LAKSHYA JEE

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Electric Charges and Field

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Today's GOALS!

- Motion of charge particle in electric field

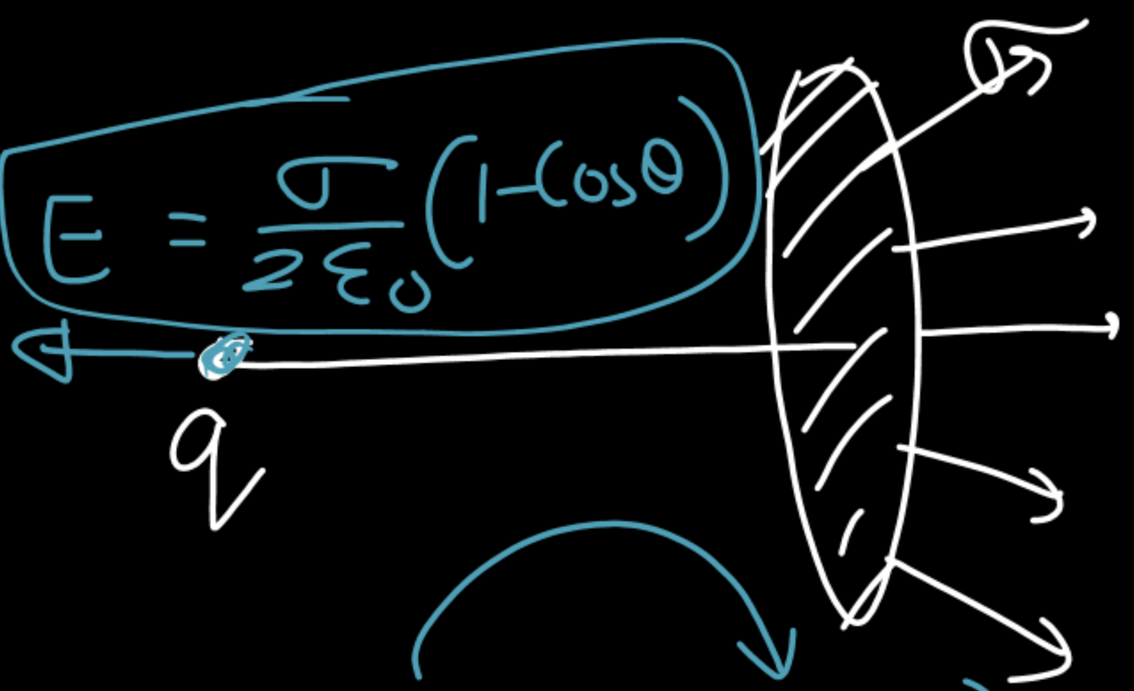
Doubts

3. A charge q is placed at some distance along the axis of a uniformly charged disc of surface charge density σ. The flux due to the charge q through the disc is ϕ. The electric force on charge q exerted by the disc is

(a) $\sigma\phi$ (b) $\frac{\sigma\phi}{4\pi}$

(c) $\frac{\sigma\phi}{2\pi}$ (d) $\frac{\sigma\phi}{3\pi}$

$$E = \frac{\sigma}{2\varepsilon_0}(1-\cos\theta)$$

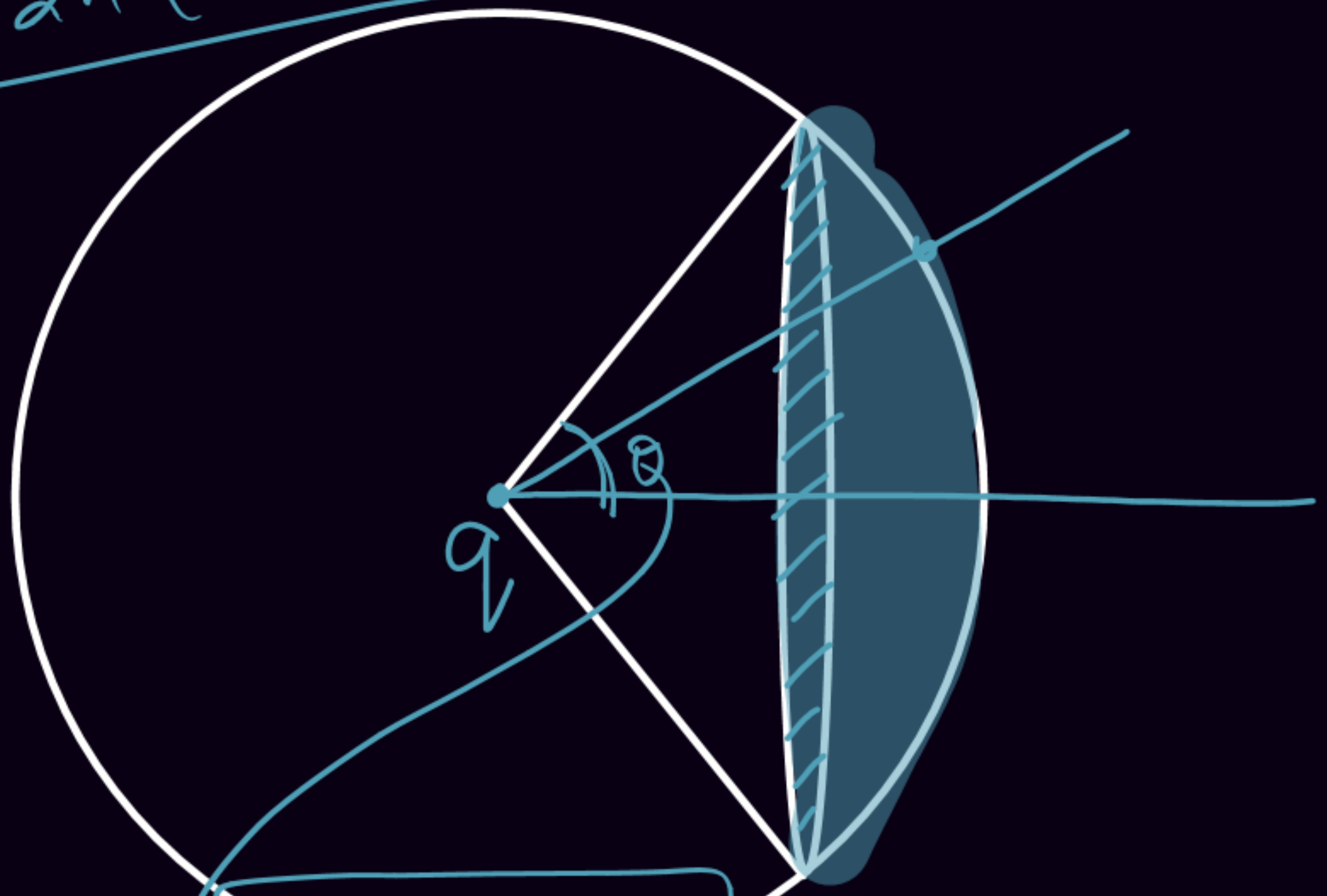


$$\boxed{\phi = \frac{q}{2\varepsilon_0}(1-\cos\theta)}$$

$$F = \frac{q\sigma}{2\varepsilon_0}(1-\cos\theta)$$

$$\boxed{F = \phi\sigma}$$

$$\boxed{\Omega = 2\pi(1-\cos\theta)}$$



plane angle

$\frac{q}{\varepsilon_0}$

① $\Phi_{\text{complete sphere}} = \frac{q}{\varepsilon_0}$

$4\pi R^2 \longrightarrow \frac{q}{\varepsilon_0}$

$\frac{\cancel{q}}{\cancel{\varepsilon_0}\, 4\pi R^2}$ $\boxed{A_{bowl}} = \frac{\cancel{q}}{2\cancel{\varepsilon_0}}(1-\cos\theta)$

$A_{Bowl} = 2\pi R^2(1-\cos\theta)$

$\Omega_{\text{Solid angle}} = \frac{A_{Bowl}}{R^2} = \boxed{2\pi(1-\cos\theta)}$

Doubts

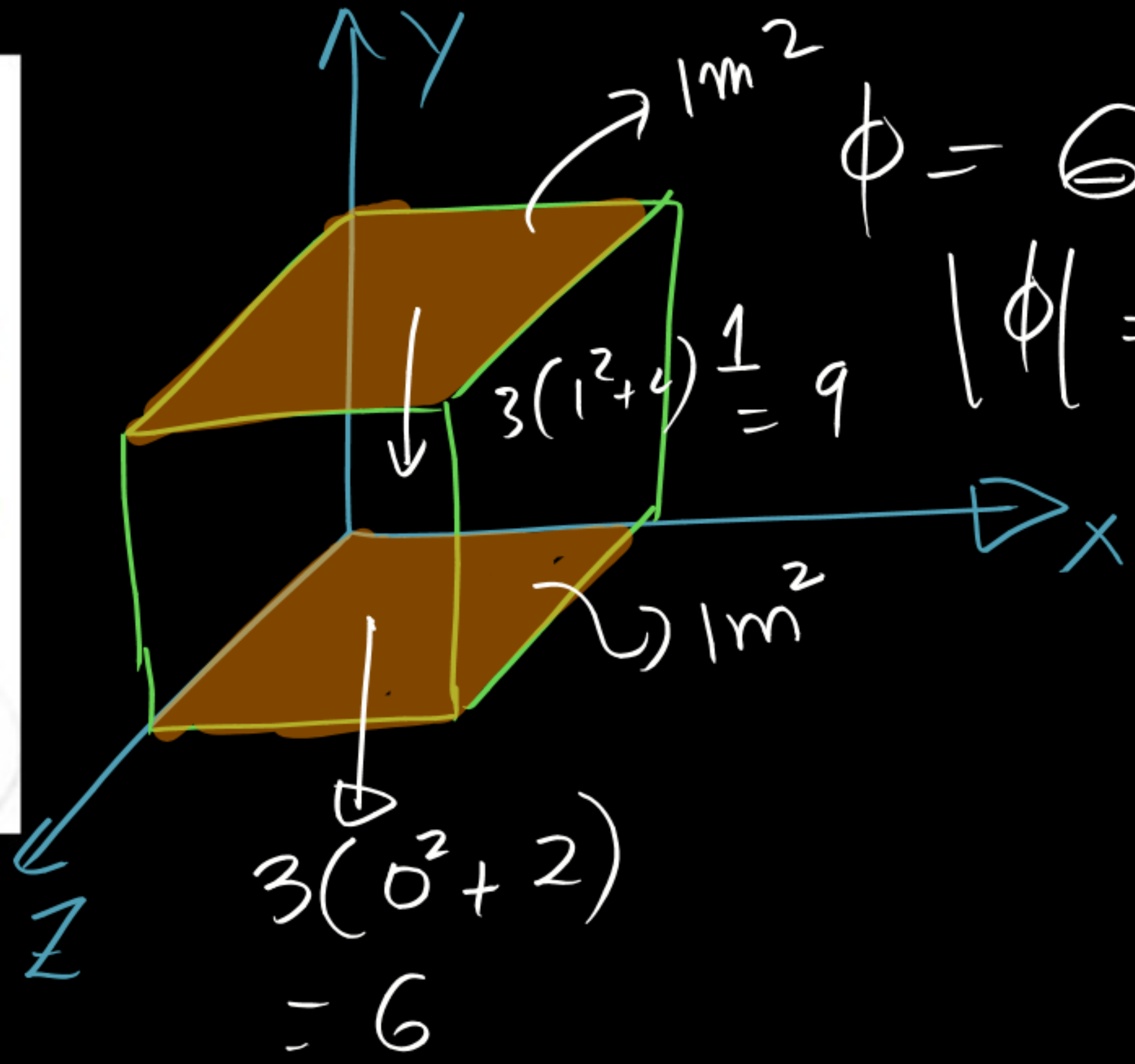
An electric field given b

$$\vec{E} = \boxed{4\hat{i}} - \boxed{3(y^2+2)\hat{j}}$$

pierces Gaussian cube of side 1 m placed at origin such that its three sides represents x, y and z axes. The net flux enclosed within the cube is

(a) 3 (b) 4

(c) 5 (d) zero



$\phi = 6 - 9$

$|\phi| = |-3|$

$$\rho = \rho_0 r \quad (0 < r \le R)$$
$$= 0 \quad (r > R)$$

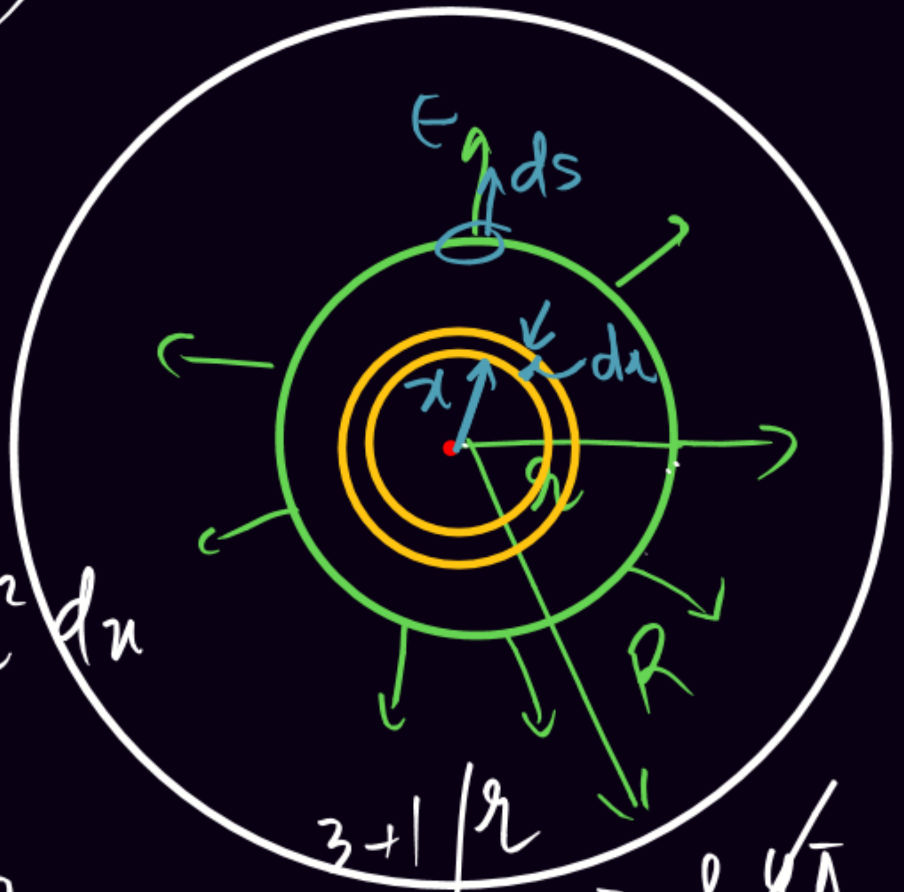
$$dV = (4\pi x^2)\,dx$$

$$\rho = \frac{dq}{dV}$$

$$dq = \rho\, dV$$

$$\int dq = \int_0^r (\rho_0 x)\, 4\pi x^2\, dx$$

$$q = \rho_0 4\pi \left[\frac{x^{3+1}}{3+1}\right]_0^r = \frac{\rho_0 \cancel{4}\pi}{\cancel{4}}\left(r^4 - 0^4\right) = \boxed{\rho_0 \pi r^4}$$

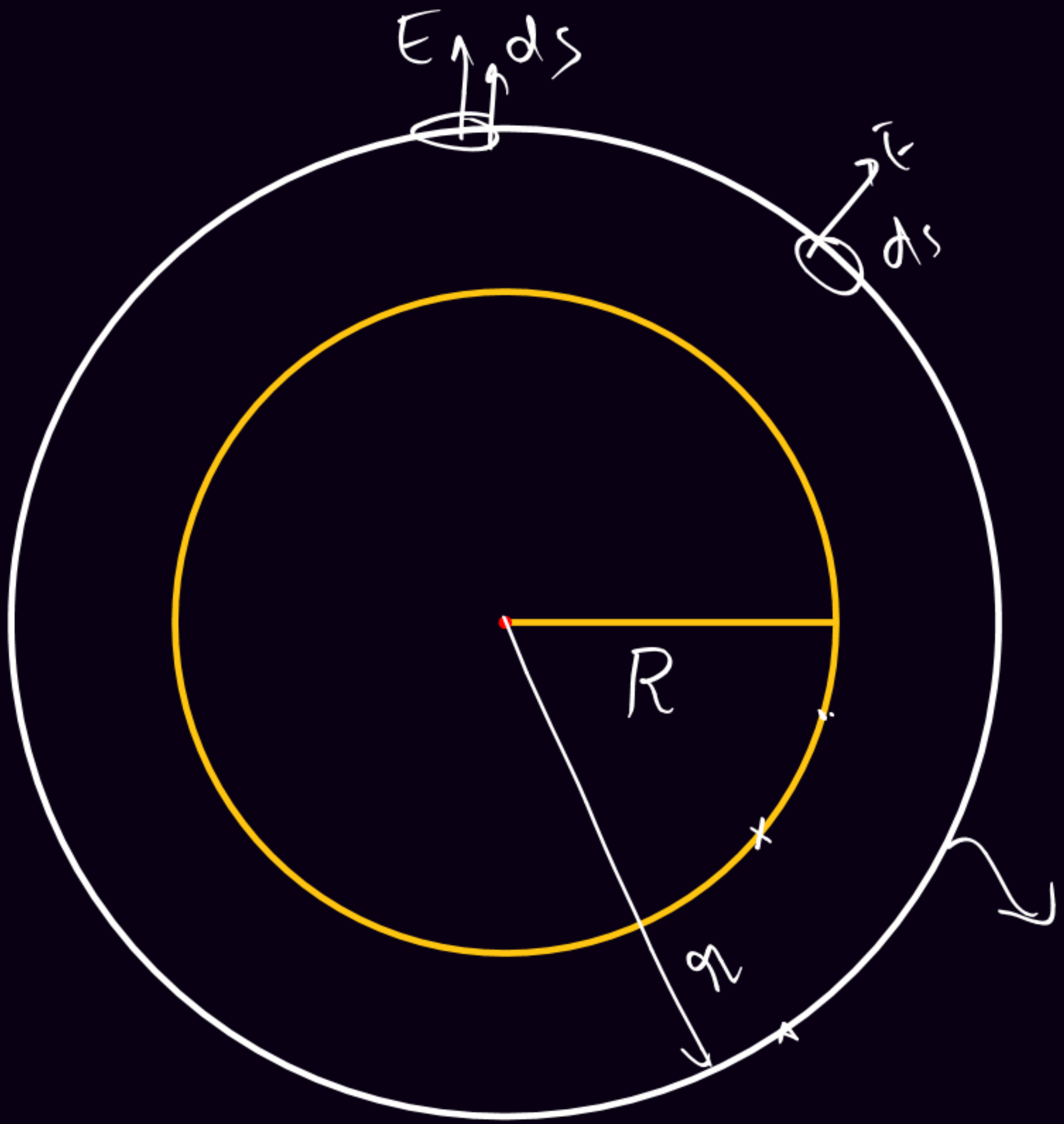


$$\oint \vec{E}\cdot d\vec{s} = \frac{q_{in}}{\varepsilon_0}$$

$$E \oint ds \cos 0 = \frac{\rho_0 \pi r^4}{\varepsilon_0}$$

$$E\, 4\cancel{\pi} \cancel{r^2} = \frac{\rho_0 \cancel{\pi} r^{\cancel{4}\,2}}{\varepsilon_0}$$

$$\boxed{E_{in} = \frac{\rho_0}{4\varepsilon_0} r^2}$$



Gaussian surface

$$E\,4\cancel{\pi}r^2 = \frac{(\rho_0 \cancel{\pi} R^4)}{\varepsilon_0}$$

$$E_{out} = \frac{\rho_0 R^4}{4\varepsilon_0 r^2}$$

$$\int dq = \int_0^R \rho_0 4\pi\, x^3\, dx$$

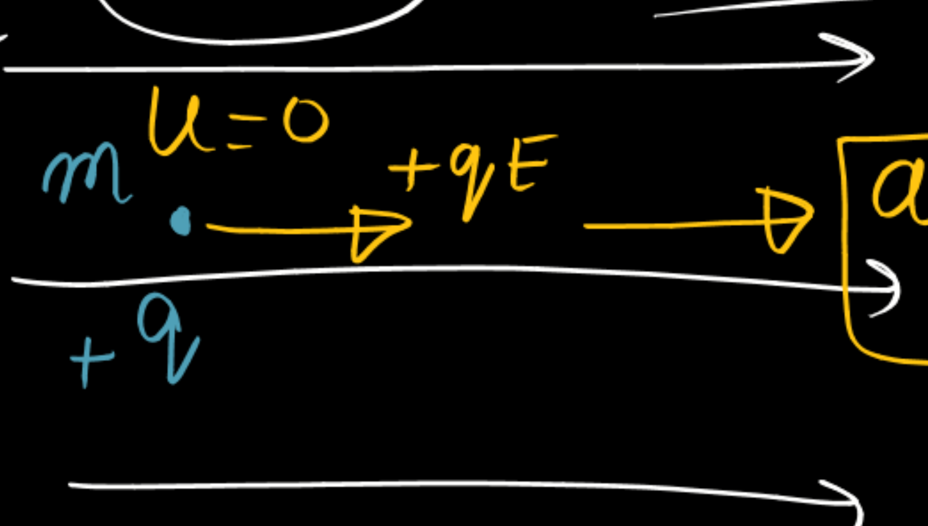
$$\rho_0 4\pi \left.\frac{x^4}{4}\right|_0^R$$

$$\boxed{\rho_0 \pi R^4}$$

Motion of charge particle in electric field

① Uniform field ② Non uniform field

Case 1 $u = 0$ straight line



$$\boxed{a = \frac{qE}{m}}$$

$F = qE$; $a = \frac{F}{m}$

$V = u + at$

$S = ut + \frac{1}{2}at^2$

$V^2 = u^2 + 2as$

Find v after time t

$V = 0 + \frac{qE}{m}t$

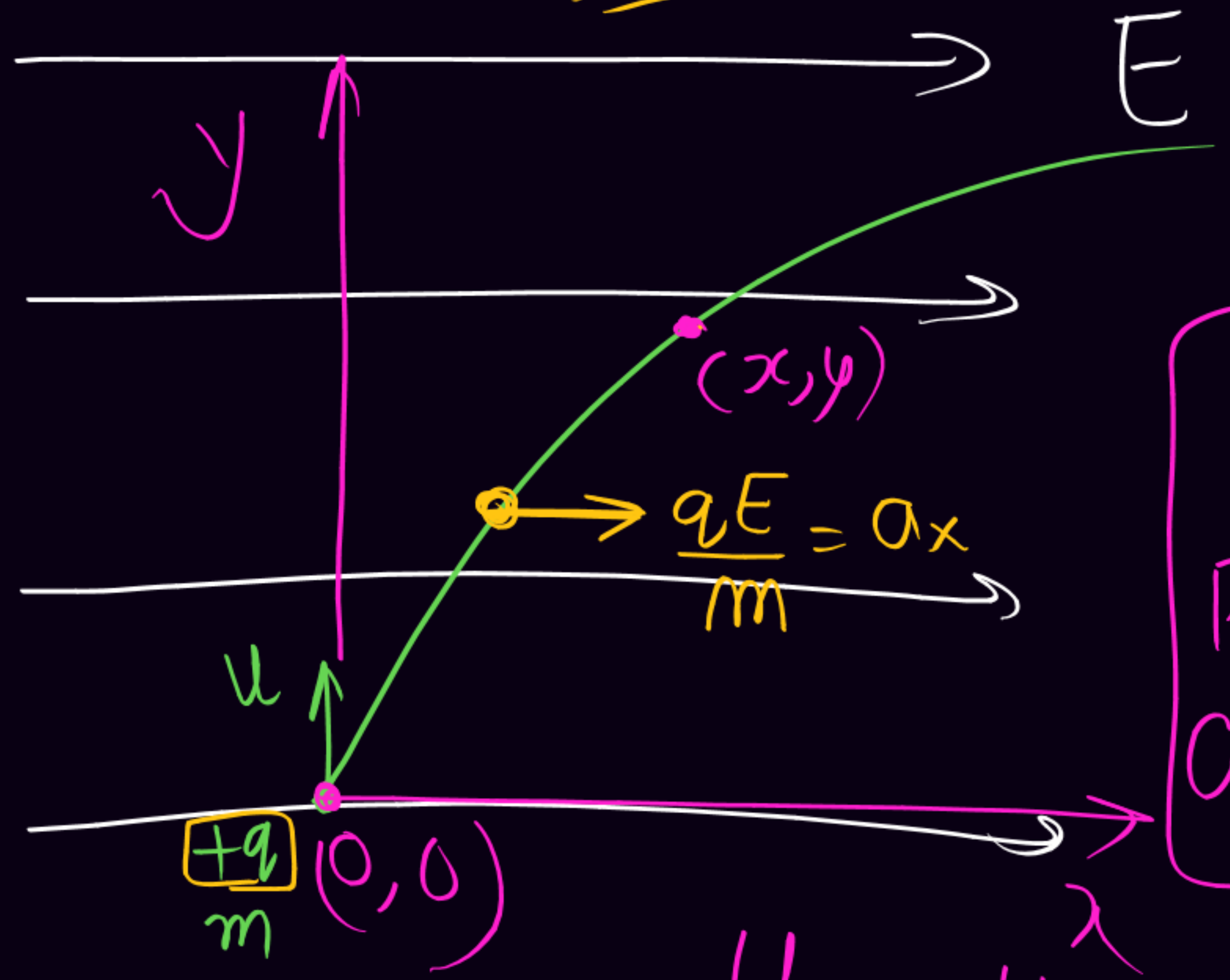
Distance travel in time t

$s = 0 + \frac{1}{2}\frac{qE}{m}t^2$

$$\boxed{s = \frac{qE}{2m}t^2}$$

Case 2

gravity is neglected



Equation of trajectory → Relation between x & y → path

- Line $y = mx + c$
- Parabola $y^2 = 4ax$
- Circle $\Rightarrow x^2 + y^2 = R^2$

$$u_y = u \qquad a_y = 0$$

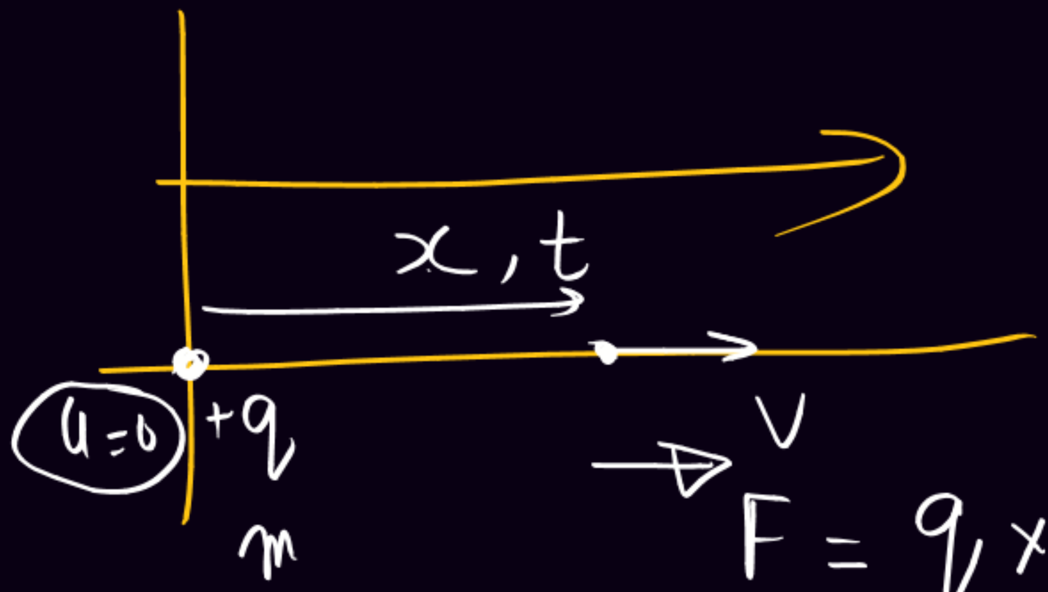
$$u_x = 0 \qquad a_x = \frac{qE}{m}$$

$$x = u_x t + \frac{1}{2} a_x t^2 \Rightarrow x = 0 + \frac{1}{2} \frac{qE}{m} t^2 \quad (1)$$

$$y = u_y t + \frac{1}{2} a_y t^2 \Rightarrow y = ut + 0 \quad (2)$$

$$x = \frac{1}{2} \frac{qE}{m} \left(\frac{y}{u} \right)^2$$

$$\boxed{y^2 = \frac{2mu^2}{qE} x}$$ ← Parabola



$E = E_0 x \hat{i}$

find (i) v as a function of $\boxed{t}$

(ii) v as a function of x.

$\rightarrow F = q \times E_0 x$

a

$\frac{dv}{dt}$ $\quad$ $v\frac{dv}{dx}$ ✓

$a = \frac{F}{m} = \left(\frac{qE_0}{m}\right)x$

$v\frac{dv}{dx} = \frac{qE_0}{m} x$

$$\int_0^v v\,dv = \frac{qE_0}{m}\int_0^x x\,dx$$

$$\frac{v^2}{\cancel{2}} = \frac{qE_0}{m}\frac{x^2}{\cancel{2}}$$

$$\boxed{v = \sqrt{\frac{qE_0}{m}}\,x}$$

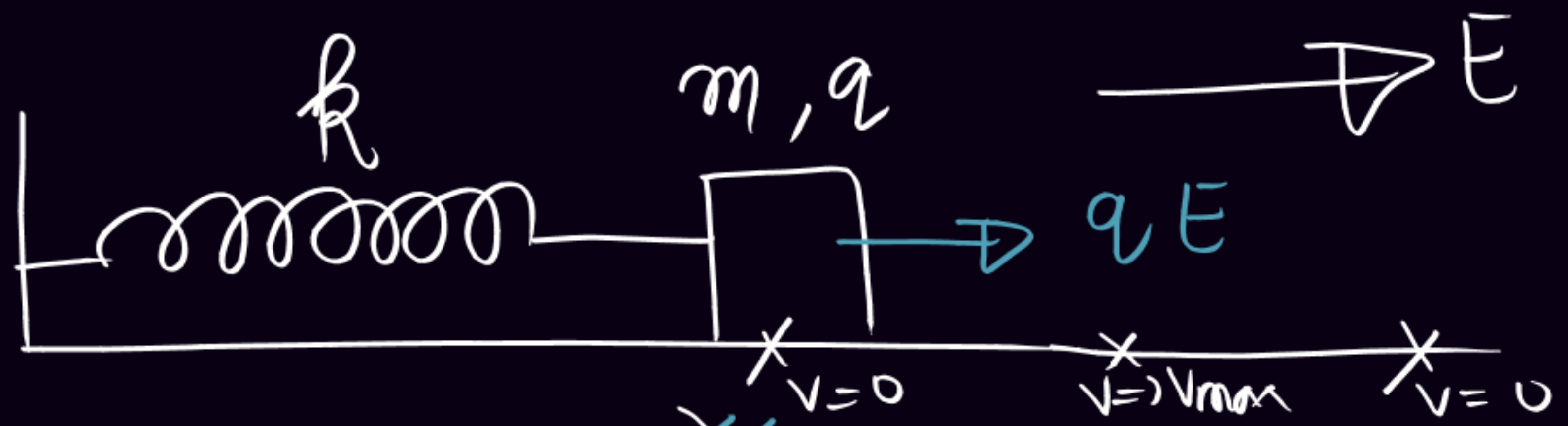
$$V = \sqrt{\frac{qE_0}{m}}\ x$$

$$\alpha = \sqrt{\frac{qE_0}{m}}$$

$$\frac{dx}{dt} = \alpha\ x$$

$$\frac{dx}{x} = \alpha\ dt$$

$$\ln x \Big|_0^x = \alpha t \Big|_0^t$$



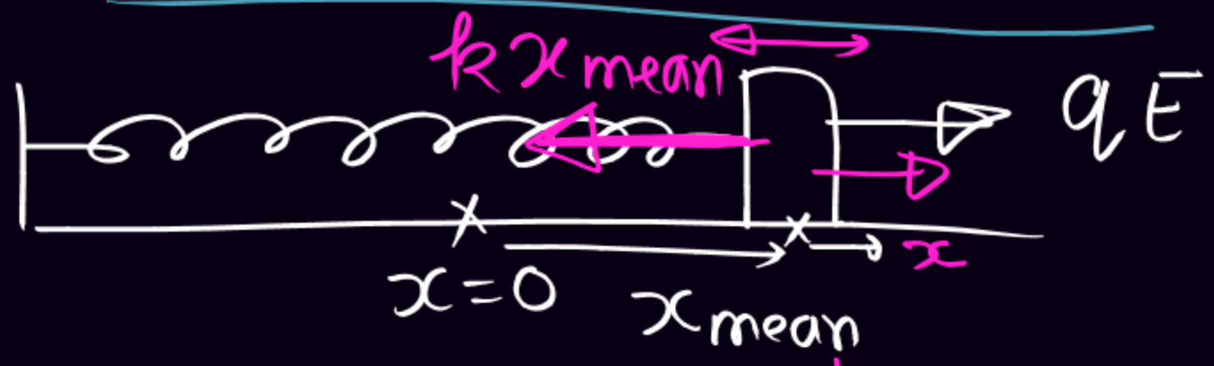
Adv:

1) Find the time period of oscillation

2) Find the amplitude of oscillation.

3) Find the maximum speed of the block

① Find mean position

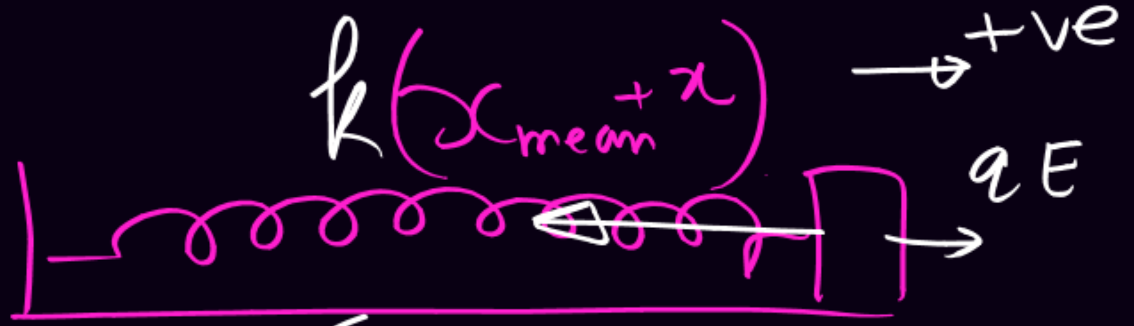


At mean position

$F_{net} = 0$

$Kx_{mean} = qE$

$$\boxed{x_{mean} = \frac{qE}{K}}$$



$$F = \cancel{qE} - k(\cancel{x_{mean}} + x)$$

$$ma = -kx$$

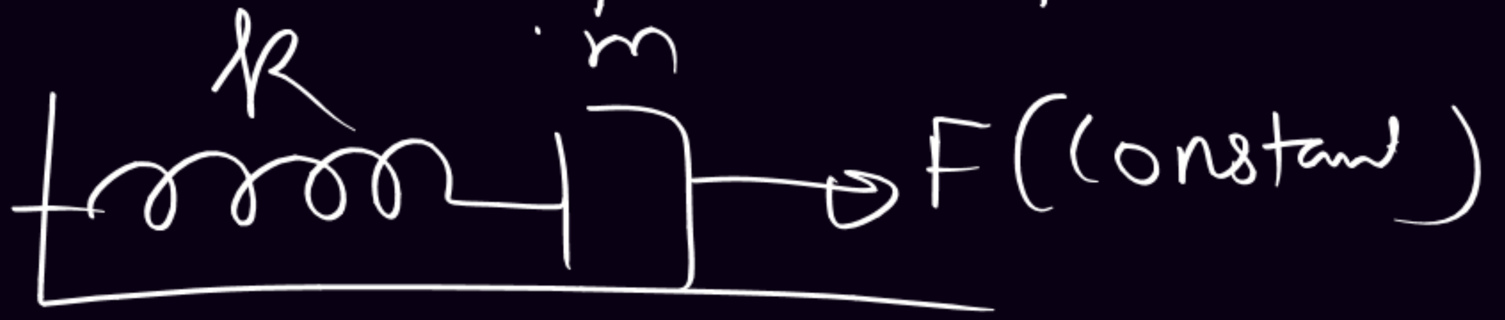
$$a = -\frac{k}{m}x$$

$$a = -\omega^2 x$$

$$\omega = \sqrt{\frac{k}{m}}$$

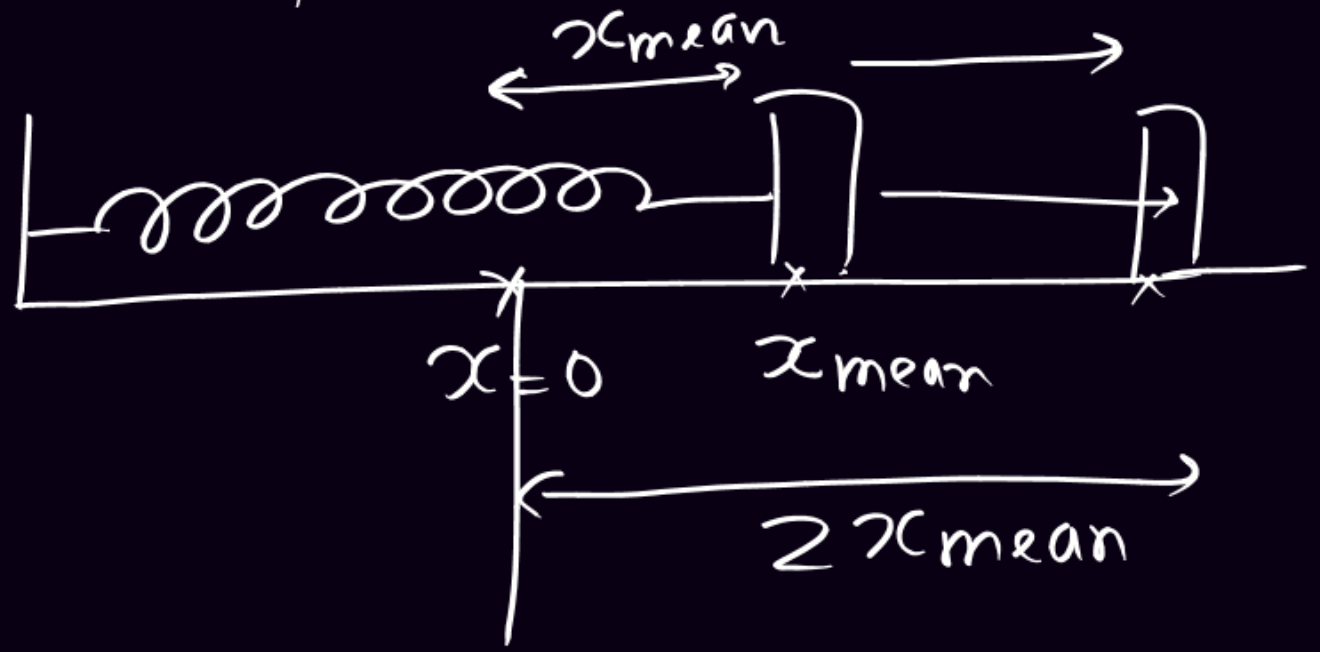
$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$ ✓

Siddhi - Time period of a spring block system is independent of the constant force.



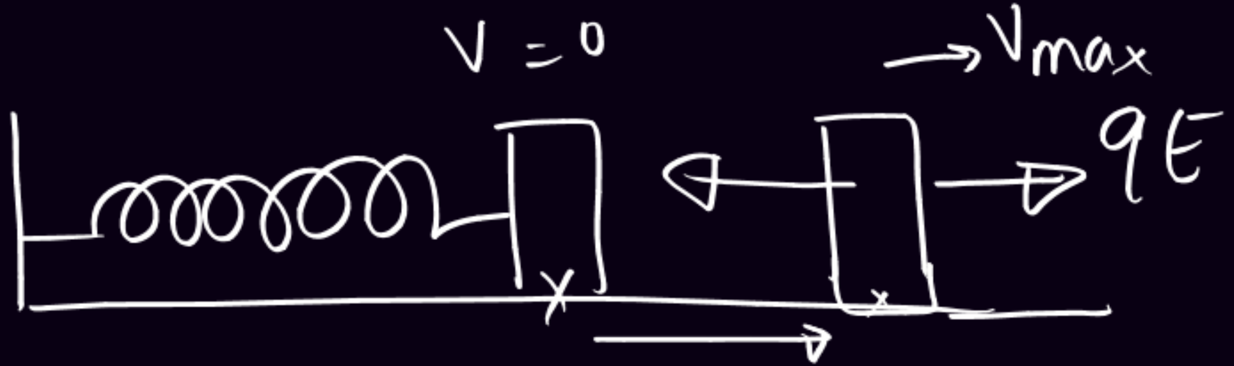
$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

Amplitude $\Rightarrow$ Maximum displacement from the mean position.



$$Amp = x_{mean} = \frac{qE}{K}$$

Siddhi 2:– In oscillations speed of the block is maximum at mean position



$x_{mean} = \frac{qE}{k}$

$\boxed{\text{Work energy th}}$

$W_{\text{all the forces}} = \Delta KE$

$(qE)\,x_{mean} - \frac{1}{2} k x_{mean}^2 = \frac{1}{2} m (v_{max}^2 - 0)$

$qE \frac{qE}{k} - \frac{1}{2} k \left(\frac{qE}{k}\right)^2 = \frac{1}{2} m v_{max}^2$ *

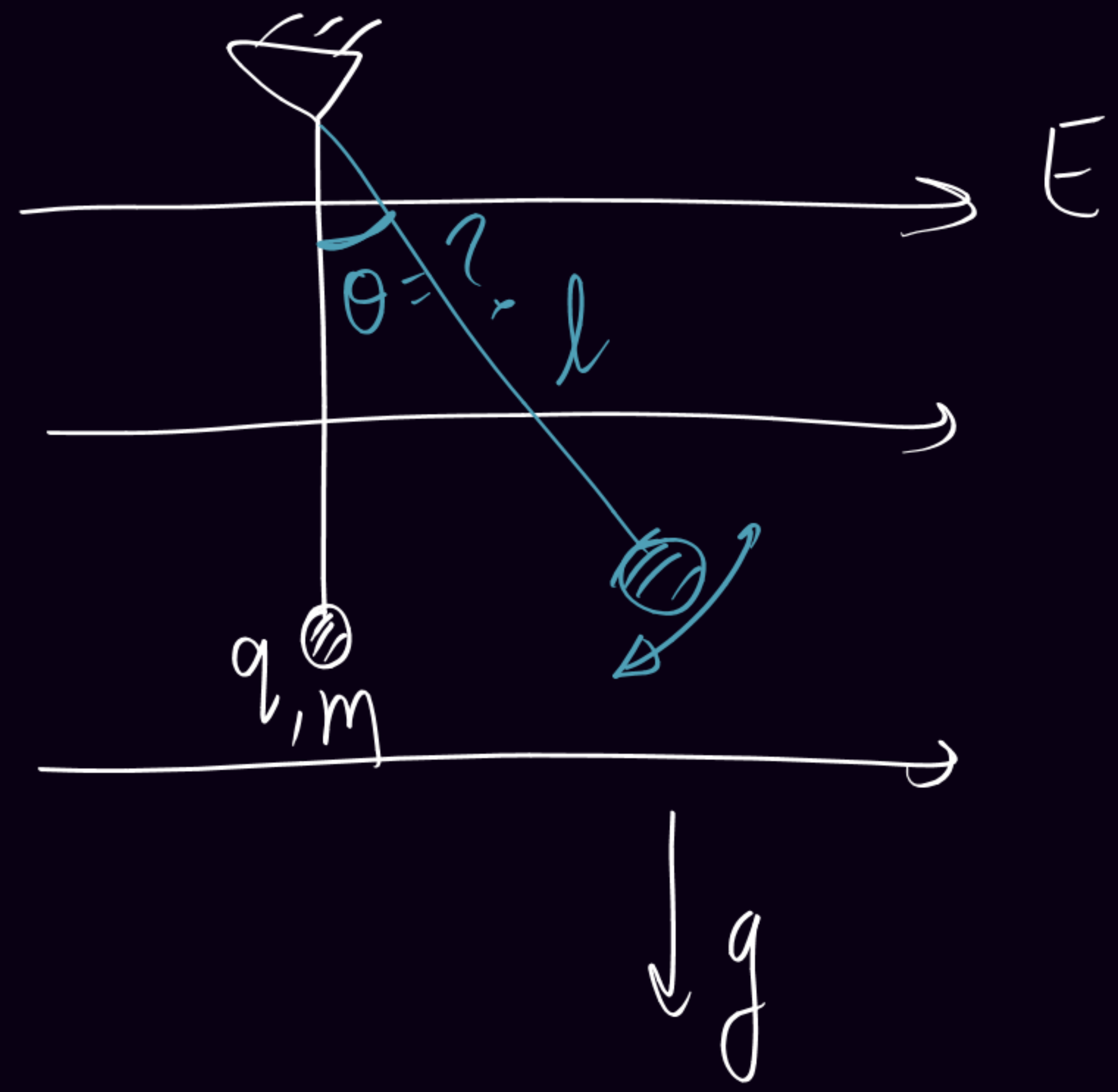
$\frac{q^2E^2}{\cancel{2}k} = \frac{1}{\cancel{2}} m v_{max}^2 \Rightarrow \boxed{v_{max} = \sqrt{\frac{q^2E^2}{km}}}$

Siddhi

$$V_{max} = A\omega \qquad \omega = \frac{2\pi}{T}$$

$$= x_{mean} \frac{\cancel{2\pi}}{\cancel{2\pi}\sqrt{\frac{m}{K}}}$$

$$= \frac{qE}{K}\sqrt{\frac{K}{m}} = \frac{qE}{\sqrt{mK}}$$



Find $\theta = ?$

$T = ?$

H.W.

Thank You Lakshyians