Math 230, Exam 1
November 18, 2018

Name: ____________________

Instructions.

- You are allowed one side of handwritten notes
- No calculators.
- It is fine to leave your answers unsimplified.
- There are 7 problems on 4 pages. Make sure your exam is complete.

Run LaTeX again to produce the table

1. Roll two dice. Let X be the sum.

[3 points] (a) Let $p_k = P(X = k)$. The possible dice rolls are given. Fill in the table for p_k.

1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

p_2	
p_3	
p_4	
p_5	
p_6	
p_7	
p_8	
p_9	
p_{10}	
p_{11}	
p_{12}	

[4 points] (b) Let $A = \{X \text{ is even}\}$ and $B = \{X \text{ is divisible by } 3\}$. Are A and B independent? Justify.

[4 points] (c) Let $C = \{X \leq 3\}$ and $D = \{X = 3\}$. Are C and D independent? Justify.

[3 points] 2. Suppose that 6 people are each assigned a whole number $1 \leq x \leq 40$. Each number is equally likely to be assigned. What is the probability that at least two people are assigned the same number? (You can leave your answer unsimplified.)

[4 points] 3. Two independent events have probabilities $\frac{1}{4}$ and $\frac{1}{3}$. What is the probability *exactly one* occurs? Hint: The answer is *not* $1/12$.

4. In the game Dreidel a 4-sided top is spun. The four equally likely outcomes are:

$$\{\text{Do nothing, Lose \$1, Gain \$2, Gain \$4}\}.$$

Let X be the amount of money exchanged after one spin.

[3 points] (a) What is EX?

[5 points] (b) What is $\text{var}(X)$?

5. Suppose Rachel flips a fair coin until *either* she gets a head, *or* she has flipped the coin 3 times.

[3 points] (a) Write all outcomes that have positive probability, and write their probability.

[3 points] (b) What is the expected number of heads she sees?

6. You flip a fair coin and win $1 if it is heads, or lose $1 if it is tails. Let $X_1, X_2, \ldots, X_n$ be random variables with $X_i = 1$ if heads is flipped and $X_i = -1$ if tails is flipped.

$$\text{Set } Y = \sum_{i=1}^{n} X_i \qquad \text{and} \qquad Z = nX_1.$$

[3 points] (a) What are $\text{var}(Y)$ and $\text{var}(Z)$? You may use the fact that $\text{var}(X_i) = 1$ for $1 \leq i \leq n$.

[3 points] (b) Explain why the two variances you calculated are different. Give some intuition. Don't just say "because the calculation says so."

7. Ash is playing Pokémon Go. There are 151 different pokémon. In a special area all pokémon are equally likely to be encountered with the rule that: **the same pokémon is never encountered twice in a row**. For example, if Pikachu was just encountered, then Pikachu will not appear in your next encounter. Instead, you are equally likely to meet any of the other **150 pokémon**.

[2 points] (a) Suppose Ash has encountered i distinct pokémon, and Y_i is the number of pokémon she encounters until she has seen $i+1$ distinct pokémon. Explain why Y_i is a geometric random variable.

[3 points] (b) What is the parameter for Y_0 and Y_i for $1 \leq i \leq 150$? (Hint: $Y_i \neq X_i$ from MAPs)

Y_0 : Y_i :

[2 points] (c) Let N be the (random) number of encounters needed for Ash to go from seeing 0 to all 151 pokémon. Write a formula for N in terms of the Y_i.

[2 points] (d) What is the expected value of N? It is okay to leave your answer as a sum.

[3 points] (e) Suppose that the "never twice in a row" rule isn't in place. So, each encounter is equally likely to be any of the 151 pokémon. Let M be the number of encounters needed to see all 151 pokémon.

Without doing any calculations, explain which ought to be larger between EM and EN.