57-A

Power Flow Calculations — Part I

Introduction to Power Flow Studies — Variables — Admittance Calculations — Equivalent Sources — Network Nodal Current Equations — Y- bus Matrix — Interation Procedure — Simple Two Node AC System — Multibus System — Types of Buses — Load Flow Equations — Gauss Interative Method — Gauss Seidel Interative Method — Acceleration Factors used in Gauss Seidel Interative Method — Newton Raphson Interative Method — *Solved Examples* — Significance of Power Flow Studies — HVDC Load Flow — Summary.

57.1. INTRODUCTION TO POWER FLOW CALCULATIONS

In 3 phase AC power system, active and reactive power flows from generators to loads *via* various Network Buses and Branches (Transmission Lines). The flow of active and reactive power is called *Power Flow or Load Flow*. The voltages of buses and their phase angles are affected by the power flow and *vice versa*. *Power flow studies deal with calculations of bus voltages, their phase angles, active and reactive power flow through various branches, generators and loads under steady state conditions.*

Active power P and reactive power Q is supplied by generators at generator buses. Active power is drawn by loads from load buses. Reactive power Q is supplied or drawn from the load buses by shunt compensation elements (shunt capacitors, shunt reactors, SVS).

The various loads draw active and reactive power from the load buses. The AC power system has tens/hundreds of Generating stations and hundreds/thousands of branches and load buses. Calculations of Power flow (Load flow) through various buses, bus voltages and phase angles, etc. are carried out by *interative process* using Digital Computer.

The method of power flow calculations has been explained in this chapter with the help of simple networks having a few generator buses and a few load buses. The same method is applicable to a multibus system.

Refer a simple Network shown in *Fig. 57.1.* The base variables for each buses are : V_k, $\angle \delta_k$, P_k and Q_k, where k is bus number 1, 2, 3 ...N. Variables for each of the six branches are active power P, reactive power Q through the branch. Current I and power factor cos ϕ. etc in generator, load and branches can be calculated from the base variables.

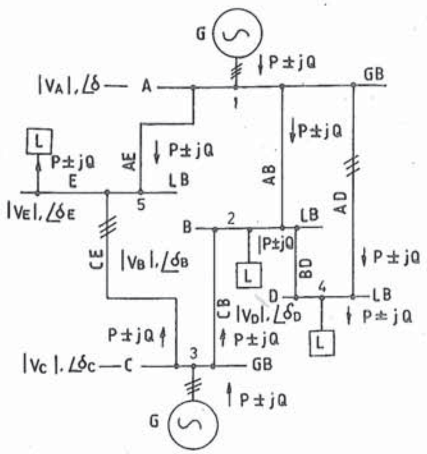


Fig. 57.1. Simplified representation of a 3-phase AC Network for Power Flow Study.

Ref. Fig. 57.1.	*Known (Specified)*	*Unknown* (To be calculated)*
Generator Buses : A, C	V, P	$\angle\delta, Q$
Load Buses : B, D, E	P, Q	$V, \angle\delta$
Branches : AE, CE, AB, CB, BD, AD		Flow : P, Q

*The objective of Power flow (Load flow) studies is to determine these unknowns for each bus.

Nature of Load Flow Problem

Electrical energy in form of 3 phase AC power flows from various *generating station buses* to various *connected load buses via* various *branches* of the transmission network. Each branch has certain admittance Y (reciprocal of impedance Z).

The power flow in a branch of a two bus network is easy to calculate as it depends on sending end bus voltage, receiving end bus voltage, load angle δ between voltage vectors V_s and V_r and the branch admittance Y (reciprocal of impedance Z). The analysis of a two bus system gives understanding about relationships between $|V_s|$ and $|V_r|$, $\angle\delta$, P and Q. Two bus system has only 2 equations which can be solved by calculator. But a total power system has thousands of buses and several thousand variables. Hence interative procedure and digital computer are essential to solve the thousands of simultaneous equations.

Power System Network has certain configuration. The bus voltages, are influenced by the power flow ($P \pm jQ$) through various branches, branch admittances and generator voltages. The flow of active and reactive power through various buses branches influences the bus voltage magnitude and their phase angles. Change in load ($P \pm jQ$) at any bus affects all the other variables of other buses and branches.

The principal variables $|V|$, $\angle\delta$, P and Q of all the buses and branches in the Network related with each other by circuit laws and can be evaluated only by solving *Simultaneously nodal current equations* for given conditions by the interation process applied to load flow equations. In Fig. 57.1, if load on bus D changes, voltage V, phase angle $\angle\delta$, P and Q at all the four network buses and six branches will change simultaneously. The objective of Power Flow Study of Network in Fig. 58.1 will be to determine the unknown variables.

Power flow studies give a systematic mathematical approach to determine the various bus voltages, their phase angles, active and reactive power flow through various branches for given *steady state conditions of the Network* and given network configuration. The power flow through the branches is known only from the printout of the successful run of Load Flow Program. Table 57.1 gives a simple example. *Power flow study of a power system deals with calculations of steady state variables for buses and branches including Voltage $|V|$, Phase angle of V, Active Power P, Reactive Power Q, Apparent Power S, Current I, Power factor cos ϕ, etc power and associated variables for steady state conditions in Network.*

Table 57.1. Example of Computer Print-out* Report of a Power Flow Study

Karnatak State Electricity Board, Banglore

Report on Power Flow Calculations for No. 2 Hubli Zonal Circle Bus Data

Bus No.	*Bus Name*	*Bus Volts*	*Bus Angle*	*Generation*		*Load*	
				P MW	*Q MVAR*	*P MW*	*Q MVAR*
1.	Belgaum	1.02	0	65	33	00	00
2.	Dharwar	0.955	– 3.9	00	00	61	30
3.	Hubli	1.04	2.0	100	48	00	00
4.	Karwar	0.923	– 8	00	00	40	10
5.	Chickodi	0.993	– 2.1	00	00	60	20
	Total Zonal Circle			165	81	161	60

End of Power Flow Program Run
No. of Interations 25

* Only for study project, not for planning and design.

57.2. NEED OF POWER FLOW STUDIES

Power flow studies are essential and very important for

— Designing a power system
— Planning of power system
— Expansion of power system
— Providing guidelines for optimum operation of power system
— Providing base data for various power system studies Refer Sec. 57.24 for further details.

57.3. OUTLINES OF THE PROCEDURE OF POWER FLOW STUDY

The Power Flow Calculations involve the following steps :

1. Single line diagram of the system is drawn.

2. The *Nodal Admittance Diagram* is drawn. The network is represented in form of its Admittance Model.

3. The *Bus Admittance Matrix* is constructed.

The steady state model of the network is presented in form of *Bus Admittance Matrix Equation* based on modal current equations and Kirchoff's current laws. The bus admittance matrix of a large network is *large and sparse*. The bus admittance matrix can be constructed as per simple building blocks procedure.

4. The *Network Buses are classified* and are given a number. The three classes are : Generator Buses, load buses and a slack bus (swing bus). Each bus is given a number k = 1, 2, ... N.

5. To begin with, the known and unknown variables are noted for each bus.

6. **Power Flow Equations** are written in terms of P, Q, V, $\angle\delta$ for kth bus. k = 2, 3, ... N.

Subscript k denotes the bus number.

7. These equations are solved by any one of the suitable interation procedure on a digital computer. The interation procedure consists of assuming certain values for the unknown variables to begin with and solving the equations to obtain the yet another revised value of the unknown variables for each bus. Interations are repeated till the *difference* between consecutive results of the same variables for each bus are within acceptable small value. At that stage the interations are stopped and final values are noted.

Superscript r denotes the interation number.

For given conditions of generation and load and power flow through various branches and voltages/phase angles of various buses are calculated by solving N number simultaneous nonlinear equations for N buses by *Interative method and digital computer solution of load flow equations.* N number bus system, to begin with, $2N$ variables are known and $2N$ are to be determined.

	Known (Specified)	*Unknown* (To be calculated)*
Generator Buses :	V, P	$\angle\delta, Q$
Load Buses :	P, Q	$V, \angle\delta$
Slack Bus (Swing Bus)⁺ :	$V, \angle\delta$	P, Q

* The objective of power flow (load flow) studies is to determine these unknowns for each bus under steady state condition.

⁺ Slack bus (with bus) is any one selected generator bus for which P and Q are not specified to begin with. Voltage magnitude and phase angle are specified. This provides for accounting for transmission losses in the branches which will be known at the end of computer run.

8. Presently one of the following two methods are preferred for solving Power Flow Equations by interation process :

— Gauss-Seidel Method — Newton-Rhapson Method

The relative merits are covered in Sec. 57.21

8. Solution of Power Flow Equations by any of the above two methods gives values of unknown principal variables for each of the N network buses. The basic variables for each bus are : Voltage magnitude $|V|$, phase angle of voltage vector : $\angle\delta$; active power P and reactive power Q.

— Out of these four principal variables for *each* bus two are known (given) and remaining two are calculated by load flow calculations.

For given conditions of generation and load, the power flow through various branches and voltages/phase angles of various buses are calculated by solving N number simultaneous nonlinear equations for N buses by *Interative Method and Digital Computer Solution of Load Flow Equations.*

— From the solution to power flow study the value of the two unknown variables for each bus are obtained and thus all the four variables for each bus are then known at the end.

— The other derived variables for the buses and branches such as current I, power factor of I, MVA etc. can then be calculated for each branch from the four principal variables of each terminal bus by applying fundamental circuit equations.

BRANCH ADMITTANCE EQUATIONS

57.4. BRANCH ADMITTANCE AND SOURCE TRANSFORMATION

In normal circuit calculations impedance Z is used, $I = V/Z$. Power flow load calculations are generally made with admittance Y parameter. $I = VY$. [Admittance Y = 1/Impedance Z]

Calculations with computer solution of power flow equations are easy with admittance parameters. [The impedance parameter calculations are used generally for short circuit calculations and are rarely used for load flow calculations.] In the following sections the Admittance form has been used for load flow calculations.

57.5. ADMITTANCE FORM OF CALCULATIONS

Reciprocal of impedance Z is called admittance Y. We have impedance triangle and admittance triangle as shown in Fig. 57.2 Y and Z are complex quantities.

$$Z = R + jX \text{ ohm} \quad Y = G + B \text{ mho} \qquad ...(57.1)$$

$$Z = \frac{1}{Y} \qquad Z = \frac{V}{I} \text{ ohm}$$

$$Y = \frac{1}{Z} \qquad Y = \frac{I}{V} \text{ mho}$$

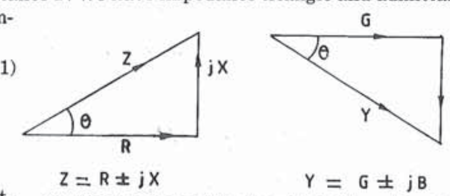


Fig. 57.2. **Impedance triangle and Admittance triangle.**

Voltage drop V_a in impedance Z_a (or admittance Y_a) by current I_a is given by :

$$V_a = IZ_a \quad I = V_a/Z_a \quad I = V_a Y_a$$

Example 57.1. *Branch Admittance and Branch Current in a two node one branch circuit shown in Fig. 57.3B, the branch impedance Z_a 3 + j4 ohm. Voltage drop in the branch (a) is V_a = 100 $\angle 0°$ V. Calculate (1) Branch current (2) Branch admittance.*

Solution.

Branch Current : $$I_a = \frac{V_a}{Z_a} = \frac{100 \angle 0°}{3 + j4} = \frac{100 \angle 0°}{5 \angle 53°} = 20 \angle -53°.$$

Branch Admittance $$Y_a = \frac{1}{Z_a} = \frac{1}{3 + j4} = \frac{(3 - j4)}{(3 + j4)(3 - j4)}$$

$$= \frac{3 - j4}{3^2 + 4^2} = \frac{3}{25} - j\frac{4}{25} = 0.12 - j0.16 \text{ mho}$$

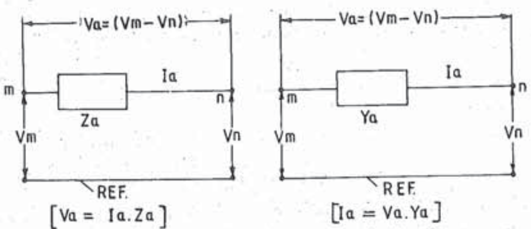


Fig. 57.3A. Branch Representation by Z or Y.

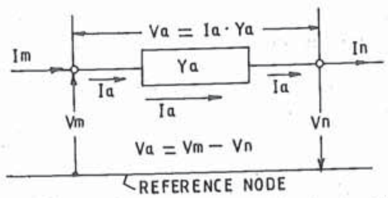


Fig. 57.3B. Branch admittance Y, Branch current I and node voltage V_m and V_n.

$$|Y_a| = \sqrt{0.12^2 + 0.14^2} = 0.2 \text{ mho}$$

$$[\angle \tan^{-1}(-0.16/0.12)] = \angle -53°$$

Check : $\quad I_a = V_a\, Y_a = 100 \angle 0° \times 0.2 \angle -53° = 20 \angle -53°$

Example 57.2. $Z = 3 + j4$ calculate Y

$$Y = \frac{1}{3+j4} = \frac{1\,(3-j4)}{(3+j4)\,(3-j4)} = \frac{3-j4}{3^2+4^2} = \frac{3-j4}{25}$$

$$= 0.12 - j0.16 = \sqrt{0.12^2 + 0.16^2} = 0.2 \text{ mho}$$

57.6. BRANCH ADMITTANCE

Ref. Fig. 57.3A. Network branch and be represented either by a branch impedance Z_a or by a branch admittance Y_a.

In Fig. 57.3B the node voltages of the terminals of branch (a) with respect to the reference node are V_m and V_n.

The current in the branch (a) is given by :

Impedance form	*Admittance form*	
$I_a = \dfrac{V_a}{Z_a}$	$I_a = V_a\, Y_a$	
$= \dfrac{(V_m - V_n)}{Z_a}$	$I_a = (V_m - V_n)\, Y_a$	...(57.2)

57.7. SOURCE TRANSFORMATION : CURRENT SOURCE —VOLTAGE SOURCE

In impedance form of representation, and active voltage source (generator) can be represented by a circuit having emf E_a in series with internal impedance of surface Z_a. The terminal voltage V_a is given by

$$E_a = V_a - I_a\, Z_a \qquad \text{...(57.3)}$$

Dividing both sides by Z_a,

$$\frac{E_a}{Z_a} = \frac{V_a}{Z_z} - \frac{I_a\, Z_a}{Z_a} \qquad \text{...(57.4)}$$

But we know, $1/Z_a = Y_a$ and $V_a/Z_a = V_a\, Y_a$. Let $E_a/Z_a = I_s$

$$\frac{E_a}{Z_a} = V_a\, Y_a - I_a \qquad \text{...(57.5)}$$

We call $E_a/Z_a = I_s$

$$\frac{E_a}{Z_a} = I_s = V_a\, Y_a - I_a \qquad \text{...(57.6)}$$

Comparing Eqns. 57.3 and 57.5, we observe that the voltage source E_a in series with Z_a can be transformed into an *equivalent* current source I_s in parallel with admittance Y_a without changing load voltage or load current. This is called source transformation.

Fig. 57.4 illustrates the source transformation. The variables on the load side shall remain unchanged.

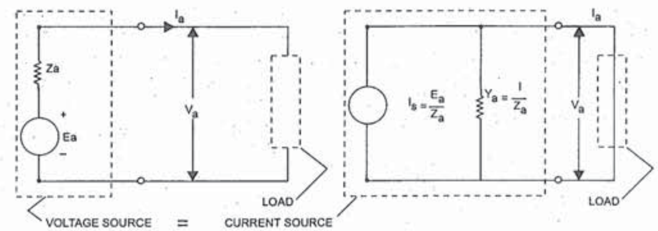


Fig. 57.4. Source transformation Current source ↔Voltage source.

Voltage source in series with source impedance = Equivalent current source in parallel with source admittance. Load current and load voltage remaining unchanged.

57.8. BUS NODAL CURRENT EQUATIONS FROM KIRCHOFF'S CURRENT LAW

Kirchoff's current law states that : "The sum of currents entering the bus from the sources, is equal to the sum of currents leaving the bus."

By applying Kirchoff's current law, the N number Bus-Nodes, N Nodal Current Equations are written for a N-bus system. These N current system. These N current equations are useful in load flow studies for formulating bus impedance matrix Y bus.

Refer a four bus network shown in Fig. 57.5. Bus number N are 1, 2, 3, 4. Branch admittances are $y_{12}, y_{23}, y_{34}, y_{24}$. Next step is to draw the Nodal Admittance Network (Fig. 57.6)

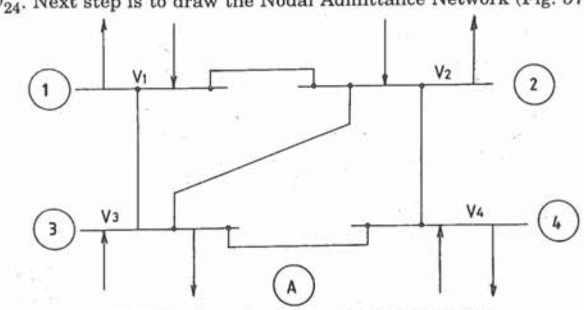


Fig. 57.5. Four bus network for nodal current equations.

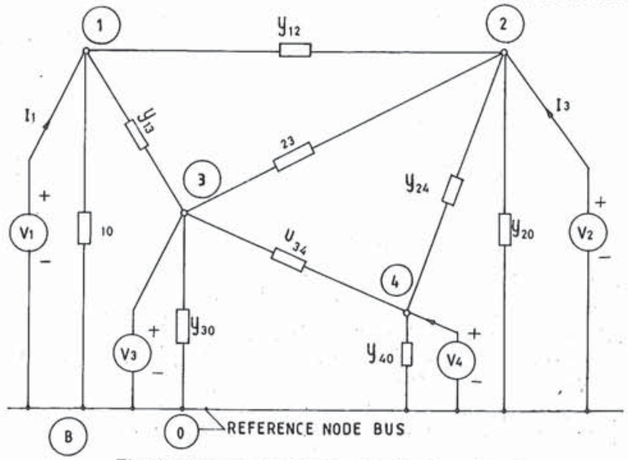


Fig. 57.6. Admittance network of the four bus network.

Next step is to write systematic Bus Current Equations and then write Bus Admittance Matrix. For bus 1, we get nodal current equation by equating current entering the bus equal to current leaving the bus are :

$$I_1 = V_1 y_{10} + (V_1 - V_2) y_{12} + (V_1 - V_3) y_{13} \qquad ...(57.7)$$

Likewise current equations are written for bus 2, 3, 4 and we get four simultaneous equations as follows :

$$I_1 = V_1 y_{10} + (V_1 - V_2) y_{12} + (V_1 - V_3) y_{13}$$
$$I_2 = V_2 y_{20} + (V_2 - V_1) y_{12} + (V_2 - V_3) y_{23} + (V_2 - V_4) y_{24}$$
$$I_3 = V_3 y_{30} + (V_3 - V_1) y_{13} + (V_3 - V_2) y_{23} + (V_3 - V_4) y_{34}$$
$$I_4 = V_4 y_{40} + (V_4 - V_2) y_{24} + (V_4 - V_3) y_{34}$$

Rearranging these equations and rewriting them in matrix form, we get :

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} y_{10}+y_{12}+y_{13} & -y_{12} & -y_{13} & 0 \\ -y_{12} & y_{20}+y_{12}+y_{23}+y_{24} & -y_{23} & -y_{24} \\ -y_{13} & -y_{23} & y_{30}+y_{13}+y_{23}+y_{34} & -y_{34} \\ 0 & -y_{24} & -y_{34} & y_{40}+y_{24}+y_{34} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} \qquad ...(57.8)$$

The admittance terms on RHS are redesignated as :

Self Admittances	*Mutual Admittances*
$Y_{11} = y_{10} + y_{12} + y_{13}$	$Y_{12} = Y_{21} = -y_{12}$
$Y_{22} = y_{20} + y_{12} + y_{23} + y_{24}$	$Y_{13} = Y_{31} = -y_{13}$
$Y_{33} = y_{30} + y_{13} + y_{23} + y_{34}$	$Y_{14} = Y_{41} = -y_{14} = 0$
$Y_{44} = y_{40} + y_{24} + y_{34}$	$Y_{23} = Y_{32} = -y_{23}$
	$Y_{24} = Y_{42} = -y_{24}$
	$Y_{23} = Y_{43} = -y_{34}$
General term : Y_{ii}	General term : Y_{ik}

Matrix equation 57.8 is written in terms of self bus admittance Y_{ii} and mutual bus admittances Y_{ik} as :

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} \qquad ...(57.9)$$

The above equation is re-written as

$$[I] = [Y_{bus}]\ [V] \qquad ...(57.10)$$

where $[I]$ is node current matrix, $[V]$ is node voltage matrix and $[Y_{bus}]$ is bus admittance matrix.

In general, for an N-Node network, the Y bus matrix is

Y_{bus} Matrix =

$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & \dots & Y_{1N} \\ Y_{21} & Y_{22} & \dots & Y_{2N} \\ \dots & \dots & \dots & \dots \\ Y_{N1} & Y_{N2} & \dots & Y_{NN} \end{bmatrix} \qquad ...(57.11)$$

Y_{bus} matrix has self bus admittance terms Y_{ii} along diagonal and mutual bus admittances terms Y_{ik} as nondiagonal.

57.9. SELF AND MUTUAL ADMITTANCE OF THE BUSES

In equation 57.11 each of the admittances Y_{ii}, $(i = 1, 2, 3, 4)$ are called self admittance of the bus or driving-point admittance of the bus. It is the algebraic sum of all the admittances terminating in that bus *e.g.*

$$Y_{11} = y_{10} + y_{12} + y_{13} \qquad ...(57.12)$$

Mutual admittance between two buses

In equation 58.11 each of the admittance Y_{ik}, $(i, k = 1, 2, 3, 4)$ is called mutual admittances or transfer admittances between bus i and k. It is the negative of sum of all the admittances connected between those two buses. We note that $(Y_{ik} = Y_{ki})$. For example, from Eqn. 57.

$$Y_{13} = Y_{31} = -y_{13} \qquad ...(57.13)$$

57.10. BUS ADMITTANCE MATRIX

General equation for N-bus network based on Kirchoff's current laws and admittance form is :

$$[I] = [Y_{bus}] * [V] \qquad ...(57.14)$$

where, $[I]$ is the N-bus current matrix, $[V]$ is the N-bus voltage matrix and, $[Y_{bus}]$ is called bus admittance matrix and Eqn. 57.14 is written as

$$\mathbf{I} = \mathbf{Y_{bus}}\ \mathbf{V}$$

where

$$\mathbf{Y}_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & \dots & Y_{1N} \\ Y_{21} & Y_{22} & \dots & Y_{2N} \\ \dots & \dots & \dots & \dots \\ Y_{N1} & Y_{N2} & \dots & Y_{NN} \end{bmatrix} \qquad ...(57.15)$$

is called the *bus admittance matrix,* and **V** and **I** are the N-element *node voltage matrix* and *node current matrix,* respectively.

Y_{bus} matrix for N-bus network has N rows and N columns. Each of the Y terms in the rows and columns has two subscripts :

— The first subscript refers to the bus number on which the current is expressed.

— The second subscript refers to the bus number whose voltage has caused that current component.
— The terms on diagonal are self-admittances.
— All the non-diagonal terms are mutual admittances. It is seen that the current entering bus k is given by :

$$I_k = \sum_{n=1}^{N} y_{kn} V_k \quad \text{...(57.16)}$$

Meaning of Self Admittance and Mutual Admittance Elements

Self Admittance of the Node

The terms Y_{ii} ($i = 1, 2, 3, 4$) are *self admittances* of respective nodes and represent the *algebraic sum of all the admittances terminating at that node.* Each diagonal term in the Y_{bus} matrix is a self admittance term.

If we short all the other nodes except node i with the reference bus and inject current I in the particular node i bus and measure the voltage V across that node i and the reference bus the ratio I/V gives the self admittance of that node.

Thus, self admittance of nodes 2 is ($i = 2$)

$$Y_{22} = \frac{I_2}{V_2} \left| \begin{array}{l} V_2 \text{ not} = 0 \\ V_1 = V_3 = V_4 = \ldots V_N = 0 \end{array} \right. \quad \text{...(57.17)}$$

Mutual Admittance between two nodes

The mutual admittance terms (Transfer Admittance terms) are the terms Y_{ik} in Y_{bus} Matrix. All the non-diagonal terms in the Y_{bus} matrix are mutual admittance terms. Y_{ik} (i ; k = 1, 2, 3, 4, ... N)

Mutual admittance between two buses is the negative of the sum of all the admittances connected directly between those two buses.

Also, $Y_{ik} = Y_{ki}$

For measuring the mutual admittance between the two nodes, all the other nodes except one of the two nodes (i, k) are shorted with the reference bus. Current I is injected in the shorted node and the voltage V across the two nodes is measured. The ratio I/V gives the mutual admittance between the two nodes. Thus,

Mutual admittance between node 1 and 2 is : $Y_{12} = Y_{21}$

$$Y_{21} = \frac{I_2}{V_1} \left| \begin{array}{l} V_1 = \text{not} = 0 \qquad (V_1 \text{ is not shorted}) \\ V_2 = V_3 = V_4 = \ldots = V_N = 0 \end{array} \right. \quad \text{...(57.18)}$$

INTERATION METHOD EXPLAINED

57.11. INTERATION PROCESS

The method of solving simultaneous equations by starting with assumed values of unknown variables and obtaining successive better values of the same variable by repeated cycles of solution is called method of interation.

Starting from some *assumed values* of unknown variables and other given values of known variables, the algorithm equations are solved to obtain *new better values* of the same unknown variables.

These new better values of unknowns are again substituted in the same equations (Algorithms) to get yet another set of new revised values. The process of calculations of the new revised values of variables (*e.g.* Bus Voltages) by using earlier result is called *"an interation"*. Interation process is continued till the difference in results between consecutive values is too small and below certain predetermined acceptable criterion.

Consider an example in which the unknown variable is bus voltage V and, ΔV is the difference in values of V from two consecutive interations. If ΔV reduces with every next interation, the process is said to be *Convergent.* When ΔV reduces below accepted criterion, (c), the interation process is stopped.

Let $c = 0.0001V$. Then in the following table, the convergence is reached at 5th interation *e.g.* in the example solved below :

Interation	1	2	3	4	5
V pu	0.9125	0.9135	0.9138	0.9137	0.9137
ΔV		0.001	0.0003	0.0001	0

Interation stopped at the end of 5 of the interation.

If ΔV increases during successive interations, the interation is *divergent.* The interation process should be stopped and reasons for divergence should be reviewed. Start newly and check for convergence.

The repeated procedure of calculation by substituting the previously obtained value in the next set of calculations in the same set of equations is called Interation Process. The interation process is stopped when the difference ΔV in values from two consecutive interations is smaller than the selected convergence criterion (c).

when $\quad c < \Delta V \quad$ Interation is stopped ...(57.19)

In load flow studies, from the solution of the Nodal equations, voltage V_k, phase angle and current are known for given steady state power system conditions. From these solution values P_k, Q_k can be calculated for each bus (k = 1 to N)

The interations are repeated till sufficiently accurate values are obtained and further interations are not giving next better values, *i.e.* covergence is reached.

Most widely used interative methods for Load Flow Calculations are (1) Gauss-Seidel method (2) Newton Raphson method.

The digital computers are used for obtaining the solution given equations and system conditions.

The equations to be solved are formulated such that they are amenable to interative solution on digital computer. The general equations used for interation are called the Algorithm.

STEPS IN INTERATION PROCESS

57.12. STEPS IN INTERATION

1. To begin with, for the First Interation, *estimated value* (V) is assigned to the unknown bus voltage. (V) ; $r = 0$.

2. Equations (Algorithm) are solved in *First Interation* by using the assumed values (V) of unknown bus voltage say (V) ; $r = 0$. In the calculation, the other known bus variables (P, Q) are substituted. Equations are solved to obtain the new updated value of voltage called (V) ; $r = 1$.

3. The new values of bus-voltages (V) ; $r = 1$ obtained from the first Interation ($r = 1$) are used for the *Second Interation*

The equation is solved again in the second interation ($r = 2$) and yet new values of bus voltage (V) ; $r = 2$ is obtained.

Each set of calculations of the new values of the variable (e.q. bus voltages) by using earlier result is called "an interation".

The interation process is repeated (r = 1, 2, 3, 4, 5 ...) until the change (ΔV) between consecutive resulting values of V at each bus are less than the specified *convergence criterion (c).*

Possibility of Convergence

— Convergence may be achieved after several tens or hundreds of interations.

— Convergence may not be achieved at all, if the solution does not exist or if the interation process is Divergent.

Example 57.3. *2-bus System solved by Interation process.*

Refer Fig. 57.7 for a 2-bus system. Procedure is explained in following equations are obtained.

$$S_2 = V_1 I^* \qquad V = S/I^* \qquad \text{...(57.19)}$$

$$V_2 = V_1 - ZI = V_1 - Z\,\frac{S_s^*}{I_2^*}$$

$$= V_1 - Z\,\frac{S_2^*}{(V_2^{(k-1)})^*} \qquad \text{...(57.20)}$$

Procedure

1. Assume value of V_2 for the start of *first interation* and call it as ; (V_2) ; $r = 0$.

2. Substitute this *(assumed)* starting value of (V_2) in the right hand side of the Equation 57.21 and solve for (V_2). Call this V_2 obtained as a result of the first interation as (V_2) ; $r = 1$.

3. Then substitute this resulting value (V_2) ; $r = 1$ of first interation again on right hand side of the Eqn. and obtain yet new value of (V_2) $r = 2$, of the second interation $r = 2$.

4. Substitute (V_2) ; $r = 2$ in the same Equation and obtain (V_2) ; $r = 3$. (V_2) ; $r = 3$ is the result of third interation ($r = 3$), and so on.

5. Calculate the difference (ΔV) between values obtained from consecutive interations ; s $[(V_2)\,;\, r = p + 1 - (V_2)\,;\, r = p]$ and compare with the accepted convergence criterion.

6. The interation is continued till the convergence to desired precision is achieved. Let us call this interation as $r = X$. At interation $r = X$, the covergence criteria is reached and the interation process is stopped.

7. The interative process used is described by the General Equation called *the Algorithm. e.g.* Eqn. 57.20 ;

$$V_2^k = V_2 - \frac{Z\,S_2^*}{(V_2^{(k-1)})^*} \qquad \text{...(57.21)}$$

8. Last Interation is when the resulting V satisfies :

$$\Delta V = \left\{[(V_2)\,;\, r = p + 1] - [(V_2)\,;\, r = p]\right\} < 0.00001 \text{ P.U} \qquad k = 1, 2, \ldots N, \text{ bus Number}$$

$$\text{Interation number} \qquad p = 1, 2, \ldots (p + 1) = X$$

At interation $r = X$, the covergence criteria is reached and the process is stopped, computer print out is obtained. From the basic variables of the buses, the remaining variables for the buses and the branches are then calculated. The above procedure is now applied in Example 57.4.

Example 57.4. *Load Flow Calculations for 2-Bus System by Interation Procedure., A simple 2-Bus System with a short transmission line between the sending end and the receiving end shown in Fig. 57.7 has the following given data, all values in P.U. :* $V_1 = 1\angle 0°$; $Z = 0.05 + j\,0.02$; $S = P + jQ = 1.06 \mid j\,0.6$.

Determine V_2 *by interation method. Convergence criterion* $c < \Delta V$ *when Interation is stopped. Given* $c = 0.00005$ *pu.*

Solution. We use the procedure explained earlier.

Algorithm (Eqn. for the Interation Process) is

$$V_2^k = V_1 - \frac{Z\,S_2^*}{(V_2^{(k-1)})^*} \qquad \text{...(57.21)}$$

1. Assume value of V_2 for the start of first interation (int = 1) and call it as (V_2) ; $r = 0$. Let $V_2 = 1\angle 0°$ p.u.

Substituting assumed $V_2 = (V_2)$; $r = 0 = 1\angle 0°$ on RHS of the Algorithm :

$$V_2^k = V_1 - \frac{Z\,S_2^*}{(V^{(k-1)^*})}$$

$$V_1 = 1\angle 0° \qquad \text{...(given)}$$

$$V_2^k = 1\angle 0° - \frac{(0.05 + j0.02)\,(1.0 + j\,0.6)}{1\angle 0°}$$

$$V_2 = (1 - 0.05 + 0.012) - j\,(0.030 + j0.02)$$

$$V_2 = 0.962 - j\,0.05 \quad \text{... result of interation 1.}$$

This new value is used in RHS for the *next interation* to obtain yet updated V_2. Procedure is repeated

The following results are of successive interations :

Used Value of V_2 *in Interation*	*Interation Number r*	*Resulting value of,* V_2
Assumed Value : $V_2 = 1 + j0$	1	$0.962 - j\,0.05$
$0.962 - j\,0.05$	2	$0.963 - j\,0.054$
$0.963 - j\,0.054$	3	$0.9635 - j\,0.054$
$0.9635 - j\,0.054$	4	$0.9635 - j\,0.054$
	Interation Stopped,	$r = 4$

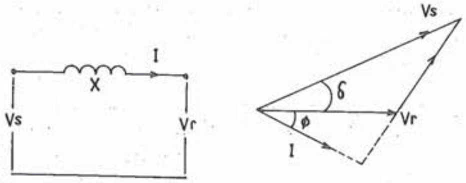
In the interation Process of this example, the acceptable convergence $\Delta V = (V_2)_4 - (V_2)_3 = 0$ has been achieved within 4 interations.

Interation	1	2	3	4
V pu	0.9625	0.9635	0.9638	0.9638
ΔV		0.001	0.0003	0

POWER FLOW IN TWO-NODE AC SYSTEMS

57.13. POWER FLOW IN A TWO BUS AC SYSTEM (SINGLE BRANCH)

Power flow study of a simple *two node AC system* gives understanding of the fundamental relationship between variables, $|V|$, $\angle\delta$, P, Q. The basic relationship are important as they apply to every branch in a multi-bus system. Let the sending end bus by subscript be (s) and receiving end bus by subscript be (r). Each node has four variables, namely P, Q, $|V|$, δ only two variables are known for each bus and the other two are unknown.



(*A*) Single line diagram (*B*) Phasor diagram

Fig. 57.7. Two node system with a short AC transmission line
(Flow P_{ac} is decided mainly by sin δ)
$|V_s|$ and $|V_r|$ magnitudes are held within narrow specified limits.

The per phase, per unit quantities are :

V_s = Sending-end voltage phasor	V_r = Receiving-end voltage phasor
V_1^* = Complex conjugate of V_s	V_r^* = Complex conjugate of V_r
$\|V\|$ = Absolute value of phasor	δ = Phase angle of phasor V
I = Sending-end current phasor	I = Receiving-end current phase
I^* = Complex conjugate of I	V^* = Complex conjugate of V
P_s = Sending end Real Power	Q_s = Sending end Reactive Power
S_s = Sending end Complex Power $= P_s + j\,Q_s$	S_r = Receiving end complex Power $= P_r + jQ_r$
S_s = Complex conjugate of S_s	S_r = Complex conjugate of S_r

δ =Power angle, angle between phasors V_s and V_r

X =Series Reactance of transmission line = $2\pi f_L$

R =Resistance of transmission line

Complex Power S, in general is given by :

$$S = P + jQ = VI^* \quad \text{voltamperes, } VA \qquad ...(57.22)$$

where, I^* is complex conjugate of phasor I ; and V is phasor voltage.

For Sending-End, we have,

$$S_s = P_s + j\,Q_s = V_s\,I^* \;\; VA \qquad ...(57.23)$$

Quantities are per phase, per unit basis. From Fig. 57, we get

$$I = \frac{1}{j\,X}(V_s - V_r) \qquad ...(57.24)$$

Therefore,

$$I^* = \frac{1}{-j\,X}(V_s^* - V_r^*) \qquad ...(57.25)$$

Substituting I^* from Eqn. 57.25 in Eqn. 57.23, we get

$$S_s = \frac{V_s}{-j\,X}(V_s^* - V_r^*) \qquad ...(57.26)$$

From phasor diagram of Fig. 57, we get

$$V_r = |V_r| \angle\delta°, \qquad \text{therefore, } V_r = |V_r|^*$$

and,

$$V_s = |V_r| \angle\delta°$$

Thus, Eqn. 57 becomes

$$S_s = \frac{|V_s|^2 - |V_r|\;|V_s|\;e^{i\delta}}{-j\,X} \qquad ...(57.27)$$

$$S_s = \frac{|V_s|\;|V_r|}{X}\sin\delta + j\frac{1}{X}\left[\,|V_s|^2 - |V_r|\;|V_r|\cos\delta\right]$$

After simplifying,

$$P_s = \text{Real Part of } S_s = \frac{1}{X}\left[\,|V_s|\;|V_r|\sin\delta\right] ...\text{Watts} \qquad ...(57.28)$$

$$Q_s = \text{Imaginary Part of } S_s = \frac{1}{X}\left[\,|V_s|^2 - |V_s|\;|V_r|\cos\delta\right] ...\text{VAr} \qquad ...(57.29)$$

Similarly, for Receiving End, we get

$$S_r = P_r + j\,Q_r = V_r\,I^* \qquad ...(57.30)$$

and

$$P_r = \text{Real Part of } S_r$$

$$P_r = \frac{1}{X}\left[\,|V_s|\;|V_r|\sin\delta\right] ...\text{W} \qquad ...(57.31)$$

The transfer of real power through the line is mainly due to load angle δ between voltage vectors. Power transfer is not much dependent on magnitudes of voltages.

$$Q_r = \frac{1}{X}\left[\,|V_s|\;|V_r|\cos\delta - |V_r|^2\right] ...\text{VAr} \qquad ...(57.32)$$

Reactive power will flow from higher voltage to lower voltage. For δ = 0, the average reactive power flow through the line is

$$Q_{av} = \frac{1}{2}\,[Q_s + Q_r]\;\text{VAr} \qquad ...(57.33)$$

$$= \frac{1}{2X}\left[\,|V_s|^2 - |V_r|^2\right] ...\text{VAr} \qquad ...(57.34)$$

The transfer of reactive power through the line is from higher voltage to lower voltage and is strongly dependent on voltage magnitudes.

Line Losses in AC Lines (P_{line}) :

If we consider line losses,

$$P_{line} = |I|^2 R \;\; ...\text{watts} \qquad ...(57.35)$$

Coming back to Equation 57

$$I^* = \frac{P + jQ}{V} \quad I = \frac{P - jQ}{V^*} \qquad ...(57.36)$$

Thus,

$$II^* = |I|^2 = \frac{P^2 + Q^2}{|V|^2}$$

Therefore, Eqn. 57.35 becomes,

$$P_{line} = \frac{(P^2 + Q^2)\,R}{|V|^2} ...\text{watts} \qquad ...(57.37)$$

The line losses are due to I^2R losses caused by flow of both real power and reactive power. Hence it is important to minimise reactive power flow through the line. To minimise line losses, the reactive power flow should be minimised by providing compensation of Q at load end.

SOLVED EXAMPLES ON TWO BUS AC SYSTEM LOAD FLOW

Example 57.5. *Load Angle. The sending end voltage $|V_s|$ for a line is 1 pu. The receiving end voltage $|V_r|$ is also 1 pu. Line reactance is j 0.05 pu. Real power flow through line is 10 pu. Calculate power angle δ between V_s and V_r vectors.*

Solution. We know,

$$P = \frac{|V_s|\;|V_r|}{X}\sin\delta \qquad ...(57.38)$$

Hence

$$\sin\delta = \frac{P}{|V_s|\;|V_r|/(X)} = \frac{10}{1 \times 1/(0.05)} = 0.5$$

Power Angle

$$\delta = \sin^{-1}(0.5) = \angle 30° \qquad \textbf{Ans.}$$

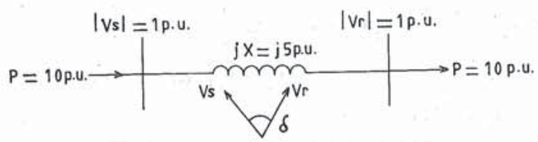


Fig. 57.8. Figure of example 57.6 and 57.7.

Example 57.6. *Active and Reactive power Flow. The sending end voltage $|V_s|$ and the receiving end voltage $|V_r|$ for a line is 1 pu. The line reactance is j 0.05 pu. Resistance is neglected. Real power flow through line is 10 pu. Calculate reactive power flow through the line state the complex power flow through the line.*

Solution. We know, $P = \dfrac{|V_s|\ |V_r|}{X} \sin \delta$

Hence $\sin \delta = \dfrac{P}{[|V_s|\ |V_r|]/(X)} = \dfrac{10}{1 \times 1/(0.05)} = 0.5$

Power Angle $\delta = \sin^{-1}(0.5) = \angle 30°$

Reactive power flow at sending end through the line is given by

$$Q_s = \frac{|V_s^2|}{x} - \frac{|V_s|\ |V_r|}{X} \cos \delta$$

$$= \frac{1^2}{0.05} - \frac{1 \times 1}{0.05} \cos 30° = +2.68 \text{ pu}$$

Reactive power flow at receiving end through the line is given by

$$Q_r = \frac{|V_s|\ |V_r|}{X} \cos \delta - \frac{|V_s|^2}{X}$$

$$= \frac{1 \times 1}{0.05} \cos 30° - \frac{1^2}{0.05} = -2.68 \text{ pu}$$

Reactive power flow through the line

$$Q_s - Q_r = 2.68 - (-2.68) = 5.36 \text{ pu}$$

Complex power flow through line $P + jQ = 10 + j\,5.36$ pu. **Ans.**

Example 56.7. *Load Angle and Power Flow in Two Bus AC System. The sending end voltage $|V_s|$ and the receiving end voltage $|V_r|$ of an AC transmission line is also 1 pu. Line reactance is j 0.05 pu resistance of line is neglected. The power angle between V_s and V_r vectors is 30 elec. Calculate the p.u. active power flow through the transmission line.*

Solution.

We know, $P = \dfrac{|V_s|\ |V_r|}{X} \sin \delta = \dfrac{1 \times 1}{0.05} \sin 30° = \dfrac{0.5}{0.05} = 10$ pu.

Example 57.8. *The reactance of a short transmission line is j 0.06 pu. The load current at receiving end is 1+ j0.6 pu and receiving end voltage is 1 ∠0° pu. Calculate (1) sending end voltage and (2) Average reactive power flow in line.*

Solution.

Sending end voltage = (Receiving end voltage) + (*IZ* drop)

$$V_s = V_r + IZ = 1\ \angle 0° + (1 + j0.6)\ (j0.6)$$

$$= 1 + j0 + j0.6 - 0.36 = 0.64 + j\,0.6 = 0.96\ \angle 3.56°$$

$$Q_{av} = \frac{1}{2}[Q_s + Q_r] = \frac{1}{2X}\left[\,|V_s|^2 - |V_r|^2\right]$$

$X = j\,0.06$, Sending and voltage $|V_s|$ are calculated above = 0.96 pu Receiving end voltage (given) = 1 pu

$$Q_{av} = \frac{1}{2\,(0.06)}[0.96^2 - 1^2] = -0.65 \text{ pu}$$

Example 57.9. *Complex power If $V = 1\ \angle 0°$ pu and $I = 1.188\ \angle -28.6°$ pu, calculate complex power S, real power P and reactive power Q.*

Solution. $S = V I^* = P + jQ \quad I = 1.88\ \angle -28.6°$ pu

$I = I = 1.188\ \angle{+28.6°}$ pu

$S = V I^* = (1\ \angle 0°)\ (1.188\ \angle{+28.6°})$ pu

$= 1.188\ \angle{+28.6°}$ pu

$= 1.188\ (\cos 28.6 + j \sin 28.6)$

$S = 1.043 + j\,0.569 = P + jQ$

Equating real part from each side, Real Power $P = 1.043$ pu Equating Imaginary part from each side, Reactive power $Q = 0.569$ pu Complex power $= P + jQ = 1.043 + j\,0.569$.

57-B

Power Flow Calculations — Part II

Gauss Seidel Method and Newton Raphson Method for Multibus AC system—Power Flow through bipolar HVDC Link.

57.14. INTRODUCTION

The principles of AC network power flow calculations by interative procedures have been covered in the earlier chapter. Various methods of power flow calculations differ from each other in respect of (1) Algorithm used (2) proceedings of the interations.

Following methods are used in power flow studies

— Gauss Seidel methods (GSM)

— Gauss Seidel methods employing acceleration factors (successive over relaxation method).

— Newton Raphson Method (NRM)

The procedure of solving Power Flow Equations of Multibus System by these methods are covered in this chapter. The procedure of power flow calculations for bipolar 2 terminal HVDC have also been explained.

57.15. POWER FLOW IN A MULTI-BUS AC SYSTEM

Consider a Nodal Admittance Model of a N-bus Network. Each bus has a corresponding node a number k $[k = 1, 2, ... N]$

— The node voltages with respect to reference node are $|V_2|$, $|V_3|$, ... V_N. The phase angles of the bus voltage are $\delta_2, \delta_3, ..., \delta_N$.

The current entering the node k is I_k.

$k = 1$ is for reference bus. The subscript denotes the node number.

The N node network can have N number of Kirchoff's nodal equations in terms of node current, node voltages *Branch Admittances*. These N equations are simplified and written in terms of *Bus Admittance Matrix*. These N equations are solved by the *Interative Procedures.* The interation number is denoted by *superscript* r *e.g.* V_k^r denotes the voltage of bus k obtained from interation r. $V_k^{(r+1)}$ denotes the voltage of bus k obtained from interation $(r + 1)$.

1. *Initial data* for a typical power flow study is

(1) Given network configuration and branch admittances

(2) Voltage levels of generator buses

(3) Generation active power and reactive power levels

(4) Active and reactive power to load buses

2. **Objective.** To determine the voltages and their phase angles for all the network buses and to calculate the unknown variables P and Q for each bus. From these calculated results the various

other quantities for buses and branches are to be calculated for steady state power flow and voltage conditions.

Initially, we draw a single line diagram of the given system and identify the generator buses and load buses.

Thus,

	Known (Specified)	*Unknown* (To be calculated)*
Generator buses	P, $\mid V \mid$	$\angle\delta$, Q
Load buses	P, Q	$\mid V \mid$, $\angle\delta$
Slack bus (Swing bus)⁺	$\mid V_1 \mid$, $\angle\delta_1$ = Ref.	P_1, Q_1

* The objective of power flow (Load flow) studies is to determine these unknowns for each bus under steady state condition.

+ Slack bus (Swing bus) is any one selected generator bus for reference voltage and reference phase angle, P and Q are not specified to begin with.

3. The *basic variables* associated with *each of the N buses are*

— Voltage magnitude $|V_k|$,

— Phase angles of voltage phasors (δ_k),

— Real power P_k,

— Reactive power Q_k

A typical *Load flow study* (power flow study, load flow calculations) gives mainly values of the following variables for specified *normal steady state* operating conditions of generation and load :

— Phasor voltages of various network buses, $V_k = |V_k| \angle\delta_k$, $k = 1$ to N for an N Bus system

— Phase-angles of bus voltage phasors $(\angle\delta_k)$

— Real power P_k, reactive power Q_k, at various network buses ($k = 1$ to N) and through branches of the electrical power system.

For each bus out of the above four variables two variables are known (given) and remaining two are unknown and are to be determined by load flow studies.

3. The *Derived Variables* are Branch current I, branch power, power factor cos ϕ of branch current, MVA etc. are calculated from the Principal Variables of terminal nodes obtained from the end results of the interation.

4. **Classification and types of Buses.** [Fig. 8.(*b*)]

In load flow studies, the network buses are classified according to the known and unknown *variables* into following three categories :

— Generator buses (P, $|V|$) or (Voltage controlled buses)

— Load buses (P, Q) Buses

— Slack bus [$|V|$-Bus or Swing bus or reference bus]. It is a bus selected for reference voltage.⁺

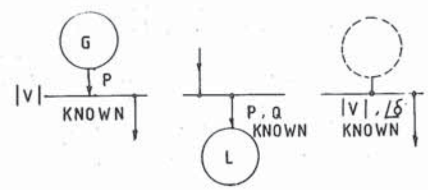


Fig. 57.8B. Types of Buses

Out of the 4 basic variables (P, Q, $|V|$, δ) for each bus, two are specified (known) variables and the other two are unknown variable (to be calculated from the power flow study). Accordingly, we have by convention real power entering the bus is positive ; real power leaving the bus is negative.

	Known (Specified)	*Unknown* (To be calculated)*
Generator buses :	P, $\lvert V \rvert$	$\angle\delta$, Q
Load buses :	P, Q	$\lvert V \rvert$, $\angle\delta$
Slack bus (Swing Bus)+	$\lvert V_1 \rvert$, $\angle 0$ = Ref.	P_1, Q_1

* The objective of power flow (load flow) studies is to determine these unknowns for each bus under steady state condition.

\+ Slack bus (Swing bus) is any one selected generator bus for reference voltage and reference phase angle. P and Q are not specified to begin with. In the further text, $k = 1$ is for slack bus.

Table 57 B.1. Classification of network buses for the power flow study

Generator Bus (P, $|V|$ Bus), (Voltage controlled Bus)

Generator bus is the bus for which the generated real power P and magnitude of generated voltage $|V|$ magnitude are known. For a generator bus real power P is injected into bus and therefore positive. The reactive power Q and the phase angle of bus voltage is to determined by solving N simultaneous equations of power flow.

Load Buses (P-Q Buses)

Load bus (PQ bus) is the bus for which real power (Active power) P and reactive power Q are specified and known and bus voltage magnitude $|V|$ and its phase angle δ are to be found. For a load bus the outgoing real power P is taken as negative

Swing bus ($|V|$ Bus) : (Reference Bus) or (Slack Bus).

It is one of the selected Generator bus at which $|V|$ and $\angle\delta$ are specified. P and Q are to be determined.

For convenience the swing bus is taken as reference with

$$V/\delta = 1 \angle 0°, \textit{ i.e. } |V| = 1 \text{ pu and } \delta = 0°$$

For a swing bus : $V = 1 + j\,0$ pu.

The transmission losses in total network remain unknown till the solution to load flow is obtained. For this reason one of the generator bus is assumed to supply for the transmission losses of real power.

57.16. PROCEDURE

- Draw single line diagram and the Admittance network. Identify the buses and branches by numbers. (Ref. Fig. 57.5 and Fig. 57.6)
- Write the power flow equations for the given network in suitable form.
- The procedures for writing power flow equations differ with the GS method and *NR* method.
- The equations are converted to the algorithm for interative solution. The procedures for writing algorithm differ with the *GS* method and *NR* method.
- Solve the equations by interative method by substituting known quantities of some variables and assumed quantities of variables under interation.
- If interation is converging, continue. If diverging, investigate the cause and take corrective action and then proceed further with interations.
- Interations are continued till the desired convergence is reached.
- Calculate derived quantities for various buses, branches.
- Analyse the results and use them.

57.17. EQUATIONS FOR POWER FLOW IN GAUSS METHOD AND GAUSS SEIDEL METHOD FOR MULTI-BUS SYSTEM

These equations co-relate the variables P, Q, V and I of various Network buses. Nodal Admittance form is preferred as it is more amenable for computer solution by interative process.

Both the Gauss and Gauss Siedel use the same power equations but the algorithms for voltage are slightly different.

57.18. GAUSS METHOD

In Gauss Method, the same values of bus voltages are used in the entire interation. The newly calculated value of bus voltage say v the next bus voltage in the *next interation*.

In this book we have used subscript k for bus number and superscript r for interation number, N is total number of buses. Thus, V_k^r would mean voltage of kth bus and rth interation. (Various text books use different subscripts and superscripts for the bus numbers and interation numbers. Applied logic should be understood for writing the Algorithm correctly.)

For the kth node (of N node network), nodal current is

$$I_k = \sum_{n=1}^{n=N} Y_{kn} V_n \qquad \text{...(57.39)}$$

which can be written as,

$$V_k = Y_{kk} V_k + \sum_{\substack{n=1 \\ n \neq k}}^{n=N} Y_{kn} V_n \qquad \text{...(57.40)}$$

N = Total number of nodes I_k = Current in k'th node

Solving for Y_k, we get

$$V_k = \frac{I_k}{Y_{kk}} - \frac{1}{Y_{kk}} \sum_{\substack{n=1 \\ n \neq k}}^{n=N} Y_{kn} V_n \qquad \text{...(57.41)}$$

N = Total number of nodes

Y_k = Bus admittance of kth node

But we have, $V_k^* I_k = S_k = P_k - jQ_k$ ...(57.42)

Hence, $$I_k = \frac{P_k - j\,Q_k}{V_k^*} \qquad \text{...(57.43)}$$

Equations 57.42 and 57.43 are applied to N nodes and a set of N nonlinear simultaneous equations are obtained as :

$$V_k = \frac{1}{Y_{kk}} \left[\frac{P_k - jQ_k}{V_k^*} - \sum_{\substack{n=1 \\ n \neq k}}^{n=N} Y_{kn} V_n \right] \qquad \text{...(57.44)}$$

for $k = 1, 2, 3, 4 \ldots N$

$k = 1$ is a swing bus with specified V. Hence in above eqn.

V_1 is taken as specified.

The set of N Equations 57.44 are called the *Power Flow Equations*.

Example 57.10. *Write load flow equations for a 4-Node network shown in Fig. 57.9A.*

Solution. In Gauss Siedel Method, the newly calculated value of bus voltage say V immediately replaces the previous value of the same bus. The newly calculated value is then used for calculating the next bus voltage in the *same interation*.

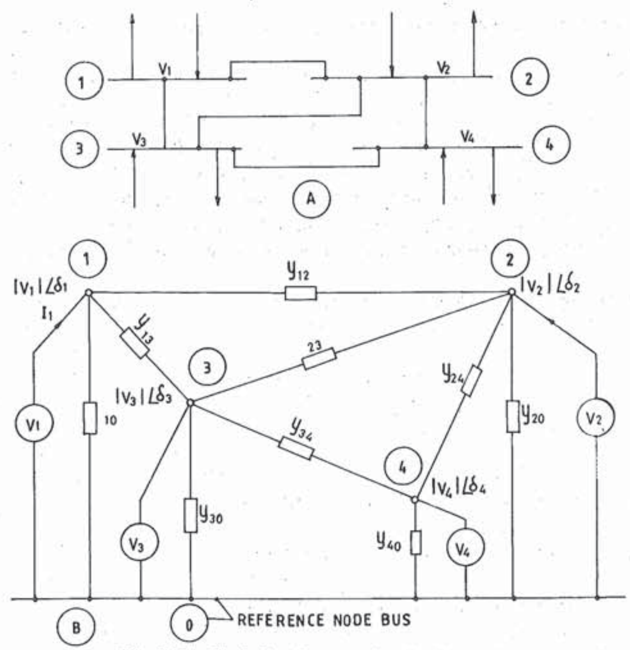


Fig. 57.9A. Single line diagram of a 4 Bus Network.
Fig. 57. 9B. A 4-Node admittance diagram for load flow study.

Algorithm from Equation 57.44 is

$$V_k = \frac{1}{Y_{kk}}\left[\frac{P_k - jQ_k}{V_k^*} - \sum_{\substack{n=1 \\ n \neq k}}^{n=N} Y_{kn} V_n\right] \qquad ...(57.45)$$

for $k = 2, 3, 4, \ldots$ N

$k = 1$ is the swing bus with known V.

k = (subscript). Bus Number $\quad V_k^*$ = complex conjugate of V_k

N = Total number of buses $\quad N$ = Total number of simultaneous nonlinear equations

P_k = Real power flow from k'th bus $\quad Q_k$ = Reactive power flow from k'th bus

r (Superscript) = Interation number, $r = 1, 2, 3, \ldots X$

At $r = X$, the specified convergence is achieved and interation is stopped.

For bus $k = 1$: Reference bus $V_1 = (1 + j\,0)$ Specified

$N = 4$. Simultaneous Non linear load flow equations got from algorithm 57.46 are :

For bus $k = 2$, $V_2 = \dfrac{1}{Y_{22}}\left[\dfrac{P_2 - jQ_2}{V_2^*} - (Y_{21} V_1 + Y_{23} V_3 + Y_{24} V_4)\right]$

For bus $k = 3$, $V_3 = \dfrac{1}{Y_{33}}\left[\dfrac{P_3 - JQ_3}{V_3^*} - (Y_{31} V_1 + Y_{32} V_2 + Y_{34} V_4)\right]$

For bus $k = 4$, $V_4 = \dfrac{1}{Y_{44}}\left[\dfrac{P_4 - jQ_4}{V_4^*} - (Y_{41} V_1 + Y_{42} V_2 + Y_{43} V_3)\right]$

57.19. SOLUTION OF POWER FLOW EQUATIONS BY GAUSS-SEIDEL METHOD

The *Algorithm* of Gauss-siedel method is obtained by modifying the power flow equation 57.44 to get the algorithm :

$$V_k = \frac{1}{Y_{kk}}\left[\frac{P_k - jQ_k}{V_k^*} - \sum_{\substack{n=1 \\ n \neq k}}^{n=N} Y_{kn} V_n\right] \qquad ...(57.45)$$

for $k = 2, 3, 4, \ldots N$

$k = 1$ is for slack bus with known voltage.

Superscript r of V_k denotes the interation number. V_3^{15} would mean voltage of bus 3 obtained from 15th interation. Algorithm 57.45 is used in for Gauss Seidel method. Note that the V_k begins with $k = 2$, as the slack bus ($k = 1$) has known voltage, so we begin the calculation with bus $k = 2$.

Proceedings of Interations by Gauss Seidel Method. The quantities on right hand side (RHS) of the Algorithm 57.45 are either initially specified quantities or initially estimated quantities. We observe that the quantity V_k for the bus voltage is on RHS and on LHS of the equation.

Initially known quantities and *assume quantities* are substituted on the RHS of the equation for interation 1. The solution of the interation gives the new corrected value of the unknown quantity V. The newly calculated corrected quantity from the interation will differ from the earlier estimated quantity used in the preceding interation. *i.e.*

$$V_k^{(r+1)} \neq V_k^r$$

where subscript k denotes bus number and superscript (r) denotes the iteration number.

The new corrected value of the corrected quantity $V_k^{(r-1)}$ is then used on RHS of the next interation to obtain yet new corrected value of V_k^r Thus, we proceed to calculate step by step the following.

Interation r	1	2	3	r
Quantity used on RHS for solving the Eqn.	V_k	V_k^1	V_k^2	V_k^{r-1}
Quantity obtained from the solution of interation	V_k^1	V_k^2	V_k^3	V_k^r

The agreement between V_k^{r-1} and V_k^r would be reached after several interations. When V_k^{r-1} is in agreement with V_k^r, the more accurate value of V_k is estimated.

Refer power flow equations for the 4 bus system shown in Fig. 57.9

Subscript $\quad k$ = bus number, $\quad k = 1, 2, 3, 4, 5, 6 \ldots N$

Superscript $\quad r$ = Interation number, $\quad r = 1, 2, 3, \ldots X$

$k = 1$ is the swing bus with known V

N = Total number of buses

N = Total number of simultaneous non linear equations.

P_k = Real power flow from k'th bus

Q_k = Reactive power flow from k'th bus

At $r = X$, the convergence is specified achieved and interation is stopped.

1. For bus $k = 1$: Reference bus $V_1 = (1 + j\,0)$ Specified
2. Computation starts with bus 2. ($k = 2$)

For 4-Bus system, $N = 4$, general equation is :

$$V_k = \frac{1}{Y_{kk}}\left[\frac{P_k - jQ_k}{V_k^*} - \sum_{\substack{n=1\\ n\neq k}}^{n=N} Y_{kn}\, V_n\right] \qquad ...(57.44)$$

For $k = 2$,

$$V_2 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_k^*} - \sum_{\substack{n=1\\ n\neq 2}}^{n=N} Y_{2n}\, V_n\right] \qquad ...(57.44)$$

Expanding RHS for the 4 Bus Network

For bus $k = 3$,

$$V_3 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^*} - (Y_{31}\, V_1 + Y_{32}\, V_2 + Y_{34}\, V_4)\right] \qquad ...(57.45)$$

Likewise for $k = 4$,

$$V_4 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{V_4^*} - (Y_{41}\, V_1 + Y_{42}\, V_2 + Y_{43}\, V_3\right]$$

The quantities on RHS are either initially specified quantities or initially estimated quantities. By substituting these quantities, the solution to Eqn. 57.44 may be obtained to get value of V_k.

Estimated value of V used on right hand side will differ from that calculated of from the interation. *i.e.* $V_k^{r-1} \neq V_k^r$.

The agreement between V_k^{r-1} and V_k^r would be reached after several interations. When V_k^{r-1} is in agreement with V_k^r, the more accurate value of V_k is éstimated.

For $k = 2$, this correct value of voltage of bus 2 is used for calculating the corrected voltage of bus 3.

Equation similar to 57.45 may be written for each bus. The interation $r + 1$ is carried out by using estimated values and known values of quantities on RHS and corrected value of V obtained in the earlier interation (r). Thus,

Substituting V_k^r

$$V_k^{(r+1)} = \frac{1}{Y_{kk}}\left[\frac{P_k - jQ_k}{V_k^r} - (Y_{k1}\, V_1^r + Y_{k2}\, V_2^r + Y_{k3}\, V_3^r + ...\right]$$

The interation process is repeated to get correct value of $V_k^{(r+1)}$.

The interation process is repeated again and again to get specified covergence.

In this method called the *Gauss Seidel Method,* the calculated value of a kth bus voltage from rth interation is used immediately for calculation of next bus voltage in the rth interation itself.

Example 57.11. *Load flow calculation of 2 bus system by Gauss Seidel interative method.*

Fig. 57.10 gives a 2-bus system with following given data :

Bus 1 : $V_1 = 1.1\ \angle 0°$ pu, *Load* $L = 1.1 + j\,0.4$ pu

Bus 2 : $V_2 =$ *To be determined, Load* $L = 0.5 + j\,0.3$ *pu*

Bus admittance : $Y_{11} = Y_{12} = 1.6\ \angle -80°$ *pu*

$Y_{12} = Y_{12} = 102\angle 100°$ *pu*

Determine the voltage and phase angle at Bus 2 by Gauss-Seidel Method.

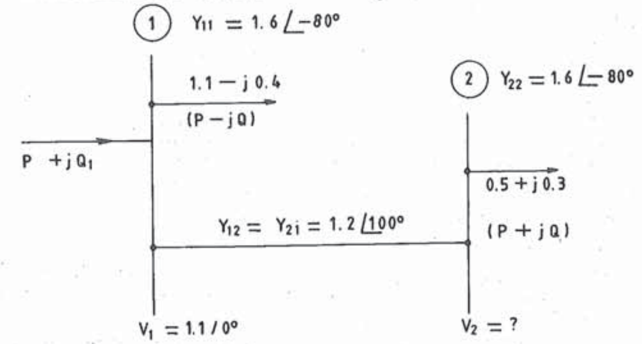


Fig. 57.10. Two bus systems of examples 57.

Solution.

Power entering the bus is positive. Power leaving the bus is negative.

Complex Power S *into* the bus 1 is

$$S_1 = (P_1 + jQ_1) - (1.1 + j\,0.4)$$
$$= (P_1 - 1.1) + j\,(Q_1 - 0.4) \text{ pu}$$

Complex power S_2 into the Bus is $S = -(0.5 - j0.3)$ p.u.

Algorithm for Gauss Seidel, from Eqn. 58.45 :

$$V_2^{(r+1)} = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^r)^*} - Y_{21}\, V_1\right] \qquad ...(1)$$

with the given data,

$$V_2^{r+1} = \frac{1}{1.6\ \angle -80°}\left[\frac{(-0.5 + j0.3)}{(V_2^0)^*} - (1.9\ \angle 100°)\ (1.1\ \angle 0°)\right]$$

$$= 0.625\ \angle 80°\left[\frac{0.583\ \angle 149°}{(V_2^0)^*} - 2.09\ \angle 100°\right] \qquad ...(2)$$

Interation 1, To Begin. Assume $V_2^0 = 1\ \angle -10°$ pu.

$$V_2^1 = 0.625\ \angle 80°\left[\frac{0.583\ \angle 149°}{(1 + \angle + 10°)} - 2.09\ \angle 100°\right] \qquad ...(2)$$

$$= (0.625\ \angle 80°)\ [(0.583\ \angle 139° - 2.09\ \angle 100°]$$
$$= 1.047\ \angle -12.6° \text{ pu.}$$

The Second Interation. Substitute V_2^1 obtained from interation 1 on RHS of Eqn. 1 to get yet new value of V_2

$$V_2^2 = 0.625\ \angle 80°\left[\frac{0.583\ \angle 149°}{(1.047\ \angle -12.06°)^*} - 2.09\ \angle 100°\right] \qquad ...(2)$$

$$V_2^2 = 1.047\ \angle -8.6° \text{ pu.} \quad \textbf{Ans.} \qquad ...(3)$$

Difference $\Delta\, V_2 = 1.047 - 1.047 = 0$, hence further interation is not required and the final result is given by (3) equation.

57.20. TREATMENT TO VOLTAGE CONTROLLED BUSES IN GAUSS-SEIDEL METHOD

We recall that the Generator Bus is Voltage controlled bus and for Generator Buses

Known	*To be determined*
$P, V_k,$	$\angle\delta, Q$

We have to treat the Generator buses differently for solving the unknowns. The Algorithm 57 is for solution of V_k which is not applicable to the generator bus with known voltage. We must arrive at a suitable Algorithm.

We write the equation for kth bus as

$$Q_k = -I_m\left[V_k \sum_{n=1}^{n=N} Y_{kn} V_n\right] \qquad ...(57.48)$$

The algorithm for the same equation is :

$$Q_k^r = -I_m\left\{V_k^{(r-1)^*}\left[\sum_{n=1}^{k-1} Y_{kn} V_n + \sum_{n=k}^{N} Y_{kn} V_n\right]\right\} \qquad ...(57.49)$$

where, I_m is imaginary part of the expression in the bracket ; superscripts r indicate number of interation, subscript indicates bus number.

Q_k^r is calculated from Eqn. 57.49 for best previous value of the buses from Eqn. 57.49. This value of Q_k is used for obtaining the new value of V_k from Equation 57.45 for voltage V_k.

57.21. ACCELERATION FACTORS (α) FOR GAUSS SEIDEL METHOD OF INTERATION

With routine interation by Gauss Seidel Method, the convergence criteria is achieved after very many interations. This results in excessive computer time. If the correct values obtained from the preceding interation are increased by using an Acceleration Factor before substituting in the next interation, the interation proceedings are accelerated and the convergence is achieved in lesser number of interations.

The factor by which the best corrected voltage result obtained from the preceding interation is multiplied before substituting the values in the next interation are called the Acceleration Factor. For example a acceleration factor $\alpha = 1.6$ may be used for real and imaginary components for calculated voltage. *Acceleration Factor is always less than 2.*

$$V^{(1)}_{2\,acc} - v_2^{(0)} + \alpha\left[V_2^{(1)} - V^{(0)}_{2\,(acc)}\right] \qquad ...(57.50)$$

α is called the Acceleration factor. Algorithm for Equation 57.50 is : For example, for interation r, for bus k :

$$V_{k\,(acc)} = V^{(r-1)}_{k\,(acc)} + \alpha\left[V_k^r - V^{(r-1)}_{K\,(acc)}\right] \qquad ...(57.51)$$

where, V_k^r = Resulting best voltage of bus k, from interation r, before applying Acceleration factor α

$V^r_{k\,(acc)}$ = Modified value of best voltage of bus k, from interation r, after applying Acceleration factor α, for using in interation $(r + 1)$.

Example 57.12. *Acceleration Factors in Gauss Seidel Method Assumed Value : assumed value of V for first interation was $V_2^0 = 1 + j0$.*

The best result of interation 1 by Gauss Seidel Method for bus 2 was $V_2^1 = 0.983664 - j\,0.032316$.

The Acceleration Factor to be used as 1.6.

Calculated the modified $V^1_{2\,(acc)}$ to be used for next interation.

Solution.

From $\qquad V^1_{2\,(acc)} = V_2^0 + \alpha\,(V_2^1 - V_2^0) \qquad ...(57.52)$

Let $\quad \alpha = 1.6$. Substituting $V_2^0 = 1$ pu and $V_2^1 = 0.983564 - j\,0.32356$

$V^1_{2\,(acc)} = 1 + 1.6\,[(0.983564 - j\,.03316) - 1]$ p.u.

$V^1_{2\,(acc)} = 0.973703 - j\,0.051706$ p.u

We use this Modified Accelerated New value for next interation instead of the earlier $V_2^1 = 0.983564 - j\,0.032316$.

NEWTON-RHAPSON METHOD

57.22. INTRODUCTION AND COMPARISON BETWEEN GAUSS SEIDEL METHOD AND NEWTON RAPHSON METHOD

The comparison between these two most widely used methods is given in the following table.

Table 57.B2. Comparison between Gauss Seidel and Newton Raphson Method

Gauss Seidel Method	Newton Raphson Method
— Well Established, Simple	— Recent
— More number of Interations	— Less number of interations
— More Computer Time	— Less Computer Time
— High Computation Cost	— Less Computation Cost
— Convergence uncertain	— Convergence certain
— Acceleration Factors used for rapid convergence	

Gauss Seidel is a simple interative method of solving N number power flow equations by interative method. There is no need of partial derivatives. The Newton Raphson Method is based on *Taylor's Series* and partial derivatives.

The Newton Raphson Method is recent, requires less number of interations to reach convergence, takes less computer time hence computation cost is low, and the convergence is certain. Today, System Study Groups of Utilities and Power System Planning Sectors prefer the Newton Raphson Method.

The Newton Raphson Method is based on *Taylor's Series* of function with two or more variables and their partial derivatives.

To begin with let us study the two equations with two variables x_1 and x_2. Thereafter the procedure may be extended to power flow equations having several variables.

57.23. TAILORS SERIES FOR TWO EQUATIONS WITH TWO VARIABLES

Consider the two functions of variables are x_1 and x_2 related by equations by following two equations

$$f\,(x_1\,;x_2) = C_1 \qquad ...(57.53)$$

$$f\,(x_1\,;x_2) = C_2 \qquad ...(57.54)$$

where, C_1 and C_2 are constants.

Solutions to these equations give values of x_1 and x_2. Let the *initial estimates give values of* x_1 and x_2 *be* x_1^0 *and* x_2^0.

Let the initial estimated solution differ from the final correct solution by $(\Delta\,x_1^0)$ and $(\Delta\,x_2^0)$.

i.e. correct solution—initial estimated solution = Δx

$$f_1 (x_1^0 + \Delta x_1^0 \,;\, x_2^0 + \Delta x_2^0) = C_1 \qquad ...(57.55)$$

$$f_2 (x_1^0 + \Delta x_1^0 \,;\, x_2^0 + \Delta x_2^0) = C_2 \qquad ...(57.56)$$

The Left Hand Side of these two equations is expanded by *Taylor's Series*, to get :

$$f_1 (x_1^{(0)}, x_2^{(0)} + \Delta x_1^{(0)} \left.\frac{\delta f_1}{\partial x_1}\right|_{x_1^{(0)}} + \Delta x_2^{(0)} \left.\frac{\partial f_1}{\partial x_2}\right|_{x_2^{(0)}} + \ldots = C_1 \qquad ...(57.57)$$

$$f_2 (x_1^{(0)}, x_2^{(0)} + \Delta x_1^{(0)} \left.\frac{\partial f_2}{\partial x_1}\right|_{x_1^{(0)}} + \Delta x_2^{(0)} \left.\frac{\partial f_2}{\partial x_2}\right|_{x_2^{(0)}} + \ldots = C_2 \qquad ...(57.58)$$

Partial derivatives of second and higher order are neglected. The results are written in matrix form as :

$$\begin{bmatrix} C_1 - f_1 (x_1^{(0)}, x_2^{(0)}) \\ C_2 - f_2 (x_1^{(0)}, x_2^{(0)}) \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}_{x_1^{(0)}, x_2^{(0)}} \begin{bmatrix} \Delta x_1^{(0)} \\ \Delta x_2^{(0)} \end{bmatrix} \qquad ...(57.59)$$

The problem is to solve $x_1^{(0)}$ and $x_2^{(0)}$. The terms $\partial/\partial x_1$, $\partial/\partial x_2$ indicate that the partial derivative is evaluated for the estimated values of $x_1^{(0)}$ and $x_2^{(0)}$. Other partial derivative terms are also evaluated similarly. The Equation 57.59 is conveniently written as

$$\begin{bmatrix} \Delta C_1^{(0)} \\ \Delta C_2^{(0)} \end{bmatrix} = J^{(0)} \begin{bmatrix} \Delta x_1^{(0)} \\ \Delta x_2^{(0)} \end{bmatrix} \qquad ...(57.60)$$

In Eqn. 57.60, ΔC^0 and ΔC^0 are the differences as appearing on the LHS of Eqn. 57.59.

In Equation 57.60, the Square Matrix of the partial derivatives on RHS is called the Jacobian J_0 (of the initial Estimates $x_1^{(0)}$ and $x_2^{(0)}$).

$$J_0 = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix}_{x_1^0 \,;\, x_2^0} \qquad ...(57.61)$$

And the Equation 57.61 is written in short as :

$$\begin{bmatrix} \Delta C_1^0 \\ \Delta C_2^0 \end{bmatrix} = J_0 \begin{bmatrix} \Delta x_1^{(0)} \\ \Delta x_2^{(0)} \end{bmatrix} \qquad ...(57.62)$$

The solution of the matrix equation 57.62 gives $x_1^{(0)}$ and $x_2^{(0)}$. The better estimates of the solution is given by

$$x_1^{(1)} = x_1^{(0)} + \Delta x_1^{(0)} \qquad ...(57.63)$$

$$x_2^{(1)} = x_2^{(0)} + \Delta x_2^{(0)} \qquad ...(57.64)$$

Repeating the process of interations, with these values, we get yet better estimated values. The Δx_1 and Δx_2 becomes smaller and smaller with every interation and finally the interation process is stopped when Δx_1 and Δx_2 are lesser then the pre-selected convergence criterion *c*.

To apply the Newton-Raphson Method described above to a Load Flow Problem, P_k and Q_k are specified for all the buses except the swing bus.

C_1 corresponds to Specified P_k of Buses ... except for swing bus.

C_2 corresponds to Specified Q_k of Buses ... except for swing bus.

Calculated *P*'s of buses corresponds to $f_1 (x_1^0, x_2^0)$

Calculated *Q*'s of buses corresponds to $f_2 (x_1^0, x_2^0)$

ΔP and ΔQ correspond to Δx and Δx and Δx. The details are described in the next section.

57.24. NEWTON RAPHSON METHOD APPLIED TO LOAD FLOW PROBLEM

Consider an *N* Bus Power System for load flow problem, by NR Method.

Let the,

*k*th bus have the phasor voltage $V_k = |V_k| \angle \delta_k$

N buses have phasor voltages $V_n = |V_n| \angle \delta_n$

The bus admittance of *k*th bus is $Y_{kn} = |Y_{kn}| \angle \theta_{kn}$

From Eqns. 57.39 and 57.43 ;

$$P_k - jQ_k = \sum_{n=1}^{N} |V_k V_n Y_{kn}| \underline{/\theta_{kn} + \delta_n - \delta_k} \qquad ...(57.65)$$

$$P_k = \sum_{n=1}^{N} |V_k V_n Y_{kn}| \cos (\theta_{kn} + \delta_n - \delta_k) \qquad ...(57.66)$$

$$Q_k = \sum_{n=1}^{N} |V_k V_n Y_{kn}| \sin (\theta_{kn} + \delta_n - \delta_k) \qquad ...(57.67)$$

where, P_k = Real part of RHS in Eqn. 57.65

$$P_k = |V_k V_n Y_{kn}| \cos (\theta_{kn} + \delta_n - \delta_k) \qquad ...(57.66)$$

Q_k = Imaginary part of RHS in Eqn. 58.65

$$Q_k = |V_k V_n Y_{kn}| \sin (\theta_{kn} + \delta_n - \delta_k) \qquad ...(57.67)$$

For the swing bus, $|V|$ and δ are specified and are omitted from the interative solution.

We first estimate *V* and δ for each bus except the swing bus. The specified *P* and *Q* correspond to C_1 and C_2 in Eqn. 57.57, 57.58 and 57.59.

The calculated *P*'s and *Q*'s represent $f_1 (x_1^0, x_2^0)$ and $f_2 (x_1^0, x_2^0)$. Let subscript k_s be for *k*'th bus and *specified value.* (of *P* or *Q*) and subscript k_c be for *k*'th bus and *calculated value* (of *P* or *Q*).

Having *P* and *Q* specified for every bus (except a swing bus) corresponds to knowing C_1 and C_2 in (57.59). We first estimate *V* and δ for each bus except the swing bus, for which they are known. We then substitute these estimated values, which correspond to the estimated values for x_1 and x_2, in (57.66) and (57.67) to calculate *P*'s and *Q*'s that correspond to $f_1 (x_1^{(0)}, x_2^{(0)})$ and $f_2 (x_1^{(0)}, x_2^{(0)})$.

Further we compute,

$$\Delta P_k^{(0)} = P_{ks} - P_{kc}^{(0)} \qquad ...(57.68)$$

$$\Delta Q_k^{(0)} = Q_{ks} - Q_{kc}^{(0)} \qquad ...(57.69)$$

Where subscripts *s* and *c* for attached to bus subscript *k* represent *specified* and *calculated.* These correspond to the LHS of Eqn. 57.60 [ΔC_1 ΔC_2]

Matrix Equation for a 3-Bus System corresponding to Eqn. 57.59 and Eqn. 57.60, omitting Swing Bus 1 :

$$\begin{bmatrix} \Delta P_2^{(0)} \\ \Delta P_3^{(0)} \\ \Delta Q_2^{(0)} \\ \Delta Q_3^{(0} \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial |V_2|} & \frac{\partial P_2}{\partial |V_3|} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial |V_2|} & \frac{\partial P_3}{\partial |V_3|} \\ \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & \frac{\partial Q_2}{\partial |V_2|} & \frac{\partial Q_2}{\partial |V_3|} \\ \frac{\partial Q_3}{\partial \delta_2} & \frac{\partial Q_3}{\partial \delta_3} & \frac{\partial Q_3}{\partial |V_2|} & \frac{\partial Q_3}{\partial |V_3|} \end{bmatrix}_{\substack{\Delta\delta_2^{(0)},\, \Delta\delta_3^{(0)} \\ \Delta V_2^{(0)},\, \Delta V_3^{(0)}}} \begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \\ \Delta\, |V_2^{(0)}| \\ \Delta\, |V_3^{(0)}| \end{bmatrix} \quad ...(59.70)$$

The Square Matrix of Partial Derivatives is the Jacobian. The elements of the Jacobian are determined by taking partial derivatives of P_k and Q_k and substituting there in the voltages assumed for the first interation *or* the voltages calculated from the previous interation.

The Equation 57.70 can be solved by inverting the Jacobian.

$$\begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \\ \Delta\, |V_2^{(0)}| \\ \Delta\, |V_3^{(0)}| \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial |V_2|} & \frac{\partial P_2}{\partial |V_2|} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial |V_2|} & \frac{\partial P_3}{\partial |V_3|} \\ \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & \frac{\partial Q_2}{\partial |V_2|} & \frac{\partial Q_2}{\partial |V_3|} \\ \frac{\partial Q_3}{\partial \delta_2} & \frac{\partial Q_3}{\partial \delta_3} & \frac{\partial Q_3}{\partial |V_2|} & \frac{\partial Q_3}{\partial |V_3|} \end{bmatrix}_{\substack{\Delta\delta_2^{(0)},\, \Delta\delta_3^{(0)} \\ \Delta V_2^{(0)},\, \Delta V_3^{(0)}}} \begin{bmatrix} \Delta P_2^{(0)} \\ \Delta P_3^{(0)} \\ \Delta Q_2^{(0)} \\ \Delta Q_3^{(0} \end{bmatrix}$$

Equation (57.71) is solved to get values of $\Delta\delta_k^{(0)}$ and $\Delta\,|V^{(0)}|$. The values determined for $\Delta\delta_k^{(0)}$ and $\Delta V_k^{(0)}$ are added to the previous estimates of V and δ to obtain *new estimates with which the next iteration is started. The interation process is repeated until the values in either column matrix are as small as desired convergence.*

The resulting values of $\Delta\,\delta_2^{(0)}$ and $\Delta\,V_k^{(0)}$ are added to the previously estimated values of V and δ to obtain yet new estimated values of V and δ. With which the next interation is started. The process is repeated until the values in either column matrix are as small as the selected convergence criteria.

Example 57.13. Newton Raphson Method. *A three bus system shown in Fig. 57.11 has following bus admittance matrix :*

$$Y_{bus} = \begin{bmatrix} 24.23\,\angle{-75.95^\circ} & 12.13\,\angle{104.04^\circ} & 12.13\,\angle{104.04^\circ} \\ 12.13\,\angle{104.04^\circ} & 24.23\,\angle{-75.95^\circ} & 12.13\,\angle{104.04^\circ} \\ 12.13\,\angle{104.04^\circ} & 12.13\,\angle{104.04^\circ} & 24.23\,\angle{-75.95^\circ} \end{bmatrix} p.u.$$

The per unit bus voltage of Bus 1 and 2 are indicated in the Figure. Per unit power flows into/from the buses are indicated by arrows. Calculate Voltage of Bus 2 by the Newton Raphson Method.

Solution.

The unknown quantity is the voltage of Bus 2. Let the initial estimate of V be $V = 1 + j0$ pu. Then from Eqn. 57.66 , we get $P_2^{(0)}$ as

$$P_2^{(0)} = |V_2^{(0)}|\ |V_1^{(0)}|\ |Y_{21}| \cos(\theta_{21} + \delta_1^{(0)} - \delta_2^{(0)}) + |V_2^{(0)}|^2\ |Y_{22}| \cos\theta_{22}$$
$$+ |V_2^{(0)}|\ |V_3^{(0)}|\ |Y_{23}| \cos(\theta_{23} + \delta_3^{(0)} - \delta_2^{(0)})$$

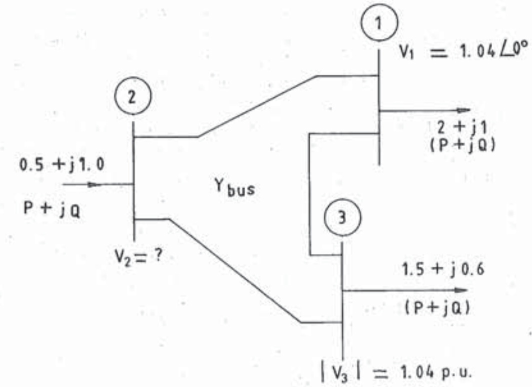


Fig. 57.11. Load flow of 3 bus network by NR method.

$$= (1)\ (1.04)\ (12.31) \cos 104.04^\circ + (1)\ (24.23) \cos(-75.95^\circ)$$
$$+ \mid (1)\ (1.04)\ (12.31) \cos 104.04^\circ = -0.34 \text{ pu.}$$

Likewise, $P_3^{(0)} = |V_3^{(0)}|\ |V_1^{(0)}|\ |Y_{31}| \cos(\theta_{31} + \delta_1^{(0)} - \delta_3^{(0)}) + |V_3^{(0)}|\ |V_2^{(0)}|\ |Y_{32}|$

$$\times \cos\left(\theta_{32} + \delta_2^{(0)} - \delta_3^{(0)} - \delta_3^{(0)}\right) + |V_3^{(0)}|^2\ |Y_{33}| \cos\theta_{33}$$
$$= (1.04)\ (1.04)\ (12.31) \cos 104.04^\circ + (1.04)\ (12.31) \cos 104.04^\circ$$
$$+ (1.04)^2\ (24.23) \cos(-75.95^\circ) = 0.026 \text{ pu}$$

Likewise from Eqn. (57.67) we get Q as,

$$Q_2^{(0)} = -\ |V_2^{(0)}|\ |V_1^{(0)}|\ |Y_{21}| \sin(\theta_{21} + \delta_1^{(0)} - \delta_2^{(0)}) - |V_2^{(0)}|^2\ |Y_{22}| \sin\theta_{22}$$
$$-\ |V_2^{(0)}|\ |V_3^{(0)}| \sin(\theta_{23} + \delta_2^{(0)} - \delta_3^{(0)})$$
$$= -(1)\ (1.04)\ (12.31) \sin 104.04^\circ - (1)\ (24.23) \sin(-75.95^\circ)$$
$$-(1)\ (1.04)\ (12.31) \sin 104.04^\circ = -1.34 \text{ pu.}$$

From (57.68), $\Delta P_2^{(0)} = 0.5 - (-0.34) = 0.82$ pu

$\Delta P_3^{(0)} = -1.5 - 0.026 = -1.526$ pu

Similarly, from (57.69) $\Delta Q_2^{(0)} = 1 - (-1.33) = 2.33$ pu

For the given three-bus system (with V_3 known), (Eqn. 57.70) becomes

$$\begin{bmatrix} \Delta P_2^{(0)} \\ \Delta P_3^{(0)} \\ \Delta Q_2^{(0)} \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial |V_2|} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial |V_2|} \\ \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & \frac{\partial Q_2}{\partial |V_2|} \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \\ \Delta\, |V_2^{(0)}| \end{bmatrix} \quad ....(A)$$

Differentiating P_k from Eqns. 57.66 and 57.67 and substituting numerical values in Eqn. A above gives.

$$\begin{bmatrix} 0.83 \\ -1.526 \\ 2.33 \end{bmatrix} = \begin{bmatrix} 24.27 & -12.23 & 5.64 \\ -12.23 & 24.95 & -3.05 \\ -6.11 & 3.05 & 22.54 \end{bmatrix} \begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \\ \Delta\, |V_2^{(0)}| \end{bmatrix} \quad ....(B)$$

We recall the Matrix Inversion principles to three Matrices

$$A = BC \quad \text{then } B^{-1}A = C$$

By applying this principle Eqn. B re-written as,

$$\begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \\ \delta\,|V_2^{(0)}| \end{bmatrix} = \begin{bmatrix} 24.47 & -12.23 & 5.64 \\ -12.23 & 24.95 & -3.05 \\ -6.11 & 3.05 & 22.54 \end{bmatrix}^{-1} \begin{bmatrix} 0.83 \\ -1.526 \\ 2.33 \end{bmatrix} \qquad ...(C)$$

This gives (C) as

$$\begin{bmatrix} \Delta\delta_2^{(0)} \\ \Delta\delta_3^{(0)} \\ \Delta\,|V_2|^{(0)} \end{bmatrix} = \begin{bmatrix} 0.05179 & 0.02653 & -0.00937 \\ 0.02666 & 0.05309 & 0.00051 \\ 0.01043 & -0.00001 & 0.04176 \end{bmatrix} \begin{bmatrix} 0.83 \\ -1.526 \\ 2.33 \end{bmatrix} \qquad ...(D)$$

Solving for $\Delta\,|V_2^{(0)}|$, the solution of Eqn. gives

$$\Delta\,|V_2^{(0)}| = (0.01043)\,(0.83) + (0.00001)\,(1.526) + (0.04176)\,(2.33) = 0.106 \text{ pu}$$

Thus, $\qquad |V_2^{(1)}| = 1 + 0.106 = 1.106$ pu

This procedure is repeated until convergence is reached.

Finally, we get $\qquad V_2 = 1.081 \angle -1.36°$ pu. **Ans.**

57.25. IMPORTANCE AND OBJECTIVES OF POWER FLOW STUDIES

The Power Flow Studies. (Load Flow Studies) are the essential and most important part of power system studies. Power system engineers need data about the bus voltage, active and reactive power flowing though the branches under steady state. Such data is obtained from load flow study. Load flow studies are extremely important and essential for Power System Planning, Designing, Expansion design and for providing guidelines to Control Room Operating Engineers in following activities.

— Load flow studies are the basis for power system planning, designing and expansion and providing operating instructions to control room operators. (Ref. Ch. 53)
— Evaluation the operating performance of a power system under normal-steady state.
— Providing operating instructions to generating station and substation control rooms for loading, reactive power compensation, relay settings, tap-setting, and switching sequence. Selecting the optimum settings of Over Current Relays.
— Analysing the effect of rearranging the circuits on the load flows, bus voltages.
— Preparing software for on line operation, control and monitoring of the power system.
— Analysing the effect of temporary loss of generating station or transmission path on the load flow.
— Determining the effect of compensation of reactive power on bus voltages.
— Calculation of line losses for various load flow conditions.
— Planning expansion of Network. Introducing HVDC line, interconnection, EHV AC line.
— *Obtaining initial input data for various other power system studies* such as : Economic Load Despatch, Reactive power and voltage control, State Estimation, Fault Calculations, Generation Planning, Transmission Planning, etc.

POWER FLOW THROUGH TWO TERMINAL HVDC LINK

57.26. POWER FLOW THROUGH A BIPOLAR TWO TERMINAL HVDC LINK

The power flow calculation for an HVDC Link is quite different from that for an AC Link. Refer Fig. 57.12.

Let, U_d = DC voltage, pole to ground at the mid point of line pole, kV, DC

U_{d1} = DC voltage, pole to ground at Rectifier—End of line pole, kV, DC

U_{d2} = DC voltage, pole to ground at Inverter—End of line pole, kV, DC

Then, $U_d = (U_{d1} + U_{d2})/2$ ... kV DC, pole to ground corresponding pole to pole voltage is $(2\,U_k)$ ...kV DC

Same current I_d flows through the pole 1 and Pole 2 line conductors.

I_d = Current in DC line pole, kA

Then average DC power P_d flowing through one line pole is

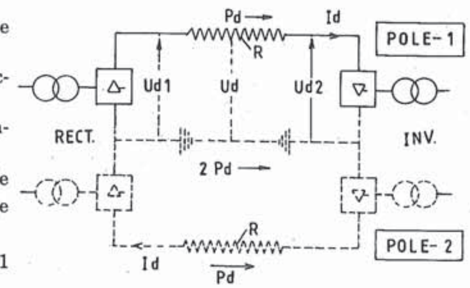


Fig. 57.12. Power flow through a bipolar 2-terminal HVDC link [Flow P_{dc} is dictated mainly by $(U_{d1} - U_{d2})/R$].

$$P_d = U_d \times I_d = \frac{U_{d1} + U_{d2}}{2} \times I_d \text{ ...MW/pole} \qquad ...(57.74)$$

Bipolar power $\qquad = 2 \times P_d$ ...MW $\qquad ...(57.75)$

Rectifier (Sending) End power : $P_{dr} = U_{d1} \times I_d$ ...MW pole

where U_{d1} is rectifier end DC voltage, pole to ground, kV DC, pole to Ground

Inverter (Receiving) End Power : $P_{d1} = U_{d2} \times I_d$

where U_{d2} is rectifier end DC voltage, pole to ground, kV

$$U_{d1} - U_{d2} = \text{Voltage drop in one DC line pole}$$

$$P_{dr} - P_{di} = P_{loss} = \text{ line loss in one DC line pole.}$$

$$P_{loss} = (Id)^2\,R$$

Bipolar line loss $\qquad = 2 \times P_{loss} = 2\,(I_d)^2\,R$

where R = line pole resistance of one line conductor.

$$U_{d1} - U_{d2} = \text{Voltage drop in DC line pole} = I_d \cdot R \text{...kV DC.}$$

where, I_d is in kA and R in ohm.

Note. In power calculations U_{dc} in kV and I_{dc} in kA gives P_{dc} in MW.

Example 57.14. Power Flow through Bipolar 2-Terminal HVDC Line

Ref. Fig. 57.12. A 840 km long bipolar 2 Terminal Rihand Delhi HVDC link has following operating conditions ;

— Sending end DC pole voltage (Rihand), $U_{d1} = 520$ kV DC, Pole to earth.
— Receiving end DC Pole Voltage (Delhi), $U_{d2} = 490$ kV DC, Pole to earth
— Current in the line pole conductor, $I_d = 1$ ...kA DC

Both the line poles are transferring equal power at equal voltage conditions. *Calculate* the following :

1. Power Flow per pole from Rihand to Delhi $\qquad$ 2. Bipolar power flow

3. Line Loss per pole in bipolar line
4. Bipolar line loss
5. Reactive power flow through the DC line.

Solution.

DC line pole voltage at middle of line length :

$$U_d = \frac{U_{d1} + U_{d2}}{2} = \frac{510 + 490}{2} = 500\ldots \text{ kV DC pole to ground}$$

$$I_d = \text{Current in a HVDC pole} = \text{given as } 1 \text{ kA}$$

The power flow *per pole,* through line pole at mid point of line

$$P_d = U_d \times I_d \ldots \text{MW} \qquad \ldots(1)$$

$$= 500 \times 1 = 500 \ldots \text{MW per pole}$$

where, P_d = Power flow per pole through the HVDC link, MW

U_d = Average of pole to ground sending—end voltage U_{d1} and receiving end voltage U_{d2}, kV

$$\text{Bipolar power flow} = 2 \times P_d = 2 \times 500 = 1000 \text{ MW} \qquad \ldots(2)$$

R = DC resistance of HVDC line pole between sending end and receiving end, ohm

$$\text{Given : } I_d = 1 \text{ kA} = \frac{U_{d1} - U_{d2}}{R} = \frac{510 - 490}{R} = \frac{20}{R} = 1 \ldots \text{kA}$$

$$R = 20/1 = 20 \text{ ohm}$$

$$\text{Line loss per pole } P_{loss} = (I_d)^2 R \ldots \text{MW per pole} \qquad \ldots(5)$$

$$= 1 \times 1 \times 20 = 20 \text{ MW per pole}$$

$$\text{Total Bipolar line loss} = 2 \times (I_d)^2 R \ldots \text{MW for 2 poles} \qquad \ldots(6)$$

$$= 40 \text{ MW for 2 poles}$$

Sending-end power per pole :

$$P_{dr} = U_{d1} \times I_d = 510 \; kV \times 1 \text{ kA} = 510 \text{ MW/pole}$$

Receiving-end power per pole :

$$P_{di} = U_{d2} \times I_d = 490 \text{ kV} \times 1 \text{ kA} = 490 \text{ MW/pole}$$

$$\text{Line loss per pole} = P_{dr} - P_{di} = 510 - 490 = 20 \text{ MW (Check)}$$

Steady state reactive power flow through HVDC line = 0

Since, I_d and V_d are with same phase angle along the line length.

Example 57.15. Load flow through Bipolar 2-Terminal HVDC Line.

A bipolar 2-terminal HVDC link delivers 1000 MW DC at ± 500 kV at receiving end. The line losses in 2 poles are 60 MW.

Calculate following :

1. Sending end power
2. Power in the middle of line
3. Line resistance per pole
4. Sending end DC voltage
5. Receiving end DC voltage
6. DC voltage in the middle of line.

Solution.

Rectifier (sending) end bipole power

= Inverter (Receiving) end bipole Power + bipole line loss

$$2\,P_{dr} = 2P_{di} + 2P_L$$

$$= 1000 + 60 = 1060 \text{ MW}$$

$$\text{Power in the middle of bipole line} = [P_{dr} + P_{di}]/2$$

$$\fallingdotseq [1060 + 1000]/2 = 1030 \text{ MW}$$

Rectifier end DC voltage = Rectifier DC power/DC current

DC line current is same at rectifier end to inverter-end. Inverter end DC current is

$$I_d = P_d/U_d = 1000/1000 = 1000 \text{ A} = I_{dr}$$

Rectifier sending end bipolar DC voltage

$$= P_d/I_d = 1060/1000 = 1060 \text{ kV DC}$$

Rectifier sending end bipole DC voltage = 1060 kV DC pole to pole

Rectifier sending end DC pole to ground voltage

$$= \pm 530 \text{ kV DC pole to pole}$$

DC voltage in middle of line pole to ground

$$= [530 + 500]/2 = 515 \text{ kV DC}$$

Line resistance per pole

$$= (U_{dr} - U_{di})/I_d = (530 - 500)/1 = 30/1 = 30 \text{ ohm}$$

Note. In power calculations U_{dc} in kV and I_{dc} in kA gives P_{dc} in MW.

57.27. IMPORTANT CONCLUSIONS ABOUT POWER FLOW THROUGH AC AND HVDC LINKS

1. Operation and Control principles of AC Link and HVDC Link are quite different. In AC line power transfer is due to power angle δ between V_s and V_r phasors. The increase in load brings about increase in δ automatically. However no precise and fast control of power is possible through a particular AC line in an AC network.

In an *HVDC Link* the power flow is

$$P_d = U_d \times I_d = \frac{U_{d1} + U_{d2}}{2} \times I_d \ldots \text{MW} \qquad \ldots(57.74)$$

$$P_d = U_d \times \frac{U_{d1} - U_{d2}}{R} \ldots \text{MW} \qquad \ldots(57.75)$$

In these Eqns. U_d in kV, I_d in kA and P_d in MW.

Since R is small, a small variation in $(U_{d1} - U_{d2})$ gives a significant change in DC power flow. DC power flow can be controlled precisely and rapidly by controlling $(U_{d1} - U_{d2})$, by tap changers of converter transformers (slow control 10 seconds) and by control of phase angles of converter valves firing simultaneously from both the ends of the HVDC Link Rapid control (tens of milliseconds). Power flow can be set at precise level. Power flow through a particular DC link can be changed at a rate of 30 MW/min over a wide range (*e.g.* 200 MW to 1500 MW).

2. Transfer of Real Power P_s (watts) through a branch of an AC network dependent mainly on power angle δ, between V_s and V_r phasors. The angle *o* is therefore called *Power angle or Load angle.* This angle increases with flow of *P* through the branch. In an AC line, power flow through a branch *does not* depend much on the magnitude difference $|V_s| - |V_r|$.

Power Flow in *HVDC line* however *depends* mainly on the magnitude difference $|U_{d1}| - |U_{d2}|$, and there is no question of load angle δ in case of HVDC lines.

3. Maximum power transfer (steady state stability limit) of an AC line occurs when power angle δ = 90°. Neglecting line losses,

$$\text{for } \delta = 90°\ldots, \qquad P_{max} = \frac{|V_s|\;|V_r|}{x} \angle \ldots \text{W} \qquad \ldots(57.9)$$

Beyond P_{max} the transmission link stability is lost. HVDC transmission link has no such limit based on power angle. Maximum HVDC power is usually dictated by thermal limit of thyristor-converter and converter transformers.

4. In AC lines, the reactive power Q flows in the direction from higher voltage to lower voltage. In HVDC line there is no continuous flow of reactive power.

5. HVDC line losses are less than 5% of power transfer ; HVDC line has no I_R line losses due to reactive power flow. Hence line losses are less than equivalent AC line carrying same power.

Whereas losses in AC line are about 15 to 20% high transmission losses in AC line are due to additional reactive power flow.

57.28. POWER FLOW THROUGH AC LINE AND PARALLEL HVDC LINE

Refer Fig. 57.12B illustrating the power flow through HVDC line and parallel AC line.

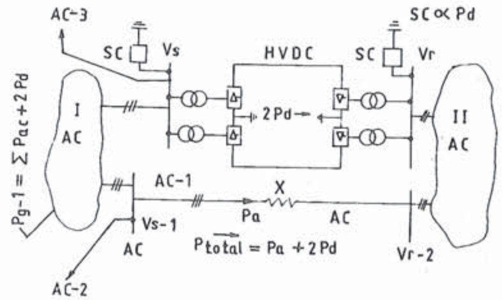


Fig. 57.12. B Power flow through HVDC line and parallel ac line.

Total power transfer from AC-1 to AC-2 is the sum of P_a and $2P_{dc}$.

$$P \text{ total} = P_a + 2\,P_d$$

where, $P_a =$ power flow through AC line $= \dfrac{|V_{s1}| \cdot |V_{r2}|}{X} \sin \delta,$

P_a cannot be easily and rapidly changed.

$\delta =$ Phase angle between V_{s-1} and V_{r-2} vectors

$2P_d =$ Power trough Bipolar HVDC line $= 2U_d \times I_d$

$U_d =$ pole to ground voltage and i_d = DC pole current

$2P_d$ can be easily and rapidly changed.

Power flow $2P_d$ from AC network 1 to AC Network 2 *via* bipolar DC line can be precisely controlled by means of HVDC Link.

Power flow P_a from AC Network 1 to AC Network 2 *via* AC line cannot be changed easily and rapidly as increased generation in Network 1 would result in increased power flow trough various AC lines such as AC 1, AC 2, AC 3. The increase in power flow is shared by various outgoing lines in accordance with the loads at receiving ends and resulting phase power angles δ_1, δ_2, δ_3 of the AC lines.

Voltage control. As the power $2P_d$ through DC line is increased, the Leading shunt compensation (SC) at receiving end should be increased to regulate V_r. Amount of shunt compensation required is about 60% of $2P_d$.

Total power generator. When AC Network 1 is sending power to other parts of the Network, the general equation is :

$$P_{g-1} = \sum P_{ac} + 2P_{dc} \qquad ...(57.76)$$

where, $P_{g-1} =$ The total power generated in Network 1

$\Sigma P_{ac} =$ Total power transmitted through various AC lines

$2P_{dc} =$ Total power transmitted through bipolar HVDC line

$$P_{ac} = P_g - 2\,P_{dc} = P_{ac1} + P_{ac2} + P_{ac3}$$

where, P_{ac1}, P_{ac2} are the power flow through various AC lines.

The increase or decrease in $2P_{dc}$ can be independently achieved by HVDC control. This is a unique advantage of HVDC Link over AC link. Neglecting the losses, the total power generated is equal to total power transmitted, P_g remaining the same, the increase in P_{dc} results in corresponding decrease in ΣP_{ac}. The power flows through various AC lines (P_{ac1}, P_{ac2} etc) are determined by means of power flow calculations for the total network.

Example 57.16. Parallel AC line and HVDC line. *A super thermal power plant is generating 6000 MW power. Out of the total generation, the Bipolar HVDC line transmits 2000 MW to a remote load centre and the remaining power is transmitted through four AC lines. The power flow through the three AC lines is 1000 MW each. Calculate the power flow through the fourth AC line.*

Solution.

$$P_{g-1} = \Sigma\, P_{ac} + 2P_{dc}$$

where, P_{g-1} is total generation, ΣP_{ac} is total power flow through all the AC lines and $2P_{dc}$ is bipolar HVDC power flow.

$$P_{g-1} = 6000 = \Sigma\, P_{ac} + 2000 \text{ MW}$$

Therefore,

$$\Sigma\, P_{ac} = 6000 - 2000 = 4000 \text{ MW}$$

$$\Sigma\, P_{ac} = P_{ac-1} + P_{ac2} + P_{ac-3} + P_{ac4}$$

$$\Sigma\, P_{ac} = 4000 \text{ MW} = P_{ac1} + P_{ac2} + P_{ac3} + P_{ac4}$$

$$= 1000 + 1000 + 1000 + P_{ac4}$$

$$P_{ac4} = 1000 \text{ MW} \quad \textbf{Ans.}$$

Example 57.17. Parallel AC line and HVDC line. *A super thermal power plant is generating 6000 MW power. Out of the total generation, the Bipolar HVDC line transmits 4000 MW to a remote load centre and the remaining power is transmitted through four AC lines equally. What would be the power flow through each of the four AC lines, assuming they share the load equally.*

Solution. $\Sigma\, P_{ac} = P_{g-1} - 2P_{dc}$

where P_{g-1} is total generation, ΣP_{ac} is total power flow through all the AC lines and $2P_{dc}$ is bipolar HVDC power flow.

$$P_{ac} = 6000 - 4000 = 2000 \text{ MW}$$

Power flow through each AC line = 2000/4 = 500 MW.

SUMMARY

The calculation of steady state P and Q flow through various branches of the Network and the resulting bus voltages and their phase angles is called *Power Flow Study* or *Load Flow Study*.

The *Principal variables* associated with *each of the N buses* are :

— Voltage magnitude $|V_k|$, — Phase angles of voltage phasors (δ_k),

— Real power P_k, — Reactive power Q_k

(subscript k for the bus number, k = 1, 2, 3, ... N for an N bus system. The *Derived Variables* for branches are current I. Power Factor cos ϕ, MVA etc., these are calculated from the Principal Variables obtained from the end results of the interation).

For each bus generally two of the above four principal variables are specified and other two are to be determined from Load flow calculations. The two unknown variables for each of the N buses are calculated by solving N simultaneous non-linear load flow equations co-relating P_k, Q_k, V_k for $k = 1$ to N bus number.

— Gauss seidel interative method (GS Method)
— Successive over relaxation method. (GS method employing acceleration factors)
— Newton-Raphson interative method. (NR method)

The power flow calculations of an N-Bus system involves solution to N number of simultaneous nonlinear power flow equations, with generally two unknowns for each of N Busses. The solution should satisfy network power flow equations. Modern load flow studies are performed by solving the network power flow equations by *Interative Procedure and use of Digital Computer/Personal Computer.*

Starting from the *assumed values* of unknown variables and given values of other variables, the load flow equations are solved to obtain new better values of the same unknown variables.

These new better values of unknowns are again substituted in the same equations (Algorithms) to get yet an another set of new revised values. The process of calculations of the new revised values of variables (*e.g.* Bus Voltages) by using earlier result is called *"an interation"*.

The interations are repeated till sufficiently accurate values are obtained and further interations are not giving next better values, *i.e.* convergence is reached. The equations used for interative solution are called *Algorithm.* The Gauss Seidel Method and Newton Raphson Method are used for interative solution of load flow problems.

Power flow calculations are essential for system design, system expansion, relay coordination, power system analysis and operation guidelines, and for input data for Stability Studies.

Power flow through HVDC link can be controlled precisely, rapidly and to required level through particular HVDC Link. *The increase or decrease in $2P_{dc}$ can be independently achieved by HVDC control.* This is a unique advantage of HVDC Link over AC link. Neglecting the losses, the total power generated is equal to total power transmitted, P_g remaining the same, the increase in P_{dc} results in corresponding decrease in ΣP_{ac}.

QUESTIONS

1. Explain the meaning of bus admittance matrix Y bus, with the help of a 3 bus network.
2. Distinguish between Load Bus, Generator Bus and Slack Bus in a Power Flow Study. State the known variables and unknown variables for each of the type of bus.
3. Write a general equation for voltage V_k of kth node of a N node power system, in terms of P_k, Q_k, Y_{kk}, V_k and Y_{kn} and V_n. Write the 3 equations for voltages of V_2, V_3 and V_4 a 4 Bus Network considering V_1 as Slack bus voltage.
4. In a 2 bus system, bus 1 is supplied by generator G_1 and bus 2 by generator G_2. The real power supplied by G_1 is 5.0 pu and by G_2 is 5.0 p.u., $|V_1| = 1.0$ pu. ; $|V_2|$ is 1.1 pu.

   Each bus has complex local load, equal to $S_1 = S_2 = 3 + j\,4$ pu. The two buses are connected by branch 1-2 having series reactance of 0.08 p.u. Calculate the total complex load on each bus.

   Ans. $8 + j\,5.7$ pu.
5. Explain the Gauss Seidel interation method for power flow solution.

6. Explain the information obtained from a typical power flow study and the significance of power flow studies.
7. Give comparison between Gauss siedel method and newton Raphson method of power flow studies.
8. Explain the significance of following equations for a branch in an AC network with respect to control of power flow through a branch of an AC network. How can the power through the branch be increased ?

$$P_s = \frac{1}{2}\,|V_s|\;|V_r|\;\sin\delta$$

9. Derive expression for power flow through a pole of a 2 terminal bipolar HVDC system having sending end DC voltage U_{d1} per pole, receiving end DC voltage U_{d2} per pole, line resistance r per pole and current in pole conductor I_d. Derive expression for P_d at sending end, P_d at receiving end and power loss in line pole.

   A bipolar 2 terminal HVDC line pole has resistance of 10 ohms, DC current in line pole is 1000 A, receiving end DC voltage is 400 kV DC pole to ground. Calculate : Line loss per pole, sending end DC power per pole, Sending end DC voltage per pole and bipolar DC power at middle of the HVDC line length.
10. A generating station generates 5000 MW. The power is transmitted through one bipolar HVDC line and three AC lines. The HVDC line transmits total 3000 MW. The two AC lines transmit totally 1500 MW. Calculate the power transmitted through the third AC line.

    Explain why HVDC line is preferred to AC line for better power flow control.