
Student Name : _____ Student Number: _____

MIDTERM ONE

READ THE DIRECTIONS

ONCE YOU START, MAKE SURE YOUR EXAM HAS 5 PAGES (including the coverage)

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Total	50	

- Show all work for full credit.
- **BOX** in your answer to each question.
- Unless otherwise indicated always use EXACT numbers. (i.e $\sqrt{\pi}$ instead of 1.77).
- You may use the fact that $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$.
- You may use a scientific calculator during this examination; graphing calculators and other electronic devices are not allowed and should be turned off for the duration of the exam.
- If you use trial-and-error, a guess-and-check method, or numerical approximation when an exact method is available, you will not receive full credit.
- You may use one double-sided, hand-written, 8.5 by 11 inch page of notes.
- You have 60 minutes to complete the exam - so, spend on average ≤ 15 minutes per page.

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1. Evaluate the following limits. If the limit does not exist write 'DNE' and explain why. Showing no work, or just applying a formula will receive zero points.

(a) (3 points) $\lim_{r \rightarrow 1} \frac{r^2}{(r-1)^3}$

(b) (3 points) $\lim_{x \rightarrow \infty} \frac{\sqrt[3]{8x^6 + 3x^2 - 7} + x^2}{3x^2 - 7x}$

(c) (4 points) $\lim_{x \rightarrow 0} \frac{\pi \sin^3(x)}{x(1 - \cos(x))}$

(d) (4 points) $\lim_{h \rightarrow 0} \frac{(\frac{\pi}{2} + h) \cos(\frac{\pi}{2} + h) - (\frac{\pi}{2}) \cos(\frac{\pi}{2})}{h}$. *Hint: Limit definition of derivative.*

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2. Using the derivative rules (Power, Product, Quotient, etc...) find the derivatives of the following functions.

You do not need to simplify your answers!

(a) (4 points) $f(x) = (\sqrt{x} + 1)e^x \cos x$

(b) (4 points) $g(x) = \frac{\sin x + \tan x}{x^e + e^x}$

3. (5 points) $f(x) = \frac{1}{\sqrt{x+1}}$. Using the **LIMIT DEFINITION OF THE DERIVATIVE** find $f'(1)$.

4. Let $F(x) = \begin{cases} xe^x, & x > 1 \\ \frac{e}{2}x^2 + \frac{1}{2}e^x, & x \leq 1 \end{cases}$.

(a) (3 points) Is F continuous at $x = 1$? *You must show your work and explain with a sentence.*

(b) (4 points) Is F differentiable at $x = 1$? *You must show your work and explain with a sentence.*

5. Matt is grinding a rail on his skateboard with position $p(t) = \frac{1}{3}t^3 - 2t^2 + 3t + 10$.

(a) (2 points) Find Matt's average velocity on the time interval $[0, 1]$.

(b) (3 points) Find all times when Matt is neither moving left nor right.

6. (2 points) Write the equation of the tangent line to the circle $(x - 1)^2 + y^2 = 4$ at $(2, \sqrt{3})$?

7. Let $f(x) = 2x^2$ and $g(x) = x^2 + 1$.

(a) (2 points) What is the slope and y-intercept of the tangent line to $f(x)$ at $x = a$.

(b) (2 points) What is the slope and y-intercept of the tangent line to $g(x)$ at $x = b$.

(c) (2 points) If a line is simultaneously tangent to $f(x)$ at $x = a$ and $g(x)$ at $x = b$. Write two equations involving only a and b that a and b must satisfy.

(d) (3 points) Find all values of a and b so that the tangent line to $f(x)$ at $x = a$ is simultaneously tangent to $g(x)$ at $x = b$.