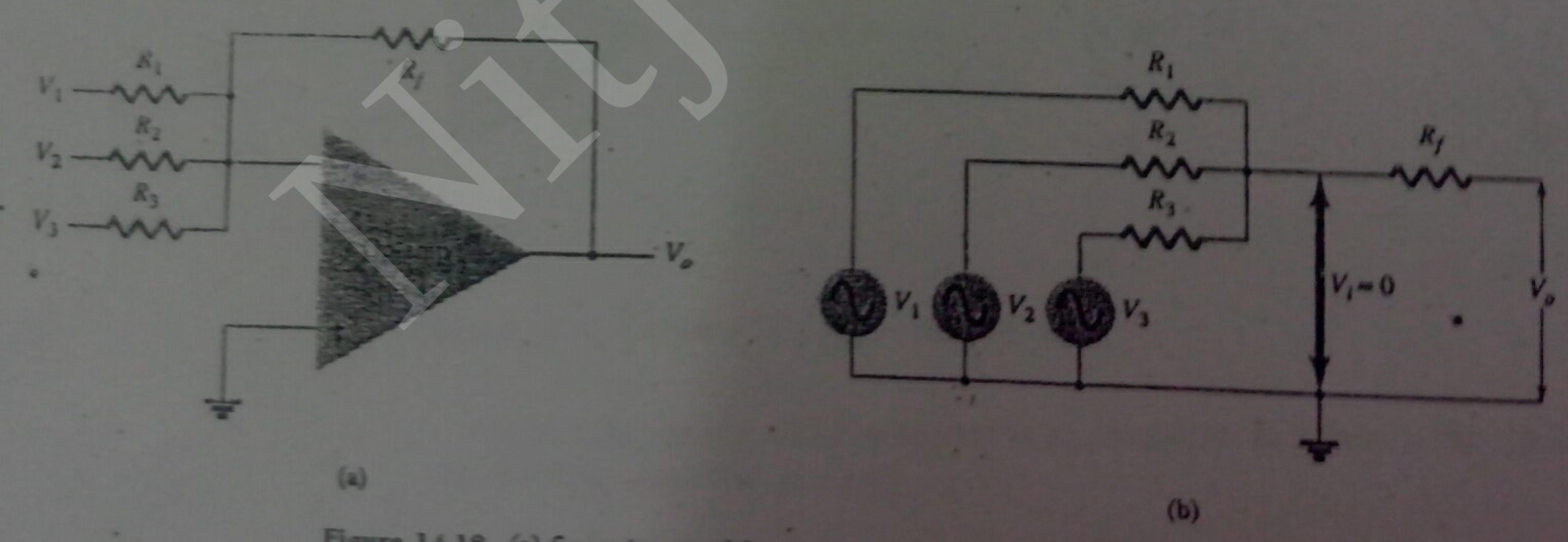

Figure 14.18 (a) Summing amplifier; (b) virtual-ground equivalent circuit.

EXAMPLE 14.5

Calculate the output voltage of an op-amp summing amplifier for the following sets of voltages and resistors. (Use $R_f = 1\ \text{M}\Omega$ in all cases.)

(a) $V_1 = +1$ V, $V_2 = +2$ V, $V_3 = +3$ V, $R_1 = 500$ kΩ, $R_2 = 1$ MΩ, $R_3 = 1$ MΩ.

(b) $V_1 = -2$ V, $V_2 = +3$ V, $V_3 = +1$ V, $R_1 = 200$ kΩ, $R_2 = 500$ kΩ, $R_3 =$ 1 MΩ.

Solution

Using Eq. (14.11):

(a) $V_o = -\left[\dfrac{1000\ \text{k}\Omega}{500\ \text{k}\Omega}(+1\ \text{V}) + \dfrac{1000\ \text{k}\Omega}{1000\ \text{k}\Omega}(+2\ \text{V}) + \dfrac{1000\ \text{k}\Omega}{1000\ \text{k}\Omega}(+3\ \text{V})\right]$

$= -[2(1\ \text{V}) + 1(2\ \text{V}) + 1(3\ \text{V})] = -7\ \text{V}$

(b) $V_o = -\left[\dfrac{1000\ \text{k}\Omega}{200\ \text{k}\Omega}(-2\ \text{V}) + \dfrac{1000\ \text{k}\Omega}{500\ \text{k}\Omega}(+3\ \text{V}) + \dfrac{1000\ \text{k}\Omega}{1000\ \text{k}\Omega}(+1\ \text{V})\right]$

$= -[5(-2\ \text{V}) + 2(3\ \text{V}) + 1(1\ \text{V})] = +3\ \text{V}$

Integrator

So far the input and feedback components have been resistors. If the feedback component used is a capacitor, as shown in Fig. 14.19a, the resulting connection is called an *integrator*. The virtual-ground equivalent circuit (Fig. 14.19b) shows that an expression for the voltage between input and output can be derived in terms of the current I, from input to output. Recall that virtual ground means that we can consider the voltage at the junction of R and X_C to be ground (since $V_i \approx 0$ V) but that no current goes into ground at that point. The capacitive impedance can be expressed as

$$X_C = \frac{1}{j\omega C} = \frac{1}{sC}$$

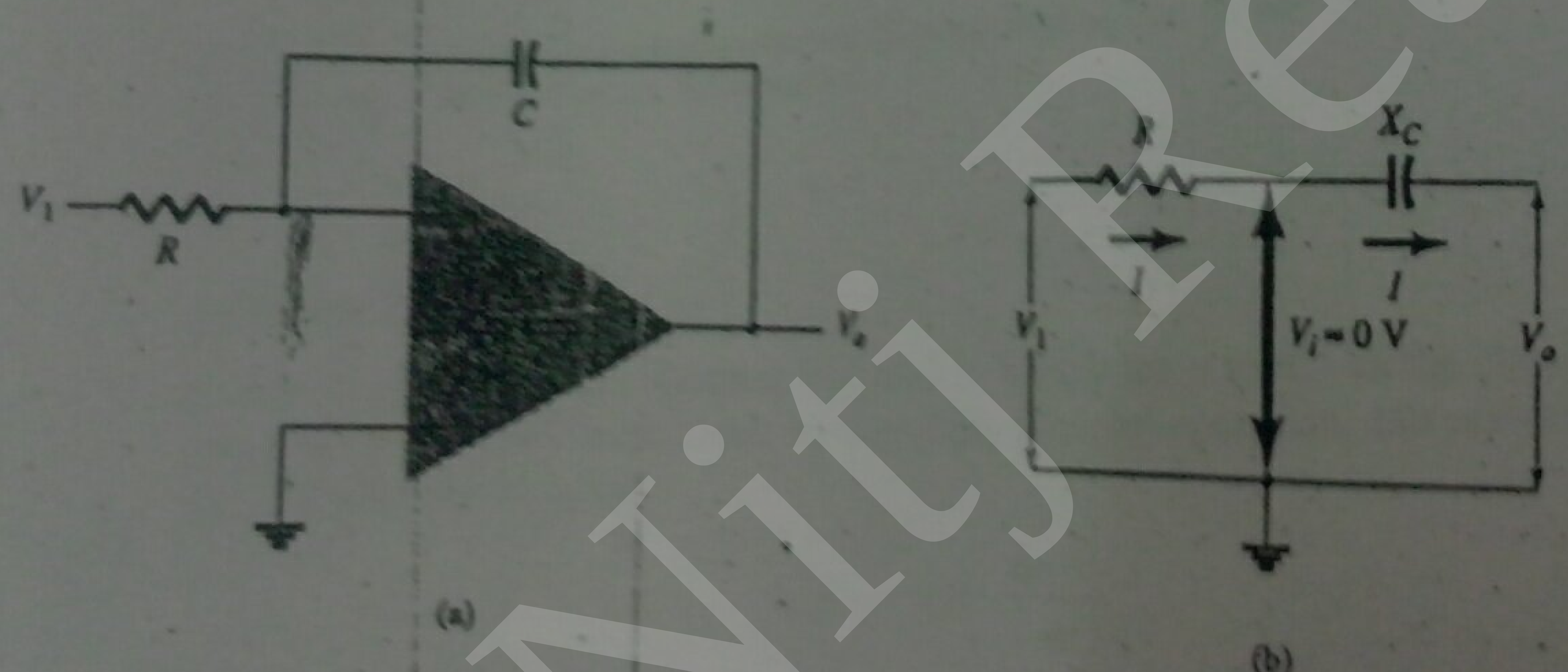

Figure 14.19 Integrator.

where $s = j\omega$ is in the Laplace notation.* Solving for V_o/V_i yields

$$I = \frac{V_1}{R} = -\frac{V_o}{X_C} = \frac{-V_o}{1/sC} = -sCV_o$$

$$\frac{V_o}{V_1} = \frac{-1}{sCR} \qquad (14.12)$$

The expression above can be rewritten in the time domain as

$$v_o(t) = -\frac{1}{RC}\int v_1(t)\,dt \qquad (14.13)$$

*Laplace notation allows expressing differential or integral operations which are part of calculus in algebraic form using the operator s. Readers unfamiliar with calculus should ignore the steps leading to Eq. (14.13) and follow the physical meaning used thereafter.

Equation (14.13) shows that the output is the integral of the input, with an inversion and scale multiplier of $1/RC$. The ability to integrate a given signal provides the analog computer with the ability to solve differential equations and therefore provides the ability to electrically solve analogs of physical system operation.

The integration operation is one of summation, summing the area under a waveform or curve over a period of time. If a fixed voltage is applied as input to an integrator circuit, Eq. (14.13) shows that the output voltage grows over a period of time, providing a ramp voltage. Equation (14.13) can thus be understood to show that the output voltage ramp (for a fixed input voltage) is opposite in polarity to the input voltage and is multiplied by the factor $1/RC$. While the circuit of Fig. 14.19 can operate on many varied types of input signals, the following examples will use only a fixed input voltage, resulting in a ramp output voltage.

As an example, consider an input voltage, $V_1 = 1$ V, to the integrator circuit of Fig. 14.20a. The scale factor of $1/RC$ is

$$-\frac{1}{RC} = \frac{1}{(1\ \text{M}\Omega)(1\ \mu\text{F})} = -1$$

so that the output is a negative ramp voltage as shown in Fig. 14.20b. If the scale factor is changed by making $R = 100$ kΩ, for example, then

$$-\frac{1}{RC} = \frac{1}{(100\ \text{k}\Omega)(1\ \mu\text{F})} = -10$$

and the output is then a steeper ramp voltage, as shown in Fig. 14.20c.

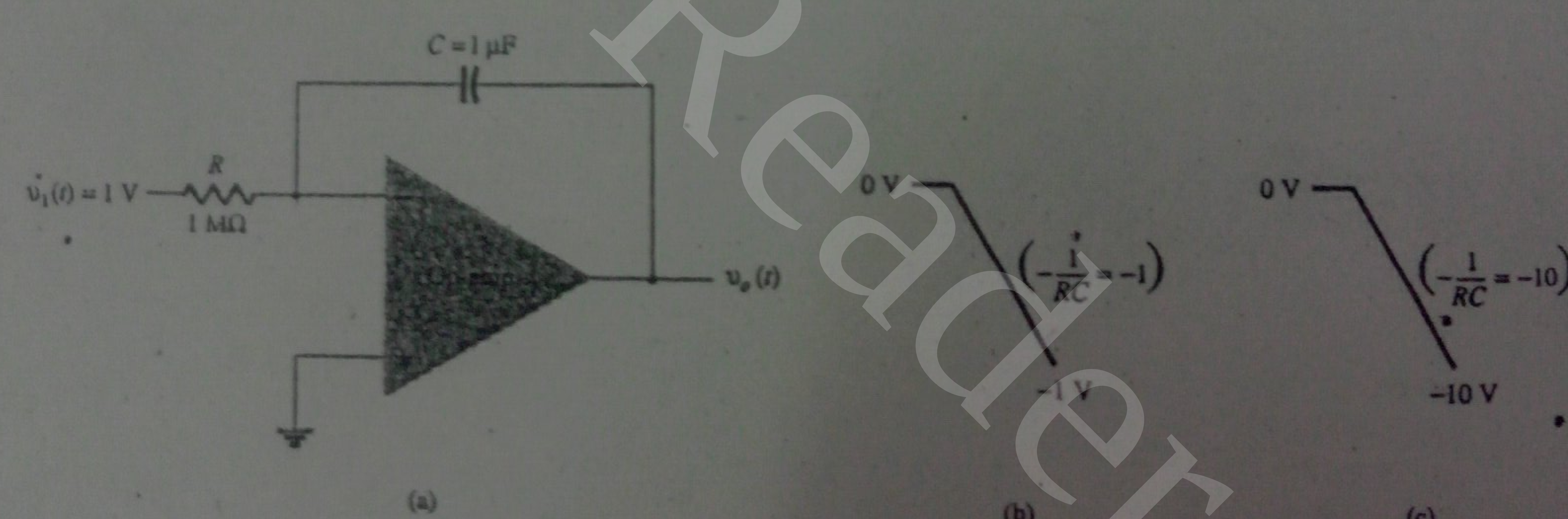

Figure 14.20 Operation of integrator with step input.

More than one input may be applied to an integrator, as shown in Fig. 14.21, with the resulting operation given by

$$v_o(t) = -\left[\frac{1}{R_1C}\int v_1(t)\,dt + \frac{1}{R_2C}\int v_2(t)\,dt + \frac{1}{R_3C}\int v_3(t)\,dt\right] \quad (14.14)$$

An example of a summing integrator as used in an analog computer is given in Fig. 14.21. The actual circuit is shown with input resistors and feedback capacitor; whereas the analog-computer representation indicates only the scale factor for each input.

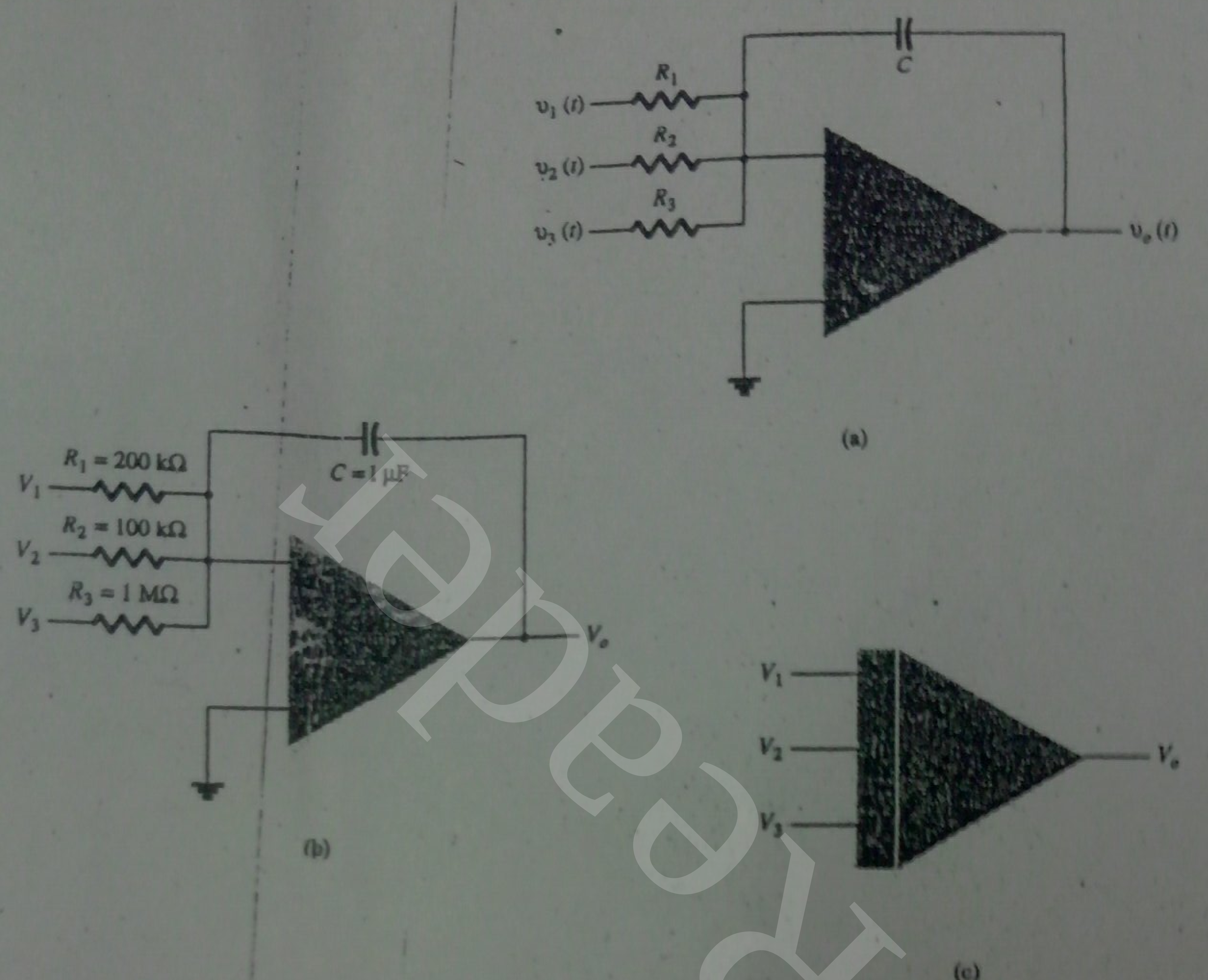

Figure 14.21 (a) Summing-integrator circuit; (b) component values; (c) analog-computer, integrator-circuit representation.

Differentiator

A differentiator circuit is shown in Fig. 14.22. While not as useful as the circuit forms covered above, the differentiator does provide a useful operation, the resulting relation for the circuit being

$$v_o(t) = -RC\,\frac{dv_1(t)}{dt} \tag{14.15}$$

where the scale factor is $-RC$.

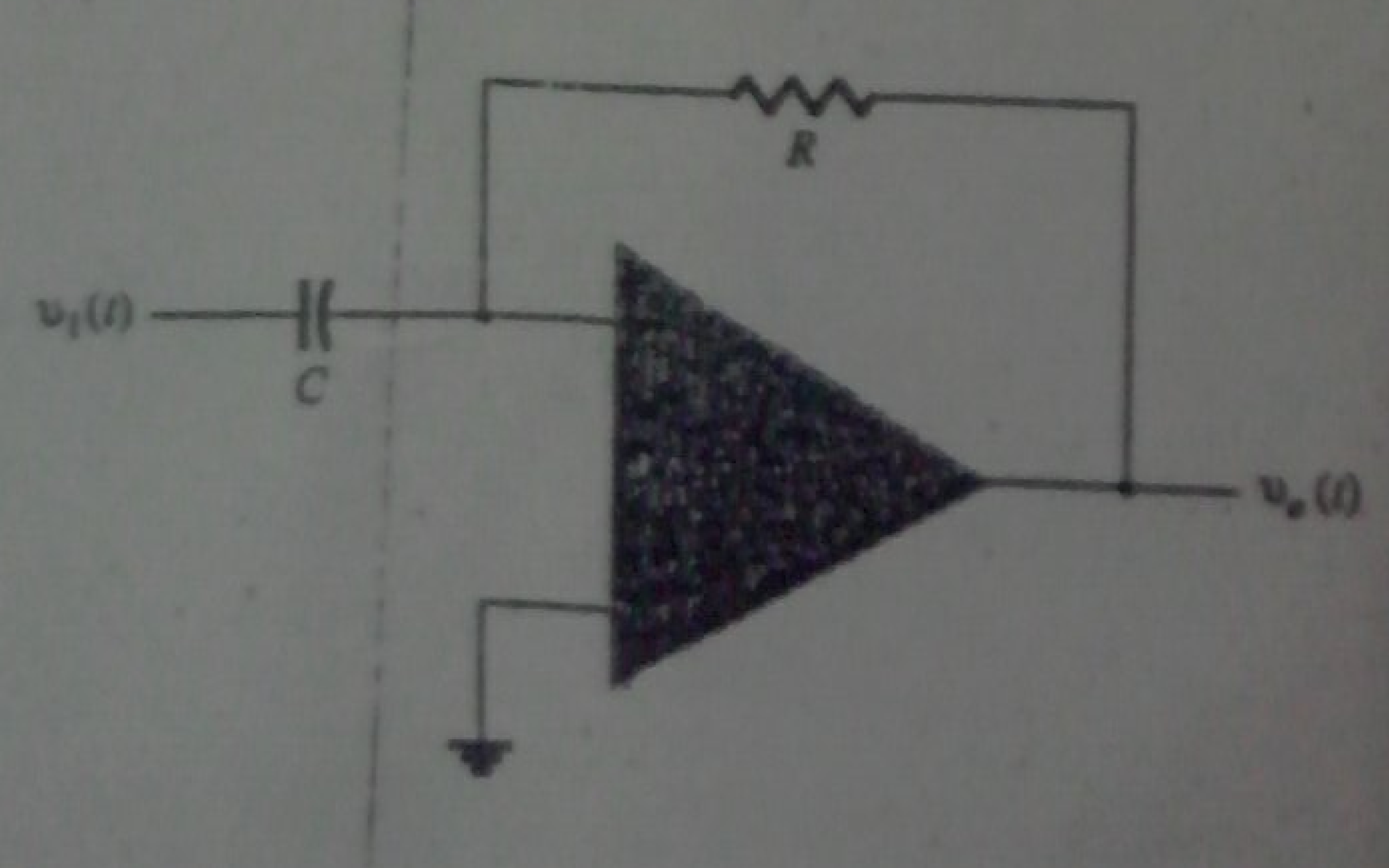

Figure 14.22 Differentiator circuit.