Optimal (double) taxation with tax evasion and firm growth

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Abstract

Tax evasion is in general a nuisance for governments, which must devote resources to fight it to ensure that taxpayers pay their taxes. However, if taxpayers invest avoided taxes in a productive way, governments can also benefit from evasion by taxing the outcome of taxpayers' investments. Moreover, by auditing past tax declarations, governments can still recover avoided taxes from the past while still benefiting from the result of past evasion. This amounts to a form of double taxation. This paper models tax evasion by firms in a dynamic setting where firms have incentives to invest all their assets. It shows that the optimal policy for the government is not to reduce evasion to zero, even when all enforcement parameters are free. In practice, evasion functions as a loan from the government to the taxpayer, where expected fines work as interest rates. The incentives outlined in this paper are likely to hold for small, financially constrained firms with high growth potential.

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1 Introduction: Firm growth, taxes and financial constraints

In its ability to tax economic activity, the government is a similar to a shareholder of the whole economy: it can collect part of the revenues produced by individuals or firms. Consequently, tax revenues benefit from economic growth, and excessive taxation may be counterproductive for raising revenue. Increasing taxes affects the behavior of agents, encouraging them to produce less or to evade taxes, and at some point a marginal increase in tax rates may reduce tax revenues. This phenomenon is commonly referred to as "Laffer curve", in honor of the American economist Arthur Laffer. There are several different theoretical foundations for the Laffer curve. In this paper, I provide another one: the idea that taxation and enforcement may affect firm growth. I argue that financially constrained firms with growth potential may use tax evasion to alleviate their financial constraint and expand investments. The evade amount is not entirely lost to the government, which can recover it through tax inspections (enforcement action), typically happening after the evaded amount has already been spent. In fact, besides having a "shareholder" claim on economic activity, the government also operates implicitly as an implicit "lender" when agents evade taxes.

Firms may grow faster by evading taxes. The evaded amounts are additional profits, which can be reinvested in the firm's activities and make it grow. If caught by the tax authority, however, the firm must pay a fine on the evaded amount. The returns on the evaded amount invested productively must be weighed against the potential cost of the penalty. But for firms with high growth potential and limited access to financial markets, cheating on taxes may be a way to ease current budget constraints and invest in productive activity. Indeed, as James Andreoni (1992) put it once, evasion in a multi-period setting may function in a similar way to a loan: the firm can raise current revenue by cutting on tax expenditure, but has an expected future payment of a fine. Even if this expected future payment – the "interest" on the loan – is high, firms may find it interesting to take it.

The government may also take advantage of firms evading taxes to grow. First, because firm growth raises the size of the tax base in later periods. A larger firm pays more taxes. Second, because operating as a "lender" also gives it the opportunity to collect "interests" on the amount evaded, by running a tax audit. A tax audit in this setting gives rise to a kind of double taxation. When a firm gets audited, it must pay to the government a fine relative to the evaded amounts in previous period, but it also pays taxes based on its current size, which would be smaller if the firm had not evaded previously. The government thus benefits from evasion, but also forces compliance.

I illustrate this mechanism in a two-period dynamic model where a firm has high growth potential but has limited assets. The firm can evade taxes in both periods, but can be audited only in the second period, which happens with positive probability and implies a fine proportional to the amount evaded in the first period. This is a very standard tax evasion model following on the steps of Allingham and Sandmo (1972) and Yitzhaki (1974), adapted for a risk-neutral decision maker as in Cremer and Gahvari (1993), in a dynamic setting as in Andreoni (1992). The link between tax evasion and financial constraint has been raised in the literature by Andreoni (1992) in his model of personal income tax evasion. Gatti and Honorati (2008) and Alm, Liu, and Zhang (2018) have documented a positive correlation between evasion and lack of access to financial markets in developing economies.

There are good reasons to suppose that firms, and in particular small firms, are financially constrained. Even when they have high expected revenues in future periods, and only need liquidity to reach that stage, financial institutions may hesitate to lend due to asymmetric information problems. Lack of observability of the quality of projects, lack of enforcement of promises or limited contracting capacity lead to the fact that firms cannot borrow freely in financial markets[*] This is particularly true for smaller firms, with little collateral and track record. It is also more likely to hold in developing countries, where financial markets are less developed and enforcement of contracts is weaker.

In the model proposed in this paper, firms have a limited amount of assets that they can invest in a technology with decreasing marginal returns. They have a high growth potential, so that it is in their interest to invest everything they can, and only then distribute dividends. As already mentioned, they can boost their investments by evading taxes. When they do so, they contract a debt with the government, which they may pay with interest if they get audited in the future. The cost of this expected payment will determine the extent to which firms wish to engage in this risky activity of evasion.

In the model, firms have a technology with positive and decreasing marginal returns, so that smaller firms face very high marginal returns and tend to evade more. For them, the cost of paying a penalty on a marginal evaded unit is lower than the marginal benefit of expanding capacity. This leads to the fact that compliance improves for larger firms, a fact that is corroborated in the empirical literature about firm tax evasion, such as Pomeranz (2015) for Chile, Brockmeyer and Hernandez (2017) and Bachas and Soto (2021) for Costa Rica, Naritomi (2019) and Ulyssea (2018) for Brazil.

In the model, firms have strong incentives to evade, including when probability of audit is

[*]There are two maind types of asymmetric information problems: adverse selection and moral hazard. Each of them may lead to credit constraints. Albuquerque and Hopenhayn (2004) and Clementi and Hopenhayn (2006) are models of adverse selection and moral hazard where financial institutions propose a dynamic contract to a firm. Although the two contracts have important differences, in both models the firm is credit constrained and can only borrow up to a certain limit.

extremely high (even 100%). The reason, again, is that evasion eases their financial constraint. Even though expected payment on the evaded amount is greater than 100% of the evaded value, it may be worth doing it. The excessive expected payment is the counterpart of interests in a standard loan. The government, on the other hand, wants the firm to evade. Tax evasion provides the government with cheap finance, since it allows the government to tax firms that are larger in the second period, but still allows them to recover the evaded amounts with penalties. As the model shows, even when the government has a lump-sum tax at its disposal, it still may use distortionary taxation to take advantage of this double taxation opportunity.

2 Model of the behavior of the firm

To illustrate how growth incentives affect tax compliance, I propose a simple dynamic model with two periods. The firm maximizes expected dividends over two periods, and chooses compliance levels at each period, x_1 and x_2 over the firm value $\pi(A_1)$ and $\pi(A_2)$, derived from assets A_1 and A_2. The function $\pi(A_1)$ is increasing and concave. The government is free to set different taxes for each period, τ_1 and τ_2, and audits a proportion p of firms only in the second period. During the audit both periods are verified. If the declared amounts $x_1\pi(A_1)$ and $x_2\pi(A_2)$ are inferior to the truth, the taxpayer must pay the evaded taxes plus a proportional fine ϕ. One key feature of this model is that audits occur only in the second period with probability p, and check tax liability in both periods. This is similar to Andreoni (1992), but in his model is no taxation on income in the second period in his model and therefore also no verification of second period income. Defining as y_1 and y_2 the net value of the firm in each period, the time discount rate β and the share α of distributed dividents in the first period, the firm's problem can be formulated as follows:

$$\begin{aligned} \tilde{\Pi} &\equiv \max_{x_1,x_2,\alpha} \Pi \\ \Pi &= \alpha y_1 + \mathbb{E}[y_2] \end{aligned} \tag{1}$$

$$\begin{aligned} y_1 &= \pi(A_1)(1-\tau_1 x_1) \\ \mathbb{E}[y_2] &= (1-p)\Big(\pi(A_2)(1-\tau_2 x_2)\Big) + p\Big(\pi(A_2)(1-\tau_2(x_2+(1-x_2)\phi) - \pi(A_1)\tau_1(1-x_1)\phi\Big) \\ A_2 &= (1-\alpha)\pi(A_1)(1-\tau_1 x_1) \end{aligned}$$

In this formulation, I abstract from any problem related to time discounting. Moreover, I make the following assumptions:

Assumption 1. ***Taxation of excessive marginal returns***: *Only excessive marginal returns are taxed. This means that* $\pi'(A_t)(1-\tau_t) \geq 1$*, which implies that* $\pi(A_t)(1-\tau_t) \geq A_t$*. The latter can be interpreted as "no wealth taxation". This assumption puts upper bounds on the level of* τ_t *that the government can set at each period* $t \in \{1,2\}$*.*

The maximization of this problem by the firm implies the following facts. First, assumption 1 implies that the ratio α of dividends distributed in period 1 is equal to 0. This means that the firm uses the first period to accumulated assets and grow, and it is not worth to forgo growth in exchange of first period consumption. The second result regards first period compliance: depending on the firm's initial asset size A_1, firms will be *informal* (compliance $x_1 = 0$), *evaders* ($x_1 \in (0,1)$) or *compliers* ($x_1 = 1$). Larger firms comply more. In the second period, evasion will follow a bang-bang rule: if expected penalty is high, firms comply ($x_2 = 1$), else they will evade totally and run the risk of paying the fine ($x_2 = 0$).

This problem is solved by backwards induction. Determining the compliance level in the second period, x_2 is a static problem:

$$\frac{\partial \Pi}{\partial x_2} = \pi(A_2)\tau_2(p\phi - 1) \tag{2}$$

Since the entrepreneur is risk-neutral, the problem is lineas in x_2 and compliance in second period is either 1 or 0 depending on the values of the enforcement parameters. There is full compliance if $p\phi > 1$ and no compliance (i.e. full evasion) if $p\phi < 1$.

The dynamic problem appears as the entrepreneur chooses the compliance level in the first period, because this affects outcomes in the following period. The first order condition for x_1 yields:

$$\frac{d\Pi}{dx_1} = \pi(A_1)\tau_1 \left\{ \underbrace{\phi p}_{\text{lower expected fine}} \quad \underbrace{-\pi'(A_2)(1-\tau_2(x_2+(1-x_2)p\phi)}_{\text{lower expected profits}} \right\} \tag{3}$$

A marginal increase in compliance dx_1 in the first period reduces the expected fines paid in the second period (a gain to the firm) but decreases its profits in the second period, since it can grow less. The optimal compliance level is the one that makes the marginal gains (in terms of lower fines) equal to the marginal costs (in terms of lower second period profits). The optimum is achieved when:

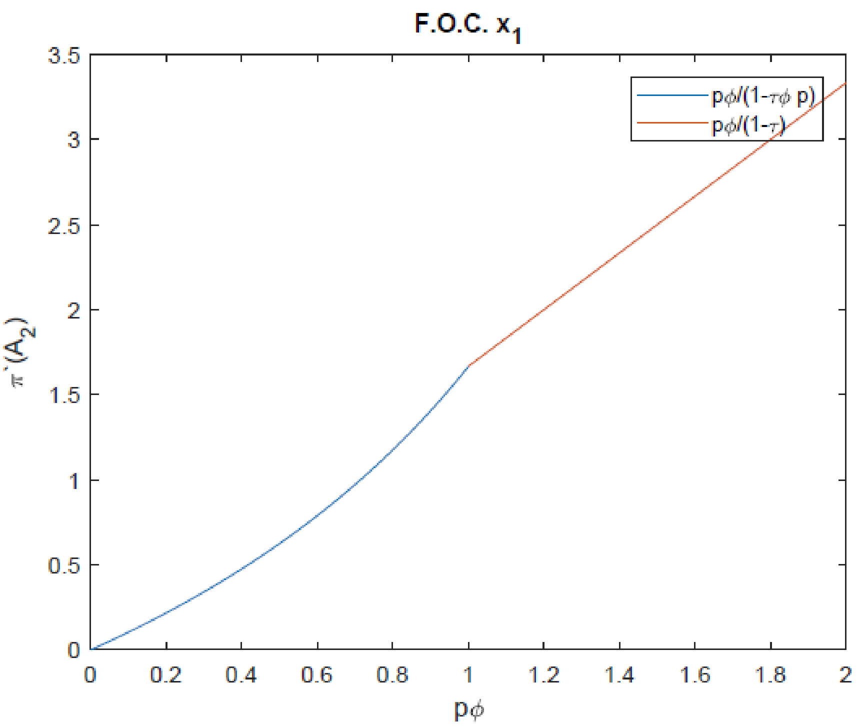
$$\pi'(A_2) = \frac{\phi p}{1-\tau_2(x_2+(1-x_2)p\phi)} \tag{4}$$

The right hand side of this equation is a constant that depends solely on the parameters of the tax system. As can be seen from the first order condition with respect to x_2 (compliance in period $t = 2$), if $p\phi > 1$ we have $x_2 = 1$ and if $p\phi < 1$ the optimum is full evasion, $x_2 = 0$. Therefore, the first order condition in equation 4 can be rewritten as:

$$\pi'(A_2) = \begin{cases} \frac{\phi p}{1-\tau_2} & \text{if } p\phi > 1 \\ \frac{\phi p}{1-\tau_2 p\phi} & \text{if } p\phi < 1 \end{cases} \tag{5}$$

This equation maps all possible values of $p\phi$ to the correspondent optimal A_2. The marginal return to capital $\pi'(A_2)$ is monotonically increasing in ϕp, despite the discontinuity that happens at $\phi p = 1$, as illustrated in figure 1. This means that the optimal level of assets A_2 is decreasing in the enforcement parameters.

Figure 1: Mapping of expected fine to marginal benefits at the optimum



There is one unique level of A_2 that is associated with the first order condition, which I call A_2^*. This level determines the behavior of the firm with regard to evasion in the first period, that is, the amount of compliance x_1.

$$A_2^* = \begin{cases} \pi'^{-1}[\frac{\phi p}{1-\tau_2}] & \text{if } p\phi > 1 \\ \pi'^{-1}[\frac{\phi p}{1-\tau_2 p\phi}] & \text{if } p\phi < 1 \end{cases} \tag{6}$$

To achieve A_2^*, the firm can decide the level of compliance x_1. More compliance (higher x_1) means higher costs in the first period and less assets in the second period. The problem is that x_1 is bounded between 0 and 1, and this sets boundaries on the possible range of A_2^* that can be feasible. If a firm is very small for example, and the value for A_2^* is very high relative to the initial size, the firm will not reach it even if it evades fully.

Define $\bar{x_1}$ as the level of x_1 such that the first order condition holds with equality. The compliance rule can be stated as follows:

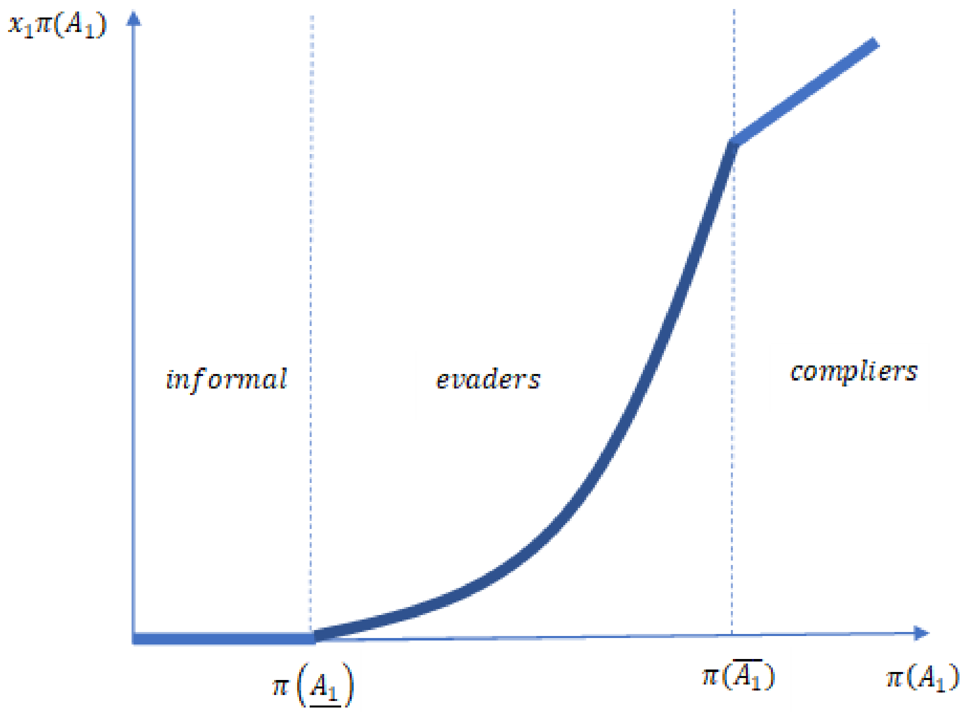
$$x_1 = \begin{cases} 0 & \text{if } \pi(A_1) \leq A_2^* \\ x_1^* & \text{if } \pi(A_1)(1-\tau_1) \leq A_2^* \leq \pi(A_1) \\ 1 & \text{if } \pi(A_1)(1-\tau_1) \geq A_2^* \end{cases} \tag{7}$$

Notice also that since A_2^* is a constant, equation 7 also defines unique thresholds of A_1 that define in which of the three categories the firm belongs: informal, evader or compliant. These thresholds are $\underline{A_1}$ and $\bar{A}_1$, such that:

$$x_1 = \begin{cases} 0 & \text{if } A_1 \leq \underline{A_1} \\ x_1^* & \text{if } \underline{A_1} < A_1 \leq \bar{A}_1 \\ 1 & \text{if } \bar{A}_1 < A_1 \end{cases} \tag{8}$$

In the end the government observes $x_1\pi(A_1)$ declared by the firm, which follows a schedule with respect to the possible values of A_1 as in figure 2:

Figure 2: Optimal evasion rates and initial firm size



Though the analysis in this paper can be done for low of high levels of $p\phi$, the analogy of evasion with a loan becomes more interesting in the case where $p\phi > 1$. In fact, in standard models of tax evasion, this situation tends to lead to full compliance of firms or individuals. However, in the current model this is not necessarily the case, since this cost is weighed against the return of evading taxes and re-investing. For simplicity, the remainder of this paper will make the assumption that $p\phi > 1$.

Assumption 2. ***Positive interest rates:*** $p\phi > 1$. *Costs to evasion are high enough so that the cost of the expected penalty is greater than the value of the initial tax liability.*

Moreover, I will assume that A_1 is such that the firm is an *evader*, that is its compliance level x_1 lies strictly between 0 and 1 and is determined as an interior solution to the maximization problem.

Assumption 3. ***Evader:*** $A_1 \in (\underline{A}_1, \bar{A}_1)$*, such that* $x_1 = x_1^*$.

2.1 Comparative statics for the firm's problem

Compliance increases monotonically with size, simply because the benefit from evading is decreasing with size due to the concavity of the profit function. Apart from the two polar cases in which there is no evasion (small firms) or full compliance (large firms), there is partial compliance x_1^*. Partial compliance increases with the firm' initial size and with the probability of being audited, as expected. Full differentiation of the first order condition (equation 4) gives:

$$\frac{dx_1^*}{dA_1} = \frac{\pi'(A_1)}{\pi(A_1)} \frac{1-\tau_1 x_1^*}{\tau_1} > 0 \tag{9}$$

Unambiguously, larger firms comply more with taxes than smaller firms, for a given technology. This result is compatible with stylized facts documented in the literature of tax evasion by firms, in particular in developing countries, as mentioned in the first section. In this model, this happens because a larger firm needs to evade less than a small firm to achieve the same size in the second period. A large firm would be risking too much downside by growing beyond the target A_2 given by the first order condition. Since they both want the same target A_2 (for a given technology), the smaller firm has to evade more.

Compliance also unambiguously increases with the probability of being audited, as would be expected. Higher probability of penalty increases compliance in the first period, because it reduces the marginal return of evasion.

$$\frac{dx_1^*}{dp} = -\frac{\phi}{1-\tau_2} \frac{1}{\pi''(A_2)\tau_1\pi(A_1)} = \frac{1}{\varepsilon_{\pi'}} \frac{1-x_1\tau_1}{p\tau_1} > 0 \tag{10}$$

Where $\varepsilon_{\pi'} \equiv -\frac{\pi''(A_2)}{\pi'(A_2)} A_2$ is the elasticity of the marginal returns to assets. The second equality uses the fact that $\pi'(A_2) = p\phi/(1-\tau 2)$ and $A_2 = \pi(A_1)(1-x_1\tau_1)$.

Tax rates also have an impact on compliance levels. However, first period tax rates τ_1 have no impact on the target size A_2 of assets in the second period. Increasing taxes in the first period means indeed that the firm will evade more to achieve that target.

$$\frac{dx_1^*}{d\tau_1} = -\frac{x_1^*}{\tau_1} < 0 \tag{11}$$

Second period taxes τ_2 have no impact on second period decisions, as already discussed, since x_2 depends only on $p\phi$ being greater or smaller than 1. However, they have an impact on the target level A_2. This yields an expression that is very similar to the derivative of x_1^* with respect to p, seen above in equation 10:

$$\frac{dx_1^*}{d\tau_2} = -\frac{\pi'(A_2)}{\pi''(A_2)} \frac{1}{\tau_1\pi(A_1)(1-\tau_2)} = \frac{1}{\varepsilon_{\pi'}} \frac{1-x_1\tau_1}{(1-\tau_2)\tau_1} > 0 \tag{12}$$

By reducing net returns on second period profits, τ_2 increases first period compliance unambiguously. Finally, we can check the sensitivity of x_1^* with respect to the penalty rate ϕ.

$$\frac{dx_1^*}{d\phi} = \frac{1}{\varepsilon_{\pi'}} \frac{1 - x_1\tau_1}{\phi\tau_1} > 0 \tag{13}$$

Equations 10, 11, 12 and 13 are useful to solve the government's problem, presented next.

3 The problem of a revenue maximizing government

The government raises revenues over the two periods using taxes and audits. Audits in this setting give the government the chance to tax twice the same tax liability. The reason is that that firms use evaded taxes to increase their size in the second period, which also increases tax liability in the second period. If the firm complies in the second period, the government benefits from the firms' evasion, because it taxes a larger firm. By auditing a firm that grew thanks to evasion, the tax authority makes sure that the full liability of the second period is taxed, and also the full liability in the first period. However, this amounts to double taxation, since the firm would have had another size in the second period if it had paid the full liability in the first period. This is illustrated formally in what follows. Define the government's revenues over two periods for a certain firm of initial size A_1 as $\mathcal{G}(A_1)$:

$$\mathcal{G}(A_1) = \underbrace{\tau_1 x_1 \pi(A_1) + \tau_2 x_2 \pi(A_2)}_{\text{tax revenues}} + \underbrace{p\phi(\tau_1(1-x_1)\pi(A_1) + \tau_2(1-x_2)\pi(A_2))}_{\text{audit revenues}} \tag{14}$$

Where x_1, x_2 are defined in the firm's maximization problem. Assume for simplicity that there is no cost of carrying out an audit. This assumption allows us to treat p and ϕ as equivalent. Indeed, what matters in the problem is $p\phi$, which will henceforth be treated as a single parameter. The maximization problem of the government is:

$$\begin{aligned} &\max_{\tau_1,\tau_2,p\phi} \mathcal{G}(A_1) = \tau_1 x_1 \pi(A_1) + \tau_2 \pi(A_2) + p\phi(\tau_1(1-x_1)\pi(A_1) - p\psi \\ &\text{s.t.} \qquad \pi'(A_1)(1-\tau_1) \geq 1 \\ &\qquad\qquad \pi'(A_2)(1-\tau_2) \geq 1 \end{aligned} \tag{15}$$

Taking the (unrealistic) assumption that the government knows what is the intial size A_1 of the firms, maximizing $\mathcal{G}$ gives optimal values for all policy parameters: τ_1, τ_2, p and ϕ. Taking first order conditions yields the following expressions:

$$\frac{d\mathcal{G}}{d\tau_1} = x_1\pi(A_1) + \tau_1\frac{dx_1}{d\tau_1}\pi(A_1) + \tau_2 x_2\pi'(A_2)\frac{dA_2}{d\tau_1} + p\phi(1-x_1)\pi(A_1) - p\phi\tau_1\frac{dx_1}{d\tau_1}\pi(A_1) \tag{16}$$

This expression is simplified by differentiating the first order condition of te firm's problem, equation 4, and getting $dA_2/d\tau_1 = 0$ and $dx_1/d\tau_1 = -x_1/\tau_1$.

$$\frac{d\mathcal{G}}{d\tau_1} = \begin{cases} p\phi\pi(A_1) & \text{if } x_1 = x^* \text{ or } x_1 = 0 \\ \pi(A_1) & \text{if } x_1 = 1 \end{cases} \tag{17}$$

This result means that as long as τ_1 respects the assumption of no wealth taxation, increasing first period taxes always raises more revenue, because it does not change the incentives to grow into the second period but increases revenues from penalties (for evaders and informal) or from first period taxation (for compliers). It follows that the revenue-maximizing tax rate in the first period is the highest possible, that is, the one such that $\pi'(A_1)(1-\tau_1) = 1$, or $\tau_1 = 1 - \pi'(A_1)^{-1}$. Indeed, τ_1 works as an interest rate in the implicit loan taken by the firm, and as a lender, the government benefits from setting it to the highest level possible (i.e., the higher level at which the firm is willing to borrow money).

As for the second period tax τ_2, the first order condition yields:

$$\begin{aligned} \frac{\partial\mathcal{G}}{\partial\tau_2} &= \tau_1\frac{dx_1}{d\tau_2}\pi(A_1) + \pi(A_2) + \tau_2\pi'(A_2)\frac{dA_2}{d\tau_2} - p\phi\frac{dx_1}{d\tau_2}\tau_1\pi(A_1) \\ &= \underbrace{\pi(A_2)\big(x_2 + (1-x_2)p\phi\big)}_{\text{direct pos. effect}} - \underbrace{\frac{A_2}{(1-\tau_2)}\Big(\frac{\pi'(A_2)-1}{\varepsilon_{\pi'}}\Big)}_{\text{indirect size effect}} \end{aligned} \tag{18}$$

The above equation is equal to zero if and only if:

$$\tau_2 = 1 - \frac{\pi'(A_2)-1}{\varepsilon_{\pi'}}\frac{A_2}{\pi(A_2)\big(x_2 + (1-x_2)p\phi\big)} \tag{19}$$

The revenue-maximizing government faces different incentives for the optimization of τ_1 and τ_2. Whereas raising τ_1 is always revenue increasing, this is not the case of τ_2, since higher tax rates in the second period reduce the firms' incentives to grow.

The first order condition of the government's problem with respect to the expected penalty for evasion $p\phi$ is given by:

$$\frac{\partial \mathcal{G}}{\partial p\phi} = \tau_1 \pi(A_1)\frac{\partial x_1}{\partial p}\Big(1 - \frac{p\phi}{1-\tau_2} + \frac{\phi(1-x_1)}{dx_1/dp}\Big)$$
$$= \underbrace{\pi(A_1)\tau_1(1-x_1) + \pi(A_2)(1-x_2)\tau_2}_{\text{additional audit revenues}} - \underbrace{-\frac{A_2}{p\phi}\Big(\frac{\pi'(A_2)-1}{\varepsilon_{\pi'}}\Big)}_{\text{lower tax revenues in second period}} \tag{20}$$

An increase in the probability of audits increases unambiguously the audit revenues, but decreases the tax revenues in the second period. The decrease in second period taxes comes from the fact that the firm is discouraged from evading in the first period, and therefore achieves a smaller size in the second period. The tax base is lower, yielding less taxes to the government.

This expression is equal to zero if and only if:

$$p\phi = \frac{A_2 \frac{\pi'(A_2)-1}{\varepsilon_{\pi'}}}{\pi(A_1)\tau_1(1-x_1) + \pi(A_2)(1-x_2)\tau_2} \tag{21}$$

In this problem, it is not optimal for the government to maximize compliance by setting the punishment ϕ to infinity, for example. In fact, increasing the penalty increases compliance, but discourages the firm from growing. A revenue maximizing government prefers firms to grow before taxing them, and uses the punishment to recover part of the evaded amount used to invest, that is, the part that was implicitly borrowed by the firm.

4 Discussion and conclusion

In this model, a firm evades to ease its financial constraint. The incentives to evade come from the fact that the marginal return on a unit of evaded tax is greater than the marginal penalty that it will have to pay on this amount. A firm evades until the expected marginal profit from evasion is equal to the marginal expected penalty payment, as is common in any classical model of tax evasion. The contribution relative to the literature is that it sheds light on the dynamic incentives that arise from the possibility of investing evaded resources productively.

One striking feature of the model is that firms may have the incentive to evade taxes even if audit probabilities are very high, even if it is equal to 100%, if the marginal return of an investent is high enough. The reason for is that firms use evasion as a loan from the government, where the expected penalty ($p\phi$) take the role of interests on this loan. Firms that have a high revenue potential in the second period find it economically advantageous to take this loan and pay the interest. This result echoes Andreoni (1992), who also found that some financially constrained individuals would evade personal income tax to smooth consumption, even if audit probability in the second period was 100%.

The other point made in this model is that evasion in this setting may provide a form of cheap finance for the government. The government benefits from evasion by taxing second period revenues, but it can still claim first period evaded tax liability. For this reason, the government would like to induce evasion in the first period. By doing that, it spurs firm growth, and still accumulates a credit with the companies, which it can claim by auditing them. The consequence is that governments may use distortionary taxation even in a setting with no information asymmetry and if a lump sum tax is available. Although governments in practice grant tax holidays for some taxes to nascent companies, taxing them more (short of making capital return negative) raises expected government revenues by increasing the government's claim on evaded taxes in the economy.

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Appendices

A Derivation of proposition 1

Proof. The results above are derived by using the derivatives of x_1 with respect to each parameter (equations 10, 11, 12 and 13), and the derivatives of $\tilde{\Pi}$ and $\mathcal{G}$ with respect to each parameter. Thanks to the envelope theorem we can simply write the derivatives of $\tilde{\Pi}$ with respect to the parameters as:

$$\frac{d\tilde{\Pi}}{d\tau_1} = \frac{\partial\tilde{\Pi}}{\partial\tau_1} = -p\phi(1-x_1)\pi(A_1) \tag{22}$$

$$\frac{d\tilde{\Pi}}{d\tau_2} = \frac{\partial\tilde{\Pi}}{\partial\tau_2} = -\pi(A_2) \tag{23}$$

$$\frac{d\tilde{\Pi}}{dp} = \frac{\partial\tilde{\Pi}}{\partial p} = \phi(1-x_1)\tau_1\pi(A_1) \tag{24}$$

$$\frac{d\tilde{\Pi}}{d\phi} = \frac{\partial\tilde{\Pi}}{\partial\phi} = p(1-x_1)\tau_1\pi(A_1) \tag{25}$$

The derivatives of $\mathcal{G}$ are:

$$\begin{aligned}\frac{d\mathcal{G}}{d\tau_1} &= x_1\pi(A_1) + \tau_1\frac{dx_1}{d\tau_1}\pi(A_1) + \tau_2 x_2\pi'(A_2)\frac{dA_2}{d\tau_1} + p\phi(1-x_1)\pi(A_1) - p\phi\tau_1\frac{dx_1}{d\tau_1}\pi(A_1)\\ &= p\phi\pi(A_1)\end{aligned} \tag{26}$$

$$\begin{aligned}\frac{\mathcal{G}}{d\tau_2} &= \tau_1\pi(A_1)\frac{dx_1}{d\tau_2}(1-\tau_2\pi'(A_2)-p\phi) + \pi(A_2)\\ &= \frac{A_2}{\varepsilon_{\pi'}(1-\tau_2)}(1-\pi'(A_2)) + \pi(A_2)\end{aligned} \tag{27}$$

$$
\begin{aligned}
\frac{\mathcal{G}}{dp} &= \tau_1\pi(A_1)\frac{dx_1}{dp}(1-\tau_2\pi'(A_2)-p\phi)+\phi(1-x_1)\tau_1\pi(A_1)-\psi \\
&= \frac{A_2}{\varepsilon_{\pi'}p}(1-\pi'(A_2))+\phi(1-x_1)\tau_1\pi(A_1)-\psi
\end{aligned} \tag{28}
$$

$$
\frac{d\mathcal{G}}{d\phi} = \frac{A_2}{\varepsilon_{\pi'}\phi}(1-\pi'(A_2))+p(1-x_1)\tau_1\pi(A_1) \tag{29}
$$

Setting $\lambda = 1$ as a consequence of lump-sum taxes, we get that the expressions for the derivative of the objective function are just the sums of the derivatives of $\tilde{\Pi}$ and $\mathcal{G}$:

$$
\{\tau_1\} \qquad \frac{d\mathcal{L}}{d\tau_2} = -(1-x_1)+1 > 0 \tag{30}
$$

The above expressions shows that it is always advantageous to increase τ_1, since it induces the firm to evade more, raising the possibility of double taxation via audits in the second period. This double taxation is a cheap way to finance the government.

$$
\begin{aligned}
\{\tau_2\} \qquad \frac{d\mathcal{L}}{d\tau_2} &= \frac{A_2}{\varepsilon_{\pi'}(1-\tau_2)}(1-\pi'(A_2)) = 0 \\
&\text{iff } 1-\pi'(A_2) = 0 \\
&\text{iff } 1-\frac{p\phi}{1-\tau_2} = 0 \\
&\text{iff } \tau_2 = \phi p - 1
\end{aligned} \tag{31}
$$

As we will see below, τ_2 and ϕ have interdependent values, but are not determined. Therefore, we can simply set $\tau_2 = 0$ as one possible solution to the problem.

$$
\begin{aligned}
\{\phi\} \qquad \frac{d\mathcal{L}}{d\phi} &== \frac{A_2}{\varepsilon_{\pi'}\phi}(1-\pi'(A_2)) = 0 \\
&\text{iff } 1-\pi'(A_2) = 0 \\
&\text{iff } 1-\frac{p\phi}{1-\tau_2} = 0 \\
&\text{iff } \tau_2 = \phi p - 1
\end{aligned} \tag{32}
$$

This is exactly the same expression for the optimality condition of τ_2. Since we set $\tau_2 = 0$, it follows that $\phi = p^{-1}$. We can now find the value for p.

$$
\begin{aligned}
\{p\} \qquad & \frac{d\mathcal{L}}{dp} = \frac{A_2}{\varepsilon_{\pi'} p}(1 - \pi'(A_2)) - \psi = 0 \\
& \text{iff } p = \frac{A_2}{\varepsilon_{\pi'} \psi}
\end{aligned} \tag{33}
$$

$\square$