124A MidTerm One REVIEW OUTLINE ©

Matt J.

Outline

1. **Limits**
   (a) Bag of Tricks
      i. Conjugate
      ii. Just Plug In
      iii. Left and Right Limits
      iv. Divide Out
      v. Factor and Cancel
      vi. Limit Laws
      vii. Business with $\frac{\sin x}{x}$
      viii. Horizontal Asymptotes
   (b) Continuity
   (c) What functions are guaranteed continuous?
   (d) Piecewise functions
   (e) Vertical Asymptotes
   (f) Limit Definition of Derivative
2. **Explicit Derivatives**
   (a) Drawing graph of f' versus f
   (b) The derivative as a function, interpreting units
   (c) Finding tangents to curves
   (d) Product Rule, Quotient Rule
   (e) Derivatives of Elementary Functions

Practice Problems

1. **Background Stuff**

   (a) Circles

      i. Algebraic Formula $(x-a)^2 + (y-b)^2 = r^2$.

      ii. Parametric equations
$$x(t) = A\cos(\omega t - \theta) + B$$
$$y(t) = A\sin(\omega t - \theta) + C$$

      iii. The fact that tangent lines are perpendicular to radial segments on the circle.

      iv. Be able to look at an expression and identify that it is a the limit definition of the tangent to a circle. (i.e. compute the following limit by using only geometry...)
$$\lim_{h\to -0} \frac{\sqrt{25-(3+h)^2}-4}{h}$$

      v. A flashlight is attached to the edge of a rotating wheel of radius 8 meters. The wheel rotates counterclockwise at a rate of $\frac{\pi}{3}$ rad/sec and the flashlight begins at the position $(8,0)$. Assume that the light beam of the flashlight always points in a direction tangential to the circle. Let $P(t)$ be the coordinates o the flashlight at time t seconds and $Q(t)$ the place where the light beam crosses the x-axis. *(a) Write a formula for $Q(t)$; (b) Write an equation for the tangent line at time t.*

2. **Limits**

   (a) Bag of Tricks

      i. Conjugate
$$\lim_{x\to 2} \frac{\sqrt{6-x}-2}{\sqrt{3-x}-1}$$

      ii. Just Plug In
$$\lim_{\sigma\to e^2} \frac{\ln\sigma}{e^{\sqrt{\sigma}}}$$

      iii. Left and Right Limits
$$\lim_{x\to\pi^-} \frac{e^{\sin\frac{x}{2}}\cos x}{\pi - x}$$
$$\lim_{x\to\pi^+} \frac{e^{\sin\frac{x}{2}}\cos x}{\pi - x}$$

      iv. Divide Out
$$\lim_{x\to\infty} \frac{\sqrt{4x^2-1}}{\sqrt{x^2-x+3}}$$

v. Factor and Cancel

$$\lim_{\theta \to 0} \frac{\sin^2 \theta}{1 - \cos \theta}$$

$$\lim_{x \to 2} \frac{(x-2)\sin(\pi x)}{\sqrt{x^2 - 4}}$$

vi. Limit Product Law

$$\lim_{x \to 0} \frac{\sin^2 x}{x}$$

vii. Take limit inside a Continuous Function

$$\lim_{t \to \infty} e^{\sin(\frac{\pi}{t})}$$

$$\lim_{t \to \infty} \cos(e^{-t})$$

viii. Business with $\frac{\sin x}{x}$

$$\lim_{\theta \to 0} \frac{\sin(\pi \theta^2)\sin(\theta)}{e\theta^3}$$

ix. Horizontal Asymptotes

Find the horizontal asymptotes of the following functions: (Note: f(x) is difficult)

$$g(x) = \frac{x^3 - 3}{4x^3 + 3}$$

$$f(x) = \frac{e^x(\sin x - 1)}{e^{x^2}}$$

(b) Continuity

A function f is continuous when for all points, a, in the domain of f it holds that

$$\lim_{x \to a^-} f(x) = f(a) = \lim_{x \to a^+} f(x)$$

(c) What functions are guaranteed continuous?

Where is the function $g(t) = \sin(e^{x^2 - \cos x}) + 70$ continuous?

(d) Piecewise functions

$$g(y) = \begin{cases} e^y - \sin(\pi y), & y < 1 \\ e^y - \ln y, & y \geq 1 \end{cases}$$

Is g continuous?

(e) Vertical Asymptotes

Basically you get a vertical asymptote when you are dividing something that is nonzero by zero. Find the vertical asymptotes of the following function:

$$h(t) = \frac{t^3 - t + 1}{\sin(\frac{\pi}{2}t)(t^2 + 3t + 2)}; \quad 0 \le t \le 6$$

(f) Limit Definition of Derivative

Using the limit definition of the derivative calculate the following derivates

$$f(x) = \frac{1}{x^2}$$

$$g(t) = t^2 - t + 2$$

3. **Explicit Derivatives**

(a) Drawing graph of f' versus f

(b) Properties of Tangents to Curves

Find the equation of the tangent line at $x = \pi$ to the following function:

$$g(x) = \sin x + \cos x$$

Find the equation of the tangent line at $x = 0$ to the following function:

$$h(x) = \frac{e^x + 5x}{x^2 + 4}$$

Find the equation of the tangent line to $f(x) = x^2 + 2x + 3$ which passes through $(1, 1)$.

(c) Product Rule, Quotient Rule

Calculate the following derivatives using rules (not the limit definition)

$$f(x) = x^3 \ln x \sin x \cos x$$

$$h(z) = \frac{\sin z + z^4}{\sqrt{z} - 3z^{4/5}}$$

$$g(t) = \frac{\sin^2(t)}{\cos t}$$

(d) Trig and ln derivatives

Know the derivatives of $\sin x, \cos x, \tan x, \sec x,$ *etc...*