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Epistemic Complexity and Knowledge Construction

Morphogenesis, Symbolic Dynamics and Beyond

Springer

Arturo Carsetti
Facoltà di Lettere e Filosofia
Università di Roma – Tor Vergata
V. Lariana 7
00199 Roma
Italy

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In remembrance of my mother, now and everywhere. In the realm of a Lacemaker whose intended artwork allows the emergence of new possible forms of vision and life.

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Chapter 1
Complexity, Self-Organization and Natural Evolution

1.1 Entropy and the "Intermediate State"

At the level of natural evolution, when observing phenomena which are neither purely ordered nor purely random, but characterized by a plot of sophisticated organization modules, we are normally observing an intermediate situation between a complete absence of constraints, on the one hand, and the highest degree of redundancy, on the other. In this sense, optimal organization should be considered an effective compromise (Schroedinger's aperiodic crystal) between the greatest possible variability and the highest possible specificity. Given the deep structure underlying the surface message, this compromise inevitably articulates itself in accordance with a dynamical dimension proper to a self-organizing process, i.e. a process on the basis of which changes in organizational modules are apparently brought about by a program which is not predetermined, but produced by the encounter between the realization of specific virtualities unfolding within the evolving system, and the action, partly determined by specific coagulum functions, of generative (and selective) information fluxes emerging at the level of the reality external to the system. In principle, this process may for a variety of factors cause a progressive reduction in the conditions of possible high redundancy proper to the initial phase, together with a successive sudden increase in symbolic potential variability. This will therefore allow a further extension of the internal regulatory factors' range of action, in conjunction with the appearance of new constraints and regenerated organizational forms, so that the increase in variability will be matched by increased specificity and reduced disorder, and with no paradox or contradiction involved. It is precisely this near-contextual increase in variability and specificity that we refer to in speaking of an internal "opening up" of a coupled system furnished with self-organizing modules. In other words, when at the level of life specific thresholds are crossed, the bases of variability are extended, while, simultaneously, the conditions favourable for extremely complex organizational phenomena begin to be realized.

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The variability-specificity dialectics characterizing self-organization appears, on a more general level, to be strictly connected to the evolution of an amply-acknowledged function: thermodynamic entropy. We know, for example, how, under certain conditions relative to the articulation at the co-evolutive level of a specific self-organization process, physical entropy can increase as disorder simultaneously decreases, particularly when the number of microstates is not constant and increases more rapidly than the number relative to S (entropy).

What this evinces, however, is the essential relation, at the entropic level, between the cognitive agent's information about the system, on the one hand, and the increase in entropy, on the other. Thus, to the coupled evolution of the system and the environment, and the variability-specificity dialectics, a further element must be added, i.e., the cognitive agent considered as a factor of primary reference, the functional role of which it is necessary to clarify. In terms of effectiveness, the calculations of entropy would seem, within the realm of statistical mechanics, to depend on what is known of the system and what measurements can be performed on it. The probability distribution changes essentially if the distribution is to be applied to a different number of accessible states from that originally calculated. In this case the information can increase entropy in real terms.

Let us consider, as an example, the common definition of entropy as a measure of disorder and let us point out, in accordance with Bais and Doyne Farmer,[1] that the thermodynamic entropy is indifferent to whether motions are microscopic or macroscopic: it only counts the number of accessible states and their probabilities. But if we define probabilities that depend only on the positions of the particles and not on their velocities we obtain an entropy associated with a new set of probabilities which we might call the "spatial order entropy", an entropy that articulates quite differently from the thermodynamic entropy.

Given that, according to the theoreticians of "coarse graining", a cognitive agent is unable to determine the details of microscopic dynamics beyond a certain scale of size, to describe events on a smaller scale it will be necessary to perform the necessary calculations on the basis of average behaviour. It is the average of total probability for groups of microscopic states which is usually calculated, and this average probability is attributed to each microscopic state, maintaining constant $n(t)$, the number of accessible states.

It is to this type of manipulation, the nucleus of coarse graining, that entropy increase in statistical mechanics is to be attributed according to Landsberg, for example.[2] Here, in his opinion, we can identify the real emergence of what is virtually time's arrow.

As is well known, the coarse graining approach sinks its roots in the entropy definition as outlined successively by Boltzmann and Gibbs.[3] The formula:

$$S = -k \sum_i P_i \log_e P_i \tag{1.1}$$

is essentially Boltzmann's definition for the statistical entropy of a system; k is a numerical constant; if $P_i = 1/W$ where W represents the number of distinct microstates we directly have $S = k \log W$. From this formula the first and second laws of thermodynamics can be readily derived. It defines the entropy S absolutely. Classical thermodynamics, on the other hand, offers no such possibility since only the entropy difference $dS = dQ/T$ is defined at the level of macroscopic thermodynamics. Gibbs generalized Boltzmann's entropy, introducing the use of general ensembles to represent systems of actual interest. A general ensemble is directly taken to be a collection of mental copies of the system of interest. Such replicas are regarded as distributed among all the available microscopic states consistent with the thermodynamic coordinates characterizing the system. From many points of view, the idea of general ensemble as outlined by Gibbs represents a theoretical tool capable of giving precision to notions of probability in the form of frequency interpretation.

In classical statistical mechanics a microscopic state of a system of elements each having f degrees of freedom is specified by a point in a 2Nf dimensional phase space (where N is the number of elements). The phase space is often denoted as Γ–space, Gibbs gave the first foundation of a general theory of statistical mechanics in terms of Γ–space. In his conception a Γ–space was regarded as populated with moving phase points representing the microscopic states successively attained by the virtual replicas of the system of interest. Let ρ be a density function, $\rho(P, t)$ such that $n\rho\delta\Gamma$ is the number, δn, of systems in the volume element $\delta\Gamma$ centred on some point P at time t and such that the integral of $n\rho\delta\Gamma$ over the whole space is equal to n. The density ρ (P,t) may be regarded as the probability per unit-volume of the Γ-space that a system picked at random from the ensemble will be in the particular region $\delta\Gamma$.

Liouville's theorem tells us that ρ remains constant in the close vicinity of any one of the systems of the ensemble as the system moves within the Γ-space. In this way Gibbs defined the following function:

$$S' = -k \int \rho \log_e \rho \, d\Gamma \tag{1.2}$$

Analyzing the properties of the function S', Gibbs immediately realized the problem created by Liouville's theorem. Since the theorem holds for every representative point P, the integral relative to S' remains constant in time. According to the theorem it follows that the entropy S' cannot increase during the gradual unfolding of an irreversible process. Citing Gibbs, the paradox arising looks as follows: let us take into consideration the spreading of a quantity of colour during the stirring of a liquid: although Liouville's theorem shows that the density of the colouring matter at any same point of the liquid will be unchanged, at the same time no fact is more familiar to us than that stirring tends to bring a liquid to a state of uniform mixture. This could be explained by saying that if we were to look at the system on the very fine scale of the filaments, we would see no mixing, but if we were obliged to look at it according to a much coarser scale, an approach to uniformity would be apparent. Hence the beginning, at the coarse graining level, of a subjective approach to the understanding of irreversibility. According to this point of view, the increase in entropy does not describe the system in itself; on the contrary it concerns our lack of information with respect to the real system.

Although, at first glance, Gibbs appears to use the paradigm to endorse the birth of a subjective interpretation of coarse graining, it is important to read this correctly within the context of his general acceptance of frequency interpretation. Actually, Gibbs conceives of entropy essentially as a statistical concept based on the idea of a representative ensemble. Since the given ensemble is chosen in such a way as to reflect all the possible partial information we may have of the system of actual interest, it is clear that Gibbs' theoretical definition of entropy is based on the formulation of an extremely close link between entropy and information. An objective interpretation of the probability calculus which perceives the probabilities as limiting relative frequencies, as obtained by means of an increasingly wide (but still finite) number of repeated experiments, is completely in line with Gibbs' firm belief in a possible *a priori* specification of probabilities concerning the distribution of phase points on the basis of objective frequency data.

Jaynes, however, in the 1950s,[4] pointed out that it would never be possible to determine the true entropy of a natural language such as, for example, English, simply on the basis of frequency-observation, however accurate, because of the astronomical figures involved. At a general level, it is impossible to base probability on frequency: indeed, it should always be remembered that all frequency-calculations presuppose specific assumptions and previous probability evaluations. It is thus necessary, in his opinion, to return to Gibbs' original intuition, rejecting the successive objective evaluations based on frequency interpretation: canonical distribution represents only our state of awareness when we are in possession of partial information obtained from specific macroscopic calculations. In this sense, thermodynamic system's entropy has to be seen as a measure of the extent of ignorance of a person whose only knowledge of the system's microstate consists of the values of the macroscopic quantities X_i defining the thermodynamic state of the system itself. This constitutes an "objective" quantity in that it is a function only of the X_i, and in no way depends on the inclinations of the single subjects. Using other types of quantity or arbitrary assumptions, which are not justified by the real information in our possession, would in fact constitute an essentially "subjective" process of projection. From all this Jaynes derives the following basic tenets: (1) the entropy of statistical mechanics measures our degree of ignorance as regards the true unknown microstate; (2) it is not the physical process, but our ability to follow it, which is irreversible.

Gibbs's closest followers roundly criticize the pure subjectivism Jaynes ultimately endorses, while realizing the need to provide an epistemological answer to the *aporia* he evidences. In the opinion of Tolman,[5] to cite one example, this is to be found in Gibbs' original objectivism, drawing on Popper's findings to delineate a method which is objective but conjectural, able to embrace both loyalty to the reality of things, and the imaginative daring of Gibbs' philosophy at its most creative.

It is these ideas of Tolman and the Ehrenfests' which really allow a more detailed theoretical analysis of coarse-graining techniques, enabling in turn a more sophisticated account of the relationship between the notions of coarse- and fine-grained density. On this theoretical basis, it is then possible to review the whole question of irreversibility as originally set out by Gibbs.

In the classical treatment of the generalized H-theorem the Γ-space was divided into regions, each of volume ν. The volume of a particular region is relative to the limits of accuracy available to the cognitive agent. In this sense, the actual numerical value of a coarse grained entropy is a function of the chosen size ν. With regard to this important discrimination let us define a coarse grained density.

$$\bar{\rho}(\nu, P, t) = \frac{1}{\nu}\int_{\nu} \rho(P, t)\, \mathrm{d}\Gamma \tag{1.3}$$

where ρ has the same meaning as above and the integral is over the volume of a specific region. Now the coarse grained density in a particular region is the mean of the fine grained densities in the vicinity of the point P on which the region is centred. We are thus able to define a quantity $H' = \int \bar{\rho} \log_e \bar{\rho}\, \mathrm{d}\Gamma$. $\bar{\rho}$, unlike ρ, is not subject to the restriction relative to Liouville's theorem governing the behavior of ρ. It follows that H' need not remain constant in time as required by this theorem in the previous case. This means that the constant volume ν, containing the original points, has changed its shape and has developed "filaments" in accordance with Gibbs' colour example. It follows that at later times there will be regions of Γ-space where $\rho \neq \bar{\rho}$. It seems reasonable to expect a high probability for H' to diminish due to a continued spreading over the accessible region of Γ-space until a condition of equilibrium is attained. This result, even if more realistic and uncontroversial, can again be read simply as a loss of information or, more precisely, as a decrease with time of our information as to the conditions of the system of interest. In this sense, even if the theoretical achievements obtained by the coarse graining theoreticians actually give an answer to Gibbs' *aporia* concerning the objective nature of irreversibility, the *querelle* between the conjectural objectivism advocated by Tolman and the pure subjectivism advocated by Jaynes appears, on the basis of this particular deepening of the analysis, to be still unsettled.

Besides Tolman's conjectural objectivism we can also distinguish, however, a strong form of objectivism which is an ideal counter to the pure subjectivism as advocated by Jaynes: the conceptual approach to coarse graining and, more generally, irreversibility, as outlined by Brussels School.

Indeed, at a basically specular level, Prigogine,[6] too, bases himself on Gibbs, Tolman and the Ehrenfests, finally concluding that coarse graining requires radical revision, albeit within the framework of a theoretical conception which is strongly objective. In his opinion irreversibility must be closely linked to the concept of dynamical instability. Taking a number of concepts outlined by Tolman and the Ehrenfests, Prigogine works towards the definition of a distribution function deriving from that introduced above: a new function which, Prigogine insists, must in no way be even partially arbitrary. More essentially, its "coarseness" must include the breaking down of the symmetry of the initial equations of motion. In other words, Prigogine maintains that probability should always be considered as objective, and the subtending non-locality be taken as a reflection of the instability of motion of the dynamical systems subject to the second law of thermodynamics.

The starting point of Prigogine's innovative reading of the technique of coarse graining is the analysis of the evolution of a conservative dynamical system in an appropriate Γ–space. As we have just said, at the level of these systems, volume is preserved in phase space. The dynamics can be expressed in terms of a unitary operator U_t acting on the distribution function: $\rho_t(\omega) = U_t\ \rho_0(\omega)$. The U_t form a dynamical group: $U_t U_s = U_{t+s}$ (t, s positive or negative). Given the flagrant contradiction between the laws of conservative dynamical systems and the second law of thermodynamics, many physicists, as we have seen, have regarded the second law as arising from some approximation that we add to the laws of dynamics. Prigogine, on the other hand, maintains that irreversibility plays a fundamental role in Nature and proceeds to outline the general properties of dissipative dynamical systems by introducing in Γ a distribution function ${}_P\bar{\rho}(\omega)$ which, in contrast to classical mechanics, evolves according to a Markov process: ${}_P\bar{\rho}_t(\omega) = W_t\,{}_P\bar{\rho}_0(\omega)$. W_t satisfies the semigroup condition: $W_t W_s = W_{t+s}$ (t, $s \geq 0$). This condition, combined with the Markovian properties of the process, drives the system to equilibrium or to some non-equilibrium state characterized by suitable attractors when specific constraints are introduced. Hence the possibility of delineating a transformation ${}_P\bar{\rho} = \Lambda\rho$ where the transformation operator Λ acting on ρ depends on the laws of dynamics. In this sense, the symmetry-breaking transformation Λ links dynamical and probabilistic processes. What then evolves, in dissipative systems, is no longer the initial distribution function, but a transform of this function, the transformation itself emerging as determined by dynamical laws. The basis of Prigogine's rereading (in a creative sense) of the original intuitions by Tolman and the Ehrenfests is first of all represented by the introduction of this kind of transformation.

While, in effect, the move from ρ to $\bar{\rho}$ in coarse graining was determined by a basic arbitrariness in coarse-graining distribution, the move from ρ to ${}_P\bar{\rho}$, as traced by Prigogine, is countersigned by the existence of a transformation presenting decidedly Markovian characteristics. Its existence can be rigorously demonstrated for an important class of dynamical systems which are highly unstable and possessing SCI (sensitivity to initial conditions). The essential elements characterizing this transformation are: (1) a high degree of instability, or the definite existence of a randomness intrinsic to the system; (2) the breakdown of time symmetry.

On the general level, the function of transformation introduced above appears to be a decreasing function of the internal time Λ(T), which corresponds to a non-local description and specifically characterizes the evolution of unstable dynamical systems. In the now classic baker transformation, the internal time is directly measured starting from the "fragmentation" of the distribution function. By applying this transformation it emerges that, at a given time, ρ and ${}_P\bar{\rho}$ are generally constituted of both past and future elements, within the realm of the internal time Λ (T). In mathematical terms, as glanced at above, all this corresponds to the move from a group to a semi-group. All the same, as Prigogine goes on to say, it is impossible to deduce the time-direction from specific laws. Here it is a primitive concept: a prerequisite for all forms of life. The objectivity of irreversible evolution is thus compounded by the primitive nature of the distinction between past and future, and the circle of the evolution of life is completed within a single link.

Thus, to return to the problem of coarse graining, we can now easily realize how at the basis of Prigogine’s reasoning is the very specific consideration that in the case of unstable dynamical systems, arbitrarily small differences in initial conditions are amplified exponentially. This means that, to ensure a particular level of precision as regards relative predictions at linearly increasing time-intervals, we need levels of precision about the initial conditions which increase exponentially. But each finite observation inevitably obliges us to cut off the digital expansion for all coordinates somewhere: an essential and inevitable form of coarse graining. Due to the sensitive dependence on initial conditions, any ignorance about seemingly insignificant digits spreads with time towards significant digits, leading to an essentially unpredictable behavior. It is no longer possible to reduce the description made in terms of sets; on the contrary, it constitutes the departure point for all further considerations. The distribution functions and statistical concepts thus emerge as the only conceptual tool available to us, and objective insofar as it is the basis of all possible prediction activity. Behind the primary nature of the distribution functions, then, we again discover the existence of human cognitive activity evolving in finite terms at a necessarily macroscopic level. Far from moving Prigogine towards a subjective dimension, these conditions, in the general harmony of his theory, represent the guarantee that the effected coarse graining is non-arbitrary in nature, but it is the reflection, within the awareness proper to thought (intended as an evolved form of dissipative structure) of the inseparable link objectively existing between instability, on the one hand, and probabilistic prediction, on the other. If no original instability existed as the evolutionary basis of our being, we would not be predictive beings; at the same time, given that we are geared to operate cognitively, it is impossible not to take the primitive and undeniable fact of instability as our departure point.

It will be obvious that the necessary transition from instability to probability also leads to irreversibility: to the distinction between past and future. Here, too, Prigogine underlines that in entering the realm of probability and abandoning the world of trajectories, no loss of information takes place, as the subjectivists would have it. What determines temporal evolution is the type of instability and process concerned; it is the chaotic nature proper to the real process which makes the transition to a probabilistic description possible and necessary. This is the reason we can effectively affirm, as stated above, that the concept of probability must be generated from within the dynamics, and thereby acquire its objective nature. Prigogine thus departs from a scrupulous analysis of the coarse graining approach to arrive at a creative transformation of its formalism, neatly and sharply reversing the epistemological evaluation of the techniques initially elaborated by Tolman and the Ehrenfests. The basic theoretical point is a new conception of humanity-Nature relationship; we are now faced with a unitary evolution, and no longer with two separate worlds characterized by the continuous formulation, at the cognitive level, of hypotheses and conjectures concerning possible relations of correspondence. The individual who predicts finds within him/herself the reasons for that instability and intrinsic randomness lying behind the emergence of dissipative structures of which awareness, at the cognitive level, is only a particularly evolved form.

From a general point of view, entropy production, in general, shows a double function: it can lead to disorder, but it is often, equally a mechanism for producing order. The order associated with the emergence of dissipative structures under non-equilibrium conditions cannot, however, be directly and exclusively related to a decrease of entropy at a local level. As Nicolis affirms,[7] order seems to be, first of all, a compromise between fluctuations, on the one hand, and processes of stabilization, on the other, a compromise that appears to be characterized by the presence of a very high level of sophistication. If we destroy this delicate balance, we have as a result an erratic state or a "homeostatic" fossil-like state in which full uniformity is imposed. In this sense, fluctuations appear as the physical counterpart of mutation, whereas the search for stability plays the role of selection.

In far-from-equilibrium situations there is a multiplicity of stable states. This multiplicity can be represented, for instance, by utilizing a bifurcation diagram. Near the bifurcation point the system has a choice between the branches. Thus, for some critical values of the control parameter, new solutions emerge, new fixed points will appear. The bifurcation is a source of innovation; it generates solutions displaying broken symmetries. A symmetry breaking in an undifferentiated medium can, for instance, enable particular unities to "read" their environment in a different way, to differentiate, that is, into specialized unities. In this sense stability appears closely anchored to the realization of a process of differentiation and of a process of functional "partition". Such processes manifest themselves as a real source of order. However, the ability of systems of finite size to maintain a stable broken symmetry is always limited, first of all because of the existence of fluctuations. In this way it is the successive series of the realized compromises between fluctuations and stability that enables a biological system to "tame" chance.

At the level of the phenomenon of bifurcation the evolution of the one-variable system takes place in a one-dimensional phase-space. The only invariant sets that can exist in this space are fixed points. When we pass from one-variable dissipative systems to two-variable systems and to dynamical systems evolving in phase spaces of dimension greater than two, more intricate transition phenomena arise. We can have fixed points and limit-cycles, but we can also have strange attractors. These latter are formed by sets of points that have a fractal dimension. The system is driven to the attractor. The behavior of the system is very sensitive to initial conditions: the distance between neighboring trajectories grows exponentially in time. In this way we cannot predict the final outcome with certitude, whatever the actual finite precision of our information might be. To ensure some fixed level of precision we need, as we have just said, exponentially increasing information about initial conditions. But every finite observation can produce only finite information. In this sense, irreversibility rises from an objective limitation on measurability. The randomness is not the result of experimental imperfections or of a complicated environment that we cannot control, but is rather deeply rooted, as Prigogine explicitly remarks, in the dynamics of deterministic systems involving few variables. The actual determination of this dynamics is performed, in an objective way, by a "finite" observer. It is the connection between chaotic dynamics and measurements performed by a cognitive agent that generates "information" or, more precisely, algorithmic information.

From a general point of view, the dissipative systems show how the irreversibility can be transformed into a pattern and how it is possible to exclude purely random sequences as well as periodic sequences. Actually, a deviation from equiprobability acts as an extremely efficient selector of some particular sequences out of the set of all possible random sequences. A dissipative system realizes exactly such a deviation. In this way non-equilibrium, linked to particular conditions of observability, appears as the natural landscape in which selection (and evolution) can take place.

The dynamical systems of which we are speaking about can be defined, under appropriate constraints, in a discrete space in which the evolution shows itself as a Markovian process. From an effective point of view, the state space of systems giving rise to a chaotic dynamics is essentially continuous. At this level the deviations from the mean cannot be seen as localized events. However, in many cases chaotic dynamics can be presented in the form of a Markov chain utilizing well-founded tools offered by the traditional information theory. This fact leads to a shift from physical entropy to informational entropy. We enter, according to Nicolis, into a new level of abstraction characterized by the presence of symbolic dynamics. It is at the level of this new kind of dynamics that we can recognize the presence of information-rich structures generated spontaneously by a given dynamical system. It is exactly in this sense that we can affirm that non-equilibrium processes with chaotic dynamics generate information: that information, in particular, which, once combined with the action of (natural) selection, appears, for Prigogine and Nicolis, to characterize coherently the hidden articulation of the intermediate state. At the level of this shift we are really faced with the existence of a precise link between two different kinds of randomness: physical randomness and algorithmic randomness; hence the necessity to analyze the functional nature of this particular connection in order to identify the co-evolutive and mysterious path that characterizes the evolution proper to the "intermediate state". But, how can we accomplish this difficult task? Which mathematical tools and models must we employ? Complexity is the key: it represents the adequate bridge between algorithmic entropy and physical entropy, and really offers, as we shall see, a possible explanation of the role played by the cognitive agent at the level of natural evolution.

1.2 Algorithmic Complexity and Self-Referentiality

According to Chaitin,[8] the algorithmic complexity of a binary string s is formally defined as the length of the shortest program, itself written as binary string s^*, that reproduces s when executed on a computer. A "program", in this context, is simply an algorithm, a step-by-step procedure that has been coded into binary form. A string s is said to be algorithmically random if its algorithmic complexity is maximal, comparable to the length of the string: in other words, it cannot be compressed by employing a more concise description other than by writing out s in its entirety. The code that replicates the string is the string itself. Algorithmic randomness is, therefore, consonant with randomness in the usual sense of maximum entropy and of the theory of normal numbers. Random sequences must define normal numbers even if not every normal number is random.

As we have said before, randomness in the information-theoretic sense of maximum entropy is normally framed in terms of the probabilities of the source symbols. By contrast, the complexity definition of randomness makes no recourse to probability, and depends only on the availability of a mechanical procedure for computing a binary string. Moreover, the complexity definition is independent of the provenance of the pattern relative to the given string s. It looks only at the string itself, as merely a succession of digits.

With respect to this conceptual frame, a string is random if: (a) each digit is generated by some mechanism in an unpredictable manner; the randomness resides, in other words, in the very disorder of the generating process; (b) its entropy is maximal and recognition of the string elicits total surprise; (c) no prescribed program of shorter length can dictate its successive digits.

From a general point of view, traditional information theory states that messages from an information source that is not completely random can be compressed. In this sense, the afore-mentioned definition of randomness can be directly considered as the converse of this fundamental theorem of statistical information theory: if lack of randomness in a message allows it to be coded into a shorter sequence, then the random messages must be those that cannot be coded into shorter messages. From a general point of view, we can affirm that a string s is random iff $H(s)$ is approximately equal to $|s| + H(|s|)$. An infinite string z is random iff there is a c such that $H(z_n) > n - \mathrm{c}$ for all n, where z_n denotes the first n bits of z.

Let us, now, consider a specific program p of length $|p|$. The probability that p will be generated by applying a random process of two random variables $|p|$ times is $2^{-|p|}$.

Let s be an object encodable as binary string and let $S = \{s_i\}$ be a set of such objects s_i, then the algorithmic probability P is defined by:

$$P(s) = \sum_{U(p)=s} 2^{-|p|} \tag{1.4}$$

$$P(S) = \sum_{s_i \in S} P(s_i) = \sum_{U(p)=S} 2^{-|p|} \tag{1.5}$$

$$\Omega = \sum_{U(p)\downarrow} 2^{-|p|} \tag{1.6}$$

Ω is the halting probability (with null free data). Ω is random and, in the limit of infinite computing time, can be obtained in the limit from below by a computable algorithm.

$P_{C'}(s)$ and $\Omega_{C'}$ can be defined for any (not necessarily universal) computer U$'$ by substituting U$'$ for U in (1.4) and (1.6). There are infinitely many programs contributing to the sum in (1.4), but the dominating term in the sum for $P(s)$ stems from the canonical program s^*. It can be shown that there are few minimal programs contributing substantially to the sums (1.4), (1.5) and (1.6). Thus the probabilities to produce a specific object s as well as the halting probability can also be defined by taking into account only the canonical programs.

Let s be an object encodable as binary number and let $S = \{s_i\}$ be a set of such objects s_i, then the algorithmic probability P^* can also be defined by:

$$P^*(s) = 2^{-|s^*|} = 2^{-H(s)} \tag{1.7}$$

Finally we can state the incompleteness theorem of Chaitin:

Theorem 1. *Any recursively axiomatizable formalized theory enables one to determine only finitely many digits of* Ω.

It is worth underlying that Ω is Δ_2^0. We have also to remark that Ω is relative to the chosen universal machine U and thus to a particular coding of Turing machines that is used. As is well known, Δ_2^0 sets have a very natural computational characterization, based on the idea of computability in the limit; this is the notion of "trial and error predicate" as developed by Putnam.[9] In this sense, Ω can be represented by a trial and error predicate.

Chaitin is indeed aware of the fact that Ω is computable in the limit: in his opinion, with computations in the limit, which is equivalent to having an oracle for the halting problem, Ω seems quite understandable: it becomes a computable sequence. From a more general point of view, as Chaitin and Calude directly affirm,[10] Ω is "computably enumerable". We have, moreover, to recognize that, at the level of extreme undecidability, the incompleteness results arising from Ω are in no way the "strongest". Ω is just one among various undecidable sets.

The fact that Ω is unpredictable and incompressible follows from its compact encoding of the halting problem. Because the first n bits of Ω solve the halting problem for all programs of n bits or fewer, they constitute an "axiom" sufficient to prove the incompressibility of all incompressible integers of n bits or fewer. If we consider the axioms of a formal theory to be encoded as a single finite bit string and the rules of inference to be an algorithm for enumerating the theorems given the axioms, by an n-bit theory we can indicate the set of theorems deduced from an n-bit axiom. Remembering that the information content of knowing the first n bits of Ω is $\geq n - \mathrm{c}$ and that the information content of knowing any n bits of Ω is $\geq n - \mathrm{c}$, we are finally able to prove that if a theory has H (Axiom) $<n$, then it can yield at most $n + \mathrm{c}$ (scattered) bits of Ω. (A theory yields a bit of Ω when it enables us to determine its position and its 0/1 value).

What we have just said may be summarized, from another point of view, by saying that as we precisely and consistently specify the methods of reasoning permitted, we determine an upper bound to the complexity of our results. In other words, we cannot prove the consistency of a particular system by methods of proof which are restricted enough to be represented within the system itself.

In this sense, in Chaitin's conception, we have to step outside the methods of reasoning accepted by Hilbert: in particular, as indicated by Gödel, we have to extend Hilbert's finitary standpoint by admitting the necessary utilization of certain abstract concepts in addition to the merely combinatorial concepts referring to symbols. For this purpose we must count as abstract those concepts not comprising properties or relations of concrete objects but which are concerned with thought-constructions (connected to the choice of particular measures, the invention of new methods, the identification of the sense of symbols, etc.).

However, once a certain level of complexity is reached, it is impossible to consider in advance the regularities and the internal evolution that certain phenomenon might exhibit. They can be viewed only from the point of view of some higher-level language, a language that, as a matter of fact, may in turn be helpful to define new regularities. Information content is not only a property of the information source, but also depends on the ability of the cognitive agent to construct sequential tests, to find adequate distance functions and to create new languages. In other words, it is not possible to define a form of absolute complexity with respect to a formalized theory. In this context the crucial point is that introducing new axioms with reference to the information-theoretic limits imposed by incompleteness theorems, implies resorting to abstract concepts that can be considered as "evidence" inspected not by the senses, but by the intellectual tools of the human brain.

So far, from an informational and algorithmic point of view, it has proved very difficult to outline, with any consistency, the internal genesis and the structure of the aforesaid operations on abstract concepts that live in the logical space of higher-level languages. In any case, we are perfectly aware that it is only through a precise process of approximation that the regularities that live at the deep level of the real processes can reveal themselves within a confirmational and "coupled" context, progressively showing their semantic information content. What we need to do is to exploit new distance functions defined with reference to higher-level languages, in order to identify new patterns of data and to "guide" the process of revelation of these data by means of a specific approximation process and a continuously renewed identification of fixed-points. This process of revelation may, in turn, in a confirmational (and selective) context, determine a series of cut-offs at the level of the development of the intellectual structures, constraining in this way, from the outside, the path of inner conceptual creativity.

In this sense, we have to each time individuate and "prime" the "good" new axioms, the good new methods, the suitable new measures and so on. It is, for instance, no longer possible simply to add the "discovered" undecidable statements to the original given system. We need new and, at the same time, concrete axioms, well suited in order to attain our conceptual goals.

We are faced with a kind of self-organization process necessarily linked to an increasing form of conceptual awareness, an awareness that, in some way, provides the tools for an effective control of the process itself. It is at the level of such a self-organizing process that we can identify, at the second-order level and in a co-evolutionary landscape, the birth of new domains of individuals etc. The good new axioms must be natural and concrete; they must reveal themselves as linked to the emergence of ever-new structures, to the holistic growth of the original system and, at the same time, to its "opening up" from the interior at the deep level. In this way an entire net of links with other systems and an articulated set of new constraints will enter the stage. New forms of coherence and new paths of assimilation will appear: hence the important role played at this level by selection activities and by the articulation of precise forms of co-evolution.

From a conceptual point of view, the algorithmic information theory shows why it is possible, with no trace of contradiction, to speak of the non-existence of finite algorithms in relation to problems which are, however, well defined in terms of uniqueness and existence. This non-existence constitutes a non-eliminable basic given, just as the existence Prigogine perceives of a randomness based on dynamics, is a basic given at the physical level. Symbolic dynamics show us how, behind the extensive use of the results of the algorithmic information theory, particularly in the field of the models of dynamical systems, and, more in general, of physical processes, there exists the stratagem of reducing randomness living at the level of the continuum to that which operates in the field of the discrete. Hence, at the level of, for example, the phase space and the representation of a dynamical system through this space, it is possible, by taking the correct steps, to make a binary digital string correspond to each orbit and to incorporate all the results of the algorithmic information theory into the construction of the model. This strategy actually corresponds to an underlying conceptual design of a very general nature, which basically considers the theoretical models of physical processes as actual computational processes within which the state of the system is identified by means of a complexion W in the phase space, while physical laws are referred to specific algorithms. In computational language, the action of the algorithms corresponds to specific Turing processes; in this sense it is apparently possible to modulate the corresponding programs by means of strings of binary digits. Within the frame of a dynamical system, the result will be programs describing the algorithm of evolution and strings corresponding to the states. This will then provide the possibility of using the results of the algorithmic information theory to define, at an abstract level, the limits of the models used, their predictive capacity, etc. Physical and mathematical randomness thus reveal themselves as even more directly interconnected, and the concepts of algorithmic incompressibility and exponential complexity emerge as being essentially subtended to the reality of physical randomness. The whole, classic question concerning intractability, NP-completeness, and so on appears thus directly linked to the reality of the evolution of dynamical systems. The statement that chaos is deterministic randomness is even more precisely formulated if the laws of chaos are predicated on the basis of the traditional undecidability theory. For example, it is already possible to demonstrate how in specific dynamical systems, chaos is simply the effect of the undecidability proper to a recursive, system-related algorithm. At an even more general level, we can state that deterministic, chaos-generating systems may be considered as set-ups able to act as models (in a logical-mathematical sense) for formal systems (algorithms) within which it is possible to trace the trajectory of a mathematically realized randomness. In other words, as already stated, it is possible to consider the randomness defined at an algorithmic-informational level, as the counterpart and theoretical foundation of physical randomness connected to the articulation of well-defined processes – i.e.

set-ups. This last kind of randomness, however, must be considered as absolute, not only because it satisfies the requisites of objectivity, coherence and intrinsic incompressibility, but also because the cognitive agent is able to define it principally by stating the limitation theorems concerning the formal system within which randomness is recognized as such, particularly with regard to that specific system which, at an algorithmic and programming level, assists, as we have just said, in the "guided" construction of the corresponding set-up. Indeed, when the cognitive agent becomes aware of the fact that a particular axiomatic system allows him to define a given number as random, but is still unable to resort to its formal tools in order to calculate that number or to outline a proof of its purely casual nature, s/he sees a concrete foreshadowing of an absolutely random string which, out of all possible messages, will be that which is most redolent of information precisely because all redundancy has been eliminated.

Let us point out that an analysis of mathematically defined randomness also gives full prominence to the concept of self-referentiality in its construction. As we have seen, it would be impossible to speak of Ω if the system were not sufficiently powerful and self-referential. Even at a physical level, on the other hand, a cognitive agent attempting to determine the complexity of an evolving system by means of the measurements utilized at the cognitive level also needs information concerning the research tools applied. Thus, the construction of the concept of algorithmic entropy also represents one of the necessary steps whereby the theme of self-referentiality is completely revealed, and most markedly at the moment when the cognitive agent, *via* Gödel-Turing, becomes aware of the possible extension and limits of the most sophisticated methods of analysis and measurement the agent possesses.

We saw, on the other hand, how also Prigogine underlines the illusory nature of the world of the trajectories envisaged by classical dynamics, and posits the cognitive agent as actor-spectator, characterized by a form of ongoing self-referentiality and, in general, by the precise awareness of constituting the historical product of a Nature with which the agent interacts, modifying it while personally being modified, together with his/her whole cognitive apparatus.

These considerations again lead us, almost at the closing of an ideal circle, to the problem of coarse graining from which we started out. On the basis of the material examined, Gibbs' *aporia* would now obviously appear as strictly linked with both the phenomenon of deterministic chaos and the problematics concerning undecidability theory. As Prigogine may have been the first to demonstrate (albeit without the full awareness that only a detailed understanding of the formal aspects of NP-completeness etc. can give), the use of coarse graining techniques once more finely and accurately honed, again represents the obligatory trajectory if we are to reach an effective definition of intrinsic randomness characterizing unstable phenomena at the physical level. From a general point of view, as Grassberger points out,[11] when studying a dynamical system the cognitive agent is not only unable to deal with too many variables, but also to cope with infinitely precise numbers. If we accept that space-time is continuous, this means that the cognitive agent is always obliged to cut off the digital expansions for all coordinates somewhere. In the case of deterministic systems characterized by a sensitive dependence on initial conditions, this reduction, together with any form of imprecision, however apparently insignificant, in the expansion of digits, will with time lead to forms of actual unpredictability which cannot be avoided by taking a more precise measurement of the initial conditions. When the attractor is chaotic the dilating and contracting fibers bring about the progressive elimination of the initial information. The result is that, in a short time, the initial indeterminacy spreads, as it were, through the entire attractor, and no casual link any longer exists between past and future. A measure of this process is that concerning topological entropy and metrical entropy. At the metrical level, the average rate at which the information is generated will give some measure of the emerging chaos, to put it in rather general terms. In other words, as time passes and the trajectories diverge exponentially, it will be necessary to introduce new information in order to retain some trace of the system's future state. Let us indicate by the constant K the information flow rate in the limit of nearly error-free measurements. For predictable systems, one would have $K = 0$; for Brownian motion one has $K = \infty$: even perfect knowledge of the state at some instant would not be sufficient to predict it in the near future. In this sense, a strange attractor appears to be characterized by two essential invariants: its information dimension D, and its entropy K. For a system in conditions of deterministic chaos: $K > 0$.

Within this frame of reference, the fundamental point to be underlined is as follows: distinguishing between pure stochastic processes and deterministic chaos is only possible insofar as we are able to apply a precise coarse-graining technique, a distinction which would not be viable if it were not possible to consider the limit of infinitely fine coarse-graining. The chief indication of the existence of determinism is based on the fact that effective Kolmogorov entropy flow rates and effective dimensions stay finite in the limit of fine coarse graining. Prigogine's theory thus tallies to an extent with the results of Kolmogorov's complexity theory, demonstrating the central role of the conceptual evolution of coarse-graining technique in the development of the contemporary theory of complexity. The analysis of this technique, as clearly emerges from the above considerations, shows us that it possesses an objective character and that it is also anchored both to a precisely determined relationship between the cognitive agent and the Source, and the existence of self-referential conditions which effectively help to determine the "emergence" of specific limitation factors.

1.3 Cellular Automata and Self-Organization

When we take into consideration metric entropy, we can easily realize that the particular link between selection, information and complexity investigated so far is also present at the level of description of the dynamical behaviour of a system characterized by the presence of a lattice of identical Boolean components interacting locally in space and by the presence of specific threshold phenomena. At the level of such a network some global computation patterns can arise as a function of time. This well known fact is based on the existence of particular attractors of the underlying dynamical network: depending on the initial stimulus the system will end in different basins of attraction. The aforesaid dynamical system considered as a network can be directly considered as a cellular automaton. Such an automaton can be looked upon as a very large net of automata that may have more than two dimensions (but equally only one). This net consists of points-sites that are connected to other points of the net by lines-edges: the points are normally considered as finite automata and the lines as the input or output wires of these automata. A single automaton is called a cell.

An immediate generalization of the concept of cellular automaton is represented by the concept of neural network. At the level of this type of generalization we put the cells at the vertices of a digraph with arbitrary communication links so that both communication and local transitions may depend non-uniformly on neighbour sites. The evolution of the network is assumed to take place in discrete time steps. Moreover, this kind of network, in order to be able to "simulate" a Turing machine – in order, that is, to possess full computational ability- requires a finite number of cells, a number that, however, dynamically increase without bound. We can prove that every cellular automaton is a neural network (NN) and that neural networks are at least as powerful as Turing machines.

As regards the architecture of neural networks, we have, first of all, to distinguish feed forward networks from recurrent networks. Within the class of recurrent networks one of the most powerful and elegant systems is represented by the Hopfield network,[12] a network characterized by the presence of non-linear computing cells, symmetric synaptic connections and a lot of feedback. It embodies a precise physical principle: that of storing information in a dynamically stable configuration. For example, in this kind of model the attractors might be thought of as memories held by the neural network. Such a system, equipped with attractors, naturally classifies and generalizes. All initial states in the same basin converge to the same attractor and hence are classified as identical. The Hopfield model can be seen, thus, as a Content-Addressable Memory which involves the recall of a stored pattern by presenting a partial or distorted version of it to the memory.

The Hopfield model and, in general, recurrent neural networks show themselves as powerful recipes in order to simulate important biological structures of interest at the level of cognitive activities (associative memories, pattern recognition systems and so on), but they also appear as being deeply rooted in statistical physics. Actually, a Hopfield network, considered as a large assembly of identical cells, each characterized by an internal state and interconnected by synapses of varying weights in accordance with a dynamic rule, is very close to some simple models of magnetic materials encountered in statistical mechanics, in particular to the spin-glass model. If, for instance, we compare the equations that characterize the spin-glass model with those of the Hopfield model we immediately perceive that there is a precise isomorphism between these two models.

It is, however, necessary to underline that only if synapses are symmetric does there exist a precise mapping between the steady dynamic states of a neural network (like the Hopfield network) and the thermodynamically stable states of a corresponding Hamiltonian model. But, from an effective point of view, there also exist random networks with weak connectivity and essentially no synaptic pairs. Hence the necessity of the construction of dilute asymmetric neural networks like, for instance, the dilute Hopfield-Hebb network. A Boolean emulation of this last kind of network is represented by a network which is randomly connected, with no underlying lattice. Each node of the network is constituted by a Boolean function $F_i(g_{i_1}, \ldots, g_{i_c})$ where F_i takes the values ± 1 and the g_{i_r} are the states ± 1 of the cells, i_r, $r = 1, \ldots\ c$, feeding cell i. We assume, moreover, that each cell is fed by exactly c others. The dynamics is given by: $g(t+1) = F_i(g_{i_1}(t), \ldots, g_{i_c}(t))$. What we actually have is a Boolean network.

When the F_i are randomly chosen quenched functions the model corresponds to the model introduced by Kauffman[13] as a model of biological order and self-organization at the level of complex living systems. The Kauffman model is a sort of Boolean analogue of the SK spin-glass model but, at the same time, constitutes an ideal recipe in order to study that specific randomly induced complexity which, from a neo-Darwinian point of view, appears as the real essence of natural evolution. According to this model, networks are made up of binary on-off variables. A network has N such variables. Each variable is regulated by a number of the variables in the networks which serve as its inputs. A Boolean function specifies the activity of the regulated variable at the next moment for each of the possible combinations of current activities of the input variables.

Let K stand for the number of input variables regulating a given binary element. The number of combinations of states of the K inputs is just 2^{K}. Since there are two choices for each combination of states of the K inputs, the total number of Boolean functions is $2^{2^{\mathrm{K}}}$. Each of the possible combinations of activities or inactivities of the elements of the network corresponds to one state of the entire network. The system passes through a succession of states called a trajectory. Since the system is deterministic and must always pass from a state to the same successor state, the system will cycle repeatedly around a state cycle. The state cycles are the dynamical attractors of the Boolean network: the set of states flowing into one state cycle constitutes the basin of attraction of that state cycle. In this context we can individuate, first of all, two extreme cases: the network $\mathrm{K} = \mathrm{N}$ and the network $\mathrm{K} = 2$. In the first case each element receives inputs from all elements. In these maximally disordered systems the successor to each state is a completely random choice among the 2^{N} possible states. The attractors of these systems increase in length exponentially as N increases. These networks are characterized by a big sensitivity to initial conditions. However, these systems manifest a precise form of regularity. The number of cycles, hence basins of attractors, is small: N/e.

In the second case the scenario is completely different. The sensitivity to initial conditions is very low; each attractor is stable to most minimal perturbations. We find that most mutations only alter attractors slightly. This class of systems exhibits small attractors, homeostasis and correlated landscapes. Between the two extreme cases there is a third intermediate case. Thus, as Kauffman remarks, we can distinguish particular structures that live on the edge of chaos, at the interface existing between chaos and order. These networks appear able to tune their internal structures and coupling to one another so as to optimize their adaptability and to perform efficient computations utilizing the mutations.

From a general point of view, we can therefore affirm, according to Kauffman's main thesis, that the transition between order and chaos appears as an attractor for the evolutionary dynamics of networks which exhibit adaptation. Actually, it is plausible that systems in the solid regime near the edge of chaos possess an internal organization able to permit successful computations as well as a "coherent" form of knowledge. Complex living systems normally "read" their world in an intelligent way, developing their inner potentialities step by step. But Boolean networks also really classify. In order, however, that classification may be coherent and stable the "knower" should not be chaotic nor should its classification. Hence the necessity that the knowing system is in solid regime perhaps near the edge of chaos. In such a theoretical landscape, learning itself may be seen, according to Kauffman, as a fundamental mechanism which converts chaotic attractors to orderly ones.

In this way, in the passage from the theory of dissipative systems to the theory of self-organizing Boolean networks, we can really perceive the development of a continuous and coherent line of research devoted to the individuation of the general principles that characterize the ultimate nature of the intermediate state. Step by step, we have explored a "strange" conceptual texture whose yarns, following Prigogine, Nicolis, Kauffman, Hopfield etc., are represented by randomness and asymmetry, information and selection, trajectories and attractors, chaos and adaptation, memory and classification, knower and known, learning and evolution. With reference to the development of this particular line of research (a line of research that, from a historical point of view, has largely coincided with the same unfolding of some of the central concepts of contemporary complexity theory) it constitutes no surprise that a given model like, for instance, the Hopfield model is isomorphic with the spin-glass model and, contemporarily, constitutes a main achievement in the field of neural networks and cognitive sciences. Analogously, is no surprise that the Kauffman model works so well, from an effective point of view, in so many different fields of research: from statistical physics to molecular biology to evolutionary theory.

To many respects, the roots of this character of theoretical interconnection and multiple functionality, lie, in actual fact, in the same nucleus of the scientific program developed by Kauffman (as well as of the scientific programs outlined by Prigogine, Nicolis, Anderson, Hopfield etc.). Kauffman's principal aim consists precisely in delineating, in perspective, a new type of statistical mechanics, one that defines ensembles of dynamical systems and seeks generic behaviors in distinct ensembles: a type of statistical mechanics that reveals itself, moreover, as indissolubly linked both to contemporary studies on the dynamical behavior of dissipative systems and the outlining of adequate Boolean models for biological structures.

This sort of program appears to be largely coherent with that particular accomplishment of the Boltzmann program delineated by Prigogine in his last works. According to Prigogine, irreversibility, as we have said, is a primary, original property. Irreversible processes play an essential role in inscribing Time into matter and in producing information. In this sense, irreversibility requires a precise revision of classical dynamics, an extension, in particular, of this dynamics incorporating probability, instability and chaos. The microscopic description of the universe can effectively be made only in terms of unstable dynamical systems. In the light of these considerations, we thus clearly realize how the progressive (and tentative) extension of the conceptual tools of statistical mechanics actually joins up with an innovative exploration of the "nature" of cellular automata and random Boolean networks and, at the same time, with a wider description of the behaviour of non-linear dynamical systems and dissipative systems.

We must, however, underline that throughout the unfolding of this whole theoretical research program, as specifically realized for instance by Kauffman, we have been constantly anchored, from a logical point of view, at the basic level, to propositional Boolean frames. Actually, the random Boolean networks that we have considered until now, even if they reach the "status" of cellular automata, constitute the final result of a specific assembly of particular, more simple basic components. These components can be seen either as finite automata (or as automata accepting Boolean languages) or as particular forms of Boolean algebras whose generators are represented by binary inputs or propositional variables. A random Boolean network is a massively parallel-processing system linking the activities of thousands or millions of such variables, variables that act according to a "propositional" fashion. Each cell in the Kauffman model is, effectively, anchored to a Boolean function. From a computational point of view, this fact is perfectly compatible with the framework of the traditional information theory (which, as Shannon clearly shows, is essentially characterized in propositional or monadic terms) and with the algebraic theory of Markov automata.

If, however, we plan to build a complex cellular network able to simulate, in a coherent way, the biological activities of neural nets and, in general, some of the functions performed by the cognitive structures (as well as, at a different level, by the organization of genomic regulatory system), we are necessarily obliged to abandon the exclusive and unique reference to Boolean propositional frames and threshold circuits having a cyclic character.

It is quite evident, for instance, that by resorting to simple Boolean functions for describing the rules of regulatory interactions between genes we cannot attain an adequate explanation of the regulatory activities concerning the expression and the evolution of the genome. As we shall see in the following pages, in accordance with the theoretical modules introduced at the level of the alternative splicing, the levels of gene expression appear to depend on complex modulations induced by specific binding proteins. Thus, we have to utilize multiple-valued functions instead of Boolean functions in order to build a concrete model, what is more we also have to link the action performed by these multiple-valued functions to the dialectics between the computing membrane and the genome sequence and we have to identify the transcription factors at work as well as the links existing between these very factors and RNA, microRNA and so on. The last results in this field of research, as recently published in the frame of the ENCODE project, are astonishing with respect to the game at stake and the involved combinatorial power. A living cell is not a molecular computer that configures itself as part of the execution of its code in accordance to the main thesis advocated by von Neumann. It really self-organizes; what is more the aforementioned execution must be performed with respect to meaning in action.

Thus, we must delineate the architecture of more complex cellular automata which, at the basis of their sophisticated behavior, do not possess simple automata accepting Boolean languages or, in general, simple Boolean mono-dimensional frames. We have to introduce, from the beginning, at the basic level, more complex algebraic structures and we have to outline complex cooperative activities emerging from the organic interconnection of these specific kinds of structures. We have, in other words, to resort to particular Boolean frames not restricted to the propositional level, but having a predicative and, in perspective, an intensional and holistic character.

With respect to the first level, the predicative one, the algebraic structures which we speak about should be characterized not only in terms of simple Boolean algebras, propositional Boolean gates and so on, but also in terms of Halmos algebras, classically considered as Boolean algebras with additional algebraic operations including quantifier operations as well as operations from some semigroup related to a specific given variety. In this sense, we have to consider, in the first instance, at the level of the aforesaid complex structures, the combined presence of cylindrifications and interconnected constraints. We have, therefore, to graft onto the corpus of these particular structures the specific conditions relative to the recurrent nets.

From an intensional point of view, we should, in turn, distinguish a series of levels characterized by precise types according to a well-specified hierarchical and "vertical" dimension. These levels are linked to precise algebraic transformations operating on the previous steps through the successive imposition of particular constraints. In this sense we should distinguish an entire hierarchy of collections of types of entities.

Moreover, it is necessary to note that if we need a new statistical mechanics as well as a new level of possible description, i.e. the outlining of a new kind of dynamics, as advocated by Prigogine and Nicolis, we certainly also need a theory of semantic information underlying this specific type of dynamics and its possible extensions. Only if we reach an adequate description of the semantic aspects of the deep information processes shall we be able, for instance, to understand the inner articulation of molecular language and, at the same time, the distinguishing features of that natural (and semantic) "order" that inscribes itself in this very language. However, it is precisely in order to explore the aforesaid semantic aspects of depth information that we have to take into account semantic information measures articulating not only at the propositional or monadic level (as we actually do according to the traditional information theory) but also at the predicative and second-order level. In fact, as is evident, what best distinguishes meaning's activity, from a technical point of view, at the level of biological functions and natural order, is its capacity of articulating in accordance with a continuous and autonomous emergence of ever-new linguistic roots, an emergence that can be taken into account only by resorting to explanatory models operating both at the first-order level and second-order level.

Indeed as Atlan correctly remarks,[14] in a natural self-organizing system (a biological one, characterized by the existence of precise cognitive activities) the goal has not been set from the outside. What is self-organizing is the function itself with its meaning. The origin of meaning in the organization, of the system is an emergent property. In this sense, the origin of meaning and its progressive unfolding appear closely connected to precise linguistic and logical operations and to well-defined embodiment procedures. These operations and procedures normally induce continuous processes of inner reorganization at the level of the system. The behavior of the net, in other words, possesses a meaning not only to the extent that it will result autonomous and to the extent that we can inspect a continuous revelation of hidden properties within it, but also to the extent that it will prove capable of being intentionally linked to a continuous production of possible new interpretation acts and reorganization schemes as well as to the progressive individuation of "good" measures. The state space relative to these computational, self-observational and intentional activities cannot be confined, in this sense, on logical grounds, only within the boundaries of a nested articulation of propositional Boolean frames. On the contrary, in order to give an explanation, within a general theory of cellular nets, for such complex phenomena, we will be obliged to resort to particular informational tools that reveal themselves as essentially linked to the articulation of predicative and higher-order level languages, to the outlining of a multidimensional information theory and to the definition of an adequate intensional and second-order semantics. Therefore, this state space will reveal itself as indissolubly linked to the progressive exploitation of specific, non-standard procedures.

In other words, in the light of recent research in the field of self-organization theory,[15] Kauffman's theory of self-organizing nets, even if it offers a first, partially satisfactory modeling of the processes of mutation, differentiation and selection as they articulate at the level of a simple biochemical network, does not appear capable of giving an adequate explanation for the processes of creative evolution: for the processes, that is, that characterize, at the biological level, the progressive unfolding of the complex semantic content proper to the original replicative source. A replicative biological source that, actually, articulates in an autonomous and synthetic way, constructing its own structure while transmitting its message. The transmission content is represented by the progressive "unfolding" of this very source, of the instructions concerning, each time, its actual self-realization, its primary operational closure. This closure realizes, in turn, its own "invariance" and, at the same time, its metamorphosis by means of replication and by means of a continuous production of varied complexity. The resulting embodiment cannot be seen as a simple output determined only by the mapping function; on the contrary, the transmission of the semantic information content appears to be also linked to a process of revelation and "renewal" of itself. A renewal that, at the teleonomical level, appears determined by a plot of cognitive procedures in accordance with the actual individuation of specific limitation factors.

We are effectively faced with a complex parallel dissipative network within which we can distinguish, first of all, the presence of a model-inscription, of a multiplicity of interpretation functions and of a representation surface. This network is open to the flux of deep information and is constrained by the selective pressures. The role of the attractors takes place in the background of this intricate series of processes; it cannot concern only a single component of the cycle of the metamorphosis.

In this sense, the self-organizing and cognitive activities articulating at the biological level express themselves necessarily in a functional and linguistic space that cannot be described only by means of models effectively and directly originating in an extensional and mono-dimensional articulation of automata accepting simple Boolean languages, even if the entire net in which these automata are included is capable of expressing, as a whole, a full computational ability. In particular, at the level of this type of activities it is necessary to perform a precise exploration of the feedback processes linking the hierarchical and intensional growth of the whole system with the progressive and contemporary increase of the inner complexity of its basic components, an increase that is characterized by an incessant "emergence" of new predicates and generators.

So when we try, for instance, to outline a theory of meaning at the level of such a space, the connected semantics cannot, evidently, be defined according to the traditional paths of standard extensional semantics. It will be necessary to outline a more powerful type of semantics: a semantics of processes and not of states, a semantics within which it will be possible to take into consideration intensional types, holistic traits and procedures, recurrent structures, a nested and multidimensional construction of individuals and so on.

We need, in other words, a self-organization theory capable, in perspective, of expressing itself at an intensional and hyper-intensional level; we have, for instance, to extend the standard framework relative to the traditional Boolean models in order to build a new and more general kind of theoretical concept: the concept of self-organizing model. The design of such a new kind of semantics, if successful, will necessarily lead us, as a consequence, to perceive the possibility of outlining a new and more powerful theory of cellular automata, of automata in particular, that will manifest themselves as coupled models of creative and functional processes. We shall no longer be only in contact with classification systems or associative memories or simple self-organizing nets, but with a possible modeling of precise biological activities, with biochemical networks or biochemical simulation automata capable of self-organizing, as coupled systems, their emergent behaviour including their same simulation functions.

As already mentioned, the objective and absolute nature of randomness as it emerges when the randomness is systematically constructed with respect to formal systems and set-ups articulating within the boundaries of first-order languages, would be seen as strictly connected, on a mathematical level, with the insoluble nature of the halting problem and the definition of a non-computable, real number. It was also mentioned above that entering the world of randomness necessarily requires the previous recognition of the existence within the system of specific self-referential and coupled processes. On the other hand, we have seen, at the physical level, the impossibility of deducing the past-future distinction from reversible laws of time. Time direction appears to be a primitive concept supported by the cognitive agents and inscribed in their primary cognitive constitution, that of beings immersed in the Nature they describe and whose historical product they are. In this sense, the discovery of irreversible processes acquires the sense of a formal acknowledgment of the limits linked to any possible manipulation.

This conception of physical randomness, refining and creatively interpreting the original coarse-graining methods, send us back to Prigogine's original delineation of the probability-dynamics relationship. Indeed, in Prigogine's opinion, measuring both implies and essentially determines a situation of irreversibility which only becomes more epistemologically explicit when causality is added. The description in dynamical terms, then, can be seen as the result of a very specific process: the production of a physical object for a scientific subject it both excludes and implies.

Scientific work thus emerges as a question of progressive construction. Symbolic Dynamics must give an intelligible account of the spontaneous appearance of cognitive systems; on the other hand, the recognition of the need of causal explanations and the work of cognitive modeling should be considered as the modalities characterizing the activity proper to a natural (and evolutive) organism. The cognitive agent-dissipative structures circle is thus closed, the macroscopic world appearing able to offer the observer the departure point for his cognitive constructions.

The type of intelligibility offered by dissipative structures, Prigogine states, in terms of a system which self-organizes through a series of bifurcations, can be applied both to the development of life and the working of the brain. Intelligence is an evolved form of dissipative structure that offers an objective justification of the causal dimension of dynamics, permitting a form of awareness of the extension and limits of the apparatus which has gradually evolved. It is this which is behind Prigogine's revisitation of the basic principles of Piaget's genetic epistemology, particularly of Piaget's precise awareness according to which the cognitive processes appear simultaneously to be the result of the organic self-regulation whose mechanisms they reflect, and the most widely differentiated organs of this regulation in its interactions with the outside.

This revisitation, in all its classic circularity, is perhaps the clearest indication of Prigogine's covert epistemological projection in defining the objective character of randomness, while at the same time pointing to its limits. We have seen that there is no possible doubt concerning the objective and absolute nature of randomness as it occurs in first-order languages. It is the result of a precise logical and informational construction linked to the proof of specific incompleteness theorems. In terms of the quasi-empirical conception of information theory, the levels of incompressibility appear, at the present state of knowledge, to be able to arrange themselves in an enormous variety of intermediate stages according to continuous variations in class and context of reference. This in no way, of course, justifies considering first-order language randomness as anything less than absolute. Unlike the case of the generators of pseudo-random numbers, where we move from one

Q-class of reference to another while remaining within a particular axiomatic and unitary system, random reals present us with a crossroads for both knowledge and "physical" constructibility. Moving beyond the Pillars of Hercules represented, for example, by Ω, and tracing new axioms would involve a traumatic leap in the hierarchy of complexity; it would mean the determining of a new system of axioms, the jettisoning of old, finitistic methods and the loss of the model's classic invariance features. At an informational level, it would mean renouncing any ideal of a mono-dimensional and direct characterizing of information and recognizing that no direct equivalence can be posited between neg-entropy and information.

The Brave New World that then opens before us will no longer possess the requisites of reflexivity and completeness characterizing the preceding levels. And in this world, and successive universes, there would be no further columns of Hercules to mark so precisely, as in the case of Ω, the shift from one level to another in the complexity hierarchy. However, the convergence processes which gradually emerge in these new universes will be able in turn to be represented, within relatively approximate limits, at the first-order level, which will therefore prove itself an effective construction-table for further degrees of complexity. It is the move from the old order to new universes which testifies *in primis* to the absolute nature of randomness articulating at the mono-dimensional level. And although this is far from excluding successive proofs of the compressibility of strings previously deemed random, these proofs will inevitably have to be based on logical orders characterized by the absence of the features of the worlds' completeness and invariance: on orders which are chiefly characterized by a real dialectical interaction between cognitive and structural processes.

In these new worlds it will no longer be possible to consider the process of justification and awareness in which randomness finds expression, in any absolute sense, as a sort of definite fixed point of the coupled system source-cognitive agent. It is, however, precisely this kind of conception, as we have seen, which permeates, to many respects, Prigogine's methodological formulation of the problem of the relationship between intelligibility, on the one hand, and objective randomness, on the other and leads him to rediscover the classic character of circularity subtending Piaget's genetic epistemology, a character which is linked, as we know, to a specific reversibility frame.

In Prigogine's case, as a result of the afore-mentioned projection and the rooting of probability in dynamics, it is as if the cognitive agent reached a point of awareness (able to close the global epistemological cycle), and shut him/herself within a pre-established and unchanging apparatus of rules and axioms anchored to an essentially finitistic dimension. This cognitive agent will thus be precluded from glimpsing the existence of a randomness which is methodologically different, as situated at a deeper level than that produced by the dissolution of the frame of limits recognizable when the above-mentioned apparatus is utilized and "criticized" at the logical and methodological level. On the contrary, we know that, within the boundaries of a selective and coupled frame, the cognitive agent's methods can possibly evolve beyond the old finite limits and prime the revelation of new relations as well as of new and adequate axioms, thus ultimately accessing higher orders of complexity previously concealed also because other, partial structures, judged random in an absolute sense, had been privileged.

The logic characterizing an evolution of this kind, based on the individuation and the correlated action of concepts articulating at the second-order level will, however, as stated above, require constant reference to new forms of informational interaction between the external source and the cognitive agent and to the effective construction of new, non-mono-dimensional measures articulating within a frame of reference characterized, on the logical plane, by essential incompleteness.

The concept of objectivity will clearly have to change. The new conceptual structures, the new interpretation modules will be manifest to the cognitive agent only after a journey into the realms of abstract and conceptual evidence, as well as of the possible which springs from existing, achieved constructions: i.e., only when these modules, by means of possible invention and suitable de-repression processes, will have coagulated in the fibres of the evolving cognitive network proper to the cognitive agent him/herself, although also in dependence of the selective activities performed by the external source.

Only, then, insofar as they manage to trigger precise processes of de-repression of latent information, will the cognitive agent be able to apply these modules as instruments of objective awareness of the world. Where these conditions obtain, it will be possible to view the new conceptual structures both as analytical forms of reading and constitution of natural processes simultaneously. The basic point to underline here is that this whole process cannot come about, at the higher levels, in a purely "spontaneous" and physical way – the effect, for example, of isolated chaotic events and their mere composition. It can only be posited within a specific and coupled relationship between the source and the cognitive agent, a coupled process characterized by the progressive appearance of new forms of reflection, imagination, and self-regulation: i.e., the appearance of repeatedly new cognitive models.

The role of these models will then emerge as of seminal importance in the evolution of living systems. The contact with conceptual evidence, the journey through the realms of the "possible", and the exploration of the memory of life and the world of History represent the keystone for the effective emergence of that specific action of coagulum which alone can trigger the de-repression of the latent potentialities proper to the source. A particular embodiment of these specific tools appears able to prime, for instance, new forms of conceptual "reading" of the information content hidden in the text provided by the molecular language.

These tools do not constitute only a kind of abstract representation and, what's more, are not "neutral". In consequence of the choice of one specific model we shall have, as a result, some specific irreversible performances: the realization that is of a particular action of "guide" with respect to the unfolding of depth information. In this sense, true cognition appears to be necessarily connected with successful forms of reading, with those forms, in particular, that permit a specific coherent unfolding of the deep information content.

In the light of these considerations, if a given model appears as successful, it will attain a form of real (albeit partial) autonomy. It will present itself, that is, as a self-organizing and autonomous structure and not as a simple form of cognitive awareness: true cognition appears as a recovery and as a guide, as a form of simulated (or artificial) life. The simulation models realized this way, if valid, materialize, therefore, as creative channels, as autonomous functional systems. In other words, in order to become aware of the secret aspects of the self-organizing original source we must construct, in turn, at the surface level, cognitive models capable of simulating the external reality and even of simulating themselves as autonomous unities. We must attain the construction of simulation automata able to realize even a form of simulation of simulation.

In order to achieve this difficult task we have to reach a new level of objective knowledge. In particular, at the level of scientific semantics, we must not only search for a finer-grained cognitive representation of the real events, but we also have to go into the real (physical and biological) generative (and symbolic) dynamics of the events. We have to transform our logical investigation tools into the same roots of a new possible development of the entire coupled system. It is, precisely, in this sense, that we need the outlining of an adequate semantics of processes.

With respect to this general theoretical framework, the real problem, then, doesn't consist in referring, each time, to the simple existence of an intermediate state or to the individuation of the successive realizations of the different possible compromises between fluctuations and stability. The real problem is to follow the living "contours" of the evolution of the underlying generative dynamics, in order to be, finally, able to prime, in a coherent way, the unfolding of depth information. But this is, precisely, one of the primary reasons that have lead to the actual outlining of simulation models at the level of biological sciences: to determine, that is, the conditions for the unfolding and the recovery of the deep information content of the source and contemporarily to become a sort of arch and gridiron for the construction and the revelation of the Other (the creative life) through the constraints relative to the successful realization of the paths of simulation. In this sense, the construction of new algorithms, able to indicate the invention of further cognitive models, will emerge as one of the essential moves both for a better understanding of the complexity of living systems and a more sophisticated articulation of their actual evolution. Self-organization should be considered first of all as the secret (and teleonomical) "arch" between meaningful complexity, on the one hand, and selection by (generative) elimination, on the other.

Notes

1. Cf. Bais F.A. & Doyne Farmer J. (2008), "The physics of information" in *Philosophy of Information*, (P. Adriaans, & J. Van Benthem eds.), Amsterdam: Elsevier, 609–684.
2. Cf. Landsberg P.T. (1978), *Thermodynamics and Statistical Mechanics*, London: Oxford U.P.
3. Cf. Gibbs J.W. (1902), *Elementary Principles in Statistical Mechanics*, New Haven. (New York: Dover, 1960).
4. Cf. Jaynes E.T. "Information theory and statistical mechanics", (I and II), *Physical Review*, 106(1957), 4: 620–630 and 108(1957), 2: 171–190.

5. Cf. Tolman R.C. (1938), *The Principles of Statistical Mechanics*, Oxford: Oxford U.P. From a general point of view cf. also Denbigh K.G. & Denbigh J.S. (1985), *Entropy in Relation to Incomplete Knowledge*, Cambridge: Cambridge University Press.
6. Cf. Prigogine I. (1980), *From Being to Becoming*, San Francisco: W.H. Freeman and Prigogine I. (1993), *Les lois du Chaos*, Paris: Flammarion.
7. Cf. Nicolis G. & Prigogine I. (1989), *Exploring Complexity*, New York: W.H. Freeman.
8. Cf. Chaitin G. (1987), *Algorithmic Information Theory*, Cambridge: Cambridge University Press.
9. Cf. Putnam H. (1965), "Trial and error predicate and the solution to a problem of Mostowski", *Journal of Symbolic Logic*, 30: 49–57.
10. Cf. Chaitin G. & Calude C. (1999), "Mathematics/randomness everywhere", *Nature*, 400: 319–320.
11. Cf. Grassberger, P. (1984), "Information Aspects of Strange Attractors", in *Proceedings of NATO Workshop on Chaos in Astrophysics* (J. Pergand et al. eds.), Florida.
12. Cf. Hopfield J.J. (1982), "Neural networks and physical systems with emergent collective computational abilities", *Proceedings of the National Academy of Sciences*, 79: 2254–2258.
13. Cf. Kauffmann S.A. (1993), *The Origins of Order*, New York: Oxford U.P.
14. Cf. Atlan H. (1992), "Self-organizing networks: weak, strong and intentional, the role of their underdetermination", *La Nuova Critica*, 19–20: 51–71.
15. Cf. Carsetti A. (2000), "Randomness, information and meaningful complexity: Some remarks about the emergence of biological structures", *La Nuova Critica*, 36: 47–109.

Chapter 2
Embodiment Processes and Biological Computing

2.1 The Game of Life and the Alternative Splicing

As we have just remarked, at the biological (and creative) level, the original, developmental and selective source, while transmits and applies its message, constructs its own structure. The transmission content is represented by the progressive "revelation" through forms of the very source, of the self-organizing (and emotional) "instructions" concerning, each time, its actual realization at the surface level and its primary operational closure. This closure realizes its own invariance and, at the same time, its metamorphosis by means of the full unfolding of a specific embodiment process, by means of replication and by means of a continuous production of varied complexity. The final result of this process cannot be seen as a simple output: "the phenome" (according to the terminology proposed by H. Atlan[1]) completely determined by an input string (the genome). It by no means represents, however, an output determined only by a mapping function, so that the resulting structure appears as not encoded at all in the input string. On the contrary, the transmission of the information content on behalf of the source appears to be a process of realization, revelation and "renewal" of itself, a process realized, in particular, in accordance with the conditions proper to a coupled system: as a matter of fact, at the biological level the function necessarily self-organizes together with its meaning thereby avoiding (as linked to the stake) Scylla (the simple dissipation) and Carybdis (the pure crystallization). Hence the necessity at the level of life of the continuously renewed realization of a specific compromise: the "aperiodic crystal", as Schrödinger called that particular intermediate state represented by DNA.

Within this theoretical framework, the action of the *telos* is principally instrumental in enabling a progressive renewal of original creativity; the victory over the forces of dissipation primarily depends on the capacity on behalf of the source to manage to realize a targeted (and constrained) development, at the formal level, of its own potentialities: the *telos* constitutes the method and the *Via* of this particular unfolding thus animating the deep patterns of the embodiment process. The need for meaning selection to intervene derives from this, enacting a Medusa-like use

A. Carsetti, *Epistemic Complexity and Knowledge Construction: Morphogenesis, Symbolic Dynamics and Beyond*, Theory and Decision Library A 45,
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of the reflexivity tools to fix the lines of a realm of necessity able to guarantee the achievement and the permanence of an ever-new (and fruitful) equilibrium. Ariadne prepares the kingdom of the "ruler" by helping in offering adequate semantic constraints, in fixing and petrifying, thus outlining the necessary support for the new "conception". Through successful embodiment the model therefore realizes within itself the deep content of the original incompressibility while opening it to the dominion of meaning and "rational perception".

Actually, the DNA appears as the receptacle of information "programmed" by natural selection. It represents the first (and basic) achievement of the embodiment process, along the successive expression of the laws of the "inscription" and in view of the full unfolding of the realm of necessity, the actual triumph of the" ruler". It articulates through the cellular growth that is taking place according to the constraints imposed by the selection performed in ambient meaning and by the *bricolage* operated with respect to the preexisting structures. It is along this peculiar channel that the flux of deep creativity may, therefore, express itself and tune, in an innovative way, its original incompressibility according to the emergence of different stages of functional construction and to the correlated realization of a specific "nesting" process at the semantic level.

Within the frame of this particular kind of metamorphosis the genome must be seen as neither a program nor a set of "data". It appears, on the contrary, definable, in theoretical terms, as a "model", a model finally inducing the full articulation of a "ruler" working as a recipe. Both the interpretation function and the representation apparatus concerning that particular cellular machinery represented by the activity of proteins make essential reference to this kind of model. We are effectively faced with a complex cellular (and living) "network" within which we can individuate, first of all, the presence of a specific process of "inscription" as well as of an interpretation function operating at the level of surface representation. This network is intrinsically open to the fluxes of deep creativity and results constrained by the selective pressures acting in ambient meaning. The role of the attractors takes place in the background of this intricate series of processes; it cannot only concern a single component of the cycle of the metamorphosis. The genome expressing itself gives rise to a surface structure, to the body of that particular factory represented by the living cell of which the genome itself comes to inhabit the nucleus (at the level of eukaryotic cells) acting as the template and the forge of the transformation.

The genome expresses itself into a given phenotype in a complex way. Actually, the genome sequence codes for its own translating machinery, it determines the birth of a cellular machinery responsible, in turn, for gene regulation and expression. A particular gene, for instance, codes for RNA polymerase whose function is to transcribe the genes into messenger RNA. Without RNA polymerase there is no messenger RNA, we are faced with the absence of cellular life. However, RNA polymerase is necessary for its very synthesis because it transcribes its gene. Hence the essential circularity that characterizes living organism.[2] The cellular machinery "represents", step by step, the genome into an organism realizing the final stage of what we have called the embodiment process. In this sense, the genome and the cellular machinery really interact by establishing an evolving and coupled network:

as we shall see one of the key results of this interaction is represented by the continuous engraving (through selection) at the level of the organisms of specific formats: among them we can distinguish, first of all, the formats relative to the architectures of sensorial perception.

As Bray correctly remarks: "In unicellular organisms, protein-based circuits act in place of a nervous system to control behaviour; in the larger and more complicated cells of plants and animals, many thousands of proteins functionally connected to each other carry information from the plasma membrane to the genome. The imprint of the environment on the concentration and activity of many thousands of proteins in a cell is in effect a memory trace, like a 'random access memory' containing ever-changing information about the cell's surroundings. Because of their high degree of interconnection, systems of interacting proteins act as neural networks trained by evolution to respond appropriately to patterns of extracellular stimuli. The 'wiring' of these networks depends on diffusion-limited encounters between molecules, and for this and other reasons they have unique features not found in conventional computer-based neural networks".[3]

A common feature of protein circuits in living cells is represented by their ability to integrate multiple inputs. We can find the most astonishing evidence of this combinatorial capacity at the level of the regulation of DNA transcription in eukaryotic cells. Actually, a typical gene in a multicellular organism requires the assembly of a transcriptional complex composed of enzymes, transcription factors and gene regulatory proteins. Because these components are drawn from a very large pool of candidates, an extremely large variety of different transcriptional complexes, each with a different 'blend' of proteins, is possible.

Gene regulation was first studied most accurately in relatively simple bacterial systems. Most bacterial RNA transcripts are said to be colinear, with DNA directly encoding them. In other words, there is a one-to-one correspondence of bases between the gene and the mRNA transcribed from the gene (excepting $5'$ and $3'$ noncoding regions). However, in 1977, several groups of researchers identified a series of RNA molecules that they termed "mosaics," each of which contained sequences from noncontiguous sites in the viral genome.[4] These mosaics were found late in viral infection. Studies of early infection revealed long primary RNA transcripts that contained all of the sequences from the late RNAs, as well as other specific sequences: the introns, i.e. those stretches of DNA, which get transcribed into RNA but not translated into proteins. As is well known, those stretches of DNA that do code for amino acids in the proteins are called exons. The human genome is estimated to contain some 180,000 exons. With a current estimate of 21,000 genes, the average exon content of our genes is about 9. In general, introns tend to be much longer than exons. An average eukaryotic exon is only 140 nucleotides long, but one human intron stretches for 480,000 nucleotides. Removal of the introns – and splicing the exons together – are among the essential steps in synthesizing mRNA. Early in the course of splicing research, yet another surprising discovery was made; specifically, researchers noticed that not only was pre-mRNA punctuated by introns that needed to be excised, but also that alternative patterns of splicing within a single pre-mRNA molecule could yield different functional mRNAs.[5]

The first example of alternative splicing was defined in the adenovirus in 1977 when it was possible to demonstrate that one pre-mRNA molecule could be spliced at different junctions to result in a variety of mature mRNA molecules, each containing different combinations of exons. An example of a gene with an impressive number of alternative splicing patterns is the *Dscam* gene from *Drosophila*, which is involved in guiding embryonic nerves to their targets during formation of the fly's nervous system. Examination of the *Dscam* sequence reveals such a large number of introns that differential splicing could, in theory, create a staggering 38,000 different mRNAs. As Schmucker et al. showed in 2000, this ability to create so many mRNAs may provide the diversity necessary for forming a complex structure such as the nervous system.[6] In general, the existence of multiple mRNA transcripts within single genes may account for the complexity of some organisms, such as humans, that have relatively few genes. Alternative splicing exactly provides a mechanism for producing a wide variety of proteins from a small number of genes. While we humans may turn out to have only some 21,000 genes, we probably make at least 10 times that number of different proteins. It is now estimated that 92–94% of our genes produce pre-mRNAs that are alternatively-spliced. What is more, in the case, for instance, of the *Dscam* proteins, we can observe that they are used to establish a unique identity for each neuron. Each developing neuron synthesizes a dozen or so *Dscam* mRNAs out of the thousands of possibilities. Which ones are selected may appear to be simply a matter of chance, but because of the great number of possibilities, each neuron will most likely end up with a unique set of a dozen or so *Dscam* proteins. As each developing neuron in the central nervous system sprouts dendrites and an axon, these express its unique collection of *Dscam* proteins. If the various extensions of a single neuron should meet each other in the tangled web that is the hallmark of nervous tissue, they are repelled. In this way, thousands of different neurons can coexist in intimate contact without the danger of nonfunctional contacts between the various extensions of the same neuron.[7]

At a basic level, regulation includes splice-site recognition by the spliceosome, which is mediated by various proteins. Additional regulatory levels include environmental changes that affect splice-site choice and the relationship among transcription by RNA polymerase II (RNAPII), nucleosome occupancy and splicing. The existence of introns and differential splicing helps to explain how new genes are synthesized during evolution. As a matter of fact, splicing makes genes more "modular," allowing new combinations of exons to be arranged during evolution. Furthermore, new exons can be inserted into old introns, creating new proteins without disrupting the function of the old gene. In this sense, the definition of alternative exons is very important for understanding the links between splicing and evolution.

From a speculative point of view, we can assume that the analysis of the ongoing integration process as performed by the ruler on the basis of the activity of the different protein circuits and the involved assimilation procedures, can lead, at the level of the computing (and self-organizing) membrane, to the identification of a specific grammar capable of grasping the structure hidden in the final result of the integration process reducing it to a coherent and minimal set of instructions (as it happens, for instance, in the case of percolation phenomena). This identification presupposes the utilization of specific tools (reflexive, simulative, compositional etc.) living in a logical space characterized by the presence of specific second-order methods and non-standard procedures.[8] By exploiting these procedures at the level of the afore-mentioned analysis of the ongoing integration process, it is then possible to individuate specific (and hierarchical) schemes of programs connecting a variety of regulatory factors able to capture the deep functional structure proper to the effected synthesis. These schemes in turn can be utilized by the developmental source at the regulatory level in order to modulate (and construct) new patterns of expression of its deep information content at the surface level. In this way the action expressed by the polymerases comes to be influenced (and determined) by a variety of secondary "contractions". These patterns appear as indissolubly linked to the new game that is opening between exons and introns and between regulatory DNA and structural DNA. Here specific protoprograms get into the action at the deep level of the Bios: we can actually identify the ultimate roots relative to the possible unfolding of a new embodiment process as related to the birth of new proteins and renewed transcription units. Hence the necessity of recognizing at the level of DNA transcription and related procedures, the existence of a general meta-scheme in action concerning the progressive unfolding of a true editing process as well as of a precise self-modulation activity. The work performed by means of the alternative splicing constitutes one of the essential threads of this very activity. Utilizing a metaphor, we could say that it is actually Marsyas as architect (and craftsman) who modulates the inner "sense" of the *Via*, helping thus in following this very sense along its secret paths: he works for his "resurrection", but for the Other and in the Other, also exploiting the story-telling of the resurrection itself along the sweep of his artwork. On the one hand, we have a sensory (and computational) system that evolves and self-organizes by means of the realization of specific computations and assimilations as performed at the membrane level, on the other hand we have, on the contrary, the individuation-invention of intended schemes of programs, the individuation, in particular, of specific grids relative to different transcription complexes each with a different blend of proteins. It is the order of the binding proteins involved in these grids (or signal arrays) that determines and primes (in accordance with the action expressed by the different polymerases) the synthesis of new proteins when they are considered as necessary by the coupled system, on the basis of the estimates made, in order to assure its biological equilibrium as well as to give the correct answers to the challenges imposed by the environment. In accordance with our metaphor (and with the ancient tale), it is only through the incisions operated on his skin that Marsyas can come to listen to the song of the God. Along the assimilation process the membrane dreams of possible worlds while the God provides the suitable incisions. In general, these incisions are only possible because the membrane is able to read and simulate itself. In other words, it is only the correct exploration of the second-order realm that can allow the true hearing of the song. I do not send only messages signaling the need for the production of proteins already put into yard, I can also offer the right "suggestions" for the start of the production along the assembly line and in accordance with the analysis performed at the second-order level, of one or more new proteins on the basis of an alternative splicing of the exons. Hence the importance of the editing work. It is only with reference to such a dialectics that the new coder can each time take his moves in a creative but intentional way. The new eyes that come to open, the new arising life of the flesh appear therefore as related to the measurements put in place by the craftsman by means of the simulation tools and the ongoing narration, in order to perform an adequate recovery process on the basis of the outlining of the correct gridiron. In this way the coder will then be able to present himself to himself up to fixate and identify himself as an observing system in action. We are faced with an inner self-modulation of the coupled system: the alternative splicing represents one of the essential tools relative to the self-organization of the channel. In this sense, the dialectics between coder and ruler really plays a fundamental role: it is in view of a continuous synthesis of new proteins and new structures as well as of the accomplishment, each time, of a correct assimilation process.

When we look at Nature as inventor and architect and, in general, as morphogenesis in action, we see that the molecular systems that rearrange DNA normally process molecules in accordance with the presentation of a grid of signals. As is well known retroviral recombination is one of the genetic mechanisms that utilize such grids; at this level we can inspect, in particular, the synthesis of a single DNA molecule from two or more molecules of viral RNA. This task is carried out by an enzyme, the retroviral reverse transcriptase which is "guided" by a multiplicity of signal grids in the original RNA. As the transcriptase synthesizes the replica from its template, it may fall under the action of particular grids that trigger specific template switches that present themselves as the key event of retroviral recombination. If we generalize these results taking into consideration more complex techniques as, for instance, DNA shuffling, we can go more deeply into the secret reasons of this kind of recombination. Exon shuffling is a process in which a new exon is inserted into an existing gene or an exon is duplicated in the same gene.[9] It characterizes eukaryotic evolution and has contributed substantially to the complexity of the proteome. Exon shuffling creates new combinations of exons by intronic recombination which is the recombination between two non-homologous sequences or between short homologous sequences that induce genomic rearrangements. The mechanism includes the insertion of introns at positions that correspond to the boundaries of a protein domain, tandem duplication resulting from recombination in the inserted introns, and the transfer of introns to a different, non-homologous gene by intronic recombination. The diversity that can be generated by duplicated exons is strikingly shown by the Down syndrome cell adhesion molecule (*Dscam*) gene in *D. melanogaster*. The multiple, mutually exclusive exons of *Dscam* lead to enormous numbers of splice variants. The secret of this astonishing combinatorial power lies exactly in the intended individuation each time of the adequate grid.

At the level of the ruler, meaning expresses itself within the contours of an acting body as characterized by the successive articulation of "intelligent" protein circuits. It posits itself as living information which witnesses the continual amalgamation of observing systems into itself (Medusa in action, then – but finally Hercules at the crossroads of the two possible directions: dissipation and crystallization, questioning himself about a possible realization of the equilibrium of his creative body, as envisaged in his paintings by Annibale Carracci). Coding for proteins is strictly connected with the achieved embodiment as well as with the fixing and the enlightening of an intrinsically semantic information able to assure the progressive realization of a specific "nesting" process. In this sense, the computing membrane operates in accordance with the principles proper to a self-organizing model. From a general point of view, we can assert that the afore-mentioned realization of natural organisms coincides, in some way, with a process of successive "adjunction" of different observing systems. The realm of necessity (Nature in action) pulsates through perceptual acts. It is the ruler that marks the perimeter of the Temple of life, of the semantic information at work.

With respect to this frame of reference, when we look, for instance, at the last results obtained by the ENCODE scientists as regards the regulatory action put in place at the level of DNA, we can easily realize that transcription factors bind in a combinatorial fashion to specify the on-and-off of genes and that the ensemble of these very factors forms a regulatory network constituting the wiring diagram of the cell.[10] In this way, a limited set of transcription factors is able to organize the large diversity of gene-expression patterns in different cell types and conditions. The co-association of the different transcription factors is highly context-specific and can be organized in accordance with a stratified hierarchy. It results in a grid capable of inducing each time a specific set of secondary "contractions" at the transcription level. The transcription factors orchestrate gene activity from moment to moment according to specific basic rules on the basis of the application of particular self-organizing modules. The first map of the regulatory protein docking sites on the human genome as obtained in the frame of ENCODE Project reveals the dictionary of DNA words and permits a first sketch of the genome's programming language. There is an enormous variety of features that contribute to gene regulation. Every cell type uses different combination and permutation of these features to generate and "individuate" in a self-organizing way its unique biology. In this context, we need to not only individuate the secret reason of the orchestration put in place, but also to participate in it: we have to be actors and spectators at the same time as advocated by Prigogine.

The genome determining the expression of a cellular machinery, determines the birth of both an apparatus and a surface program embedded in that apparatus. Effectively, the apparatus doesn't appear to be an interpreter with a given program, it appears rather as a parallel computing network (but working at the surface level) with a precise evolving internal dynamics, a network able, moreover, to represent and reflect itself (and express, still within itself, its own truth predicate). The program "embedded" in this apparatus concerns the general frame of the connections and constraints progressively arising, its exclusive capacity to express (and canalize by attractors) a specific coordination activity within the boundaries of the becoming net. This capacity, on the other hand, can be "crystallized" on the basis of specific operations of self-representation and abstraction, so that it can be, finally, seen as the very "image" of the functional (but embodied) programs (synthetic forms in action) through which the apparatus progressively self-organizes expressing its autonomy. Here we can find the real roots of an effective identification process through adequate "enlightening". At this process-level we can find, on the one hand, information which codes for proteins, and, on the other hand, an operational closure which leads to the individuation of a "self"; from this derives the effective possibility of a dominion of necessary abstraction scanned by the "time" of the embodiment and supported by the cellular body in action. While Narcissus goes beyond the mirror and petrifies arriving to inhabit the waters of meaning and acting as a fixed point, Marsyas, instead, burns itself giving rise to productive abstraction. Narcissus offering the right attractors (along the first sense of the *Via*) allows the rising of that holistic enlightening that preludes to the new conception, i.e. to that particular breath of meaning that leads to the birth of Marsyas (taking on the guise of the Lord of the garlands).

By fixing the invariants by virtue of its positing itself as method in action, and by realizing itself as matrix for the "correct" retractions, the *telos* ultimately articulates as the editor of the process of "incarnation". It offers the first stage in constituting a Temple of life and observation and in drawing the very distinction between self and non-self. In this way, following the victory over the dissipative forces, a dominion of information, as connected to a precise level of neg-entropy, can be established. Meaning assumes precise contours insofar as it is able to express itself through the ruler, but in accordance with the action of the *telos*, thus helping in realizing a precise "inscription" process within the space belonging to the Temple of life. The Temple's columns flank the passage to the standard (in a precise mathematical sense, in accordance for instance with the intuitions of Reeb's school[11]), marking the successive adjunction of different observing systems which posit themselves as the real supporters of the ongoing construction. These acts of observation, then, are not only linked to decision and conventions, but more centrally to the unfolding of a process of incarnation and the dialectical relationship between ruler and coder as well as to the continuous emergence, at the evolutionary level, of new formats of sensibility.

Modifying and nurturing natural Reality, they constitute the support-structure of the rhythm by which it is traversed and scanned. To recover and define new acts is to assist in building a new Temple, and to effect a new incarnation by renewing the ongoing dialectical process: hence an observation which generates observation and amplifies the confines of the dominion of life. When Athena looks at (and incorporates) new means of observation which accrue, the realm of necessity is amplified, and that of Nature along with it. The possibilities of the falsification as well as the definition of new pile-works are multiplied, permitting the palace of knowledge to grow up, albeit only on the basis of a sacrifice and the actual construction of the Temple of life (the Cistern). Athena (as Medusa in action) assumes a body and a shape in determining the new ruler, and is thus able to inscribe herself within the space of this very Temple.

Through the unfolding of this continuous metamorphosis it is possible for the system to recognize (and pursue) the secret paths of the intentional information characterizing its intrinsic development as programmed by natural selection. The final result is a source that assumes a replicative capacity commensurate with a precise invariance and with the constitution of specific forms inhabiting life. It inscribes itself as enacting information and as a hereditary principle in action, as a source of varied complexity but compared with a hereditary apparatus which as such self-organizes in view of possible regeneration. The source which undergoes this specific metamorphosis process on the basis first of all of self-reflection opens out then towards a self-replication process which is targeted and part of a co-evolutionary path. The *telos* has to fix and be fixed in a "mask" to allow the source (at the end of the ongoing metamorphosis) to burst out and become enacting information, to express itself by means of living and moving programs: hence a developmental source that reveals itself as both productive and intentional, which helps to reject simple dissipation and progressively determines (starting from itself) the emergence of "strange objects" (according to Monod's definition).

2.2 The Interface Between Ruler and Coder

It is precisely with reference to the afore-mentioned process of metamorphosis and to the correlated embedded "program" that the genome acts as a model. A model that must not be considered only from a logical and semantical point of view (in a denotational sense), but also from a biological and functional point of view. As a model, that is, considered as acting information + intentionality, finally inducing the emergence of the ruler in action. In order to describe the functional nature of this particular model as well as of the link existing at the biological level between form and information, the resolution, however, of at least of three orders of problems results indispensable: (1) the outlining of a statistical mechanics at the biological level concerning genes and macromolecules and no more only atoms and molecules, able, moreover, to take into consideration the role of the self-organization forces; (2) the outlining of a semantic information theory taking into consideration the concept of observational meaning: the meaning as connected, at the same time, to a process, to an observer and to a hierarchical and embedded representation process; (3) the outlining of new measures with respect to the very concept of biological information. We need measures capable of taking into the consideration the growth processes, the statistical fluctuations living at the microscopic level etc. The Shannonian measure concerns essentially stationary processes articulating in a one-dimensional landscape, on the contrary, a true measure of information for life and hereditary structures should concern semantic information at work as connected to the action of specific, self-organizing coupled systems.

The model is the "temporary" receptacle of the biological functions and the replicative life; in particular, it appears, as we have just said, as the receptacle of information programmed by natural selection. The genome, in other words, is a model for a series of biological actions and symmetry breakings, for the realization of a complex path whose goal is represented by the attainment, on behalf of the apparatus, of a sufficiently complete functional autonomy at the surface level (within a dynamic ambient meaning). The interpretation function relative to this kind of model appears exactly to concern the actual realization of the embodiment process through the progressive execution of meaning. The realm of the bodies in action is the realm of a ruler able to settle the contours of meaningful information. In this sense, as Maynard Smith correctly remarks,[12] a DNA molecule has a particular sequence because it specifies a particular protein: it contains information concerning proteins and specifies a form that articulates as synthesis in action. DNA and (regulatory) proteins carry instructions for the development of the organism; in particular genomic information is meaningful in that it generates an organism able to survive in the environment in which selection has acted. In turn, the organisms act as vehicles capable of permitting the source the successive realization of its own "renewal". The source "channels" itself through the *telos* finally articulating itself as a model: we are faced with intentional information at work, really immersed in its meaning. In this way the ruler is continuously connected to the action of the coder as well as to the progressive articulation of a specific nesting process at the semantic level. Everything is marked by a complex regulatory interplay involving a continuous and circular link between depth information and surface information and between meaning and incompressibility.

The coupling constituted by the apparatus and the surface "program" possesses, as we have just remarked, specific capacities of emergent self-representation and articulated invention; it paves, in particular, the way to the unfolding of a possible simulation activity. This coupling realizes itself in a rehearsal stage in strict connection with the model and interacting with it. If the coupling succeeds (under the guidance expressed by the *telos*) in realizing adequate simulation programs able to "open" and support the successive expression of protoprograms living at the level of deep creativity (if, that is, the teleonomic "dream" of the model comes true), it will grow and gain a form of functional autonomy. So the ancient model will become mere data and a new functional order will irrupt (exactly at the moment when the apparatus starts to behave as a pure interpreter, processing DNA as data with reference to a surface program which results nearly completely "exteriorized"). Life is a self-organizing process supported by the continuous transaction of two originary forces: incompressibility and meaning. It is at this level that we are necessarily faced with the riddle concerning morphogenesis. In this context the simulation work, if successful, constitutes each time the basis for the emergence of the new coder.

The coder imparting intentionality allows information to be articulated as semantic, to be immersed in operant meaning (i.e., to sanction the birth of an apparatus able to perceive according to the truth). The progressive realization of the embodiment, of an apparatus able to feed meaning, corresponds to the coding in action. But the original source will manage to code because the *telos* was able to "follow" (and execute) meaning in action in an adequate way. Insofar as the DNA constitutes itself as a model *via* the embodiment process, the model at work necessarily reveals itself as intentional (self-organizing, in perspective, as a possible biological basis of a specific perceptual activity). Hence a source that through the *Via* manages to code and perceive but with respect to the progressive articulation and the "adjunction" of specific "observers" that continuously join and inhabit the

Natura naturata. Then, it will be finally possible the rising of a new "conception" at the level of the effective closure of operant meaning as well as the birth of specific simulation procedures articulating as a whole on the basis of the development of body's "intelligence". The source that posits itself as model renders itself to life but in accordance with the truth. Only the *telos* capable of reflecting itself into the truth will be able to offer the source real intentionality: hence the intrinsic circularity between form and information.

From an objective point of view, the inscription process possesses a self-limiting character with respect to the infinite potentialities of expression which are present in the original source. Moreover, the model, at the beginning, is "blind" (from the point of view of categorial intuition) like the Minotaur as envisaged by Picasso in Vollard Suite. The Minotaur through the embodiment process must be filled by intuitions and semantic constraints, only in this way he can progressively manage to open his eyes. In order to become a suitable channel for the successive revelation of the deep information (incompressibility) living in the source, the model must not simply replicate itself: it also has to prepare and utilize specific generative tools in order to afford for a representation process possibly capable of allowing the source to express its inner creativity in a new and more complex way. It necessarily self-organizes within an ambient meaning on the basis of *telos*' activity but in order to permit the renewal and the recovery of the very tools of original incompressibility. The original source can transmit and apply itself creatively, only to the extent that it succeeds in canalizing, each time, by exploiting first of all the "inscription" process, the emergence of a new type of semantic order which selectively inscribes itself by programs into the becoming fibres of the channel and which shows itself as a tool for a new "moment", for a new expression of the deep creativity proper to the very source. The crucial step, along this difficult path, is represented by the actual development (through the complete realization of the embodiment and at the level of a non-replicative realm) of a specific process of recovery. The essential aim of this kind of process consists in extracting and outlining, according to different stages of exactness and completeness, a simulation in formal terms (a sort of simulation machine) of the functional procedures which constitute from within that particular biological network that finally presents itself as the concrete achievement of the embodiment process (and as an observing system[13]). The opening of the eyes at the level of the Minotaur coincides with his petrifying and fixing as an observing system and with his becoming a mortal being, a body of computations and assimilations. In contrast, the simulation implemented by Marsyas leads to the building of an artwork (on a conceptual level) that allows the entry into the realm of abstraction as a prodrome for the renewal of his very being as Sylenus. The simulation activity is first of all in view of a correct assimilation of the external cues. We assimilate in order to give the correct answers and to develop our potentialities also modifying our very roots. Narcissus has to fix his image by invariants in order that meaning can arrange itself in accordance with a holistic enlightening finally accepting him in its interior as sleeping Endymion. Hence the possibility of the occurrence of a new conception, the possible birth of a simulation process capable of ensuring a self-renewal of the system but in the Other, and in the overcoming of itself as inventor and craftsman, an overcoming which finally coincides with the ultimate expression of the ruler.

2.3 The Recipe at Work: The Role of the Simulation Tools at the Evolutionary Level

The biological advantage represented by the capacity of simulation developed on behalf of the nervous system, along the course of evolution, is well known. The existence of this kind of mechanism transcends the genetic scheme of Darwinian selection. Normally the simulation activity is considered as the only available instrument in order to make adequate predictions about the behavior of non-linear dynamical systems. In this way the organism can possibly avoid difficulties, conflicts etc. and maintain its stability. The problematics, however, is more subtle. The evolving (and living) net, actually, simulates itself (through emergent self-representation processes) in order, first of all, to *think* of itself "from the outside" and in order to offer, finally, the abstract programs emerging from the simulation machine to the deep generative fluxes as a *recipe* for a new, coherent and productive revelation of their informational content. So the simulation activity appears biologically programmed (for what concerns its primary aim) not only to control and prevent disturbing events possibly occurring in an external reality, but also to offer a possible "guide" to the processes of emergent (and intentional) information in order to avoid forms of incoherence, collapses from the inside etc. and to express new patterns of action consistently. Thus, we have to distinguish once again between a selection performed in ambient incompressibility and a selection performed in ambient meaning. It is this last kind of selection that supports the new irruption, the irruption of a new "order" (of the invariants characterizing the emerging chaos) at the level of the realized metamorphosis: hence the new coder in action. It is the simulation work that permits the assimilation of external cues. Narcissus has to fix himself in the image by invariants in order to prime the nesting process relative to meaning in action, hence the possibility of the rising of a new conception.

In this sense, for instance, the simulation machine, considered as a schematic assembly of simulation programs identified also by means of specific assimilation procedures, can allow, on the basis of the utilization of reflexive and non-standard tools, virtual elements and relations living at the level of the original source to be individuated, so that it will be possible to test, at a selective level, new forms of composition and synthesis of these same elements and relations (but along their effective emergence). If this attempt is successful, the ancient model will become (in the limit) pure data and, at the same time, new forms of inscription will graft themselves on its connective tissue. Then, new generative fluxes and new (correlated) emergent properties will appear, new patterns of coherence will rise and, contemporarily, possible conflicts between the invariance characterizing the ancient inscription and the emergent expression of new generative principles will be canalized according to renewed and evolving correlation functions. The simulation activity appears devoted, first of all, not to guarantee a given fixed invariance, but to prepare the emergence of new and more complex forms of invariance at the creative (and chaotic) level: thus, at the level of the ruler, meaning appears to construct itself, step by step, as a "productive" channel. It self-regulates and finally appears not only as a ruler in action, but also as a co-evolving channel, as a reality indissolubly linked to a specific nesting process in accordance with the truth. In this sense, the original biological source attains its own invariance not because it reflects a given, fixed order (an order that, in the background of the dissipation process, could only present itself as the order or law of Chance), but because it succeeds in developing, each time, the necessary tools for its representation at the surface level so that new levels of deep incompressibility can, finally, express themselves by means of the emergence of new functional (and living) "ideas". These ideas will represent the "intentional" stakes able to canalize the real development of the capacity of productive regeneration of the source, the new "moments" of a Time considered, contemporarily, both as construction and as recovery. They reveal themselves completely only at the moment of the opening-up of the new coder and represent the true results of the application of specific conceptual procedures. They flourish as sensitivity in action. Thus life and cognition appear as indissolubly intertwined.

Invariance is the fruit of the move to the standard, but in view of the new irruption. The difficulty lies in recovering each time the correct path in accordance with the dialectics in action: Hercules at the crossroads of dissipation and crystallization. I must inscribe myself (to tap my flesh, as in the ancient tale) because the Other could succeed in writing and dictating to me in the interior of my soul and simultaneously to "ascend": hence the need for a new concept of information as *productive* complexity. What changes now is the relationship with the incompressibility: the information is no longer a given, with regard to which a measure can be defined; no longer the result of a hinge-action defined within itself. The actions of measuring, observing and inventing enter the realm of incompressibility, determining its internal evolution according to the unfolding of an increasingly diverse dialectics among the original selective forces and between invariance and morphogenesis. This explains why the link between depth information and surface information has to be re-identified each time, and why the range of the instruments of the theory of incompressibility has to be extended in a continuous way. In this sense, as we have just seen, at the level of a biological network acting as a ruler, it is necessary to postulate the existence not only of pathways of observation, but also of simulation, in view of the possible opening up of a new language. Hence the necessity of the recourse to the mathematics of non-standard: life is within life and precedes it, but in proportion to the correct construction of the Artwork, and the successive edification of a Nature which can then open to new patterns of perceptual activity.

In the frame of this new theoretical perspective we need more adequate measures of meaningful complexity, also capable, for instance, of taking into account the dynamic and interactive aspects of depth information. We have, in particular, to outline new models for the interface existing between the observer and the observed system. At the level of these kinds of models, emergence (in a co-evolutionary landscape) and truth (in an intensional setting), for many aspects, will necessarily coincide. Moreover, a coupled system in this theoretical perspective must be considered as a multiplicative unit: a source-forge for other coupled systems. With respect to these kinds of systems we have, first of all, to recognize the presence of a computational apparatus able, in particular, to operate on symbols and schemes. Actually, we well know, for instance, how it is possible to define a particular O-machine, the halting function machine, that has as its "classical" part the universal Turing machine specified by Turing in 1936 and as its "non classical" part a primitive operation that returns the values of Turing's halting function H (x, y). The halting function machine can compute many functions that are not Turing computable. In particular all functions reducible to the halting function and/or the primitives of an ordinary Turing machine are computable by the halting function machine.

If we take into consideration an O-machine we can easily realize that each additional primitive operation of an O-machine is an "operation on symbols" and we know that the notion of a symbol-processing machine is more general than the notion of Turing machine. In this sense, we can think of the functions that are solvable by a first-order oracle machine as being harder that those solvable by Turing machine. From a more general point of view, oracles should be considered as information with possible error that take time to consult[14]: a physical experiment directly presents itself as an external device to the Turing machine. We are faced with a first individuation of a coupled system, as originally envisaged by Turing. Besides the distinction of degree, we also have to underlie that if it is true that self-reflection (as well as limitation procedures) is part of biological functioning and that self-reflection inscribes itself in the very structure of the different undecidables, it is also true that we can distinguish, at the level of the afore-mentioned computational apparatus, many other "intellectual" instruments. Among these instruments we can individuate, for instance, the coagulum functions, the simulation recipes, specific operations on schemes etc.

In the light of these considerations, we can think of a "biological" computer considered not as a simple and isolated TM but as a complex processing system essentially characterized by the existence of a continuous coupling between its deep generative basis and the representational and "perceptual" level and between its global behaviour (as an "intelligent" and autonomous device) and the surrounding environment. At the level of such a mysterious machine, by means of simulation activities and inventive procedures, as these latter articulate themselves in a "landscape" governed by the effective articulation of specific selective pressures, one can prime, in a mediate way (by means of patterns of selection through elimination and partial insertion which act on the mechanisms of regulation of the informational fluxes which live at the basic level) the firing of hidden connections, the "fusion" of specific circuits, the rising of particular interactions between different processing streams and the autonomous evolution of new forms of action. This evolution can, in turn, prime, at the surface level, the concatenated imposition of specific constraints by elimination, the appearance of new textures of rules (as well as of new invariants) and the birth of particular assimilation processes.

With reference to this particular "landscape", the constraints imposed by the selective pressures (operating in ambient meaning) at the level of the dynamics proper to the original (dissipative) source can, actually, permit a more complex (and targeted) canalization of the generative fluxes at the stake. In particular, they can allow the unfolding of particular potentialities, the full expression of productive principles never before revealed and, consequently, the effective construction of new autonomous processes of production of specific invariants. Thus, within the frame of the intermediate state, the biological source of information can establish, by means, first of all, of the dialectical link between variation, selection and differentiation, the necessary conditions for the realization, along its own fibres, of specific functions of "synthesis" and semantic organization specifically oriented to a *new* expression of the deep content of the original creative fluxes, an expression that at the molecular level articulates through the constitution of specific forms of life. On the other hand, the selective pressures operating in ambient incompressibility can mould the semantic complexity allowing for the emergence of new stable symmetry breakings and new constraints. This moulded complexity, this specialized net of constraints, constitutes the necessary condition for the emergence of new forms of semantic order. Starting from this emergence and from the individuation of the correct grid new generative principles will be canalized; there will be, consequently, the possibility to encapsulate these principles into matter. This fact, in turn, will permit new sources of invariance and dissipation to be generated. In strict connection to this successive generation, specific variations of the boundary conditions and new patterns of selection will rise in a circular way. This complex dialectics could not be possible, however, without the complete unfolding of *telos*' activity. Actually, only the execution of meaning on behalf of the *telos* may permit the source to assume intentionality and the meaning to become productive. As we have just said, the coder imparting intentionality, under *telos'* guidance, permits information to be articulated as semantic: hence the full development of a body of meaning able to see according to the truth.

By exploring the territories of simulation, the system opens to the paths of morphogenesis: on the one hand, we have invariance but with respect to meaning in action, on the other hand, we are faced with morphogenesis but in accordance to a continuous recovery of the roots proper to the original creativity. In this sense, from a biological point of view, the genome, no more considered only as a pure replication system but also as a self-organizing system devoted to the expression and the "renewal" of the character of creative generation proper to the source, appears to articulate its construction activity according to precise and differentiated stages of development. Its production of varied complexity (as it is obtained on the basis of the progressive unfolding of the coder) is moulded by specific selective pressures according to particular creative arrangements of nets of constraints: finally, for its part, the ruler can determine, in ambient meaning, (under *telos*' guidance) the progressive expression of a specific channel; it paves the way for a further partial revelation of new forms of incompressibility. It self-organizes as a ruler acting, at the same time, as a recipe. The new order possibly arising in consequence of this very revelation will inscribe itself within and over the old structure, within and over the biochemical "fibres" of the last revealed order. In this sense, we can remark how it is impossible, in principle, according to these considerations, to recreate present life by simulation. Man can only make himself as a tool in order to determine, utilizing suitable simulation models, a further and different expression of the Bios.

The depth information (incompressibility) of which we speak about is not a particular structure or an order or a set of correlations. It is the "place" (the locus of action) of the regulative principles, of the *ideas* that arise in the background of the irruption of new chaos. These *regulative* invariants become an order only when they succeed in expressing themselves as a specific act of synthesis within the frame represented by the net of constraints determined by the progressive realization of an adequate self-organizing channel. In other words, this type of synthesis can take place only with reference, first of all, to the dialectics in action between the production of varied complexity, on the one hand, and the progressive nesting of meaning's selective activity, on the other hand. The actual manifestation of this dialectics, of this interactive dynamics, shows itself as a process of production of meaningful information, a process which necessarily articulates through the successive appearance of specific patterns of order and coherence as well as of specific attractors. We have, moreover, to underline, once more, that it is, precisely, the autonomous capacity, at the level of the channel of articulating according to these patterns in a correct way (but exploiting, step by step, the suitable paths and "reflecting" the general scheme of these very paths in order to build an adequate simulation program), that can, finally, open the way to the revelation of unexpressed proto-programs living at the level of the deep generative fluxes as well as to the subsequent transformation of the generative principles into specific properties able to generate the information intrinsic to the dynamical systems. As we have just said, only in this way can the coder offer real intentionality to the biological source.

2.4 Reflexive Domains vs. Self-Organizing Domains

The original source of depth information, once encapsulated according to the "inscription" (once encapsulated, that is, into matter as a biological replicative structure) realizes the incarnation of this very inscription, along the progressive (but constrained) unfolding, at the surface level, of specific dissipative activities. After the accomplishment of the embodiment process (and the achieved identification of the territories governed by the ruler), the system, by means of specific reflexive procedures (as well as of other instruments of analysis), proceeds, therefore, to outline, in simulative terms, a recovery of the entire process. If the system succeeds in extracting and simulating the secret "message" concerning the inscription process, i.e., in reconstructing the "memory" in the suitable forms, we shall have as a result a potential primer for all those functional processes which will possibly lead to the revelation of new programs of synthesis and action articulating at the deep level (with respect to the definite closure of the process of encapsulation). Along this difficult path the fundamental step, at the beginning, is represented by the realization of the "inscription" process. Actually, as we have just remarked, the original source in order to reach a form of stable (non dissipative) expression and realize a specific recovery process must encapsulate itself into specific generative "properties". These properties must emerge as "inscribed" in the system's physical matter in such a way as to allow the varied complexity (produced by the properties themselves) to be generated repeatedly and autonomously. This will prove possible only insofar as the generative principles are stored (and incarnated) within matter (through the ongoing canalization and the progressive construction of the "mask") with respect to the formal arch of their effective realization. The "external" action of these principles must, then, necessarily be read and recognized by the coupled system as it has developed internally (within the ongoing metamorphosis and the constant interaction with the circumscription activity expressed by meaning in action), thereby allowing the system to trace, within the contours proper to the "shadow", a representation of the principles' effected action on a surface (and formal) level, i.e. to trace a representation-programme realized in the conditions of the system's evolution to be inscribed in its memory (in order to render itself to the *Via* in a complete way). This representation will then reproduce the previously-formed canalization modules within the system and can then be directly applied (as advocated by Prigogine), by means of specific transformations operated within the boundaries of the system, to information-flows which can be pure or mixed, i.e. not necessarily marked by the presence of the afore mentioned modules. A process of assimilation is thus produced which is mediated and indirect by means of which, through the effected reconstruction of the action of the original generative nuclei, the system can manage to objectively store the information needed to render the varied complexity formation mechanism both stable and autonomous, in accordance with increasing levels of sophistication. This means not only that a (potentially) infinitary source can manage to reflect itself in a finite output, and observe itself as an active synthetic unit, but also, and not least, that a system can be constructed which is self-reflecting and, through this kind of reflection, able to set up a process of self-reproduction and even of regeneration. The passage into the "shadow" thus constitutes the prime support for full embodiment but in view of a new "conception" (as envisaged, for instance, as we have just said, at the level of the development of contemporary Non-standard Analysis). The structure which manifests itself through the fixed points guarantees the objective existence of the primary informational flows, of the "ideal objects" that become embodied in an operant organization. Insofar as it manifests itself through fixed points in the language concerning the properties at work, the ideal infinitary structure appears to itself and to the observer as an entity possessing a finitary character which is still, however, anchored to its original real state through continuous approximations. We are actually faced with a reflexive domain, a domain able to express and reflect within itself the structure of the "ideas" and performances which constitute its texture, as well as to express, still within itself, its own truth predicate. It is by referring to such a domain that the subsequent building of an autonomous system can be articulated which is able to represent within itself not only the framework of its procedures as well as its own categorial apparatus, but which also reveals itself able to suggest, in a not predetermined way, the lines for an innovative extension of this very apparatus.

As is well known, from a mathematical standpoint a first example of a reflexive domain (although well limited with respect to the width of its functional apparatus) is the D_∞ domain, which represents the first mathematical model of the λ-calculus. The D_∞ domain is isomorphic with its own function space and is thus described by the equation: $D_\infty = [D_\infty \to D_\infty]$.[14] As is evident, however, the self-organizing domains we have mentioned before really possess a more general character. Actually, they must posit themselves as models of much more sophisticated and intertwined calculi and grammars; more specifically, they should be able to draw the lines of complex functional (and living) configurations also capable of taking into account the semantics of natural languages. Nonetheless, the languages linked to these wider domains cannot but have their starting point in the language concerning a grammar of a pure type, as the λ-calculus is. In this respect, therefore, the problem to be addressed first of all is not only to show how it is possible to construct a domain as D_∞ in a consistent way but how it is then possible to extend it along a multiplicity of embedded levels in accordance with the specimen proper to the biological organization. In order to do this we shall be directly obliged, for instance, to make essential reference to the construction-exploration at the second-order level of new models of real line. As we shall see, Non-standard models and Non-standard Analysis will enter the simulative "areas" of our intelligent apparatus in a constructive way.

By storing and stabilizing information, by transforming processes into properties, and operating a precise assimilation of the original generative principles, the system can really offer deep fluxes a varied complexity which is stable and autonomous. In this way it can activate from within, through processes of continuous reorganization, what, at maximum-complexity levels, will prove to be a precise and targeted action of coagulum allowing ever-new aspects of in-depth information to emerge. When processes are transformed into properties, when the original creativity is articulated along the pathways of particular *eigenforms* and the *telos* begins to act, once again, as synthesis and ongoing "confluence", we are actually faced, at the biological level, with the presentation of a specific process of "mirroring" in the water stream of the emerging meaning. The source becomes intentional insofar as the *telos* posits itself as achieved coder: hence the possibility for a source to "channel" itself intentionally in order to attain true invariance. Here we can inspect semantic information in action. At the evolutionary level, before exploring the regions of in-depth informational complexity, it is first necessary for a biological autonomous unity to recognize itself with respect to its own meaning in accordance with the sense proper to the *Via*. Opening up to life and self-realization, and finding oneself at higher levels of depth information are in fact possible only if adequate canalization is found and if the *telos* really flows into the correct execution of meaning, with the objective of assuring intentionality and vision. In other words, exploration which is adequate in the realm of necessity seems to be the indispensable condition for a creative rediscovery of chaos (and its invariants) The genomic information gives rise to organisms able to recognize the actual world and themselves in the world on the basis of specific formats, but principally to allow the original information to be preserved and transmitted along the route of the possible recovery of its own roots. For this to occur, “self-recognition” is necessary at each stage with respect to the operant meaning, allowing the system-source to perceive itself through the eyes of Narcissus who is “added” to meaning as the observer of oneself (drowning in his very meaning). This is the true path of (natural) selection in ambient meaning. Perceiving at the biological level is the way to release and determine new selection, thus allowing for the generation of new and more suitable organisms: but this will only be possible if meaning is pursued (and executed) correctly. Through the different organisms which manage to see according to the operant meaning and in agreement with the truth, information can go so far as to realize the “inscription” process and provide an in-depth articulation in harmony with meaning’s selective activity. Once again, this will allow the original creativity to open and fix itself in order to achieve further levels of complexity: the circularity between form and information directly presents itself as a major engine of evolution.

Notes

1. Cf. Atlan H. (2000), “Self-organizing networks: weak, strong and intentional, the role of their underdetermination” in *Functional Models of Cognition*, (A. Carsetti ed.), Dordrecht, Kluwer A. P., 127–143.
2. Cf. Kourilsky P. (1987), *Les artisans de l’hérédité*, Paris, O. Jacob.
3. Cf. Bray D. (1995), “Protein molecules as computational elements in living cells”, *Nature,* 376: 307–312.
4. Cf. Berget S.M. et al. (1977), “Spliced segments at the 5′ adenovirus 2 late mRNA”, *Proceedings of the National Academy of Sciences*, 74: 3171–3175.
5. Cf. Darnell J.E. Jr. (1978), “Implication of RNA-RNA splicing in evolution of eukaryotic cells”, *Science*, 202: 1257–1260.
6. Cf. Schmuker D. et al. (2000), “*Drosophila Dscam* is an axon guidance receptor exhibiting extraordinary molecular diversity”, *Cell*, 101: 671–684.
7. Cf. in first instance and for more information: Keren H., Lev-Maor G. & Ast G. (2010), “Alternative splicing and evolution: diversification, exon definition and function”, *Nature Reviews. Genetics*, 1: 345–355.
8. Cf. for more details with respect to the mathematical formalism: Carsetti A. (2012), “ Eigenforms, Natural Computing and Morphogenesis”, Paper presented to the Turing Centenary Conference, CiE 2012.
9. Cf. Gerstein M.B. et al. (2012), “Architecture of the human regulatory network derived from ENCODE data”, *Nature*, 489: 91–100.
10. Cf. Reeb G. (1979), *L’analyse non-standard vieille de soixante ans?* Strasbourg, IRMA.
11. Cf. Maynard Smith J. (2000), “The concept of information in biology”, *Philosophy of Science*, 67: 177–194.

12. Cf. Carsetti A. (2000), “Randomness, information and meaningful complexity: Some remarks about the emergence of biological structures”, *La Nuova Critica*, 36: 47–109.
13. Cf. Beggs E., Costa J.F. & Tucker J.W. (2010), “Physical oracles: The turing machine and the wheatstone bridge”, *Studia Logica*: 35–56.
14. Cf. Scott D. (1980), “Relating theories of the lambda calculus” in *To H.B. Curry: Essays on Combinatory Logic, Lambda Calculus and Formalism*, (P. Seldin & R. Hindley eds.), Academic Press, New York, 403–450.

Chapter 3
Randomness, Semantic Information and Limitation Procedures

3.1 Logic and Probability: The Role of Constituents

Let L be a classical (two-valued) first-order language that contains a family $\sqcap$ of predicates. From a general point of view, the set F of all formulas at the level of the two-valued predicate calculus becomes, as is well known, a Boolean algebra after identification of equivalent formulas.

In particular, from a narrower point of view, let $\sqcap$ be a finite family of monadic predicates: P_1, $P_2 \ldots$ P_m. By $\varepsilon(\sqcap)$ we denote the smallest set of the compound predicates including $\sqcap$ and closed with respect to the application of the rules concerning the connectives $\neg$, &, v.

Then we define a *congruence* (or equivalence) relation ζ between the elements of $\varepsilon(\sqcap)$. The Boolean ring $\bar{\varepsilon}$ $(\sqcap)$ associated to the preboolean set $\varepsilon(\sqcap)$, that is the quotient of $\varepsilon(\sqcap)$ modulo the congruence relation ζ, will be called the *monadic algebra* generated by $\sqcap$. In our monadic case $\bar{\varepsilon}(\sqcap)$ is *atomic* and the atoms are "represented" by the *Q-predicates* (i.e. the elements of the algebra are classes of equivalence and not simple predicates). As is well known, an arbitrary Q-predicate can be said to be of the form:

$$(\pm)\,P_1(x)\,\&\,(\pm)\,P_2(x)\,\&\ldots\&\,(\pm)\,P_k(x), \tag{3.1}$$

where each symbol $(\pm)$ may again be replaced either by $\neg$ or by a blank in all the different combinations. The number of Q-predicates is therefore $2^k = K$.

Starting from k *generators* there are, from a general point of view, 2^k possible minimal polynomials each of which can either be present or absent in a given canonical form. There are therefore 2^{2^k} possible distinct canonical forms and consequently a *free Boolean algebra* with k generators has exactly 2^{2^k} different elements. If, however, a Boolean algebra with k generators is not free, then it will have less than 2^{2^k} elements. In other words if, for instance, we have only two generators: a and b and if they satisfy the additional relation or constraint: $a \geq b$,

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the total lattice telescopes. In this sense, the real problem is to individuate the nature of (possibly hidden) relations existing between the generators.

By resorting to the generators and relations it is, then, possible to outline a model, at the monadic level, partially different from the traditional urn model. A model, that is, based rather than on a domain of individuals with given properties, on a field of generators and a set of relations. We can outline, in other words, a *frame* based on (1) the specification of some subbasic elements (the *generators*); (2) the derivation from these elements of all possible joins of meets of subbasics; (3) the specification of certain *relations* to hold between expressions of step two.

Such a framework represents a very general method of Universal Algebra. Within it, we have the possibility to outline a particular frame with generators only, and no relations. This frame (or better the *presentation* relative to it) will be called *free*. By extension, the algebra it presents will be called free. The algebra $\bar{\varepsilon}\,(\sqcap)$, for instance, that we have considered before is a ring free on G. G is an independent generator and card $(G) =$ card (H), where H is the set of the elementary predicates.

When we construct the normal forms at the level of monadic predicates, we are basically dealing with a series of possible worlds. With reference to these worlds it is, then, possible to insert the individuals by resorting to particular logical "insertion" techniques which may constitute a bit of (relative) novelty with respect to the traditional methods of logic. In this way, we can finally recover the logical form proper to the Carnapian state-descriptions. So, also the (monadic) urn models will appear as a particular case within these more general logical structures.

The theoretical design that we have just outlined possesses a sequential and a compositional character. We start with primitive predicates and we go along the derivation from these predicates of all possible joins of meets of these elements. In this way, we trace the design of different kinds of worlds and of their composition. This design remains, however, as we have seen, within the boundaries of a unique algebra generated by G.

The perspective changes suddenly if we take into consideration, generalizing the initial Carnapian frame of reference, models that are not relative to the design, at a compositional level, of a unique algebra but result as the product (not in the strict Boolean sense) of the interaction between a multiplicity of nuclei of generators. In this way, a net of algebras can arise, a net that can realize patterns of connectivity of a very high level. If we construct such algebras as *heterogeneous algebras* the net will reveal itself as a *net of automata*. We shall be in the presence of a Boolean net of connected automata whose primitive inputs are represented by monadic predicates. This type of structure can articulate as a sequential structure, but can also articulate as a *recurrent structure*.

If we decide to flow into the paths of a recurrent structure, some radical changes arise with respect to the traditional urn models and structure-descriptions. In particular, we must now take into account the birth of new concepts: the concept of *cycle,* the concept of *attractor*, the concept of *trajectory*.

The resulting net appears to be a *self-organizing* net. With respect to the predicates we shall no longer be dealing with the traditional forms of compositionality, along the successive development of the layers of the complexity. On the contrary, we shall be faced with forms of constrained modulation, linked to the successive constitution of specific basins of attractors. We will be able, in other words, to inspect the progressive unfolding of complex forms. It is within these abstract complex forms linked to a *generative dynamics,* which appears determined according to temporal connection schemes, that we must now insert the individuals and consequently define the different types of worlds. The concept of accessibility, the traditional alternativity relation, must also be related to the afore-mentioned complex forms.

Thus, when we assume the existence of a circular net of binary-valued elements where each element is connected to the others but not back to itself, we necessarily put ourselves within the boundaries of a landscape outlined according to a temporal dynamics scanned by the successive determination of specific constraints. This dynamics will also depends on the choices of the branches relative to the given bifurcations that have been made.

As is well known, the *schemes* from a Kantian point of view are, essentially, *time determinations.* In a self-organizing net the successive bifurcations, the recurrent mutations imposed on the primitive predicate-inputs, appear, actually, as temporal and connected determinations of information fluxes. In this sense, such determinations appear to concern, (differently from Hintikka's appraisal of Kant's primitive intuitions), not the (direct) presentation-construction of individuals but the (previous) construction of patterns of constraints, of clusters of selective choices. Hence the essential link with the traditional contemporary definitions of complexity at the propositional (and monadic) level.

Thus, insofar as the afore-mentioned determinations of time articulate themselves modulating the action of the generators in a recurrent way, the self-organizing nets present themselves progressively as frozen surface images of an original informational Source, as a tool, in particular, for the further construction-revelation of its inner "creativity" (as it articulates at the monadic level). The process of self-organization that characterizes the models appears, in other words, strictly connected to the mysterious paths of the expression-construction of the source: it appears as a sort of arch and gridiron for the inner revelation (and the recovery) of the "Other" (the Source).

The modulation of the structure of the predicate-inputs changes progressively with time. It will be "fixed" when the process has attained the attractor. It is the attractor that individuates each time the real "form" of the (complex) attribute. In this sense, the deep meaning appears as relative to the action expressed by specific semantic *fixed-points*, by particular subtended functions. The fixed-points of the dynamics represent the "true" revelation of that specific tuning that successively characterizes and moulds the predicates. As we have just said, such a revelation can realize, however, itself only through successive determinations of time, through the action of specific temporal schemes.

According to these lines of thought we have, thus, the possibility to outline, at the monadic level, a new particular kind of model: a self-organizing structure not bound to sets and individuals, (with relative attributes) but to generators and fluxes of tuned information. In this new theoretical framework the simple reference to possible worlds (as in Frege, for instance) in order to take into account the structure of intensionality is no longer sufficient. One has also to resort to the relation of alternativity and, in particular, to the dynamics of the constraints, to the recurrent paths of the informational flow as well as to the role played by the observer, i.e. to the interplay existing between intervening and change. At the level of this type of structure, we find, in particular, an essential plot between the successive "presentation" of the constraints and the action of the schemes, on the one hand, and the articulated design of mutations, cancellations and contractions of the predicates-inputs, on the other.

Until now we have outlined, first of all, a framework relative to a monadic algebra considered as a free ring on G (the set of elementary predicates that constitutes the independent generator). This kind of abstract structure possesses, at the same time, a generative and "propositional" character. At the monadic level it is actually possible to "graft" the individuals onto this abstract structure (extending, for certain aspects, Carnap's original suggestions), obtaining, thus, as a result, something like the traditional state-descriptions.

Then, we have seen how this kind of abstract structure can be *naturaliter* included within the more general framework relative to Boolean nets and how it is possible to articulate, also at the monadic level (considered, however, as a simple and finite extension of the propositional structures), the classical principles of the theory of recurrent networks. From a speculative point of view we have, in particular, pointed out some consequences arising from this kind of articulation, the possibility, for instance, of outlining, within the frame of recurrent connections, specific instances of self-organizing structures.

Let us remark, however, that the monadic algebra $\bar{\varepsilon}$ ($\sqcap$) that we have introduced before is not a true functional monadic algebra. In fact, it is a propositional algebra with generators that are represented by predicates, an algebra, however, that can be extended to a functional monadic algebra. We have not identified, for instance, the real action of the quantifier at the level of this kind of algebra. But, what happens when we get into consideration a true functional monadic algebra? In which way can we define the role played by the individuals at the level of this algebra? In which way is it possible to outline coherent measures of information with respect to this particular mathematical "environment"?

When we enter the realm relative to monadic first-order logic, we have at our disposal, in particular from a Carnapian point of view, the following resources of expression:

(i) A number of one-place predicates (properties): P_1 (x), P_2 (x), … P_k (x);
(ii) Names or other free singular terms: $a_1, a_2, \ldots, b_1, b_2, \ldots$;
(iii) Connectives: ¬, &, v, etc.;
(iv) Quantifiers: (Ex) ('there is at least one individual, call it x, such that') and (x) ('each individual, call it x, is such that') together with the variables bound to them (in this instance x).

Out of these we can form atomic statements concerning individuals of the form P_i (a_j), propositional combinations of these, results of generalizing upon these, propositional combinations of these generalizations and atomic statements, etc.

Let us, now, take into consideration the following interrogatives: (i) in which way is it possible to identify the "true" possible worlds at the level of monadic first-order logic? (ii) in which way is it possible to define the atoms of a free monadic algebra?; (iii) what are the different basic symmetric cases out of which we can construct suitable measures of information at the level of monadic first-order logic?

As is evident, if we can give a precise answer to the last interrogative we shall also be able to identify the effective status of possible worlds at the monadic level and to delineate the link existing between possible worlds, on the one hand, and the atoms at the algebraic level, on the other hand. Let us remark, however, that with respect to the interrogative (iii) we can give several entirely different answers, which yield several entirely different measures of information. We can consider for instance Carnap's p+. This is obtained by extending the classical considerations concerning the propositional case to monadic first-order logic without any modifications. It is assumed that every individual in our domain ('universe of discourse') has a name. We form all the atomic statements $P_i\ (a_j)$ and deal with them precisely in the same way as with the atomic statements of propositional logic. The constituents of propositional logic will then become as *state-descriptions* (as Carnap has called them). They are of the form:

$$
\begin{aligned}
&(\pm)\, P_1(a_1)\ \&\ (\pm)\, P_2(a_1)\ \&\ldots\&\ (\pm)\, P_k(a_1)\\
&\quad \&\ (\pm)\, P_1(a_2)\ \&\ (\pm) P_2(a_2)\ \&\ldots\&\ (\pm)\, P_k(a_2)\\
&\quad \&\ldots
\end{aligned}
\tag{3.2}
$$

The probability-measure p+ is obtained by giving all these state-descriptions an equal probability *(a priori*). In this way, we can of course treat finite universes of discourse only. Infinite domains have to be dealt with as limits of sequences of finite ones. So, we can consider the existential quantifier directly as a disjunction in accordance with the principles of the classical substitutional quantification.

Relative to a given domain, every statement in the monadic first-order logic is a disjunction of a number of state-descriptions. It is true if one of these is true and false otherwise. It may be said to admit all these state-descriptions and to exclude the rest. The problem of assigning *a priori* probabilities to statements of the monadic first-order logic therefore reduces in any case to the problem of specifying the probabilities of the state-descriptions.

But are state-descriptions really those comparable and symmetric cases between which probabilities should be distributed evenly? This does not seem to be the case. Actually, for statements containing quantifiers (without any restriction to the finite case) we need better measures of logical probability and information. If we analyze the structure of the monadic first-order logic a little further, it turns out that we can find another explication for the idea of a kind of alternative different from the one outlined by Carnap.

According to Hintikka's suggestions[1] the situation is as follows. By means of the given basic predicates we can form a complete system of classification, i.e. a partition of all possible individuals into pairwise exclusive and collectively exhaustive classes. These classes are exactly defined by the Q-predicates. An arbitrary Q-predicate will be called $\mathrm{Ct_i}\,(x)$ where $\mathrm{i} = 1, 2, \ldots, \mathrm{K}$.

Q-predicates may be said to specify all the different kinds of individuals that can be defined by means of the resources of expression that were given to us. By means of them, each state-description can be expressed in the following form:

$$\mathrm{Ct_{i_1}}\,(\mathrm{a_1})\,\&\,\mathrm{Ct_{i_2}}\,(\mathrm{a_2})\,\&\ldots\&\,\mathrm{Ct_{i_j}}\,(\mathrm{a_j})\,\&\ldots \tag{3.3}$$

In other words, all that a state-description does is to place each of the individuals in our domain into one of the classes defined by the Q-predicates. A *structure-description* says less than this: it tells us how many individuals belong to each of the different Q-predicates.

In this sense, the Q-predicates permit us to outline precise descriptions of all the different kinds of worlds that we can specify by means of our resources of expression without speaking of any particular individual. In order to give such a description, all we have to do is to run through the list of all the different Q-predicates and to indicate for each of them whether it is exemplified in our domain or not. Statements of the following form give these descriptions:

$$(\pm)\,(\mathrm{E}x)\,\mathrm{Ct_1}\,(x)\,\&\,(\pm)\,(\mathrm{E}x)\,\mathrm{Ct_2}(x)\,\&\ldots\&\,(\pm)\,(\mathrm{E}x)\,\mathrm{Ct_K}\,(x) \tag{3.4}$$

The number of these is obviously $\mathrm{K} = 2^{\mathrm{k}}$.

It is also obvious how (3.4) could be rewritten in a somewhat more compact form. Instead of listing both the Q-predicates that are instantiated and those that are not, it suffices to indicate which Q-predicates are exemplified, and then to add that each individual has to have one of the Q-predicates of the first kind. In other words, each statement of form (3.4) can be rewritten as follows:

$$\begin{aligned}(\mathrm{E}x)\,\mathrm{Ct_{i_1}}\,(x)\,\&\,(\mathrm{E}x)\,\mathrm{Ct_{i_2}}\,(x)\,\&\ldots\&\,(\mathrm{E}x)\,\mathrm{Ct_{i_w}}(x)\\ \&\,(x)\,[\mathrm{Ct_{i_1}}(x)\ \mathrm{v}\ \mathrm{Ct_{i_2}}(x)\ \mathrm{v}\ldots\mathrm{v}\ \mathrm{Ct_{i_w}}(x)]\end{aligned} \tag{3.5}$$

Here $\{\mathrm{Ct_{i_1}}, \mathrm{Ct_{i_2}}, \ldots, \mathrm{Ct_{i_w}}\}$ is a subset of the set of all Q-predicates.

The new form (3.5) is more simple than (3.4) for many purposes. Actually, (3.5) is often considerably shorter than (3.4).

The statements (3.5) are called the *constituents* of monadic first order logic. Among those statements of this part of logic which contain no names (or other free singular terms) they play exactly the same role as the constituents of propositional logic played there. These statements are said to be closed. Each consistent closed statement can now be represented as a disjunction of some of the constituents. It admits all these constituents while excludes the rest. Constituents (3.5) are thus the strongest closed statements of monadic first-order logic that are not logically false. For this reason, they might be called *strong generalizations.*

The analogy between the constituents of propositional logic and the constituents of monadic first-order logic suggests using the latter in the same way the former were used at the level of the Carnapian theory of semantic information. We may give each of them an equal *a priori* probability and study the resulting measures of information. Insofar as the probabilities of state-descriptions are concerned, we are really faced with a series of possibilities.

We may distribute the *a priori* probability that a constituent receives evenly among all the state-descriptions that make it true. Alternatively we may first distribute the probability-mass of each constituent evenly among all the structure-descriptions compatible with it, and only secondly distribute the probability of each structure-description evenly among all the state-descriptions whose disjunction it is.

Both methods yield measures of probability and information, which are, in certain respects, more adequate than those proposed by Carnap and Bar Hillel (1952), particularly when they are applied to closed statements. That constituents (3.5) really describe the basic alternatives that can be formulated in our simple language is brought out by the fact that every consistent statement h of the language we are considering (by assumption h therefore contains no individual constants) can be represented in the form of a disjunction of some (maybe all) of the constituents:

$$h = \mathrm{C}_{\mathrm{i}_1} \vee \mathrm{C}_{\mathrm{i}_2} \vee \ldots \vee \mathrm{C}_{\mathrm{i}_{\mathrm{w}(h)}} \tag{3.6}$$

where $\mathrm{C}_{\mathrm{i}_1}, \mathrm{C}_{\mathrm{i}_2}, \ldots, \mathrm{C}_{\mathrm{i}_{\mathrm{w}(h)}}$ is a subset of the set of all the constituents (3.5). The set $\{\mathrm{i}_1, \mathrm{i}_2, \ldots, \mathrm{i}_{\mathrm{w}(h)}\}$ is called the index set of h, and denoted by I (h). The number w(h) is called the *width* of h. The right-hand side of (3.6) is called the *(distributive) normal form* of *h.*

If the weight of C_i is p (C_i), we have for the logical probability p (h) of h the following expression:

$$\mathrm{p}(h) = \sum_{\mathrm{i} \in \mathrm{I}(h)} \mathrm{p}(\mathrm{C}_\mathrm{i}) \tag{3.7}$$

From the probabilities, moreover, we can obtain measures of semantic information in the two ways given by (3.8) and (3.9) as outlined by Carnap (and Popper):

$$\inf(h) = \neg \log \mathrm{p}(h) \tag{3.8}$$

$$\mathrm{cont}(h) = 1 - \mathrm{p}(h) \tag{3.9}$$

As we have just said, at the propositional level, at the level, in particular, of a *free Boolean algebra* **A** on a set *G* of free *generators*, every element of **A** is a finite join (disjunction) of finite meets (conjunctions); each term of these meets is either an element of *G* or the complement one. In this way we can outline the normal forms in the customary way. However, when we pass from the algebra of propositional calculus to functional monadic algebras and, in particular, to free monadic algebras, we have the appearance of one new letter: namely E (the quantifier).

Actually, a free monadic algebra constitutes a good *model* for Aristotelian Syllogistics. This means that it also constitutes a good model for Carnapian monadic logic even if we have to consider, with regards to this last kind of logical structure, the action of specific constraints. At this proposal, let us remark, however, that a free monadic algebra considers both the propositional aspects and the monadic aspects of the calculus involved. For instance, if we take into consideration a free monadic algebra with two generators, we can immediately realize, after some simple calculations, that this algebra has $2^{2^n \times 2^{2^n-1}}$ elements and $2^n \cdot 2^{2^n-1}$ atoms. We distinguish, in particular, with respect to **A** a Boolean algebra $\mathrm{B} = \mathrm{E}\ (\mathbf{A})$. B has $2^{2^n} - 1$ atoms.

This simple example permits us to realize that at the level of classical monadic languages, the constituents, as outlined, for instance, by the theory of semantic information, take into consideration, in certain respects, only the atoms relative to B. So, while in the propositional case the probability is distributed between all the atoms of the algebra, at the monadic level the theory of semantic information takes into account, in the first instance, only the basic alternatives concerning the quantificational structure and not all the alternatives present at the level of the entire algebra.

However, we can show the existence of some particular algebras: namely rich algebras. At the level of these algebras a constant c: $\mathbf{A} \rightarrow \mathbf{A}$, is an idempotent endomorphism such that $\mathrm{c}(\mathbf{A}) = \mathrm{E}(\mathbf{A})$. Then c is said to be a *witness* for p in **A** if $\mathrm{c}p = \mathrm{E}\,p$.

We can easily realize that if **A** is finite E maps each atom of **A** onto that unique atom of E(**A**) above it. In this way, it is possible to build a precise form of classification. Resorting to the quantifier we are actually able to transform atoms in individuals-generators. In particular, it is possible to “express” the atoms of **A** in term of the atoms of B.

But, what happens, we may wonder, when, abandoning the monadic level (that can be seen, so to speak, as a sort of frontier between the propositional case and the (general) predicative structures), we take into consideration the polyadic predicates and their peculiar link with the individuals? Also at this more complex level we can resort to the “design” of a framework of a generative character (even if with many technical difficulties and limitations), but we shall immediately find ourselves faced with some radical shifts with respect to the theoretical framework considered above.

Actually, when we go into the polyadic “kingdom” and we take into consideration an applied first-order language L_D which contains m primitive binary relations: $\mathrm{P_i}\ (x, y)\ (\mathrm{i} = 1, 2, m)$ the situation changes in a radical way: it is no longer possible, for instance, to directly refer to free algebras and independent generators as in the propositional case or in a monadic generative frame, adopting a unique operational logical scheme valid for all the steps in the chain of the quantificational depth. It appears necessary, on the contrary, to introduce previously specific constraints, limitation procedures, particular textures of rules and to articulate these constraints, procedures and rules, in a co-ordinated way, along the paths of a complex, multilayered logical architecture. An example will make clear some aspects of this general statement.

Let us take into consideration the aforesaid first-order language L_D. In this language the possible relations that a given "reference-point" individual y can bear to another individual x are described by formulas of the following type:

$$\mathrm{R_i}\,(x, y) = (\pm)\,\mathrm{P_1}\,(x, y)\,\&\,(\pm)\,\mathrm{P_2}(x, y)\,\&\ldots\&\,(\pm)\,\mathrm{P}_m(x, y) \tag{3.10}$$

In this way, if we decide to ignore the relations of individuals in themselves we can describe all the possible mutual relations between two (different) "reference-point" individuals by means of the following relations that will be called *Q-relations*:

$$\mathrm{Q_w}\,(x, y) = \mathrm{R_i}(x, y)\,\&\,\mathrm{R_j}\,(y, x) \tag{3.11}$$

The Q-relations constitute a relational generalization of Carnap's Q-predicates.[2] They specify all the possible kinds of ordered pairs of "reference-point" individuals or all the possible relations between two "reference-point" individuals.

If L_D contains m primitive binary predicates $\mathrm{P_i}$ it is possible to define 4^m different Q-relations, so, if, for instance, L_D contains exactly one primitive binary predicate P (x, y) we shall have four different Q-relations. If, however, the predicate in question is a symmetric relation or is characterized (on logical grounds) by the presence of other structural properties, some of these four Q-relations will not be logically possible (or consistent). Let the number of consistent Q-relations be r.

In the language L_D we can define starting from the Q-relations, different kinds of individuals. These individuals can be specified by formulas of the following type:

$$\begin{aligned}\mathrm{Ct_k}\,(x) = {} & (\mathrm{E}\,y)\,\mathrm{Q_{i_1}}\,(x, y)\,\&\,(\mathrm{E}\,y)\,\mathrm{Q_{i_2}}\,(x, y)\,\&\ldots\&\,(\mathrm{E}\,y)\,\mathrm{Q_{i_k}}\,(x, y)\,\&\,(y)\\ & (\mathrm{Q_{i_1}}\,(x, y)\ \mathrm{v}\ldots\mathrm{v}\ \mathrm{Q_{i_k}}\,(x, y))\end{aligned} \tag{3.12}$$

These complex predicates are called *attributive constituents* (or *individual constituents* in our terminology) of the language L_D. The total number of the individual constituents $\mathrm{Ct_k}$ (x) in L_D is $z = 2^r$.

From individual constituents we can build constituents of depth two by specifying, as usual, for each individual constituent $\mathrm{Ct_k}(x)$ whether individuals of that kind exist or not. These constituents correspond to different selections of individual constituents.

From a general point of view, we define the *depth* of h as the maximal number of layers of quantifiers in any part of a statement h. In other words, we impose a finite upper bound on the number of layers of quantifiers or, which amounts to the same, on the lengths of nested sequences of quantifiers (i.e. sequences of quantifiers whose scopes are contained within the scopes of all the preceding ones). Intuitively, we impose a finite limit on the number of individuals we are allowed to consider in their relation to one another at one and the same time.

From the simple "inspection" of this example we can immediately realize, with respect to the monadic case, that at the dyadic level we have, first of all, the quasiconstant presence of structure constraints, which operate, at the level of the primitive predicates-generators.

Moreover, the number of the individual constituents is now 2^r where r is the number of Q-relations and not the number of primitive predicates as in the monadic (Carnapian) case. In other words, we are dealing with a two-step logical construction: in the first stage we define all the (possible) kinds of ordered pairs of "reference point" individuals, in the second step we define all the (possible) kinds of individuals. The resort to the existential quantifier is strictly connected to the actual definition of this double articulation.

If these additional restrictions are imposed on our statements and if we accurately distinguish each time the action of the constraints at work, the situation in any *polyadic language* (without individual constants) is very much like the situation we encountered earlier in monadic first-order languages.

In this way, at the logical level, the concept of constituent can be generalized so as to apply to this more general case, and each general statement h with depth d (or less) can be effectively transformed into a disjunction of a number of constituents with depth d (and with the same predicates as h).

The main difference is that the concept of a constituent is now relative to d. Each general statement h of depth d or less thus has a distributive normal form:

$$C_{\mathrm{i}_1}^{(d)} \vee \mathrm{C}_{\mathrm{i}_2}^{(d)} \vee \ldots \vee \mathrm{C}_{\mathrm{i}_{\mathrm{w}(h)}}^{(d)} \tag{3.13}$$

where $\{\mathrm{C}_{\mathrm{i}_1}^{(d)}, \mathrm{C}_{\mathrm{i}_2}^{(d)}, \ldots, \mathrm{C}_{\mathrm{i}_{\mathrm{w}(h)}}^{(d)}\}$ is a subset of the set of all constituents with depth d (and with the appropriate predicates). For example, each constituent $\mathrm{C_i}^{(d)}$ of depth d can be expressed as a disjunction of constituents of depth $d + e$. Such a disjunction is called the *expansion* of $\mathrm{C_i}^{(d)}$*at* depth $d + e$, and any constituent occurring in some expansion of $\mathrm{C_i}^{(d)}$ is said to be *subordinate* to $\mathrm{C_i}^{(d)}$.

According to these considerations, if we call the range of predicates the "intensional base"; a possible world shows itself simply as one conceivable way in which the predicates forming the intensional base are distributed through the universe of discourse.

That constituents really describe the basic alternatives is once again well illustrated by the fact that each sentence h with depth d (or less) can be effectively transformed into a disjunction of a number of constituents with depth d (and with the same predicates as h). Each sentence h of depth d has a distributive normal form.

In order to understand in what sense the constituents are pictures we can imagine the world as an urn from which individuals may be drawn and inspected for their primary properties. A constituent specifies what kinds of balls – individuals one can draw from this urn after one another. In this sense, a constituent $\mathrm{C_i}^{(d+e)}$ of depth $d + e$ greater than d, allows us to examine a world by means of the increased number of steps-experiments. It specifies, in other words, what kinds of sequences of individuals of length $d + e$ one can find in its models.

We can have, evidently, constituents that are inconsistent. In particular we have to distinguish between constituents that are trivially inconsistent (when the inconsistency can be inspected from the same structure of $C_i^{(d)}$) and constituents whose inconsistency results hidden in such a way that it can be brought out only by expanding $C_i^{(d)}$ into deeper normal forms. There is a close connection between the inconsistency of the constituents and the various decision problems that arise in first-order logic. Actually, to effectively locate all inconsistent constituents is to solve the *decision problem* for the first-order language with the same predicates.

We have no mechanical way of identifying all of them. However, we have certain systematic ways of locating more and more inconsistent constituents: we can expand $C_i^{(d)}$ at greater and greater depths and test the subordinate constituents for trivial inconsistency: $C_i^{(d)}$ is inconsistent iff there is a depth (say $d + e$) at which all its subordinate constituents are trivially inconsistent. This represents a version of the *completeness theorem* for first-order logic.

It is natural to give the weight zero to all trivially inconsistent constituents. But what about non-trivially inconsistent ones? Are we to assign zero weights to them? If what we have in mind is information concerning some kind of reality independent of our conceptual instruments then we have to assign zero weight. We are thinking of a form of *depth information*.

But we have just seen that we cannot calculate measures of depth information effectively. So we need measures of information in which the impossibility of deciding which statements are consistent is taken into account. We need, in other words, measures of *surface information*. In this sense, we can assign a non-zero weight to all constituents that are not trivially inconsistent.

When we move from a given depth d to the next, the weight of a constituent $C_i^{(d)}$ of this depth is divided between all those subordinate constituents which are not trivially inconsistent. If we eliminate a given constituent the information relative to the statement increases. In this sense, the surface information of a statement always grows when any non-trivially inconsistent constituent turns out to be inconsistent.

In brief, surface information of a statement grows when we move to a more deeply based measure of information (depth $d + 1$ instead of d) only if some of the possibilities the statement seemed to admit of in the first place gets eliminated at this move.

We have the following result as stated by Hintikka[2]:

$$\lim_{e \to \infty} \inf^{(d+e)}(h) = \inf^{\text{depth}}(h) \tag{3.14}$$

where h is a statement of depth d.

This equation shows that depth information can be thought of as a limit of surface information at infinite depth. We have as a result that the notion of surface information gives a measure that can be increased by means of logical and mathematical investigations. We have to continuously construct new individuals considered together in an argument: this fact constitutes an increase in depth. The better we master the ramifications of our conceptual system (the more surface information we have) the more of the merely apparent alternatives we can eliminate. Thus, in a given message information concerning the world and information concerning our own concepts are, as Hintikka correctly remarks, inextricably intertwined.

3.2 Semantic Information and Algorithmic Complexity

At the beginning of the 1960s (1963–1964) we can distinguish the presence of three kinds of information theory: the statistical information theory, the theory of semantic information and the Kolmogorov-Solomonoff complexity. With respect to the first two theories, the basic connection between probability and information can be taken to be one and the same in the two cases as indicated in (3.8).

From (3.8) we obtain at once the familiar entropy expression:

$$H\,(h) = -\sum_{\mathrm{i}\,\varepsilon\,\mathrm{I}} \mathrm{p_i}\ \log\ \mathrm{p_i} \tag{3.15}$$

for the expectation of information in a situation in which we have a number of mutually exclusive alternatives with the probabilities $\mathrm{p_i}$ ($\mathrm{i} = 1, 2 \ldots$). In general, the two theories can be said to have a certain calculus based on (3.8) and on the usual probability calculus in common.

In statistical information theory, one is typically interested in what happens in the long run in certain types of uncertainty situations that can be repeated again and again. In the theory of semantic interpretation, on the contrary, we are primarily interested in the different alternatives that we can distinguish from each other by means of the resources of expression we have at our disposal.

As we have just said, the more of these alternatives a sentence admits of, the more probable it is in some 'purely logical' sense of the word. Conversely, the fewer of these alternatives a sentence admits of, i.e. the more narrowly it restricts the possibilities that it leaves open, the more informative it clearly is. (It is also obvious that this sense of information has nothing to do with the question concerning which of these possibilities we believe or know to be effectively true. It is obvious that the sentence (h & g) can be said to be more informative than (h v g) even if we know that both of them are in fact true).

Semantic information theory accordingly arises when an attempt is made to interpret the probability measure p that underlies (3.8) as being this kind of 'purely logical' probability.

The basic problem is to find some suitable symmetry principles (and/or other "general" principles) which enable us to distinguish from each other the different possibilities that certain given resources of expression permit us to individuate.

Let us consider a simple example: the simplest one: the case represented by propositional *logic*. In this case the resources of expression are the following:

(i) A finite number of unanalyzed atomic statements: $\mathrm{A_1, A_2, \ldots, A_K}$;
(ii) Propositional connectives: ¬, &, v, etc.

The constituents give the different possibilities that we can distinguish from each other. These could be said to be of the form

$$(\pm)\,A_1\,\&\,(\pm)\,A_2\,\&\ldots\&\,(\pm)\,A_K \tag{3.16}$$

The number of different constituents will thus be 2^K.

As we have just said, each statement considered in propositional logic admits some of the alternatives described by the constituents and excludes the rest of them. It can thus be represented in the form of a disjunction of some (perhaps all) of the constituents, provided that it admits at least one of them (i.e. is consistent):

$$h = C_1 \text{ v } C_2 \text{ v} \ldots \text{ v } C_{w(h)} \tag{3.17}$$

Here h is the statement we are considering and $w(h)$ is its *width*. For an inconsistent h, we may put $w(h) = 0$.

In this simple case it is fairly straightforward to define that 'logical' concept of probability which goes together with suitable measures of information. Obviously, constituents are relative to the different symmetric 'atomic events' on which we have to base our measures of information. From this point of view, a statement h is the more probable the greater its width $w(h)$ is. Obvious symmetry considerations suggest the following definition:

$$p\,(h) = w\,(h)/2^K \tag{3.18}$$

It is readily seen that this in fact creates a finite probability field. Then, we can again identify the following measure for the information of h:

$$\inf\,(h) = -\log\, p\,(h) = -\log\,\left(w\,(h)/2^K\right) = K - \log w(h) \tag{3.19}$$

where the base of our logarithms is assumed to be 2. As we have just seen (3.19) is not the only possible definition of information in the propositional case. Another way of carrying out a different kind of measure might seem to be to use as the measure of information the relative number of alternatives it excludes. The notion so defined will be called the *content* of a statement:

$$\text{cont}\,(h) = [2^K - w(h)]/2^K = 1 - p(h) \tag{3.20}$$

This is in fact a perfectly reasonable measure of the information that h conveys. It is of course related in many ways to the earlier measure inf. One such connection is given by the equation

$$\inf\,(h) = \log\,(1/1 - \text{cont}\,(h)) \tag{3.21}$$

Willis (1970) and Solomonoff (1978) use the Eq. (3.19) as a measure of the amount of information carried by the event h and Goldman (1953) develops information theory starting from it. According to this theoretical perspective, the information content given by (3.15) may be interpreted as the cost for an optimal encoding of the theory through a binary vector given a set of data. The optimal representation is obtained through a propositional sentence in disjunctive normal form. In other words, the simplest formulas are the constituents.

Starting from the propositional case we can then build adequate measures of information for the monadic and polyadic structures. In order to do this, however, we have to identify, as we have said, each time the "true" alternatives, the correct distribution of the probability mass and the correlated action of the constraints at work.

If the link between semantic information theory and statistical information theory is, for many aspects, well established, things are different with respect to the definition of the relationships existing between semantic information theory and algorithmic information theory and between statistical information theory and algorithmic information theory.

During the decade of the 1960s several scholars independently arrived at a notion that a binary string is random if its shortest description is the string itself. Among the main protagonists in this debate is the great Russian mathematician Andrei Kolmogorov, a pupil of R. Carnap, R.J. Solomonoff and Gregory Chaitin, computer scientist.

In particular, the *algorithmic complexity* of a binary string s is formally defined as the length of the *shortest program,* itself written as binary string s^*, that reproduces s when executed on some computer. A "program", in this context, is simply an algorithm, a step-by-step procedure that has been coded into binary form:

Input:string $s^* \rightarrow$ program: carry out a set of instructions on $s^* \rightarrow$ output: string s

As we have seen in Chap. 1, a string s is said to be *algorithmically random* if its algorithmic complexity is maximal, comparable to the length of the string; in other words, it cannot be compressed by employing a more concise description other than writing out s in its entirety. The code that replicates the string is the string itself.

For instance, a sequence of zeros and ones generated by a Bernoulli 1/2-process gives a binary string s that in all likelihood cannot be described by any method shorter than simply writing out all the digits. In this case, the requisite program "copy s" has roughly the same length as s itself, because what one needs to do is to supply the computer with s. Algorithmic randomness is, therefore, consonant with randomness in the usual sense of maximum entropy and of the theory of normal numbers.

Let us also remember that the minimal program s^* must itself be random. Otherwise, there is a shorter string s^{**} that reproduces s^*, and a new program can be created that concatenates s^* and s^{**} with the command "use s^{**} to generate s^* and follow the instructions of s^*". After the machine executes s^{**}, a few more bits of program are needed to instruct the computer to position itself at the beginning of s^* and continue until s appears. In this way, the new program shows itself as shorter than s^*. So, s^* is not minimal. This contradiction establishes that s^* must indeed have maximum complexity.

There is a precise duality between the increase in entropy in one direction and an increase in complexity in the opposite direction; in this sense, information content and complexity are normally seen as complementary ways of representing randomness in an individual string. For instance, at the level of physical sciences, measurements decrease entropy by reducing the uncertainty but increase algorithmic complexity by leaving behind a disordered sequence of digits. Between one measurement and the next, the average change in entropy in one direction equals the average change in complexity in the opposite direction.

Solomonoff explored in the 1970s the relationships between his definition of entropy of a sequence based on a measure of information content and the entropy as outlined by Kolmogorov. Actually, according to the great Russian mathematician (1968) the conventional entropy employed as a measure of the information content of sequences does not appear to be completely congruent with the Kolmogorov measure based on the length of the shortest program that defines a sequence.

Finally, also following, in certain respects, the track indicated by Solomonoff, Chaitin showed in 1975 that there is a persuasive analogy between the entropy concept of information theory and the size of programs. An analogy that, as Chaitin remarks, has even led many scholars to formulate alternative definitions of program size complexity.

Chaitin takes into consideration a computer considered as a decoder equipment at the receiving end of a noiseless binary communications channel. At the same time, he thinks of its programs as code words and of the result of a computation as the decoded message. With respect to this, it is natural to require that the program-code words form what is called an "instantaneous code" so that successive messages sent across the channel can be separated. Where an instantaneous code is, as is well known, a set of strings S with the property that no string in S is a prefix of another.

In his paper Chaitin introduces three precise definitions concerning canonical programs, complexities and probabilities:

$$\textit{The canonical program}: s^* = \min p(U(p, \wedge) = s), \tag{3.22}$$

$$\textit{Complexities}: \; Hc(s) = \min |p| \, (C\ (p, \wedge) = s) \text{ (may be } \infty), \; H(s) = H_U(s), \tag{3.23}$$

$$\textit{Probabilities}: \; P_C(s) = \sum 2^{-|p|} \, (C(p, \wedge) = s)\,, \quad P\ (s) = P_U(s). \tag{3.24}$$

He remarks, in particular, that for these concepts there are two different sets of terminology, one derived from computational complexity and the other from information theory. Thus, we can distinguish the three concepts of algorithmic probability, algorithmic entropy and algorithmic complexity.

Let us consider a Turing Machine Q with three tapes (a program tape, a work tape and an output tape) and with a finite number n of states. Q is defined by an $n \times 3$ table and equipped with an oracle (we omit, of course, all the details). For each TM Q of this type $P\ (s)$ is the *algorithmic probability* that Q eventually halts with the string s written on its output tape if each square of the program tape is filled with a 0 or a 1 by a separate toss of an unbiased coin. $H(s) = -\log_2 P(s)$ is the *algorithmic entropy*. Finally, the *complexity* $I(s)$ is defined to be the least n such that for some contents of its program tape, Q eventually halts with s written on the output tape after reading precisely n squares of the program tape. In other words I is the minimum number of bits required to specify an algorithm for Q to calculate s. From the concept of complexity we can derive, as usual, the concept of conditional complexity.[3]

Starting from these definitions, Chaitin came to understand the necessity of abandoning the Hilbertian methods. As we have seen in Chap. 1, in this context the crucial point is that introducing new axioms with reference to the information-theoretic limits imposed by incompleteness theorem, obliges to make reference to abstract concepts that can be considered as "evidence" inspected not by the senses, but by the conceptual tools of the human intellect. Hence the necessity of exploiting new distance functions, defined with reference to higher-level languages, in order to identify new patterns of data and to "guide" the process of revelation of these data by means of an approximation process and a continuously renewed identification of fixed-points. In this sense, we have to individuate each time the "good" new axioms, the good new methods, the suitable new measures and so on. As we have seen, it is, for instance, no longer possible simply to add specific "discovered" undecidable statements to the original given system. We need new and, at the same time, concrete axioms. But what does this mean? Let us present some examples.

J. Paris and L. Harrington found in 1977 an example of a "natural" statement involving only finite quantities that cannot be proved true within the normal axiomatic structure of finite mathematics. Let us consider the semi-formal theory PA^+ obtained from Peano Arithmetic by adding to it all the true Π^0_1 sentences; PA^+ can decide the halting problem. On the other hand, even PA^+ cannot prove the well-known undecidable proposition of Paris and Harrington. This Π^0_2 sentence is a natural finitary version of Ramsey's theorem, a simple sentence of combinatorics. More recently, H. Friedman found a second undecidable theorem, a theorem that involves a function that grows so fast that it dwarfs the function of the Paris-Harrington theorem. In particular, Friedman's Extended Kruskal theorem is not provable in the logical system Π^1_1-CA_0. Where Π^1_1-CA_0 is the subsystem of second-order arithmetic with the Π^1_1 comprehension axiom. It is possible to say that almost all the theorems of ordinary mathematics, which are expressible in the language of second-order arithmetic are already provable in Π^1_1-CA_0. At the same time, it is well known that Π^1_1-CA_0 is equivalent to a certain formal system of first-order arithmetic with Ramsey quantifiers. We can formulate a Π^0_2 finite miniaturization of the Π^1_1 combinatorial principle concerning the Friedman's Extended Kruskal theorem.[4]

Moreover, in these last years the picture has undergone a significant change with the definition of a new Boolean relation theory that utilizes Mahlo cardinals of finite order. These are generally referred to as "small" large cardinals which are compatible with $V = L$. In the light of Friedman's considerations, general set theory in its maximal conceivable form needs large cardinal axioms. We can consider, from a general point of view, the large cardinal axioms as strengthenings of the axiom of infinity of ZFC. They are expressions of an intrinsically plausible informal reflection principle.

According to Friedman's point of view, we really need large cardinals of a new kind: they should be entirely mathematically natural and they should be concrete. At least within infinitary discrete mathematics. Most ideally, involving polynomials with integer coefficients, or even finite functions on finite sets of integers.

However, it is worth emphasizing that evidence for the truth of the large cardinal axioms being applied comes from the theory that supports them, not from their role in finitary combinatorics. For instance, the proof of Friedman's natural statement requires structure far beyond the mathematical system used for finite quantities, raising it to a very high level of undecidability. In this sense, the discovery of Friedman's theorem as well of the Paris-Harrington theorem now lead mathematicians to clearly understand the mathematical reasonableness of infinite objects such as ω^{ω} and ε_0.

As is evident and as we have just remarked, the "introduction" of new axioms in order, for instance, to settle the Continuum Hypothesis, cannot be reduced to the simple identification of a (possibly unlimited) series of undecidable statements. It is true that if we fix up a logical system by calling the undecidable statements axioms and thereby declaring them to be true, new undecidable statements will crop up and new possible structures will emerge. However, the crucial problem, as we have just said, is to introduce the "good" new axioms. For instance, the axiom asserting the existence of many Woodin cardinals permits us to provide a complete theory of the projective sets of reals: thus, it reveals itself as a good axiom, an axiom well-suited to the goals of the theory of the projective sets of reals. It has proved crucial to organizing and understanding this particular theory and it does so in a manner consistent with various other set-theoretic goals. We are faced with a kind of self-organization process necessarily linked to an increasing form of conceptual awareness, an awareness that, in some way, provides the tools for an effective control of the process itself.

It is at the level of such a self-organizing process that we can identify, at the second-order level and in a co-evolutionary landscape, the birth of new domains of individuals etc. When the good new axioms reveal themselves as natural and concrete they necessarily articulate as linked to the emergence of ever-new structures, to the holistic growth of the original system and, at the same time, to its "opening" from the interior at the deep level. In this way, new forms of coherence and new paths of assimilation will appear.

Hence the appearance, at this level, as we shall see, of new types of selection activities and the articulation of precise forms of co-evolution. But, we may wonder what types of criteria should we use in order to identify these new forms of evidence? In which way is it possible to select these criteria?

3.3 Surface Information vs. Depth Information: The Biological Computer

As we have just said, to effectively locate all inconsistent constituents is to solve the decision problems for the first-order language with the same predicates. More generally the degree of unsolvability in this case is the same as the Turing degree of the function $\varphi_{C_i(d)}\ (e)$ which indicates as a function of e how many inconsistent constituents of depth $d + e$ there are subordinate to $C_i^{(d)}$. Knowing this function would enable us to effectively locate all inconsistent constituents subordinate to $C_i^{(d)}$.

In this sense, as Hintikka remarks,[5] we can say that, from a general point of view the degree of unsolvability of any finitely axiomatizable theory is the same as the degree of some finite sum of the functions $\varphi_{C_i(d)}(e)$. It follows from a result by Hanf that every recursively enumerable degree of unsolvability can be obtained in this way. So, we cannot identify, in a mechanical way, all the inconsistent constituents, even if we have certain systematic ways of locating more and more of them. It is precisely with respect to this theoretical frame that we can again affirm that depth information can be directly considered as surface information at infinite depth.

Let us, now, investigate more accurately the complicated plot existing between depth information and surface information from the point of view of the Computability Theory. Turing showed in 1936 that the halting problem is unsolvable: there is no effective procedure or algorithm for deciding whether or not a program ever halts. If we consider a formal axiomatic system as a r.e. set of assertions in a formal language, one can immediately deduce a version of Gödel's incompleteness theorem from Turing's theorem.

Let us define the *halting set* $K_0 = \{<x,y>: \Phi_x(y) < \infty\}$. Then, we can rephrase the original Turing's theorem directly as: "The halting set K_0 is not recursive". Moreover, it is easy to show that, at the same time, K_0 is recursively enumerable.

On the other hand, in accordance with this conceptual track, we can formulate the (first) incompleteness theorem of Gödel in terms of recursive function theory. This result is due to A. Church and S.C. Kleene. In the proof diagonalization is needed to show that K_0 is not recursive.

Theorem 3.1 *There is a recursively enumerable set K_0 such that for every axiomatizable theory T that is sound and extends Peano Arithmetic, there is a number n such that the formula "$n \neq K_0$" is true but not provable in T.*

We also have the possibility to derive a new proof of the theorem resorting to Kolmogorov complexity. In this case, however, we have as a result different examples of undecidable statements.

The attempts of Church and Kleene as well as Post's version of Gödel's theorem not only show, in terms of recursive function theory, that formal axiomatic systems are incomplete but, in certain respects, they also give some hints in order to outline an information-theoretic version of Gödel's theorem. A version that, as we have just said, will be given later by Chaitin. In this version we can find precise suggestions about the possibility of introducing effective measures of the information power of formal axiomatic systems.

Let us consider, for instance, an N-bit formal axiomatic system T. Chaitin's version of Gödel's incompleteness theorem affirms that there is a program of size N which does not halt, but one cannot prove this within the formal axiomatic system. On the other hand, N bits of axioms can permit one to deduce precisely which programs of size less than N halt and which ones do not. Chaitin's version emphasizes very well one of the central aspect of the intellectual "symphony" realized by Gödel. Actually, at the heart of Gödel's incompleteness theorem is the derivation (outside T) of:

$$\mathrm{Cont}\,(T) \to \neg \mathrm{E\,y\,Proof}_T\,(\mathrm{y},\text{'}\gamma\text{'}) \tag{3.25}$$

(where T is a system based on a first-order language whose intended interpretation is over the universe of natural numbers).

Cont (T) is the sentence that affirms that there is no proof of $\underline{0} \neq \underline{0}$: ¬Ey Proof_T (y, '$\underline{0} \neq \underline{0}$') and the unprovability of the sentence γ is formally stated as ¬E y Proof_T (y, 'γ').

(3.25) can be formalized in T if T is minimally adequate. In this sense (3.25) reveals itself as a theorem of T. Thus if T is consistent, Cont (T) is not provable in it.

In other words: for any minimally adequate formal deductive system T, if T is consistent then its being consistent is expressible in T but unprovable in it.

γ is provably equivalent to Cont (T). Thus, Cont (T) shows itself as an undecidable. As such it can be added to T as an additional axiom. We obtain, in this way, a stronger acceptable system. In certain respects, one may consider Cont(T) as a totally informative message, a message consisting only of information that can be obtained in no other way. All redundancy has been cancelled from it.

It is determined by means of the explicit construction of an undecidable Π_1 formula: the fixed-point lemma associates with any formal system T, in a primitive recursive way, a formula which says of itself "I am unprovable in T".

Insofar as we manage to realize, by self-reflection on our own reasoning, that our logical and mathematical inferences can be formalized by a given formal system, we also realize that self-reflection is itself part of our mathematical reasoning. In this sense, it is also at the basis of the effective construction of the undecidable. It is precisely by means of self-reflection that we can go on to infer the consistency of the system. However, the act of self-reflection must remain outside the final result of this conceptual construction.

We can now better realize in which sense depth information can be defined but, at the same time, it cannot be effectively computed. As we have just seen, according to Hintikka, depth information can be thought of as surface information at infinite depth. In certain respects, we can simply affirm that it can be calculated by an infinite process during which one can never know how close one is to the final value. In the light of these considerations we can easily realize, therefore, that depth information can be defined according to very different ways: we can distinguish, for instance, from an epistemological point of view, at least three general approaches with respect to the definition of this concept.

According to the first approach depth information can be considered as an information concerning some kind of reality independent of our conceptual constructions. According to the second approach what we have in mind, as we have just seen, in accordance with Hintikka's suggestions, is a kind of "entity" that shares, at the same time, information concerning the world and information concerning the ramifications of our conceptual system. We are faced with an inextricable plot. We can, however, think of depth information also in accordance with a third theoretical perspective: at the level of this new perspective we are really faced with coupled systems and with the action of specific processes of self-organization. So, we need more adequate measures of meaningful complexity, capable, for instance, of taking into account also the dynamic and interactive aspects of depth information. We have, in particular, to outline new models for the interface existing between the observer and the observed system. At the level of these kinds of models, emergence (in a co-evolutionary landscape) and truth (in an intensional setting), for many aspects, will necessarily coincide. Moreover a coupled system in this theoretical perspective must be considered as a multiplicative unit: a source-forge for other coupled systems.

With respect to these kinds of systems we have, first of all, to recognize the presence of an intellectual apparatus able, in particular, to operate on symbols and schemes. Actually, as we have seen in Chap. 1, we well know, for instance, how it is possible to define a particular O-machine, the halting function machine, that has as its' classical' part the universal Turing machine specified by Turing in 1936 and as its' non classical' part a primitive operation that returns the values of Turing's halting function H (x, y). The halting function machine can compute many functions that are not Turing computable. In particular all functions reducible to the halting function and/or the primitives of an ordinary Turing machine are computable by the halting function machine.

If we take into consideration an O-machine we can easily realize that each additional primitive operation of an O-machine is an "operation on symbols" and we know that the notion of a symbol-processing machine is more general than the notion of Turing machine. In this sense, we can think of the functions that are solvable by a first-order oracle machine as being harder that those solvable by Turing machine. In 1939 Turing wrote in his ordinal logics paper a short statement about "oracle machines". In particular he introduced the definition of an oracle which can supply, on demand, the answer to the halting problem for every Turing machine. Copeland recalls in his volume of 2006 that it was Max Newman who suggested, in accordance with Turing's original ideas, the opportunity to identify the uncomputable oracle with intuition (an act of intuition as related to the human capacity to see the truth of a formally unprovable Gödel statement). The essential point of this kind of oracle is that it performs non mechanical steps. Later, from 1941 onwards Turing began to explore the inner structure of machines which modified their own rules of behavior: these machines can show features which had not been foreseen by their inventor and might be said to learn. Turing concluded that the function of the brain was that of a machine, but one so complex that it could appear as a machine capable of changing its own rules. In this way the act of intuition was linked to a continuous inner metamorphosis. This implies that the machine has to refer to programming languages quite innovative. Thus we are obliged to deepen our analysis through the investigation of the link existing between operations on symbols, operations on programs and operations on numbers. Only in this way will it be possible to introduce rules capable of changing themselves. This will necessarily lead to a renewed analysis of the morphogenetic processes also revisiting the first intuitions presented by Turing in one of his last papers.

If we consider, for instance, the programming languages as modalities for the description of processes, the development and the expression of the potential faculties hidden in the deep structure of a biological processor could be considered as the successive disclosure of extended parts of possible "silent" processes-programs, of pre-algorithmic paths never envisaged before, i.e. of pre-structures, which potentially exist within the processing system (hidden in the deep layers of the processor) and which have never before been revealed and "constructed" effectively as programs because of the absence of the conditions needed for their revelation or for their actual construction.

We have to stress, however, once more, that it is only the possible invention at the neural level of the aforesaid new tools of conceptual schematization that can prime the opening up and the constrained development of these hidden structures.

At the same time we must also underline that the aforesaid invention does not operate in a Lamarckian way, but only with reference to the progressive articulation of specific selective pressures, an articulation which results essentially coupled with the simulation activity. It is only in this way that an effective dialogue between deep information and surface reality can arise. Contemporarily there will be the breaking up, on the basis of selective activities, of the old constraints which determined the constitution of certain "replicators" and the successive appearance, along the first articulation of new self-organizing constraints, of new replication structures and of new forms of expression of deep information.

So the inventive process, the connected realization of new simulation models of a higher-order level, cannot be simply seen as a coded description of a given system according to von Neumann's theory of self-reproducing automata whereby a coded description that the processing system uses as a sort of working model of itself which, at the same time, need not be decoded.

On the contrary, this inventive process should be considered, first of all, as an inner guide for the derepression processes that live inside the system. One must not add new rules to the model-system from outside, but make it as an autopoietic model-system able to interact with the dynamics that it tries to describe in order to outline the birth of new patterns of integration able to guide, in a mediate way, a "creative" development of the deep basis which subtends such dynamics.

In other words, if we consider man as a true biological creative processor, the outlining by man of such a simulation system does not signify the construction of a simple, static representation of himself or himself in the world. The model constructed must serve, in turn, at the human level, to derepress man's deep reality, to establish, that is, the necessary conditions for the birth of a constrained dialogue with the information living in the deep dimension. This dialogue can take place only by means of the successive specification of well-defined selective processes, which concern, within the more general frame of the selective pressures present at the phenotypic level, the same simulation model. In this way, the model will progressively allow the realization of an epigenetic growth of an objective nature.

Actually, the model (if successful) can induce, as we have just said, in a mediate way, the development of specific schematic actions able to canalize generative principles lying at the deep level. To the extent that the model will succeed in acting as an indirect tool for the realization of such a development, it will reveal itself as truly autonomous. This fact will allow the model to elaborate an objective representation of itself: a form of simulation of simulation. In this way, it will also allow the actual realization of a form of (partial) creativity on behalf of the model-system, of a form of artificial life, which results in being life and intelligence: intelligence considered as an ideal Form, as the recognition of a coded "cipher" on which the self-organizing processes, living at the deep levels of the source can finally graft, in an indirect way, a coherent expression of their potentialities. According to this point of view, an autopoietic simulation model should live in strict connection to a natural and semantic dimension: a natural dimension, however, that cannot fully express itself except through the utilization of formal methods. ideal representations and particular systems of rules of production.

In this way, the real mirror represented by the design of an adequate simulation model will result to be an integrant part of a larger process of natural evolution. From a general point of view, a natural creative dimension cannot develop all its potentialities, once a certain level of complexity has been reached, without the exploitation of the inner growth of the conceptual mirror represented by the simulation model. A model which, to be a real mirror and an effective canalization factor at the cognitive level, should, however, show itself, as we have said, to be increasingly autonomous, closer and closer to the complexity of the biological roots of the natural dimension and increasingly creative, even though within the effective limits of a coupled system.

The autonomy achieved by the model, within the aforesaid limits, guarantees that the neural growth partially determined, at the human level, by the integration activity expressed by the system, has been obtained according to objective modules.

This activity which canalizes the generative principles lying in the deep semantic dimension, manifests itself indirectly and is achieved through the formation of conceptual stakes (of net of concepts) which constitute the fixed points of the brain dynamics taking place, as well as of the related occurring epigenetic growth. So the results of this activity can be seen, at the same time, as an objective representation of the creativity and the partial autonomy reached by the model and as reliable evidence of the realized unfolding of the potentialities hidden in the natural dimension.

The model, in this way, must show itself to be one that is unable to survive away from the paths of the natural dimension and as one that can become an integrant part of its evolution in accordance with precise biological constraints. So, once the model has been separated from the creative flux existing at the natural level, it will crystallize and become a simple automatism. On the contrary, if the mirror-model is connected to the creative deep dimension in a continuous way, it might be able to grow with it and continually renew its own roots according to the successive steps of the realized creative development.

In this sense, the model can reveal itself as a form of "artificial" but living intelligence and, at the same time, as a form of "true" cognition.

So far we have spoken about the role of the selective activities as they articulate at the level of the observer's mind. In this case we are really faced with a form of internal selection. But we also have another form of selective activity: the external selection. It is precisely at the joining of these two forms of selection that a coupled system takes place. So, depth information directly concerns both internal selection and external selection.

As a result of the action of the internal selection we have the birth of specific coagulum functions that prime specific forms of creative reorganization at the deep level of Reality. Depending on this reorganization particular selective pressures will appear. In this way a real process of assimilation can finally articulate according to self-organizing modules.

3.4 Non-standard Models and Limitation Procedures

Given a structure $\Psi = <D, <D_n>_{n\geq 1}, <G^{\Psi}>_{G\varepsilon O.C.}>$ and a second-order language L', we can distinguish many kinds of relations. For instance we can distinguish: (a) first and second-order relations on the universes of the structure; (b) relations into the universes of the structure; (c) definable relations of the structure using a given language, and so on.

It is important to underline that these kinds of relations which we have just referred to are not always restricted to the category of relations among individuals. In other words, not all of them are first-order relations: in this way we can realize that hidden in the structure, but definable with the second-order language in use, some relations exist that do not appear as relations among individuals but are utilized in order to define first-order relations. On the other hand, we know that in the universes of any second-order structure Ψ there are only relations among individuals; when the structure is standard all the relations among individuals are in the universes of the structure. As M. Manzano correctly remarks, in standard structures all the n-ary first-order relations on Ψ are into Ψ.[6]

In this sense, when we are faced with a standard structure the ur-elements are fixed and we cannot "inspect", with respect to the inner relational growth of the structure, the successive unfolding of some specific depth dimensions different from the simple dimension relative to the full exploitation of the "surface power" of the structure itself.

Things are different when we take into consideration structures with non-full relational universes. In order to understand the secret nature of this kind of passage it is useful to examine more carefully the problematics concerning the definition of non-standard models. As is well known, Skolem discovered the existence of non-standard models of arithmetic in the 1930s. At the end of the 1940s Henkin utilized non-standard structures in order to prove his famous weak completeness theorem for the theory of types and, at the same time, outlined a non-standard model of $\mathbf{N}^2$.

In order to present the modalities of construction of this kind of non-standard model, let us, first of all, show how to build a non-standard model of the first-order theory of Peano arithmetic ($\mathbf{N}^1$): a very well known model, which results non-isomorphic with the structure $<\mathbb{N}, 0, S>$, i.e., with the intended model of $\mathbf{N}^1$ in the natural numbers. The construction can also be carried out for the enlarged model $<\mathbb{N}, 0, S, +, \times>$.

Consider the theory $\mathbf{N}^{1*}$ which results from $\mathbf{N}^1$ by adding the individual constant "a" together with the following infinite sequence of axioms, one for each natural number:

$a \neq 0$
$a \neq S0$
$a \neq SS0$
$\ldots$

It is easy to show that the infinite set of axioms of $\mathbf{N}^{1*}$ will be consistent if $\mathbf{N}^1$ is consistent. Now, by Gödel's incompleteness theorem, any consistent set of first-order formulas has a model. But, the intended interpretation of $\mathbf{N}^1$ in the natural numbers cannot be a model of $\mathbf{N}^{1*}$. Actually, any model Q of $\mathbf{N}^{1*}$ must be a model of $\mathbf{N}^1$ and, at the same time, a model of the new formulas $a \neq 0$, $a \neq S0$, $a \neq SS0$. Therefore, the universe of Q contains non-standard numbers.

We know that for every infinite cardinal, there are at least $2^{\aleph_\circ}$ non-isomorphic models of $\mathbf{N}^1$ of that cardinality. Those models of $\mathbf{N}^1$ that are isomorphic with the intended model of $\mathbf{N}^1$ are its *standard models*. All other models are *non-standard models*.

Now, let us imagine building a particular extension of our non-standard model of the first-order theory of natural numbers, Q, that is a second-order structure capable of presenting itself as a model of $\mathbf{N}^2$. Let us call this structure Q′. It is easy to show that if the structure Q′ were, as required, a model of $\mathbf{N}^2$, it must be non-standard in the second-order sense: i.e., such that each $D_n \subseteq \mathbf{P}D^n$ and $D_m \neq \mathbf{P}D^m$ for at least one $m \geq 1$. Actually, in the universe of Q′ there are non-standard numbers. This means that the set of standard numbers is not in the universe of unary relations of Q′. Thus, the structure Q′ is non-standard in the second-order sense.

When we are in second-order logic, but we make essential reference to non-standard interpretations and allow structures with non-full relational universes, quantification only applies for the sets and relations that are present in the structure. In the general structures of Henkin, for instance, we put in the universes all sets and relations that are parametrically definable in the structure by second-order formulas. In this sense, it is not surprising that the set of standard numbers is not definable by a second-order formula in a structure having non-standard numbers. If we indicate with P. Def. (Ψ, L') the set of all parametrically Ψ -definable relations on individuals using the language L', we can say directly that a given frame Ψ is a general structure iff $D_n = \mathbf{P}D^n \cap$ P. Def. (Ψ, L').

What is important to stress again is the fact that hidden within the structure some specific relations exist, some "rules" (second-order relations) that cannot be defined as relations among individuals, but are utilized to define first-order relations (i.e., relations among individuals). As a result, we obtain a particular structure where the n-ary relation universe is a proper subset of the power set of the n-ary Cartesian product of the universe of individuals. So, whereas in the standard structures the notion of subset is fixed and an n-ary relation variable refers to any n-ary relation on the universe of individuals, in the non-standard structures, on the contrary, the notion of subset is explicitly given with respect to each model. Thus, in the case of general structures the concept of subset appears directly related to the definition of a particular kind of constructible universe, a universe that we can explore utilizing, for instance, the suggestions offered by Skolem (cf. his attempt to introduce the notion of propositional function axiomatically) or by Gödel (cf. Gödel's notion of constructible universe).

From a more general and philosophical point of view, we can say that at the level of general structures, the relations among individuals appear as submitted to a bunch of constraints, specifications and rules having a relational character, a bunch that is relative to the model which we refer to and that acts "from the outside" on the successive configurations of the first-order relations.

In other words, as we have just remarked, in the universes of any second-order frame Ψ there are only relations among individuals, but it is no longer true that all the n-ary first-order relations on Ψ are into Ψ.

These hidden relations, these particular "constraints", play a central role with respect to the genesis of our models. In particular, let us remark that as a consequence of the action performed by these constraints, the function played by the individuals living in the original universe becomes more and more complex. We are no longer faced with a form of unidimensional relational growth starting from a given set of individuals and successively exploring all the possible relations among individuals, according to a pre-established surface unfolding of the relational texture. Actually, besides this kind of unidimensional growth, further growth dimensions reveal themselves at the second-order level; specific types of development that spring from the successive articulation of the original growth in accordance with a specific dialectics. Such a dialectics precisely concerns the interplay existing between the first-order characterization of the universe of individuals and the whole field of relations and constraints acting on this universe at the second-order level. As a result of the action of the rules lying at the second-order level, new dimensions of growth, new dynamic relational textures appear. Contemporarily the original universe of individuals changes, new elements grow up and the role and nature of the ancient elements undergo a radical transformation. In this sense, the identification of new growth dimensions necessarily articulates through the successive construction of new *substrata*. The aforesaid dialectics reveals itself as linked to the utilization of specific conceptual tools: limitation procedures, identification of fixed points, processes of self-reflection and self-representation, invention of new frames by "fusion" of previously established structures, coagulum functions etc.

Moreover, as we shall see, we have to recognize the presence of specific patterns of selection and differentiation. Discovery procedures, construction processes, coagulum functions, selective pressures act as a chorus of functions in unison in order to shape the varying texture of conceptual constructions. At the level of this chorus (if successful) *omnis determinatio est negatio.* The plot of limitation procedures and cancellations of relations progressively constitutes itself as the gridiron of an intellectual order capable of allowing for the successive production of specific *gestalten.* If we are able to recognize and follow the secret path of this order, we can finally manage to illuminate the "good" structures (and to individuate, first of all the good new axioms), i.e., to "read" the progressive embodiment of the *Sinn* that selectively determines the real constitution of the events.

What we have remarked until now permits us to understand more deeply the ultimate sense of Henkin's conceptual revolution. As M. Manzano correctly remarks, Henkin arrived to prove the completeness theorem for type theory, "... by changing the semantics and hence the logic. Roughly presented, the idea is very simple: The set of validities is so wide because our class of standard structures is too small. We have been very restrictive when requiring the relational universes of any model to contain all possible relations (where "possible" means in the background set theory used as metalanguage) and we have paid a high price for it. If we also allow non-standard structures, and if we now interpret validity as being true in all general models, redefining all the semantic notions referring to this larger class of general structures, completeness (in both weak and strong senses), Löwenheim-Skolem, and all these theorems can be proved as in first-order logic".[7]

In this sense, in accordance for instance with Németi's opinion,[8] standard semantics is not logically adequate because it does not include all logically possible worlds as models. On the contrary, in Henkin's general semantics many "hidden" possibilities are progressively taken into consideration as possible models. We can have, for instance, models with or without GCH (generalized continuum hypothesis). Things are really different in the case of standard semantics.

This argument can be extended in a significant way. Actually, according to Gödel's incompleteness theorem it is possible to prove that a precise link does exist between non-standard models and formally undecidable propositions. On the other hand, we have just seen how it is possible to outline, according to Henkin's results, a model containing a non-standard number system which will satisfy all of the Peano postulates, as well as any preassigned set of further axioms. We only have to introduce a new primitive "a" and add to the given set of axioms the infinite list of formulas, $a \neq 0, a \neq S0, a \neq SS0 \ldots$

By adding a non denumerable number of primitive constants b_i^{ξ} together with all formulas $b_i^{\xi_1} \neq b_i^{\xi_2}$ for $\xi_1 \neq \xi_2$, we may even build models for which the Peano axioms are valid and which contain a number system having any given cardinal.

This kind of theoretical construction shows, as we have just said, that no mathematical axiom system can be categorical, unless it constrains its universe of elements to have some specific finite cardinal number.

The conceptual importance of the discovery of non-standard models can be well understood if we try to elucidate a precise dialectical aspect characterizing the link between non-standard models and undecidable propositions. Actually, even if Gödel's theorems indicate to us how to build certain formulas which are shown to be true but unprovable, however, there is, as Henkin remarks, no general method indicated for establishing that a given theorem cannot be proved from given axioms. Such a method is exactly supplied by the procedures of constructing step by step non-standard models for number theory in which "set" and "function" are reinterpreted. These procedures show us that in order to model thought processes in an adequate way we must explore the non standard realm on the basis, first of all, of the identification of precise fixed points and the tentative definition of new kinds of universes. For example, we know that, in accordance with Mostowski's results, Gödel's famous undecidable proposition can be simply considered as a proposition that characterizes the class of natural numbers. If we refer this proposition to a system of non-standard numbers, it will no longer be valid. In this way, we can realize that, along our exploration, we are really driving specific "conceptual" stakes into the ground of an unknown territory and that this exploration articulates itself in a co-evolutionary landscape. At the level of this particular landscape constructing and discovering appear as dialectically interrelated.

It is precisely by means of this exploration process that we can ascertain that a formal system can admit models with a universe of individuals that does not have the order type of natural numbers. Henkin explicitly quotes, as an example, a simple result, every non-standard denumerable model for the Peano axioms has the order type $\omega + (\omega^* + \omega)\eta$, where η is the type of the rationals.

If we make essential reference to non-standard structures, then the set of validities is considerably reduced. At the same time, if we interpret validity as being true in all general models, completeness, Löwenheim-Skolem theorems and other well-known theorems can be proved as in the case of first-order logic. As a matter of fact, the set of validities will coincide with the set of sentences derivable in a calculus, which is an extension of the first-order calculus. However, this kind of reduction will reveal itself as successful only if we are able to explore the non-standard realm in an intelligent and "creative" way and if the arising differentiation processes articulate in accordance with precise coherence patterns and stability factors.

From a general point of view, the limitation theorems are theorems that are based on a precise distinction between theory and metatheory, between language and metalanguage. A formal system can be considered as an "objectified" language and we well know that by means of precise arithmetical procedures the syntactical properties of a given formalized theory T can be expressed in terms of arithmetic predicates and functions.

We have just remarked, for instance, that Gödel's incompleteness theorem concerns a sentence of $\mathbf{Z}$ (where $\mathbf{Z}$ is a formal system obtained by combining

Peano's axioms for the natural numbers with the logic of type theory as developed in *Principia Mathematica*) which says of the sentence itself that it is not provable in **Z**. However, the existence of such a sentence can be identified only because we are able to arithmetize metamathematics: i.e., to replace assertions about a formal system by equivalent number-theoretic statements and to express these statements within the formal system.

In this sense, as we have said before, limitation theorems show that that particular reality (or "essence") represented by "arithmetical truth" is not exhausted in a purely syntactical concept of provability. From a more general point of view, we can directly affirm that in **Z** we cannot define the notion of truth for the system itself. In other words, by constructing a system and then treating the system as an object of our study, we create some new problems, which can be formulated but cannot be answered in the given system. Actually, every sufficiently rich formal system is always submitted to the diagonal argument, an argument that is always present in the limitation theorems and, in particular, in the Löwenheim-Skolem theorem. Let us show, as a simple consequence of this last theorem, how one can prove that no formalized set theory can give us all sets of positive integers. Let S be a standard system of set theory. Since we can enumerate the theorems of S, we can also enumerate those theorems of S each of which asserts the existence of a set of positive integers. Let us consider now the set J of positive integers such that for each m, m belongs to J iff m does not belong to the m-th set in the enumeration. By Cantor's argument, J cannot occur in the enumeration of all those sets of positive integers which can be proved to exist in S. Hence, either there is no statement in S which affirms the existence of J, or, if there is such a statement, it is not a theorem of S. In either case, there exists a set of positive integers which cannot be proved to exist in S.

In other words, the axioms of our formal system cannot give us a representation of all sets of positive integers. It is precisely in this sense that the systems containing these axioms must necessarily admit non-standard models.

Thus, the limitation procedures permit us to identify the boundaries of our intellectual constructions, to characterize, for instance, as we have just remarked, the class of natural numbers. They permit us to "see", once given a specific representation system **W**, that if **W** is normal then every predicate H (the predicates, in this particular case, can be thought of as names of properties of numbers) has a fixed point. They also permit us, for example, to identify an unlimited series of new arithmetic axioms, in the form of Gödel sentences, that one can add to the ancient axioms. Then, we can use this new system of axioms in order to solve problems that were previously undecidable.

We are faced with a particular form of mental "exploration" that, if successful, embodies in an effective construction constraining the paths of our intellectual activity. This exploration concerns the identification of new worlds, of new patterns of relations, the very characterization of new universes of individuals. We shall have, as a consequence, the progressive unfolding of an articulated process of cancellation of previously established relations and the birth of new development "languages" that are grafted on the original relational growth. As we have said before, this type of mental exploration articulates at the second-order level: it can be reduced however (if successful) at the level of many-sorted first-order logic, by means of well known logical procedures.

In a nutshell, the nucleus of this kind of reduction consists in explicitly showing in many-sorted structures what is implicitly given in second-order or in type theory. According to Post's famous thesis, any law we become completely conscious of can be mechanically constructed. So, we add to the many-sorted language membership relation symbols and to the many-sorted structures membership relations as relation constants. Throughout this reduction process, we simply consider that a second-order structure (or a type theory structure) is basically a peculiar many-sorted structure, since it has several domains. In short, we prove first of all that Henkin semantics and many-sorted first-order semantics are pretty much the same. Then, *via* Henkin semantics, we establish a form of reduction of second-order semantics to first-order semantics. Second-order logic with the Henkin semantics is, in general terms, a many-sorted logic.

However, we immediately have to emphasize that this kind of reduction does not imply that the secret "reasons" that guide, from within the mental activity, the progressive unfolding of the processes of exploration and invention can be reduced to a first-order mechanism or to a set of pre-established rules.

It is true that insofar as the aforementioned exploration process manages to embody in an effective construction that acts as a bunch of constraints and classification procedures, then we have the possibility to translate this kind of structure into a many-sorted language. But, the actual unfolding of abstract procedures that constitutes the primitive nucleus of the exploration process necessarily articulates (at least) at the second or higher-order level. As a matter of fact, the first result of this very unfolding is the birth of specific (and previously unknown) differentiation processes, as well as the successive appearance of new universes of individuals.

Let us, once again, quote Gödel, "P. Bernays has pointed out on several occasions that, in view of the fact that the consistency of a formal system cannot be proved by any deduction procedures available in the system itself, it is necessary to go beyond the framework of finitary mathematics in Hilbert's sense in order to prove the consistency of classical mathematics or even of classical number theory. Since finitary mathematics is defined as the mathematics of *concrete intuition*, this seems to imply that *abstract concepts* are needed for the proof of consistency of number theory.... By abstract concepts, in this context, are meant concepts which are essentially of the second or higher level, i.e., which do not have as their content properties or relations of *concrete objects* (such as combinations of symbols), but rather of *thought structures* or *thought contents* (e.g., proofs, meaningful propositions, and so on), where in the proofs of propositions about these mental objects insights are needed which are not derived from a reflection upon the combinatorial (space-time) properties of the symbols representing them, but rather from a reflection upon the meanings involved".[9]

In this sense, there must be proofs that are not fully formalizable at a given stage in our mental experience, but that are "evident" to us at that stage on the basis of particular arrangements of limitation procedures, of the successive identification of fixed points, of the utilization of abstract concepts, of the exploration of new universes of individuals, and so on.

In other words, there are, for instance, proofs of Con (PA) (primitive arithmetic) that require abstract concepts as well as the necessary construction of new elements, concepts, for instance, that are not immediately available to concrete intuition (Hilbert's concrete intuition as restricted to finite sign-configurations). We need, in general, not only rules, but also rules capable of changing the previously established rules. In Gödel's consistency proof, for example, we can directly see that the theory of primitive recursive functionals requires the abstract concept of a "computable function of type *t*".

Thinking in mathematical terms cannot be completely constrained within the boundaries of the syntax of a specific language. In fact, we would also need to know that the rules of this particular syntactical system are consistent. But in order to realize this, we will, by the second incompleteness theorem, as we have seen before, need to use mathematics that is not captured by the rules in question.

According to Gödel, utilizing mathematical reason we are capable of outlining and, at the same time, discovering specific abstract relations that live at the second-order level and that we utilize and explore at that stage in order to define first-order relations. We are faced with a particular "presentation" of the Fregean *Sinn*, a presentation that constrains the paths of our reasoning in a significant way. Thus, abstract and non-finitary intellectual constructions are used to formulate the syntactical rules. Once again, this is for many aspects a simple consequence of incompleteness results: mental constructions cannot be exhausted in formal concepts and purely syntactical methods. We have, in general, to utilize more and more abstract concepts in order to solve lower level problems.

The utilization at the semantic level of abstract concepts, the possibility of referring to the sense of symbols and not only to their combinatorial properties, the possibility of picking up the deep information living in mathematical structures open up new horizons with respect to our understanding of the ultimate nature of mental processes.

We are actually dealing with a kind of categorial perception (or rational perception) that does not concern simple data (relative to the inspectable evidence), but complex conceptual constructions. And we know that in Husserlian terms, meaning "shapes" the forms creatively. However, we must immediately remark that categorial perception appears to embody in a realm that is far beyond the limits of Gödel's primitive suggestions, in particular of his primitive Platonist approach.[10] Actually, at the level of the articulation of mental constructions, we are faced with the existence of precise forms of co-evolution. On the one hand, we can recognize, at the level of the aforementioned process of inventive exploration, not only the presence of forms of self-reflection but also the progressive unfolding of specific fusion and integration functions; on the other hand, we find that the *Sinn* that embodies in specific and articulated rational intuitions guides and shapes, in a selective way, the paths of the exploration. With respect to the entire relational growth, it appears to determine, by means of the definition of precise constraints, the choice of some privileged patterns of functional dependencies. As a result, we can inspect a precise spreading of the development dimensions, a selective cancellation of relations and the rising of specific differentiation processes. We are faced with a new theoretical landscape characterized by the unfolding of a precise co-evolution process, by the presence, in particular, of specific mental processes submitted to the action of well specified selective pressures, to a continuous "irruption" of depth information determining the successive appearance, at the surface level, of specific *gestalten.* This irruption, however, could not take place if we were not able to explore the non-standard realm in the right way, if we were not capable of outlining adequate non-standard models and continuously comparing our actual native competence with the simulation recipes. Selection is creative because it determines ever-new linguistic functions, ever-new processing units which support the effective articulation of new coherence patterns. And, it is precisely by means of these new patterns that we shall be able to "narrate" our inner transformation, to become aware of our mental development and, at the same time, to ascertain the objective character of the transformation undergone.

We can perceive the objective existence of abstract concepts only insofar as we transform ourselves into a sort of arch or gridiron for the articulation, at the second-order or higher-order level and in accordance with specific selective procedures, of a series of conceptual plots and fusions, a series that determines a radical transformation of our intellectual capacities. It is exactly by means of the actual reflection on the new-generated abstract constructions that I shall finally be able to inspect the realization of my autonomy, the progressive embodiment of my mental activities in a "new" unitary system.

Meaning can selectively express itself only through, (a) the determination of specific patterns of coherence, (b) the nested realization, at the conceptual level, of specific "fusion" processes, (c) a co-operative articulation of the primary informational fluxes. It shapes the forms in accordance with precise stability factors, symmetry choices, coherent contractions and ramified completions. A representation of this particular shaping exactly concerns the "narration" relative to the *Cogito* and its rules as well as to that peculiar capacity proper to the *Cogito* to change the nature of these very rules from its interior.

Notes

1. Cf. for more information: Hintikka J. (1963), "Distributive normal forms in first-order logic" in *Formal Systems and Recursive Functions* (J.M Crossley & M.A. Dummett eds.), Amsterdam: North Holland 47–90.
2. Cf. Hintikka J. (1969), *Models for Modalities*, Dordrecht: Reidel.
3. Chaitin G. (1987), *Algorithmic Information Theory,* Cambridge: Cambridge University Press.
4. Cf. for more information: Harrington L.A., Morley M.D., Scederov A. & Simpson S.G. (Eds.). (1985), *Harvey Friedman Research on the Foundations of Mathematics,* Amsterdam: North Holland.

5. Hintikka J. (1970), "Surface information and depth information" in *Information and Inference*, (J. Hintikka & P. Suppes eds.), Dordrecht: Reidel: 298–330.
6. Cf. Manzano M. (1996), *Extensions of First-order Logic*, Cambridge: Cambridge University Press.
7. Cf. Manzano M. (1996), *Extensions of First-order Logic*, Cambridge, XVI.
8. Cf. Németi I. (1981), "Non-standard dynamic logic" in *Logics of Programs*, Lecture Notes in Computer Science, (Kozen, D. et al. eds.), Berlin: Springer, 131.
9. Cf. Gödel K. (1972), in *Kurt Gödel*: *Collected Works,* I, II, III (S. Feferman et al. eds. 1986, 1990, 1995), Oxford: Oxford University Press: 271–272.
10. We distinguish with respect to Gödel's epistemology two successive approaches: the primitive Platonist approach and the phenomenological approach. For more information see: Tieszen R. (1994), "Mathematical realism and Gödel's incompleteness theorem", *Philosophia Mathematica*, 3, 177–201.

Chapter 4
Natural Language and Boolean Semantics: the Genesis of the Cognitive Code

4.1 Intensional Language and Natural Logic

A *model* for a *first-order language* L is, from an extensional point of view, a structure $\mathrm{M} = <U, I>$ where U is a non empty set, called the *domain* (or *universe*) of M and I is an *interpretation function* that assigns appropriate items constructed from U to the non-logical terminology of the language L.

L is equipped with the usual variables, a countable set of predicate constants, connectives and quantifiers, punctuation marks, a countable set of function constants, a distinguished 2-ary predicate constant: "=" (in the case that L is a first-order language with equality) and a special predicate T (the truth predicate). 0-ary predicate constants are called propositional constants; 0-ary function constants are called individual constants.

In this sense I assigns to each n-ary ($n \geq 0$) relation (or predicate) constant from L, different from "=", an n-ary relation on U, to the constant "=" (if present in L) the identity relation on U and to each n-ary ($n \geq 0$) function constant an n-ary function from U^n into U. 0-ary relations on U and 0-ary functions from U^n into U are identified with truth-values and elements of U respectively. Thus I assigns to each individual constant in L an element from U.

Given a structure M we write $|\mathrm{M}|$ to denote the domain of M. For a first-order structure M we denote by A (M) the set of *assignments* over M: i.e. the set of all functions from the set of variables into $|\mathrm{M}|$. For each structure and assignment there is, as usual, a *denotation function* that assigns a member of U to each term of the language.

The relation of *satisfaction* between models, assignments and formulas is, then, defined in the customary manner.

A formula S is said to be *satisfied* in a structure M by an assignment $a \in A$ (M) iff $V_{\mathrm{M},a}$ $(S) = 1$. Instead of writing $V_{\mathrm{M},a}$ $(S) = 1$ we write also $\mathrm{M},a \vDash S$.

A. Carsetti, *Epistemic Complexity and Knowledge Construction: Morphogenesis, Symbolic Dynamics and Beyond*, Theory and Decision Library A 45,
DOI 10.1007/978-94-007-6013-4_4,

A formula S is *satisfiable* iff M, $a \vDash S$ for some structure M and some assignment a over M. A formula S is *true* in M iff M, $a \vDash S$ for any assignment a. If S is true in M, then M is called a model of S. S is (logically) *valid* iff S is true in every structure M.

The notion of model that we have just introduced is essentially an extensional one and we well know that extensional models are inherently inadequate to represent the way things are. At the level of extensional semantics we are in front of a coarse-grained representation of properties and relations and we are not able to examine closely the intrinsic modalities of this representation, modalities which appear in any case strictly linked to a specific range of possible continuous variations of the properties and relations.

Thus, as we have just said, in order to get a more adequate form of representation, instead of working exclusively with models as defined before, we shall take a set W of new primitives into account: the *possible worlds*. The objects $w \in W$ are considered as primitive as far as we assume (in the initial stage of our considerations) that they do not have any set-theoretic structure of their own. For each $w \in W$ there is a domain U^w of individuals that are actual in w. One of the members of W is considered as representing the actual world.

Let us see how, within this conceptual framework, it is possible to get a finer-grained representation of properties and relations. In the old models the traditional interpretation function represents a given property by a set of objects-individuals. This set represents the extension of the unary predicate in the model. At the level of *possible worlds semantics* we must, on the contrary represent a property P with a function f_P that assigns to each $w \in W$ a subset of U^w, thinking of f_p (w) as the set of things that have property P in world w. In other words, an interpretation function appears as a two-argument function. In the old models, as we have seen, I assigns to each individual constant in L an element from U. Now we have to take into consideration the fact that at the level of possible worlds semantics the meaning of an individual constant is given not only by its extension, but also by the way in which this same extension is determined.

According to Frege, actually, the intension of a name must include more than just its reference. It must also include the way in which the reference is given. Thus, in place of a one – argument function I (a), we must construct a two-argument function $I(a, w)$.

In this sense, a model for a *modal first-order language* L_M is a structure $M = <U, W, R, I>$ where U is a non-empty set, called the *domain* of M, W is a non-empty set, the set of *possible worlds*, R is a binary relation of "*accessibility*" on W, I is a function which assigns to each pair consisting of an n-ary relation constant ($n \geq 0$), different from "=", and an element w of W, an n-ary relation on U, to the constant "=", if present in L_M, the identity relation on U and to each n-ary function constant ($n \geq 0$) a function from U^n into U.

An *assignment* over a structure M is specified in the usual way. The *denotation function* for this type of semantics is a simple extension of the denotation function from first-order semantics.

A formula S is said to be true in M iff $V_{M,a,w}(S) = 1$ for any $a \in$ A (M) and any $w \in W$. If S is true in M, then we say that M is a *model* of S. We can write, instead of $V_{M,a,w}(S) = 1$: M, a, $w \vDash S$. A structure M is said to be a *T-system* iff R is reflexive. If, in addition, R is transitive, M is said to be an *S4-system*. If R is an equivalence relation, then M is said to be an *S5-system*. A formula S is said to be *X-valid* (where $X \in \{T, S4, S5\}$) iff S is true in every *X-system*.

Despite the fact, however, that the introduction of possible worlds makes the description much more adequate, this same description still remains a coarse-grained one. For example, the classical problems concerning the logical equivalence that are present in the classical extensional models, arise again in the theory of possible worlds. Many sentences that are true in the same possible worlds do not possess, in fact, the same semantical content or the same sense. How will it be possible to define further parameters in order to characterize these particular sentences in an univocal way? From a more general point of view the real problem concerns the ultimate nature of possible worlds. Is there "life" on possible worlds? Is it possible to go into the inner structure (if existing) of these worlds? Is it possible to consider possible worlds as complete wholes, as maximal consistent sets or, on the contrary, do we have to see them as partial situations, short "stories"? What link exists between possible worlds and the sample-space points of the usual measure-theoretical treatment of probability calculus? As we said, in the beginning, at the moment of their introduction, the possible worlds have been considered as primitives. But very soon, after the logical analyses performed by J. Hintikka, K. Thomason, K. Fine, P. Tichy etc. the *inner modal structure* of these worlds has become the object of specific logical investigations. A precise distinction between protoworlds and worlds has been introduced. The link existing between the different worlds and between individuals and attributes has constituted the object of deep studies. In this way a precise hermeneutic work has started. The effects of this work play a central role within the present philosophical and epistemological debate concerning the ultimate "reasons" of first-order intensional semantics, and the link, for instance, existing between Model theory, Topology and Category theory.

One of the most important facet of this debate is represented by the analysis of the character of "partiality" of the possible worlds, of their inner logical articulation and of the conditions of their inner coherence. Contemporarily, besides these synchronous aspects, also the diachronical aspects concerning the "life" of the possible worlds have become an object of specific discussions. Actually, the inner coherence of the possible worlds appears strictly linked to the relation of accessibility existing between them, precisely to the action of the constraints imposed by the successive articulation of this specific relation.

From a semantical and epistemological point of view, when we enter with the tools of our analysis into the inner modal structure of possible worlds, we necessarily have to take into consideration the peculiar role played by the observer, by his capacity of determining, in a partial way, the conditions of their revelation and consistent expression. The information content is not, in other words, only a property of the source (in our case of the inner structure of possible worlds) but it also depends on the ability of the observer to perform specific sequential tests and particular logical investigations, to find adequate "distance" functions at the level, for instance, of accessibility relation. In particular, the analysis of the distance functions and of specific measure functions defined with reference to higher-level languages and multilayered temporal frames, can be exploited by the observer in order to identify new patterns of data, new regions of the world under investigation and to "guide" the process of progressive revelation of these same data. In this way, new primitives will appear, specific protoworlds will reveal themselves. The conditions of atomicity of the possible worlds will eventually change. Some new constraints, in turn, can express their hidden presence. A strange "life" lying in the deep dimension will progressively manifest itself.

In order to take into account and to face such a complex reality: i.e. to go into the paths of the inner structure of possible worlds and of their ultimate connections, it appears suitable to resort, for instance, to the introduction of non-classical logics, to "creative" logics, in particular, capable of elaborating in a finer-grained way the problem concerning the logical equivalence as well as the relationships existing between logical inference and rational inference. It is according to these theoretical tools that the "life" existing in possible worlds seems, finally, to have the possibility of entering the stage of our awareness.

If we want to give a consistent explication of the meaning of linguistic expressions, of the deep information canalized by them, we have to situate these expressions within a general theory of meaning capable of giving an adequate explanation of the actual and global flow of real information. For the theorists of Situation Semantics, for example, the information flow concerns real things, living (and cognitive) entities which interact with their environment.[1] Meaning lies in the systematic relationships existing between different kinds of real situations. These crossed relations or constraints permit a given situation to contain (and capture) information concerning other different situations. In addition we can postulate that, at the meaning level, information is "distributed" in a holistic way.

In this theoretical context the logic proper to a given natural activity (as, for instance, observational language) finally, reveals itself as nothing but the set of constraints and meaning postulates in action that govern this same activity. This kind of logic contains, however, much more constraints and postulates than those which we are aware of as human beings. Within the existing Reality a deep information exists that partially escapes us, an information that can express itself only within the dynamic and coupled frame of a universe of constraints and postulates and that contemporarily appears as linked to a series of specific and continuous observational acts. Seeing is observing my means of the mind's eyes in the light of the "irradiation" of the emergent meaning. The surfacing of meaning posits itself as a kind of government in action, it expresses itself as (and through) the logic concerning this particular (and natural) "regimentation" process, as the grid of constraints that it co-determines and as the continuous renewal of this very grid.

The logical and inferential (analytical) inquiry is precisely that particular type of cognitive activity that aims to explore "facts" in order to extract additional information implicitly contained in them, i.e. to open, in the first instance, the deep content of the original informational flow. This certainly does not exclude the validity of the utilization of the rules of classical extensional semantics. These rules, however, concern only a particular sort of constraints, only some of the modalities that are necessary to pick out deep information. So, in order to collect additional information we have to explore and introduce further constraints, to utilize ever new rules: we have to feed meaning in an adequate way. In particular, we have to feed the genesis of the Forms constituting ourselves as prototypes and joining the emerging self-organizing grid. Hence the importance of a guided "*adequatio*". This *adequatio* does not concern simple things or given structures but the specific development of a capacity: only through an adequate construction of prototypes will it be possible to realize a more coherent expression of the government in action; only if we are able to join the secret grid according to a specific replication code, will we be able to feed meaning in an adequate way. Then truth will possess our minds and we shall be able to open the eyes of our mind, to constitute ourselves as minds in action. In this sense, we have to feed meaning in an "intelligent" and guided way, hence the importance of a correct identification of the Method as well as of the construction of adequate tools at the logical level. In order to see more and more I have to support a better canalization of the original flow and to feed a more coherent "circumscription" at the meaning level. Hence specific *gestalten* will reveal themselves as natural forms through the progressive realization of my embodiment: in order to join meaning and canalize the *Sinn* I have to "fix" the emerging flow into the genesis relative to the Form, I have to give life to specific "prototypes" and I have to recognize myself by previously identifying my role in and through them. Hence the possible birth of specific *eigenforms*, of specific fixed points of cognition and life able to open to possible replication. Thus, we are really faced with one of the routes proper to epistemic complexity.

But, we may wonder: which paths do we have to follow for it to be possible to carry out a conservative extension of the logical and semantical analysis at the level of a coupled system? Do we have to introduce further parameters or indices, contextual ones for instance, in the field of standard semantics? Or would we rather extend the logical tissue of first-order logic by following the paths of stronger and richer logics? Or do we still have to enter the mysterious kingdom of non-standard models? What is the role, however, played by the observer at the level of this particular kingdom? What about, for instance, the link between observer's activity and the unfolding of the nesting process? How is it possible to realize a complete expression of the *telos* considered as a needle that outlines the drawing of the path of the secret wounds that identifies the labyrinth finally recognizing himself in it? What about the link between visual cognition and natural computing? The original development language feeds meaning but it is also segmented by means of particular semantic constraints: hence the landscape proper to the embodiment process. In a nutshell: which methods do we have to adopt and follow in order to see (and think) more deeply according to the truth?

In the second half of the 1960s some of such questions as well as other problems concerning, in general, the extensional approach to natural language, induced R. Montague to propose a conservative extension of classical Tarskian semantics capable of making the information hidden in the deep structures of natural language more visible. This extension was realized by Montague by grafting, first of all, the theory of types onto the old trunk of Tarskian semantics, by taking into consideration an articulated set of intensional indices and, finally, by resorting to an innovative utilization of the λ-calculus. As a result, as is well known, Montague succeeded in outlining a system of *Intensional Logic* capable of adequately representing significative portions of natural language[2] and its semantics. For certain aspects, this system directly allows for considering logic as a form of inner government of natural language.

The starting point of Montague's analysis possesses a precise model-theoretic character and consists of associating with each *category* C of natural languages expressions a set of *possible denotations* called T_C or the type for C defined in terms of the semantic primitives of the model. In standard first-order logic the primitives of a model are given by a non-empty universe of discourse U and by the set of the two truth values. Models may be different insofar as the chosen U will be different. But, once a universe is chosen, the types for categories are fixed. In this way, the type for Sentence is given by B (we indicate by B the set of the two truth values), and the type for individual constants and variables is given by U. The type for monadic predicates is the power set of U. In general, T_{Pn} the type for n-place predicates is the power set of n-tuples over U: U^n. (We have to remark, from the beginning, that the set B is a set with a Boolean structure, in other words, the type for sentences is a Boolean algebra.)

Extending the primitive intuitions outlined by Russell concerning the theory of types and the successive sophisticated formalization of such a theory offered by Church, Montague discovered the articulation of types directly at the level of the inner structures proper to natural language; moreover he utilized the analysis of such an articulation as a sort of picklock in order to enlighten the more secret aspects of this particular kind of cognitive structure. Once the types for the primitive categories are constructed, it is possible, then, to build the types for complex categories in terms of the types of the more simple categories. So, at the level of the classical theory of types we may, starting from B and U, define the set B^U, that is the set of all possible functions from U into B: ($U \to \mathrm{B}$). Each function f in B^U characterizes a particular set of elements of U: the set of elements of U for which f assumes value 1.

Such a function, called *characteristic function* can represent the extension of a predicate i.e.: the set of entities that are true with respect to a given predicate. The process can be naturally iterated. We can, for example, define all the functions from U into B^U : $(\mathrm{B}^U)^U$. Each function h in $(\mathrm{B}^U)^U$ individuates a relation: the relation existing between x and y iff $h\ (x)\ (y) = 1$. The logical mechanism which is at the basis of such an iterative process becomes completely clear once we define the set *Type* of logical types.

Let *Type* be a particular set that satisfies the following conditions:

1. $e \in \mathit{Type}$;
2. $t \in \mathit{Type}$;
3. if $a \in \mathit{Type}$ and $b \in \mathit{Type}$, $<a, b> \in \mathit{Type}$.

According to these conditions *Type* is a set of types; e and t represent, respectively, "entity" and "truth". $<a, b>$ represents the type of formulas from objects of type a to objects of type b.

If α is an element of a given type a, then the semantic value of α (called the *denotation* of α) belongs to a well determined set which is noted D_a and is referred to as the set of possible denotations of type a. The sets D_a are recursively defined by the following rules:

1. $D_e = U$;
2. $D_t = \mathrm{B}$;
3. $D_{<a,\, b>} = D_b{}^{Da}$.
   (The notations W^V represents the set of mappings from a set V into a set W).

The conceptual scaffolding represented by type theory allows Montague to establish two precise homomorphisms at the level of the simulation of natural language: the homomorphism between natural syntax and formal syntax and the homomorphism between formal syntax and semantics. From a formal point of view, stating these two homomorphism allows, in turn, for the introduction of the λ-calculus in the landscape represented by the theoretical frame of intensional logic.[3]

Within the framework of this theoretical construction, a model **M** is given by a four tuple $<U, W, \boldsymbol{R}, V>$ where:

1. U is a set of individual objects;
2. W is a set of possible worlds w;
3. $\boldsymbol{R}$ is a set of accessibility relations between the possible worlds;
4. V is an interpretation function: it assigns elements of U to the individual constants and it assigns sets of n-tuples of elements of U to the n-place predicate constants for each possible world w.

The *assignment function* g assigns elements of U to the individual variables of the language: For each type a, the function g assigns elements of a well defined set D_a to the variables of type a.

The *semantic value* of any expression α of the formal language is obtained recursively from the semantic values of the basic elements of the language and from the rules that define the semantics of the logical connectives, the quantifiers, the modal operators and the intension and extension operators.

The semantics of the basic elements is defined as follows:

(a) if b is a non-logical constant then

$$[b]^{\mathbf{M},\, w,\, g} =_{\mathrm{def}} V\,(w,\, b)$$

(b) if x is a variable, then

$$[x]^{\mathbf{M},\, w,\, g} =_{\mathrm{def}} g\,(x)$$

If we refer this kind of theoretical construction to the examples given earlier concerning the possible denotations associated with the different categories of natural language expressions, we can immediately understand how the multiplicity of types present in Montague's system of intensional logic may be utilized in order to give a broad representation of the meanings of the different linguistic expressions.

As is well known, at the linguistic level fundamental syntactic categories are represented by common nouns, proper nouns, determiners, transitive verbs etc. If we indicate, according to the notations adopted by Keenan and Faltz[4] by N the category of common nouns, by $\bar{N}$ the category of nouns phrases and by $\bar{N}_{\text{prop}}$ the category of proper nouns, it is easy to realize that common nouns, insofar as they are not distinguished in first-order logic from P_1's (the monadic predicates) are normally thought of as subsets of U. In other words, the set of possible denotations of N's ($=$ P_1's) is U^*: the collection of all subsets of U. So, in first-order logic $T_N = U^*$. But, taking T_N to be the power set of U implies that T_N must be considered as a power set algebra, hence as a Boolean algebra.

Even easier to define is the type for $\bar{N}_{\text{prop}}$. At the level of first-order logic, actually, the type for $\bar{N}_{\text{prop}}$ is directly taken to be U. Things are different and more problematic, however, when we try to introduce a definition in first-order logic for the type for $\bar{N}$. This kind of logic, in fact, has no such category although it does have the equivalent of a subcategory of it: that is $\bar{N}_{\text{prop}}$. The problem really concerns the same definition of that particular semantic primitive represented by U (the type, as we have just seen, for $\bar{N}_{\text{prop}}$). Actually U is simply not versatile enough to provide denotations for complex $\bar{N}$'s like, for instance, "every man".

"Imagine, for example – as Keenan and Faltz remark – a U with just two members, say John and Bill, and let *man* be interpreted as U itself (so everything which exists in that universe is a man). Then if all complex $\bar{N}$'s were to take their denotations in U, *every man* would have to denote either John or Bill, say John. Then *every man is John* would be true, which it obviously isn't since Bill is a man and he is not John."[5]

In this sense, the standard first-order logic reveals itself to be a theoretical instrument which is not strong enough to mirror, within its general frame, the subtleties proper to the syntax of natural language. Those particular subtleties that we can easily recognize when we try to cope with the action of generalized quantifiers, complex noun phrases, transitive verbs etc. As Barwise and Cooper showed in 1981 with a wealth of examples, when we try to translate the expressive power of a fragment of natural language in a logical and mathematical setting like that one that characterizes the extensional first-order logic, we are immediately faced with precise losses of meaning, misunderstandings and so on.

A solution to these particular problems was precisely proposed in 1970 by R Montague. The new conceptual scaffolding outlined by the great American logician was successively improved, criticized and enlarged by Barwise, Cooper, Kamp, Partee, Keenan, Faltz etc. In the light of Montague's first intuitions, proper nouns must not be considered as denoting elements of U but rather as denoting the collection of subsets of U which contain a fixed element. More precisely, we can refer to the following definition:

Definition 4.1. For all $x \in U$, $I_x =_{\text{def.}} \{K \subseteq U : x \in K\}$.

In other words, the entity x is represented by the set of all the sets to which x belongs. More precisely, I_x will be called the *individual* generated by x. $T_{\bar{N}prop}$ will be, then, given by: $\{I_x : x \in U\}$.

If we simply consider the elements of T_N as extensional properties the above definition permits us to think of denotation of "John" as the set of properties which hold of some fixed element of U. In this sense, the elements of U will be in a one-to-one correspondence with the set of individuals. So the proper noun "John" may be considered, according to the terminology introduced by Barwise and Cooper, as a *generalized quantifier*. In order to see, by means of an example, how it is possible to interpret proper nouns as "John" and complex $\bar{N}$'s in the same type let us simply consider that "every man" formally denotes: $\cap \{I_x : p \in I_x\}$. But, conversely, a property p is "a member of an individual I_x iff $x \in p$, since p is a subset of U and I_x is the set of all the subsets of U which contain x as a member. Thus, we might also write the denotation of *every man* above $\cap \{I_x : x \in p\}$, and analogously for *a man*".[6]

What we have just said, in a very concise way, allows us to understand that the system of intensional logic outlined by Montague constitutes effectively an innovative extension of traditional first-order logic. In this last kind of logical system proper nouns are considered as individuals and are submitted to a formal processing completely different with respect to the sort of processing normally utilized in the case of generalized quantifiers. When we assume, however (as is the case of Montague's logical system), an entity or individual $x \in U$ and define a set $I_x =_{\text{def}} \{K \subseteq U : x \in K\}$, this particular individual, for instance the particular simple *NP* "John" can be taken as denoting the family of sets which contain "John". In other words, the *NP* containing just the word "John", represented by $[\text{John}]_{NP}$ will denote the family of sets containing "John". The *NP* in question represents, thus, a quantifier.

Within the conceptual framework of Intensional Logic as outlined by Montague, the aforesaid family or collection of sets can be represented for each expression α of type e in the following way: $\lambda P\ [P\ (\alpha)]$. The generalized quantifier $\lambda P\ [P\ (\alpha)]$ codes, in other words, the same information that the individual α codes. This specific expression describes – translates, so to speak, at the deep level, the role played by a proper noun as "John", allowing us to understand the function performed by this kind of linguistic expression within the "landscape" constituted by natural language and with respect to the semantic system concerning the links existing between the observer and the surrounding world. So, if $<e, t>$ is the type of simple predicates, $\ll e, t>, t>$ will be the type of generalized quantifiers.

If we indicate by P the set of properties ($T_N =_{\text{def}} P : P$ is a complete and atomic Boolean algebra or *ca*-algebra), we can directly affirm that those particular functions called generalized quantifiers (or quantifiers of type $<1>$) denote in $[P \to \text{B}]$: the set of functions with domain P and range included in B.

The conceptual revolution underlying these intuitions may be understood in a very simple way, if we refer to an example introduced by Barwise and Cooper in 1981.[7] When we try to translate a sentence containing *NP*'s like: (a) [Some person]$_{NP}$ [sneezed]$_{VP}$ into predicate calculus, this same sentence is, normally, represented, ignoring tense, as: (b) Ex [person (x) $\wedge$ sneeze (x)]. In effect, (b) is not really a translation of (a), but of a logically equivalent, but linguistically different, sentence: "something was a person and sneezed". On the contrary, what we aim to obtain is a sentence that will be true just in case the set of sneezers (represented by $\widehat{x}$ [sneeze(x)]) contains some person, that is: (c) [Some person] $\widehat{x}$ [sneeze (x)]. In this sense, there is no mismatch between the syntax of noun phrases in a natural language like English and their usual representation in traditional predicate logic.

From a general point of view, the noun phrases act, semantically, like the logician's generalized quantifiers and even proper nouns can be treated as quantifiers. Hence the appearance, at the surface level, at the level of scientific awareness, of a linguistic universal can be expressed according to the following definition, as indicated by Barwise and Cooper: Every natural language has syntactic constituents (noun phrases) whose semantic function is to express generalized quantifiers over the domain of discourse. According to this definition we can really recognize the character of universality of noun phrases. Moreover, the universal quantifier also serves to distinguish the natural language from other languages like, for instance, the formulations of first-order predicate calculus.[8]

The rediscovery in our century of the problematics concerning linguistic universals realized under the guidance of Montague, Bach, Barwise, Cooper, Keenan etc. led many scholars in the 1980s to develop an analysis of the relationships existing between logic and linguistics in a completely renewed way. As a consequence of this kind of analytical work the laws of logic no longer appear as parts of a wider kingdom represented by abstract Mathematics, but as specific and articulated properties of natural language as well as of the use that we normally make of this kind of language. Hence the equation: laws of thought = laws of language. It is exactly the logic that, step by step, comes to reveal itself as a part of linguistics, as a kind of inner government of the natural language allowing for further and more complex inscription of the original information flow at the level of code multiplication. We are faced with a process of natural computing that channels itself as natural language in action.

From a general point of view, at the logical (and semantical) level, we are faced with two different conceptions of meaning: meaning as use and meaning as government. When the observer takes a coupled system of constraints and meaning postulates in action into consideration and identifies and posits itself as a support of this very action, we are faced with the emergence of a linguistic grid that is the expression of a government at work. Logic acts at the mental level as a sieve, hence the need for an intelligent regimentation and a coherent "irradiation" in order to observe. Things are different if we adopt the second conception: actually, at the simulation level, meaning is first of all characterized by use and inner metamorphosis, by the continuous emergence of ever-new intentionality: this is the passage from a universe of "strange" objects (and acts of seeing) to a universe of constructions (and acts of thinking). Here we can find precise categorization processes at work: beside the observational acts the Subject must also exploit the procedures of rational inference. We cannot limit ourselves to "join" the government and to feed meaning in order to observe by means of the mind's eyes the very metamorphosis of the system; we also have to posit ourselves as the real support for the continuous *emergence* of new rules within the boundaries proper to a coupled system. In this way we can build works as artifices and these works can, in turn, join (and tune) our brains.

If we want to explore the afore-mentioned first conception of meaning (and its logic) it is necessary, first of all, to realize a thorough analysis of the "inscription" process associated with it and show why the Semantics of natural language must reveal itself, at a first level, nothing more than a Boolean Semantics. As a matter of fact, in the light of this theoretical perspective, an observational language does not directly point to external objects and individuals living *in se*, on the contrary it concerns the previous construction of a pure grid of reference with the identification of a net of invariants able to prime the opening of the mind' eyes through the successive outline of a specific system of prototypes, an outline that meaning's irradiation then will fix as *in se*. In order to observe, we have to join the system, at the semantical level, according to a coupled way, in particular we have to be governed by the laws proper to the system while feeding its inner growth. Vision does not concern things or objects directly but the construction of specific and functional underlying invariants (*eigenforms*); it concerns, in particular, the *living* grid connecting them as well as the inner growth relative to this very grid of which, what is more, the observer acting as a sieve constitutes the essential support and contemporarily the emerging result. Only if I am able to individuate and "follow" the government in action, will I be able to observe the different prototypes and myself as a prototype, a prototype, in particular, able to join as an observer the very grid. However, I can observe the government only if it manifests itself within the range of the full expression of a biological code: seeing is observing in the light of meaning irradiation and in the presence of a replication code in action.

This perspective constitutes, for certain aspects, one of the theoretical (and largely hidden) bases of the careful analysis of the formal structures of natural language as developed in the 1980s by Keenan and Faltz. In 1985 these scholars came to present a nearly complete revisitation of the system of Intensional Logic outlined by Montague, enlightening many implicit assumptions subtended to this system and extending the very ideal texture of the conceptual instruments introduced by the great American logician. Keenan and Faltz start by noting that, from an ontological point of view, U is no longer the type for any category of natural language (or for L_C: the *core language* outlined by Keenan and Faltz in order to represent the basic predicate-argument structure of (first-order) English). There are no expressions which denote elements of U, even if in Montague's system denotations for common nouns, proper nouns etc. are, generally, defined in terms of U.

As Keean and Faltz remark: "U rather appears now as a kind of noumenal world of entities which "supports" the phenomenological world of individuals. U then is an essential *mystery* on this approach and the ontology which comprises it is essentially mysterious. In the extension of this approach we propose below the mysterious U is eliminated completely, though proper nouns, full noun phrases and common nouns otherwise preserve exactly the character they have on the mysterious approach".[9]

The existence of such mysterious dialectics is well testified by the fact, for instance, that a simple linguistic expression like "John" while appearing, on the one hand, linked to entities living in U, it can, on the other, be translated into a complex formula like: $(\lambda P)\,(P\,(j))$ itself interpreted as the set of properties which hold of the denotations of j.

It is, precisely, in order to cope with such difficulties that Keenan and Faltz propose the elimination of U from the ontology of the extended version of first-order logic as outlined by Montague. From a conceptual point of view, the elimination of U can be articulated in three steps: (1) it is, first of all, necessary to characterize in an alternative way T_N so that T_N can be taken as the real primitive instead of U; (2) individuals must be defined as subsets of T_N which satisfy certain conditions; (3) $T_{\bar{N}}$, must be defined as the power set of T_N and regarded, thus, as a Boolean algebra.

In the light of these precise guidelines, the interpretation of proper name "John" finally simply appears as an arbitrary element of the type for $\bar{N}_{\text{prop}}$. In extended first-order logic T_N was considered, as we have seen, as a power set Boolean algebra. Since power set Boolean algebras are a special case of Boolean algebras, namely they are complete and atomic algebras, then T_N will be taken, from the beginning, as an arbitrary complete and atomic algebra. It possesses the Boolean structure of a power set algebra, but need not be the power set of some set. In this way, Keenan and Faltz obtain, by means of simple conceptual tools, a theoretical result of great importance: the type for N is now defined solely in terms of its Boolean properties. We have, in other words, the possibility of defining individuals directly in terms of the Boolean structure of T_N eliminating all reference to U.

Let us examine, now, in which way, from an algebraic point of view, this set of assumptions can arrive at consistent formulations. In Montague's system the individual generated by $x \in U$ was characterized, as we have just seen, as: $\{K \subseteq U : x \in K\}$. But, as our authors remark, $x \in K$ iff $\{x\} \subseteq K$. So, we can define, in an equivalent way, I_x to be: $\{K \subseteq U : \{x\} \in K\}$. We have, however, to note that $\{x\}$ is an element of $U^* = 2^U$. In fact $\{x\}$ is an atom. Taking T_N as an arbitrary *ca*-algebra it is, then, possible to present an equivalent definition of individual as follows:

Definition 4.2. For B any Boolean algebra and b any atom of B, $I_b =_{\text{def}} \{p \in B\colon b \leq p\}$.

We call I_b the *individual* generated by b. We denote by I_B the set whose elements are exactly the individuals I_b.

The elimination of U leads Keenan and Faltz to directly outline a new form of ontology for L_C, an ontology that presents itself essentially as a pair: $<P, \mathrm{B}>$ where P is an arbitrary complete and atomic Boolean algebra called "the (extensional) properties" and B is the Boolean algebra 2 called "truth values". With respect to such an ontology it is, then, possible to define the types for P_0, N, $\bar{N}$, $\bar{N}_{prop}$ in the following way:

1. $T_{P_0} =_{\text{def}} \mathrm{B}$;
2. $T_N =_{\text{def}} P$;
3. $T_{\bar{N}} =_{\text{def}} P^*$;
4. $T_{\bar{N}prop} =_{\text{def}} I_P$.

We have just defined P as an algebra of properties since its elements are effectively represented by properties. In this sense, T_N will turn out to be isomorphic to the type for P_1, so, what characterizes, at the deep level, the type for N is its structure, a structure that possesses a specific Boolean character. In particular, we have to remark that in first-order logic T_N is the power set of U which is itself the set of denotations for proper nouns and, thus, constitutes the set of primitive individuals. In a correspondent way, despite the existence of the differences that we have reported above, also in the logical system outlined by Keenan and Faltz, T_N is indistinguishable from the power set of the individuals in L_C. Actually, P $(=T_N)$ is isomorphic to $(I_P)^* = 2^{I_P}$, the power set of the individuals on P. However, in L_C individuals are not considered as primitives: they are defined in terms of properties, namely as sets of properties. Taking individuals to be sets of properties provides a natural way to represent the idea that individuals possess or fail to posses certain properties according to whether these sets contain or fail to contain these properties as members. Precisely, with respect to this frame of reference, properties can be considered, as we have said, either as the interpretations of N's or as objects determined by the interpretations of P_1's.

As a consequence of the aforesaid definition of individual it is immediate to realize that each individual must contain, by construction, exactly an atom. From this it is easy, in turn, to show that an atom is a property which is contained in exactly one individual. Conversely, if p is a property which is contained in exactly one individual, then it is possible to show that p must be an atom, provided we assume that T_N is atomic. Actually, we have just seen that $I_b =_{\text{def}} \{p \in B\colon b \leq p\}$. But, we know that in a Boolean algebra B a *principal filter* is a filter generated by a single element $b \in B$, and we also know that there is a precise correspondence between the atoms of B and the principal *ultrafilters*. This fact involves that, at the level of T_N, the individuals should be precisely those subsets of T_N which satisfy the Meets, Joins and Complements conditions. In other words, the individuals must be just the I_b's defined earlier.

It is in this sense, that, with reference to the logical system outlined by the American linguists, L_C appears to satisfy a further adequacy criterion: the criterion of *extensionality*:

Definition 4.3. For all properties p and q, $p = q$ iff the set of individuals which contains q is the same as the set which contains.

T_N is the set of all sets of properties and we have just seen that the individuals are, in fact, ultrafilters. Also in Montague's approach, however, the type for common nouns is the collection of subsets of U, and constitutes, thus, a power set Boolean algebra. Individuals are, precisely, particular subsets of this algebra. In this context we have to remember, first of all, that, from an algebraic point of view: (1) every finite algebra is complete; (2) the power sets are examples of Boolean algebras par excellence. With reference to (2) we have seen earlier that a power set Boolean algebra is a special case of Boolean algebra characterized by the presence of two additional properties: atomicity and completeness. There is, however, another important property of complete Boolean algebras that is particularly useful at the level of the generation of linguistic structures, Actually, we can prove the following theorem as stated by Keenan and Faltz:

Theorem 4.1. *A complete Boolean algebra B has a set of (complete and atomic) ca-free generators iff B is isomorphic to the power set of a power set.*

For example $T_{\bar{N}}$ is extensionally $P*$, the power set of T_N, itself taken to be a *ca*-algebra and, thus, isomorphic, as we have just said, to the power set of its atoms. With respect to this conceptual frame, the set of individuals must then be considered as a set of *c*-generators for $T_{\bar{N}}$. Individuals, however, are not only a set of *c*-generators for $T_{\bar{N}}$, they are also free. $T_{\bar{N}}$ is, precisely, a *ca*-algebra which has a set of free generators. In fact, the only algebras that have a set of *ca*-free generators are just those which are isomorphic to power sets of power sets and have cardinality $2^{(2^n)}$ for some cardinal n. In other words, if we have a model of n individuals there will be at the level of the model, $2^{(2^n)}$ sets of properties. We have immediately to underline, however, that most Boolean algebras do not have a set of free generators. For instance, the types of Modifier categories are not algebras with individuals. The linguistic structures, in fact, are, in general, well differentiated and polymorphic.

From a mathematical point of view, we can express the afore-mentioned properties, in a synthetic way, as follows. Let G be a set of generators of a Boolean ring B, then the following properties are equivalent:

(a) G is independent;
(b) B is free on G

If B is n-free, card $B = 2^{2^n}$ and B possesses 2^n atoms (or 2^n ultrafilters) which are the simple polynomials on G. So, when we have a Boolean algebra that is isomorphic to a power set of a power set, we are simply faced with a particular composition of power sets according to which the cardinality of the first power set refers to the number of atoms and the cardinality of the second power set refers to the number of the elements of the entire complete Boolean algebra. This fact means that we are really dealing with a particular kind of internal transformation.

Starting from generators considered as predicates we obtain atoms interpreted as collections of properties; but, if we start with generators considered as closed individuals (i.e. as complex constructions characterized by the presence of a closure process which determines the identification of a predicative "shape") we obtain atoms as worlds-urns. Once certain conditions are given, to compose a power set with another power set can involve a precise transformation of the role played by the individuals. As a consequence of this kind of change, the corresponding filters at the algebraic level will change as well.

In this regard, let us remark that, from a theoretical point of view, what we have said until now permits us to understand that two fundamental operations are at the basis of the definition of proper names (that is of that particular grid through which we normally refer to the events in the world):

1. the construction of a complex attribute that acts as a sieve;
2. the statement that a specific class defined by partitioning the family of sets provided by the model is not void.

These specific considerations permit us, in turn, to finally realize, in a very clear way, that the conceptual scaffolding built by Barwise, Cooper, Keenan, Faltz etc. is, from an epistemological point of view, characterized by an antinominalistic vein. In effect, for a nominalist, as well as for a nominalist realist, proper nouns constitute the real and primary vehicle of reference. Their ontological land is populated by empirically distinguishable individuals. This is exactly the opposite point of view with respect to the conceptual approach advocated by Keenan and Faltz. In fact, as we have just seen, for these scholars reference may articulate only within the realm constituted by a sophisticated sequence of linguistic and conceptual constructions. Therefore, they deny any significative role for the noumenal world of primitive entities U. The universe, in perspective, is identified by means of the constraints, postulates and invariants through which it espresses its action and by means of the "observational acts" which feed its invariance and contemporarily its continuous growth at the self-organizing level. With respect to this universe, we can observe events in the world only if by embodiment and assimilation, we progressively join the government of the system determining a specific nesting as well a particular irradiation of deep meaning, In this sense, individuals are no longer held to be encounterable and primitive objects, they appear as linked to a coordinated and coherent series of logical and linguistic operations and to a universe of observational acts which articulate in accordance with these very operations. Each time, the operational closure of the acts of seeing constitutes and identifies the opening of the mind's eyes. This closure, however, is strictly linked to meaning's irradiation as it arises as a consequence of the nesting process and the correlated symmetry breakings. Thus, the grid relative to the identification of the "strange" objects while showing its invariance, directly points to the inner emerging structure of the Subject. I do not directly observe individuals by means, for instance, of intuition: I have, on the contrary, to refer to specific mathematical structures, to the successive individuation of invariants and fixed-points, to a teleological activity able to prime the nesting of meaning. As a consequence the eyes of a particular mind come to open and the I of that mind (my mind) will be able to see.

4.2 Logic and Ontology

The theoretical approach that we have just presented partially tunes Keenan and Faltz into the theory concerning the roots of reference as outlined by the great American philosopher W. V. Quine.

Quine also, in fact, ends up assimilating proper nouns into predicates. In his opinion, the words which do the referring in the sentence "Socrates is a philosopher" do not occur in the surface structure of the sentence. They can be made explicit only by the use of quantifiers and variables. So, we have: “There is something that Socratizes and is a philosopher”. In Quine’s opinion the equation “$x = a$” must be reparsed in effect: “as a predication “$x = a$” where “$= a$” is the verb, the “F” of “Fx”. Or look at it as follows. What was in words “x is Socrates” and in symbols “$x =$ Socrates” is now in words still “x is Socrates”, but the “is” ceases to be treated as a separate relative term “=”. The “is” is now treated as a copula which, as in “is mortal” and “is a man”, serves merely to give a general term the form of a verb and so suit it to predicative position. “Socrates” becomes a general term that is true of just one object, but general in being treated henceforward as grammatically admissible in predicative position and not in positions suitable for variables. It comes to play the role of the “F” of “$F a$” and ceases to play that of the “a” ”.[10]

So “x is Socrates” now has the form of “x is round”. In other words when we write in predicative terms “x Socratizes” we have, in effect, defined the attribute which works as a sieve in order to make a specific partition, in order, that is, to individuate a class. The successive and necessary step consists in giving a particular form of existence to this class, in affirming that it is not void. Here we find the peculiar role of quantifiers: (Ex) (x is Socrates) or (Ex) ($x =$ Socrates).

Quine works essentially in the landscape represented by monadic and polyadic predicates whereas Montague makes essential reference to T_N and $T_{\bar{N}}$. In any case, we have remarked before the equivalence existing between T_N and P_1 in Montague’s system. Let us also note that in the same way that T_{P_1} is isomorphic to the power set of I_P, T_{Pn} is isomorphic to the power set of $(I_P)^n = ((I_P)^n)^*$.

In this theoretical perspective, at the objectual level, the variable becomes, thus for Quine, the essence of the ontological discourse. Once a theory is formulated in quantification terms its referential objects can be said to be the values of its quantified variables. The relative clause and the quantifiers, in Quine’s view, constitute the real roots of reference. According to these considerations, it appears quite clear why it is perfectly admissible, in accordance with Quine’s opinion, to infer $(Ex)Fx$, from $F(a)$, and why it is impossible to construct $(EF)Fx$. Actually, he admits existence assumption concerning the individuals: the objects, that is, of the domain, but he doesn’t admit existence assumption concerning properties or relations. Hence a specific distrust with respect to second-order predicate logic, a distrust that is perfectly coherent with the psychogenetic reconstruction of the referential apparatus outlined by him in his work.

In this way it is possible to affirm that, in Quine’s opinion, what is recognized as being is exactly, in first instance, what we assume as true for monadic predicates (complements included). We really know (and see) only insofar as we regiment our system of the world in a consistent and adequate way. At the level of proper nouns and existence functions we have already given some hints about the standard form of a regimented language whose complementary apparatus consist in predicates, variables, quantifiers and truth functions. However, if the discoveries in the field of Quantum Mechanics should oblige us, in the future, to abandon the traditional logic of truth functions, it will be our received notion of existence which will be challenged. New forms of theoretical regimentation and new processing procedures will, thus, appear nearly at the same time.

Our ontology as well as our grammar, as Quine affirms, are ineliminable parts of our conceptual contribute to our theory of the world. And, in effect, we have seen how the passage, for instance, from the ontology characterizing Montague's system to the ontology outlined by Keenan and Faltz, marks the stages of a progressive unfolding of our knowledge of the modalities proper to human beings of understanding both the surrounding world and themselves. Thus, it doesn't appear possible to think of entities, individuals and events without specifying and constructing, in advance, a specific language that would be used in order to speak about these same entities and without fixing them as code-language in action. Speaking, actually, requires our "immersion" into the language and this immersion can be effected only by the individuation of a specific code inscription and of its invariance conditions. It is with reference to this inscription and to the action performed by the meaning postulates that a specific link between a grid, on the one hand, and an observer, on the other hand, can be established; this link precisely allows for detecting and constructing the prototypes. As we have just said, the opening of the mind's eyes coincides with the act of joining the afore-mentioned grid in accordance with the limits and the inner structure proper to a reflexive model.

However, meaning cannot be enclosed only within the realm of the invariance and its government: Meaning, according to Wittgenstein's aphorism, is also "use". When we try to merge and join together the two just mentioned different conceptions of meaning, we are immediately faced with a conceptual perspective like the "internal realism" perspective advocated by Putnam, whose principal aim, for certain aspects, is to link together the philosophical approaches developed respectively by Quine and Wittgenstein. Actually, Putnam conservatively extends the approach to the problem of reference outlined by Quine. In his opinion, to talk of "facts" without specifying the language to be used, is to talk of nothing. "Object" itself has many uses and as we creatively invent new uses of words, "we find that we can speak of "objects" that were not "values of any variable" in any language we previously spoke".[11] The notion of object becomes, then, a sort of open land, an unknown territory as well as, on the other hand, the notion of reference. The exploration of this land appears to be constrained by use and invention. But, we may wonder, is it possible to guide invention and to control use? Are there rules (inductive rules, for instance) at the level of this strange, but completely familiar land?

A possible answer to these difficult and ancient questions appears implicitly contained in the original formulation of Putnam's program. In fact, it appears quite clear that the invention of new words necessarily coincides with an extension and an inner reorganization of language in use. This extension will however, concern not only the extensional aspects of the first-order structures proper to natural language, but also the intensional aspects (not limited only to the consideration of the possible worlds) and the second-order aspects. We have to stress that, as a matter of fact, language can go beyond any limit as determined by a particular abstract conception of linguistic formal rules. As Putnam remarks: "Reason can go beyond whatever reason can formalise".[12] These considerations can be paraphrased in a different way by saying that as we precisely and consistently specify the methods of reasoning permitted at the level of a given language, we determine an upper bound to the complexity of our results. Once again we cannot prove, in other words, according to Gödel's intuitions, the consistency of a particular system by methods of proof that are restricted enough to be represented within the system itself.

So, in accordance with this conception, as revisited for instance by Chaitin, we must, once again, go over the methods of reasoning accepted by Hilbert: in particular we have to extend Hilbert's finitary standpoint (and in our case, as a consequence, the conditions of predicative activity as defined by Quine) by admitting the necessary utilization of certain abstract concepts in addition to the merely combinatorial concepts referring to symbols. As we have seen in Chap. 2, for this purpose we must count as abstract those concepts that do not comprise properties or relations of concrete objects (the inspectable evidence) but which are concerned with thought-constructions, with the articulation of the intellectual tools of invention, simulation and control proper to the human brain.

The utilization, at the semantic level, of abstract concepts, the possibility of referring to the sense of symbols and not only to their combinatorial properties, the possibility, finally, of picking out the deep information existing in things (as well as in the same language considered as an expression of a precise cognitive activity) open, as is easy to understand, new horizons at the level of the problematics of reference and oblige us to take into consideration the link between reference procedures, on the one hand, and categorization processes, on the other hand.

When, for example, we affirm: "the apples are red" we realize perfectly as speakers of a natural language, that the predication constitutes a link which is stronger than the simple conjunction; the predication, in fact, requires the "immersion" of the apples into the red. From a perceptual, linguistic and conceptual point of view, we are really dealing with an object considered as a whole, an object, that is, that possesses a precise character of "entirety", a character which preludes even more complex forms of organicity. A book, for instance, is composed of pages which themselves, insofar as they are parts depending on an organic whole, do not constitute an object. These same pages, however, become simple objects, once they are torn out of the book. In the past century, Husserl was the first to introduce a sort of logical calculus of parts and wholes. He thought that only certain "organic" wholes constitute real objects and, in this sense, he introduced the notion of "*substratum*". The dialectics existing between part and whole really "homes" in the same act of predication, namely in the language considered as action and intervening. We have to discover the secret functional aspects of such a dialectics if we want to enter the mysterious kingdom of primary linguistic expressions. But to face this kind of dialectics also means to face the problems concerning the *symbolic dynamics* that governs the dependency links, the relationships between parts and wholes. If, as we have just seen, at the ontological level we can recognize the existence of an entire sequence of expression levels of increasing complexity, the dialectics part-whole must be compared to the successive constitution of the net of generative constraints through which this kind of process articulates. It is only within the secret meanders of this symbolic dynamics that we can, finally, find the theoretical instruments capable of showing us the correct paths that we need to pursue in order to obtain a more adequate characterization of the Fregean *Sinn*.

At the moment, the aim that many scientists working in the field of natural language semantics want to attain consists, precisely, in individuating the logical forms subtended to the afore-mentioned dynamics as well as the relationships existing between this very dynamics and the processes of meaning construction. If, for instance, we insert at the level of the ontology proposed by Keenan and Faltz, the dialectics concerning the dependency links and, at the same time, decide to consider the possible worlds not as external indices or as pseudotypes, but as types in every respect, the static landscape outlined by Keenan and Faltz suddenly becomes alive.

At the level of the arising new horizons, precise algebraic structures have to be grafted onto the previous structures and complex functional spaces will unfold with reference to a specific frame of geometric interactions. In consequence of the successive transformations of the dependency links, the same "shape" of the real objects, as it is normally recognized by us, will change. The same modalities through which we regiment the universe of the events as well as the modalities through which we induce, in a cooperative way, by coagulum and selection, a specific coherent unfolding of the deep information content hidden in the external Source, will be subject to a specific transformation. So, the ontological discourse connects itself with a precise genealogical and dynamic dimension, a dimension that must be linked, however, to a specific continuous procedure of conceptual recovery if we want this same dimension to be able to continue unfolding freely, performing, its specific generative function. At the level of this dimension we are faced with the presence of unknown protoprograms and silent potentialities.

In this sense, true cognition appears, at the symbolic level, as a recovery and as a guide, as a form of realized simulation. With respect to this particular framework, the simulation activity, the construction, for instance, of an adequate semantics for natural language, presents itself as a form of interactive knowledge of the complex chain of biological realizations through which Nature reveals itself in a consistent way to our minds. To simulate is not, however, as we have seen in the previous Chapters, only a form of self-reflection or a kind of simple recovery performed by a complex cognitive net in order to represent itself at the surface level and "join" the government in action. The simulation work offers, in effect, real instruments to the semantic net in order to perform a self-description process and to outline specific procedures of control as well as a possible map of an entire series of imagination paths. The progressive exploration of these paths will allow, then, deep information to canalize in an emergent way, and to exploit new and even more complex patterns of interactive expression and action. It is exactly the framing of this particular kind of laboratory of possible emergence that will assure the successive revelation of ever new portions of this very information: that particular "irruption" of the Other which can express itself only within those particular fibres of the imagination and within that variant geometrical tissue of the living *gestalten* which characterize, in an ultimate way, at the symbolic level, the cognitive activity of the Subject.

4.3 Meaning as Use and the Unfolding of Cognitive Activity

With respect to this frame of reference, we are no longer faced with an observation activity that identifies itself as vision according to the truth but with a simulation activity and a metamorphosis of meaning which express themselves by means of use and interaction, by the continuous surfacing of new forms of intentionality and conceptual constructions. When we pass from a world of objects to a world of constructions we are no longer faced exclusively with Boolean algebras, first-order structures and observational acts, we are really faced with a dynamic and functional universe characterized by inner circularity, by self-organization and by the presence of specific categorization processes as well as of second-order structures and differentiation patterns. Moreover, at the level of this particular world, as we have just said, the role played by meaning is different; meaning now is characterized in terms of a symbolic dynamics in action and with reference to a precise simulation language. As a consequence of this particular articulation, specific limitation facts can arise at the level of the progressive unfolding of this very language. In particular, new theoretical perspectives will reveal themselves with respect to the inner self-organizing aspects of the emerging structure and to the specific constitution of the individuals inhabiting this very structure considered as individuals essentially characterized not only in terms of their properties but also in terms of their relations (and their secret "affordances" at the symbolic level).

As a matter of fact, at the level of the theoretical design of Boolean Semantics the "universe" of reference possesses a sequential and a compositional character. We start with primitive predicates and we go along the derivation from these predicates of all possible joins of meets of these same elements. In this way we trace the design of different kinds of worlds and their composition. This design remains, however, as we have just seen, within the boundaries of a unique algebra generated by G.

The perspective, however, changes suddenly if we take into consideration, generalizing the initial Carnapian frame of reference, models that are not relative to the design, at a compositional level, of a unique algebra, but are the product (not in strict Boolean sense) of the interaction between a multiplicity of nuclei of generators. In this way a net of algebras can arise, a net that can span patterns of connectivity of a very high level. If we construct such algebras as *heterogeneous algebras* the net will reveal itself as a *net of automata* (once certain specific conditions are satisfied). Once again we shall be in the presence of a Boolean net of connected automata whose primitive inputs are represented by monadic predicates.

When we enter the realm of conceptual constructions, the deep meaning appears as relative to the action performed by precise semantic *fixed-points*, to a manifold, in particular, of subtended circumscription functions and to the progressive expression of specific postulates. The fixed-points of the resulting dynamics represent the "true" revelation of that specific tuning that characterizes and identifies the predicates at work. Thus, we are obliged to outline, first of all at the monadic level, a new particular kind of model: a self-organizing (and coupled) structure not bound to sets and individuals, (with relative attributes) but to generators and flows of tuned information. In this new theoretical framework, the simple reference to possible worlds (as in Frege or Hintikka, for instance) in order to take into account the structure of intensionality is no longer sufficient. One has also to resort, in the first instance, to the dynamics of the constraints, to the identification of the indices and of the recurrent paths of the informational fluxes as well as of the role played by the inventor-craftsman, to the interplay existing between intervening and change.

Moreover, when we are faced with the polyadic realm and we come to use, for instance, primitive binary relations, we must immediately make a series of choices (and assumptions) which are relative to the structural properties of such relations. Actually, in consequence of the structural properties that characterize, precisely, the dyadic predicates (i.e. that such predicates possess in an exclusively conceptual way), some specific conjunctions of these same predicates will be shown to be inconsistent. In this sense, at the level of T_{P_2}, for instance, the process of construction and constitution of individuals is a lot more complex and articulated with respect to T_{P_1}. In this particular case, an individual as well as being considered as a collection of properties, must also be defined as a chain or collection of relationships. This means that what must be joined together will no longer consist of simple entities or sets of properties but of configurations and graphs. The conjunction, at the level of generators, should be realized, so, respecting precise constraints which are of a "geometric" character, connected, in particular, to the successive gain of configurations of "points-patches" which possess determined characteristics. The role of compatibility factors becomes particularly essential. From here both the birth of complex cancellation procedures and the introduction by construction (a construction which, since articulated in a space "guided" by the imagination, we could define as Kantian) of new individuals, in a potentially unlimited way, arise. Likewise, we would have, in a correlated way, the introduction of nested quantifiers. Thus, the role played by meaning really assumes a specific and deep relevance. As a matter of fact, at the level of this type of structure, we can individuate the existence of an essential plot between the successive "presentation" of the constraints and the action of the meaning postulates, on the one hand, and the articulated design of mutations, cancellations and contractions of the predicates-inputs that characterize the higher layers of formal constructions, on the other.

When we, finally, take into consideration the second-order structures and the general structures, things appear even more complex. As we have seen in Chap. 3, in this particular case, what is important to stress is the fact that hidden in the structure some specific relations exist, some "rules" (second-order relations) that cannot be defined as relations among individuals, but are utilized to define first-order relations (i.e., relations among individuals). As a result, we obtain a particular structure where the n-ary relation universe is a proper subset of the power set of the n-ary Cartesian product of the universe of individuals. So, whereas in the standard structures the notion of subset is fixed and an n-ary relation variable refers to any n-ary relation on the universe of individuals, in the non-standard structures, on the contrary, the notion of subset is explicitly given with respect to each model. Thus, in the case of general structures the concept of subset appears directly related to the definition of a particular kind of constructible universe, a universe that, as we have noticed before, we can explore utilizing, for instance, the suggestions offered by Skolem or by Gödel (cf. Gödel's notion of constructible universe).

Once again the plot of limitation procedures and cancellations of relations progressively reveals itself as linked to the exploitation of ever-new specific conceptual categorizations. Within the realm of general structures the original self-organizing "glove" that imposes shape on itself acts contemporarily as a real support for the code inscription and, through the nesting process, for the limitation procedures: linkage operated by the *telos* allows the abstract design-frame to emerge, connected with an emergent nucleus of creativity through which other nuclei will manage to perceive and recognize themselves. What is presented, then, is a vision by principles, a process of abstraction allowing for a new flame of invention which ignites. The file which inscribes itself as a code providing the support for the nesting process, permits a progressive and genealogical unification at the level of the activity of form-production. Hence a vision which can reflect itself as thought, and can ultimately see by principles according to specific unification and abstraction procedures. A new nucleus of individual creativity can emerge through which new postulates and axiomatic principles manage to find concrete self-expression: hence the unfolding of a production of forms which disincarnates itself in pure abstraction. In this sense, the embedding at work, in conjunction with the inscription, finally allows for operative abstraction, and a meaning emerges which is able to renew itself and find in itself the source of a new and pure vision by principles.

In the light of these considerations, it appears, finally, clear that in the passage from T_{P_1} to T_{P_2} there is not only the presence of a sort of mechanical passage from a given set of functions to another. At the level of T_{P_2}, in fact, we are faced with a particular sort of algebras within which a specific series of constraints can suddenly arise. These constraints concern, for instance, the action of particular factors: compatibility, symmetry and so on, as well as the action of well determined processes of constitution of individuals according to precise geometric guide-lines. But, as we have just said, we can also have, in addition to this specific passage, an opening up of the logical second-order spaces with the consequent arrival on the scene of precise holistic issues.

It is in the framework of this mysterious path, in itself already complicated enough, that we can individuate the progressive emergence, at the co-evolutionary level, of the categorization processes that underlie at the brain level the simulation language. It is with respect to this path that the higher functions proper to the linguistic articulation come to progressively establish themselves at the intensional level: transitive verbs, determiners of higher level etc. It is with reference to this same framework that a precise dynamics of graphs with the subsequent introduction of cycles, attractors, fixed points etc. as well as the revelation of further constraints relative to problems of fitting, consistency etc. will be, finally, able to enter the stage. Precise forms of classification and therefore precise contexts of sense will appear. In this way, specific intensional structures will begin to emerge: in particular, intensional grammars defined with reference to orders-spaces of higher level. Thus, meaning can show its immense power at the productive level.

From here comes the necessity of outlining, in the case of T_{P_2} (and, in general, in the case of second-order structures), the sophisticated dynamism of a great book of Language that presents itself at the level of the conscious representation, like an effective reality in action. A reality which also emerges through our thinking and which, at the same time, determines, first of all at the genetic level, this same thinking. We no longer have before us a static book of Reality written in linguistic and mathematical characters. We have, on the contrary, a Language in action which makes itself the Word of reality, the book in progress of linguistic constructions and which by reflecting the original pure generativity in a simulation space (of which we are, as human beings, what is more, the support and the operational channel) assumes its primary forms and represents itself to itself by means of the tools of a precise symbolic dynamics. We are no longer faced, therefore, with concrete signs-symbols but rather with complex conceptual structures which are fitted in the effective articulation of a coupled process, a process into which, alongside the aforesaid dynamics relative to configurations and graphs also specific informational valuations proper to the subject, to the structures of reflection, simulation and cognition that characterize his activity, will be inserted.

We have seen how, for Putnam, the invention of new language represents the main tool to open up Reality, to discover new horizons of meaning. The awareness that comes out from the intensional analysis of the semantic structures of natural language and of the cognitive functions that subtend these same structures, leads us to clearly understand that the problem is not only that of extracting the information living deeply in things. It is in addition that of building simulation models able to bring out the information contained in the fibres of reality in such a way that this same information irrupting into the neural circuits of elaboration proper to the subject can, finally, induce an determine the emergence of new form of conceptual order and linguistic construction. The problem is, likewise, that of supplying coagulum functions which are capable of leading the Source to nest deeply, according to stronger and more powerful moduli. The emergence process and the same creativity that has been progressively realized, will present themselves as the "story" of the performed irruption and of the nesting carried out. They will articulate as forms of conceptual insight which spread out into a story, the story, in particular, of a biological realization. In order to "open" Reality, language must be embodied as an autonomous growth so that it will be possible, in perspective, to coagulate new linguistic constructions. Hence the importance of resorting to the outlining of recurrent processes and coupled processes in order to model the brain's functions. Likewise, this is the importance of that vertical (and intensional) dimension which grows upon itself, according to those exponential coefficients which we have previously considered and which appear indissolubly linked to the appearance and the definition of ever new forms of meaning. Forms which necessarily spring up through the successive discovery-construction of new *substrata* and new dependency links according to Husserl's primitive intuitions.

Genealogical processes, recurrent processes, coupled structures, new measure spaces, new orders of acting imagination: such is the scenario within which the new information can, finally, emerge. Here is the Language in action. Herein we may recognize the birth of new forms of seeing. Herein we can find the possibility to hear of a Source which comes forth from the interior of biological structures, to dictate, like a new *daimon*, the message of its self-representation, of its "wild" autonomy and of its renewed creativity.

Cognitive activity is rooted in reality, but at the same time represents the necessary means whereby reality can embody itself in an objective way: i.e., in accordance with an in-depth nesting process and a surface unfolding of operational meaning. In this sense, the objectivity of reality is also proportionate to the autonomy reached by cognitive processes.

Within this conceptual framework, reference procedures thus appear as related to the modalities providing the successful constitution of the channel, of the actual link between operations of vision and thought. Such procedures ensure not only a "regimentation" or an adequate replica, but, on the contrary, the real constitution of a cognitive autonomy in accordance with the truth. A method thus emerges which is simultaneously project, *telos* and regulating activity: a code which becomes process, positing itself as the foundation of a constantly renewed synthesis between function and meaning. In this sense, reference procedures act as guide, mirror and canalization with respect to primary information flows and involved selective forces. They also constitute a precise support for the operations which "imprison" meaning and "inscribe" the "file" considered as an autonomous generating system. In this way, they offer themselves as the actual instruments for the constant renewal of the code, for the invention and the actual articulation of an ever-new incompressibility. Hence the possible definition of new axiomatic systems, new measure spaces, the real displaying of processes of continuous reorganization at the semantic level. Indeed, it is only through a complete, first-order "reduction" and a consequent non-standard second-order analysis that new incompressibility will actually manifest itself. Therefore, the reference procedures appear to be related to a process of multiplication of minds, as well as to a process of unification of meanings which finally emerges as vision *via* principles. Here also the possibility emerges of a connection between things that are seen and those that are unseen, between visual recognition of objects and thought concerning their secret interconnections. In other words, this is the connection between the eyes of the mind and those of meaning (the eye of the intellect), a meaning which is progressively enclosed within productive thinking and manages to express itself completely through the body's intelligence.

A functional analysis of the kind reveals even more clearly, if possible, the precise awareness that, at the level of a cognitive system, in addition to processes of rational perception, we also face specific ongoing processes of semantic categorization. It is exactly when such processes unfold in a coherent and harmonious way that the "I" not only manages to emerge as an observation system, but is also moulded by the simultaneous display of the structures of intentionality. Through the intentional vision, the "I" comes to sense the Other's thought-process emerging at the level of its interiority. The drawing thus outlined, however, is meant for the Other, for the Other's autonomy, for its emerging as objectivity and action. This enables me to think of the autonomy of the Nature that "lives" (within) me.

Notes

1. Cf. at this proposal: Barwise J. & Perry J. (1983), *Situations and Attitudes*, Cambridge, Mass.: MIT Press.
2. Cf. Montague R. (1974), *Formal Philosophy*; Selected Papers of Richard Montague, edited with an introduction by R. Thomason, New Haven: Yale University Press.
3. Cf. for more information: Dowty D., Wall R. & Peters S. (1989), *Introduction to Montague Semantics*, Dordrecht: Reidel; Thayse A. (Ed.). (1991), *From Natural Languages Processing to Logic for Expert Systems Introducing a Logic Based Approach to Artificial Intelligence*, Vol. 3, New York: Wiley.
4. Cf. Keenan E. & Faltz, L. (1985), *Boolean Semantics for Natural Language*, Dordrecht.
5. Cf. Keenan E. & Faltz L. (1985), *Boolean Semantics for Natural Language*, op.cit., 48.
6. Cf. Keenan E. & Faltz L. (1985), *Boolean Semantics for Natural Language*, op.cit., 77.
7. Cf. Barwise J. & Perry J. (1983), *Situations and Attitudes*, op.cit.
8. Cf. Barwise J. & Perry J. (1983), *Situations and Attitudes*, op.cit., 45.
9. Cf. Keenan E. & Faltz L. (1985), *Boolean Semantics for Natural Language*, op.cit., 51.
10. Cf. Quine W.V. (1960), *Word and Object*, Cambridge, 179. Mass.: MIT Press.
11. Cf. Putnam H. (1983), *Representation and Reality*, 137. Cambridge, Mass.: MIT Press.
12. Cf. Putnam H. (1983), *Representation and Reality*, op.cit., 134.

Chapter 5
Morphogenesis and the Emergence of Meaning

5.1 Eigenforms, Categorial Intuitions and Rational Perception

As we have said in Chap. 1, the frontier between order and chaos appears to be able to offer, instead of punctiform mutations, an articulated variability, also capable of guiding the biomolecular systems to unfold themselves and to manifest hidden potentialities living at deep level progressively. This unfolding takes place, in particular, in accordance with the appearance of the new textures of constraints by means of which the selective pressures finally force, in a more sophisticated way, the available complexity. From a biological point of view the alternative is not between a sole environment (Darwin) and many environments (Dover), but between a changing environment, full of potentialities and unexpressed constraints (able to utilize, moreover, the varied complexity in order to tune its mechanisms of selection) and an environment, on the contrary, devoid of an autonomous and self-organizing internal dynamics.

With reference to this particular "landscape", the constraints imposed by the selective pressures (operating in ambient meaning) at the level of the dynamics of an original cellular (dissipative) automaton can, actually, permit a more complex canalization of the informational fluxes at the stake. In particular, they can allow the unfolding of silent potentialities, the full expression of generative principles never before revealed and, consequently, the effective expression of new autonomous processes of production of varied complexity.

The genomic information gives rise to organisms able to recognize the actual world and themselves in the world on the basis of specific formats, but principally to allow the original information to be preserved and transmitted along the route of a possible nesting of its own roots. For this to occur, "self-recognition" is necessary at each stage with respect to the operant meaning, allowing us to see ourselves through the eyes of Narcissus who is "added" to meaning as the observer of oneself (drowning in his very meaning).

A. Carsetti, *Epistemic Complexity and Knowledge Construction: Morphogenesis, Symbolic Dynamics and Beyond*, Theory and Decision Library A 45,
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It is Narcissus “who” allows meaning to self-conceive and the source to realize the process relative to the self-renewing “inscription”. He can really offer his severed head and his achieved ability as “editor” only to the extent to which he recognizes himself in the inscribed design and in the reflected image. Through the coder the source assumes a reproductive capacity commensurate with a precise invariance and with the individuation of intrinsic forms which inhabit life; it inscribes itself as form and as a hereditary principle in action, as a source of varied complexity but compared with a hereditary apparatus which self-organizes as such in view of possible regeneration. The source which generates on the basis of self-reflection opens out, then, towards a self-reproduction process which is targeted and part of a co-evolutionary path. Whoever arrests and captures the reflection, fixing and freezing it, also makes him/herself into a reflection; the offering of him/herself as severed head to the fluxes is in sight of a new invariance and the possible emergence of ever-new specific properties. Master/mistress of the shadow, s/he guides the process of regeneration by opening up to the new “conception”. The inscription and the suture of the wounds operate at the level of the becoming body; and it is by a path of perceptual activity of this kind that a world articulated in properties is finally recognized. The result is a source which, having stored a pathway, is able to code and articulate itself as a set of properties and recipes, and propose itself as mirror to itself but within the contours of incoming life. Hence the possibility of a genomic information self-mediated in the architecture of proteins.

With respect to this theoretical mainframe, if we take into consideration, for instance, visual cognition, we can easily realize that vision is the end result of a construction realized in the conditions of experience. It is “direct” and organic in nature because the product of neither simple mental associations nor reversible reasoning, but, primarily, the “harmonic” and targeted articulation of specific attractors at different embedded levels. The resulting texture is experienced at the conscious level by means of self-reflection; we actually sense that it cannot be reduced to anything else, but is primary and self-constituting. We see visual objects; they have no independent existence in themselves but cannot be broken down into elementary data. Grasping the information at the visual level means managing to hear, as it were, inner speech. It means first of all capturing and “playing” each time, in an inner generative language, through progressive assimilation, selection and real metamorphosis (albeit partially and roughly) and according to “genealogical” modules, the emergent (and complex) articulation of the semantic apparatus which works at the deep level and moulds and subtends, in a mediate way, the presentation of the functional patterns at the level of the optical sieve.

What must be ensured, then, is that meaning can be extended like a thread within the file, a thread that carries the choices and the piles related to specific symmetry breakings. It is Narcissus who must donate cues in order to operate the fixing of meaning. At the end of the metamorphosis the hero will mirror himself in the motionless face of Ariadne. Now his head will be cut and a vision will arise in accordance with the truth.

In this way, it will be possible to identify a “garland”; only on the strength of this construction can an “I” posit itself together with a sieve: a sieve in particular related to the world which is becoming visible. In this sense, the world which then comes to “dance” at the level of the eyes of my mind is impregnated with meaning. The “I” that perceives it realizes itself as the fixed point of the garland with respect to the “capturing” of the thread inside the file and the genealogically-modulated articulation of the file which manages to express its invariance and become “vision” (visual thinking which is also able to inspect itself), anchoring its generativity at a deep semantic dimension. The model can shape itself as such and succeed in opening the eyes of the mind in proportion to its ability to permit the categorial to anchor itself to (and be filled by) intuition (which is not, however, static, but emerges as linked to a continuous process of metamorphosis). And it is exactly in relation to the adequate constitution of the channel that a sieve can effectively articulate itself and cogently realize its selective work at the informational level. This can only happen if the two selective forces, operating respectively in ambient meaning and in ambient incompressibility, meet, and a *telos* shapes itself autonomously so as to offer itself as guide and support for the task of both capturing and “ring-threading”. It is the (anchoring) rhythm-scanning of the labyrinth by the thread of meaning which allows for the opening of the eyes, and it is the truth, then, which determines and possesses them. Hence the construction of an “I” as a fixed point: the “I” of those eyes (an “I” that perceives and that exists in proportion to its ability to perceive (and “fix”) according to the truth). What they see is generativity in action, its surfacing rhythm being dictated intuitively. What this also produces, however, is a file that is incarnated in a body that posits itself as “my” body, or more precisely, as the body of “my” mind: hence the progressive outlining of a meaning, “my” meaning which is gradually pervaded by life.[1]

Vision as emergence aims first of all to grasp (and “play”) the paths and the modalities that determine the selective action, the modalities specifically relative to the revelation of the afore-mentioned semantic apparatus at the surface level according to different and successive phases of generality. These paths and modalities thus manage to “speak” through my own fibers. It is exactly through a similar self-organizing process, characterized by the presence of a double-selection mechanism, that the mind can partially manage to perceive (and assimilate) depth information in an objective way. The extent to which the network-model succeeds, albeit partially, in encapsulating the secret cipher of this articulation through a specific chain of programs determines the model’s ability to see with the eyes of the mind as well as the successive irruption of new patterns of creativity. To assimilate and see, the system must first “think” internally of its secret abstract “capacities”, and then posit itself as a channel (through the precise indication of forms of potential coagulum) for the process of opening and anchoring of depth information. This process then works itself gradually into the system’s fibers, *via* possible selection, in accordance with the coagulum possibilities and the meaningful connections offered successively by the system itself (as immersed in its meaning).

The revelation and channeling procedures thus emerge as an essential and integrant part of a larger and coupled process of self-organization. In connection with this process, we can ascertain the successive edification of an I-subject conceived as a progressively wrought work of abstraction, unification, and emergence. The fixed points which manage to articulate themselves within this channel, at the level of the trajectories of neural dynamics, represent the real bases on which the "I" can graft and progressively constitute itself. The I-subject (the observer) can thus perceive to the extent in which the single visual perceptions are the end result of a coupled process which, through selection, finally leads the original Source to articulate and present itself as true invariance and as "harmony" within (and through) the architectures of reflection, imagination, computation and vision, at the level of the effective constitution of a body and "its" intelligence: the body of "my" mind. These perceptions are (partially) veridical, direct, and irreducible. They exist not only in themselves, but, on the contrary, also for the "I", but simultaneously constitute the primary departure-point for every successive form of reasoning perpetrated by the agent. As an observer I shall thus witness *Natura naturata* since I have connected functional forms at the semantic level according to a successful and coherent "score". In this sense at the level of the reflexivity proper to the system the *eigenforms*[2] reveal themselves as an integrant part of that self-organization process which constitutes the real engine of visual cognition, a process that the *telos* itself can manage to "imagine" only along the progressive unfolding of its reflexive tools. Without the individuation of the "I" and the composition of the eigenforms no perceptual activity is really possible.

The multiple unfolding of the eigenforms will be tailored to the symmetry breakings that manage to be realized at the level of meaning. They come to constitute themselves as fixed points in the process of construction of the structures of the operator. In this sense, they present themselves as the real bases of my own perceptual operations and, therefore, "preside" at the identification of the objects in the world. The forms arise from the determinations of the embodiment taking place by means of the "infixions" offered by Ariadne. When the garland comes to close and embraces the Minotaur that embodies and reflects himself, we have the emergence of vision. Narcissus-Minotaur will finally be able to recognize himself as an invariant reality and a source of replication through his rendering to the "stone", his realization as being in the world and his becoming (along the metamorphosis) an integrant part of the ruler.

In accordance with these intuitions, we may tentatively consider, from the more general point of view of contemporary Self-organization theory, the network of meaningful (and "intelligent") causal "programs" living at the level of our body as a complex one which forms, articulates, and develops, functionally, within a "coupled universe" characterized by the existence of two interacting selective forces. This network gradually posits itself as the real instrument for the actual emergence of meaning and the simultaneous, if indirect, surfacing of an "acting I": as the basic instrument, in other words, for the perception of real and meaningful processes, of "objects" possessing meaning, aims, intentions, etc.: above all, of objects possessing an inner plan and linked to the progressive expression of a specific cognitive action.

As we have just said, the mechanism which "extracts" pure intuitions from the underlying formal co-ordination activity, if parallel to the development of the *telos* as editor with respect to the coder, is necessarily linked to the continuous emergence of new mathematical moves at the level of the neural system's cognitive elaboration, This consideration inviting the revisiting of a number of Kantian hypotheses. It would appear, for instance, to be necessary not only to reread Kant in an evolutionistic key (cf., e.g., K. Lorenz), but also with reference to other speculative themes like, for instance, the indissoluble link existing between life and cognition and between chance and necessity. Taking into consideration coder's action opens up a new and different relationship with the processes of mathematical invention, making it necessary, for example, to explore second-order territories, the very realm of non-standard mathematics as well as the dialectics between observer and observed reality.

Pace Kant, at the level of a biological cognitive system sensibility is not a simple interface between absolute chance and an invariant intellectual order. On the contrary, the reference procedures, if successful, are able to modulate canalization and create the basis for the appearance of ever-new frames of incompressibility through morphogenesis. This is not a question of discovering and directly exploring (according, for instance, to Putnam's conception) new "territories", but of offering ourselves as the matrix and arch through which they can spring autonomously in accordance with ever increasing levels of complexity. There is no casual autonomous process already in existence, and no possible selection and synthesis activity *via* a possible "remnant" through reference procedures considered as a form of simple regimentation. These procedures are in actual fact functional to the construction and irruption of new incompressibility: meaning, as *Forma formans*, offers the possibility of creating a holistic anchorage, and is exactly what allows the categorial apparatus to emerge and act according to a coherent "arborization". From the encounter of Ariadne with Narcissus we shall have the flowering of forms, the possibility to perceive by fixed points as well as the birth of specific perceptual structures at the level of the operator.

The new invention, which is born then shapes and opens the (new) eyes of the mind: I see as a mind because new meaning is able to articulate and take root through me. As J. Petitot correctly remarks,[3] according to Kant the pure intuitions are: "'abstraites de l'action même par laquelle l'esprit coordonne, selon des lois permanentes, ses sensations' (*Dissertation,* 177). Or, cette coordination est elle-même innée et fonctionne comme un fondement de l'acquisition".[3] In this sense, the space appears as a format, the very basis of spatial intuition is innate. According to Kant, it is a condition of a subject knowing anything that the things it knows should be unified in a single consciousness. Kant calls this consciousness the transcendental unity of apperception. Kant writes that this unity comes about "not simply through my accompanying each representation with consciousness, but only in so far as I conjoin one representation with another." (B 133, p. 153 in Kemp Smith) In this sense, all coherent consciousness, hence all knowledge of anything, presupposes not just an original unity, but original, conceptual acts of possible combination to produce such unity. This means that some concepts are *a priori*. They cannot possibly have been derived from experience, since without them there would have been no original unity of experience. Just as Kant identifies in this way the existence of *a priori* concepts living at the level of the intellect, he also identifies, as we have just said, the existence of innate forms at the level of sensibility. It is in dependence of the determinations operated by the schemes on the form of Time (in connection with the operations expressed by the categories) that we shall obtain the intuitions that populate our perception of the world.

However, at the biological level, as we have just seen, what is innate is the result of an evolutionary process and is "programmed" by natural selection. Natural selection is the coder (once linked to the emergence of meaning): at the same time at the biological level this emergence process is indissolubly correlated to the continuous construction of new formats in accordance with the unfolding of ever-new mathematics, a mathematics that necessarily moulds coder's activity. Hence the necessity of articulating and inventing a mathematics capable of engraving itself in an evolutionary landscape in accordance with the opening up of meaning. In this sense, for instance, the realms of non standard-models and non-standard analysis represent, today, a fruitful perspective in order to point out, in mathematical terms, some of the basic concepts concerning the articulation of an adequate intentional information theory. This individuation, on the other side, presents itself not only as an important theoretical achievement but also as one of the essential bases of our very evolution as intelligent organisms.

As we have said in Chap. 3, in accordance to a main thesis by Marzano, Henkin arrived to prove the completeness theorem for type theory, by changing the semantics and hence the logic. But, also at the level of Computability Theory, we have seen that D. Scott in order to outline the architecture of Scott domains[4] was obliged to change semantics. The problem was in finding a viable denotational semantics for certain theories of computability (such as the untyped lambda calculus) which resist straightforward interpretations in terms of sets and functions. The important step to find a model for the lambda calculus was to consider only those functions (on a partially ordered set) which are guaranteed to have least fixed points. The set of these functions, together with an appropriate ordering, is again a "domain" in the sense of the theory. But, as we have said before, the restriction to a subset of all available functions has another advantage: the D_∞ domain is isomorphic with its own space of functions and is thus described by the equation $D_\infty = [D_\infty \rightarrow D_\infty]$, i.e., one gets functions that can be applied to themselves. In this way, we can directly introduce some classical principles characterizing Self-organization theory. Decades later, we now know many techniques for constructing such domains as suitable objects in Cartesian closed categories, but Scott's basic intuition, that computability could be interpreted as continuity, continues to exert a decisive influence today. Hence the possibility of a deep investigation of the way in which it might be possible to identify the features characterizing a *reflexive* domain, a development language, that is, able to express and reflect within itself the structure of the "ideas" and the performances which constitute its texture, as well as to express, still within itself, its own truth predicate. It is by referring to such a domain, moreover, that the subsequent building of an autonomous system could be articulated which is not only able to represent within itself the framework of its procedures (its own categorial apparatus) but also to realize, at the same time, its embodiment. In other words, the challenge now is to outline *self-organizing* domains able to make exact reference to the realm of non-standard models. At this level meaning, as we have just said, should be considered as connected to a continuous emergence process and a specific selective activity; it should be explored by means of the tools of an adequate symbolic dynamics with respect to the autonomous opening up of successive levels of depth information. In such a domain a true autonomous system should be considered, at the same time, in accordance with the afore mentioned thesis advocated by Prigogine, as an observer (spectator) and as an agent (actor). The new mathematics linked to the self-organizing domains will show itself as a revolutionary tool for a real deepening of our knowledge of physical and biological living processes and for the possible (even if partial) trigger of their inner metamorphosis.

In denotational semantics, what programs enter into semantic correspondence with is represented by function-objects, by actual functional organizations which articulate themselves at the level of an "ideal" mind. At this level, functions live in a total, abstract field of reference, possessing a potentially infinitary character. At the level of actual computations, instead, what we have are finite, but adequate sequences of state transitions. In this respect, the attempt of an infinitary source to represent itself in a simulation system is, first of all, an attempt to reflect oneself in a finite output, as well as to "see oneself" as a synthetic unit at the cognitive level, i.e. as a sort of "I see" operating in a Kantian sense. It also means, in the end, to succeed in having a replication of oneself, in constructing a system which is able to reflect itself and to operate, through reflection, a process of re-appropriation and, at the utmost, of self-reproduction.

Here a contradiction might seem to emerge. The output is finite, it can conform an infinitary structure, but it cannot create, in turn, information. Nevertheless, the problem does not lie in the creation of information concerning potentially infinitary structures, but, rather, in its transmission, in its unfolding and in its nourishment, which is at first performed by replication, i.e. by the transmission of a finite number of rules concerning the pure replication of the structure. In this respect, a living being and its DNA do not constitute an infinitary source in the actual sense; they, rather, can be conceived of as the basis for the transmission of finite, but *adequate* and *productive* amounts of information concerning the inner building of potentially infinitary structures by means of possible replication. In this way, the theoretical instruments belonging to denotational semantics can allow us to offer, although in an approximate way to be more deeply investigated, a first characterization, on a formal level, of a system which is capable of conforming, in a finite setting, a representation of the complex functional organization living inside it at an infinitary level. The crucial point is to see in what respects and according to which level of logical depth such a system can reveal itself as the product of an activity of reflection, and can, hence, constitute itself (but in replicative terms) as a reflexive domain able to talk within itself of its own truth.

The conditions must thus to be determined under which it might be possible for the human mind to draw a formal model of itself which could then be able to represent itself in an autonomous way, after having being an occasion of reflection for the mind itself of its own structure. A model, moreover, that should be capable of constituting itself, in accordance with approximate but adequate modules, as the basis of a true self-reproduction process. To such an end, the first step to be taken –

as it has been stressed – is the deep investigation of the way in which it might be possible to identify the features characterizing a reflexive domain considered as a "language" able to express and reflect within itself the structure of the "ideas" and the performances which constitute its deep texture, as well as to express, still within itself, its own truth predicate. It is exactly by referring to such a domain that it will be possible to articulate the subsequent building of an autonomous (self-organizing) system able not only to represent within itself the framework of its procedures, but also to suggest, in a not pre-determined way, the very lines for an innovative extension of the original categorial apparatus underlying it. It appears that if it could be possible to outline, even if in an approximate and partial way, some of the essential conditions characterizing a system like this, a first operational characterization could be actually provided of the deep structure of a self-productive system articulating at the semantic (and biological) level as a true observing system.

Insofar as the ideal infinitary structure reveals itself, at the level of the language, by mean of fixed points, it appears to itself (and to the observer) as limited to a finite setting, and yet anchored through approximation to its originary reality. It is in this way that we can define a semantics for our syntax even if the domains we talk about, both at the syntactic and at the semantic level, turn out to possess, from a mathematical point of view, the same formal features. In other terms, where the human mind (or a possible artificial mind) was to turn out to be capable of reflecting the space of its own categorial organization, it will necessarily have to represent the achieved awareness within a linguistic space. This very space will present itself as a semantic framework for the syntax mentioned above. Through the procedure of the fixed points, the semantic algebra could then reveal itself, in this way, as the ultimate grounding and as a not further analyzable structure (at least at a given stage of ongoing evolution). However, in order for all this to occur, in this shift from a language to another, in this activity of continuous replication and coupled unfolding, the original source, with its infinitary character, is to be always preserved by means of successive approximations and real "adjunctions". Those adjunctions that, as we have seen in Chap. 2, can assure the continuous growth of the Temple of life.

Once the possibility of constructing a language which is able to include itself (and to grow on itself) has been ascertained, it is also necessary to wonder whether, in general, a given language can also talk about its own truth, namely whether it can include its own truth predicate. In 1975, the great logician S. Kripke[5] showed how, by using the theoretical tools of the retractions, it is possible to come to draw, in a coherent way, a language L_s containing its own truth predicate. As is well known, Tarski had shown that a language L_0 constructed on the basis of first-order predicate calculus, able to talk about its own syntax and within which all predicates are completely defined in the space of the variables, cannot contain its own truth predicate. In order to define a truth predicate for such a language, we will have to appeal to a higher level language. Each language in the chain which thus emerges will have a truth predicate for the language preceding it. Hence the birth of a hierarchy, as well as the appeal to a truth predicate which is each time new for each level. Kripke showed how, besides this orthodox approach, it is possible to draw a more sophisticated and flexible approach which allows to talk about the truth of a language within the very language itself. Let L be a language and L the language obtained by extension from L adding a monadic predicate $T(x)$ whose interpretation needs to be only partially defined. An interpretation for $T(x)$ will be given by the partial set (S_1, S_2) where S_1 is the extension of $T(x)$, S_2 is the antiextension of $T(x)$ and $T(x)$ is undefined for entities outside $S_1 \cup S_2$. $L\ (S_1, S_2)$ is the interpretation of L which results from interpreting $T(x)$ by the pair (S_1, S_2). Let S'_1 be the set of true sentences of $L\ (S_1, S_2)$ and let S'_2 be the set of all elements of the domain D which either are not sentences of $L\ (S_1, S_2)$ or are false sentences of $L\ (S_1, S_2)$. If $T(x)$ is to be interpreted as truth for the very language L containing $T(x)$ itself, we must finally have $S_1 = S'_1$ and $S_2 = S'_2$. The pair (S_1, S_2) that satisfies this condition is called a *fixed point*. Thus, the way is open, exploiting the property of monotonicity, to construct in turn a language L_s that reveals itself as a fixed point.

However, if we aim to outline specific domains able to take into account not only the realm of invariance but also the multiple territories of morphogenesis, we have to refer to domains having a more general character, domains able, in particular, to set themselves as models of much more sophisticated and intertwined calculi and grammars; they should, more specifically, turn out to be able to draw the lines of complex functional organizations such as the semantic structures of natural languages. As we have remarked in Chap. 2, the problem to be first of all addressed is to show how it is possible to construct a reflexive domain as D_∞ in a consistent way, but in accordance with a possible "creative" extension of it. Let us to stress, however, once again, that the more complex structures put in place at the second-order level in accordance with this extension (if successful) should return to be articulated in ambient reflexivity once it will be possible to identify an adequate semantics able to take into account (through adequate reduction) the new "ideas" introduced at the moment of the sudden new revelation of the coder. This, in turn, will allow observers to stand up thus supporting the emergence of meaning through the successive opening up of specific "isles" of chaos.

With respect to this theoretical mainframe, a reflexive domain should be considered not as an already-existing structure but as the actual *compositio* of an existing structure with the "creative" exploration of new possible informational paths living at higher-order levels. As we know, a reflexive space is endowed with a non-commutative and non-associative algebraic structure. It is expandable and open to evolution over time as new processes are unfolded and new forms emerge. In a reflexive domain every entity has an eigenform, i.e. fixed points of transformations are present for all transformations of the reflexive domain. According to von Foerster and L. Kauffman,[6] the objects of our experience are the fixed points of operators, these operators are the structure of our perception. In this sense, we can directly consider an object A as a fixed point for the observer O: $O(A) = A$. The object is an *eigenform*. In the process of observation, we interact with ourselves and with the world to produces stabilities that become objects of our perception. Our perceptual activity, however, is conditioned by the unfolding of the embodiment process and is linked to the cues offered by Narcissus to meaning in action. These objects attain their stability through specific mathematical tools, through the unfolding, in particular, of specific limiting processes. If $F(X)$ denotes the result of applying the operations symbolized by F to the given condition, in the time-independent case we can write $J = F\ (F\ (F(\ldots)))$. Then $F\ (J) = J$. J is said to be the eigenform for the recursion F. Every recursion has a fixed point. Let us, however, point out that the notion of reflexive domain first appeared in the work of A. Church and H. Curry in the 1920s. Actually, the key to lambda calculus is the construction of a self-reflexive language: at this level we can solve the eigenform problem without the excursion to infinity. In the Church-Curry language the two basic rules: Naming and Reflexivity allow language to refer to itself and to produce itself from itself. In such a context, every object is inherently a process and the structure of the domain as a whole comes from the relationships whose exploration constitutes the domain. Any given entity acquires its properties through its relationships with everything else.

As we have just said, the pure (classical) reflexive models do not lead to true creativity and real metamorphosis because they do not give an account for the emergence processes living at the level of meaning. In particular, these models do not take into consideration the dialectical pairing of creativity and meaning. They do not loosen the knot of the intricate relationships between invariance and morphogenesis and do not arise in relation to the actual realization of a specific embodiment. Hence the importance of making reference to theoretical tools more complex and variegated as, for instance, non-standard mathematics and complexity theory, in order to provide an adequate basis for the afore mentioned extension. Let us resort to an exemplification: the von Koch curve is a eigenform, but it is also a fractal. However, it can also be designed utilizing the sophisticated mechanisms of non-standard analysis. In this last case, we have the possibility to enter a universe of replication, which also opens to the reasons of real emergence. At this level, the growth of the linguistic domain, the correlated introduction of ever-new individuals appears strictly linked to the opening up of meaning and to a continuous unfolding of specific emergence processes with respect to this very opening. Hence the need for the introduction of precise evolutionary parameters, the very necessity, in general, to bring back the inner articulation of the eigenforms not only to the structures of "simple" perception but also to those of intentionality.

In order to capture the meaning of this last statement, let us to analyze more deeply the afore mentioned articulation of the eigenforms. As we have just said, the von Koch curve is an eigenform: as such it is a limit of a process. But we may wonder: does this limit exist? is there an invariance property for the Koch curve? If we take into consideration the similarity transformations of the Koch curve and if A is any image we can write

$$W(A) = w_1(A) \cup w_2(A) \cup w_3(A) \cup w_4(A) \tag{5.1}$$

where $W(A)$ denotes the collection of all the admissible transformations. The Koch curve is invariant under the transformations w_1 to w_4. In other words, if we apply the transformation W to the image of the Koch curve we obtain the Koch curve back again: $W\ (K) = K$. The equation also shows the self-similarity of K since $K =$

$w_1(K) \cup w_2(K) \cup w_3(K) \cup w_4(K)$ states that K is composed of four similar copies of itself. In technical terms, we may characterize the Koch curve as a fixed point of the Hutchinson operator. It is easy to show that the iteration of the operator W applied to a starting configuration A_0 yields a sequence

$$A_{k+1} = W(A_k), \quad k = 0, 1, 2, \ldots \tag{5.2}$$

which converges to the limit object, the Koch curve. It is the limit object (and not the final stages of the process) which exhibits self-similarity. But how can we prove that these fractals really exist? In 1981 Hutchinson[7] showed that the operator W which describes the collage

$$W(A) = w_1(A) \cup w_2(A) \cup w_3(A) \cup \ldots \cup w_N(A) \tag{5.3}$$

is a contraction with respect to the Hausdorff distance, in other words, there is a constant c, with $0 \leq c < 1$, such that: $h\ (W(A), W(B)) \leq c \cdot h\ (A, B)$ for all compact sets A and B in the plane. Hutchinson utilizing the Contraction Mapping Principle was finally able to show that W is a contraction but with respect to the Hausdorff distance. In this sense, the von Koch curve as well as other fractals appear as objects in the plane and there are convergent processes for them, in particular the iteration of the corresponding Hutchinson operators. In this way, relating Hutchinson first intuitions and Hausdorff results, we can finally be sure that the limit objects with the self-similarity property truly exist. In the case of an eigenform like the von Koch curve, perception and existence really manage to coincide through the progressive unfolding of the embodiment process.

The von Koch curve, however, is also an object of interest at the level of non-standard analysis.[8] Things are different at this level. A standard curve F can be parameterized by a real number $x \in [0, 1]$ developed in the counting base p in the form

$$x = 0.\, x_1 x_2 \ldots = \sum_{1}^{\infty} x_k\, p^{-k}. \tag{5.4}$$

Given its fractal dimension $D = \log p/\log q$, the length of fractal is infinite while its surface vanishes. The real coordinate x is insufficient to describe thoroughly the fractal curve F: the distance along F between two points parameterized by two different x's is infinite, while points separated by a finite distance along F correspond to the same values of x. Hence the necessity to explore another formalism able to introduce new "elements", to deal, for instance, with infinitesimal and infinite numbers, to introduce, in particular, a set "larger" than **R**. We must enter the realm of non-standard analysis. At this level the set **R** of standard numbers is a subset of ***R,** the set of hyper-real numbers. ***R** contains infinitely small and infinitely large numbers. ***N** will denote the set of hyper-natural numbers. Let us generalize the usual fractal by introducing a curve F_ω in ***R** parameterized by an hyper-real number $x^* \in {}^*[0, 1]$: $x^* = x_1/p + \ldots + x_n/p^n + \ldots + x_{\omega-m}/p^{\omega-m} + \ldots + x_\omega p^\omega$, where $\omega \in {}^*\mathbf{N}_\omega$. F_ω is not a fractal in the non-standard sense (since the fragmentation is *limited up to ω) but its standard part is identical to the usual fractal: $F \equiv st(F_\omega)$. By using an infinitely great magnifying power, F_ω can be drawn in an exact way: the fractal is no more a limit concept.

At this level perception and existence do not coincide any more. We cannot "directly" perceive the curve, we have to compute and construct it at an abstract and mathematical level utilizing the tools offered by non-standard analysis. We have to make reference to elements that are introduced and explored by means of an enlargement of **R**, and this enlargement must follow well defined non-standard (and conceptual) procedures.

With respect to this theoretical frame, it is important to underline that every limited hyperreal number r is infinitely close to exactly one real number called its *shadow* and denoted by $sh(r)$. This constitutes the standard part of r. We can think of the hyperreals as a jagged line somewhere above the real line, the peaks of this line cast shadows onto the real number line which is their standard part. Hence the birth of a continuous interweaving between standard and non-standard, here again the need to resort to a scientific semantics whenever appropriate as well as to a continuous transition from the second-order level to specific many-sorted surface structures. Here we can precisely identify the role played by self-organizing models: the reflexive models are limited because they give an account only for the relationships between perception and existence at the standard level, between creativity and meaning as limited to the realm of invariance. The self-organizing models, on the contrary, aim to examine the work done by the ruler (as well as by the connected "editor") with respect to that particular transition point provided by the link existing between the realm of invariance and the paths of morphogenesis.

In this sense, it is necessary to refer to new models of the real-number line, these models, moreover, should also be objectively identified (as in the case of Harthong-Reeb theoretical construction[9]) with reference to the relationship existing between "seeing" and "thinking" as well as to the ongoing processes of construction of our neural circuitry. Hence the necessity of a continuous exploration, in evolutionary terms, of those particular modules intersecting mathematical investigations and epigenetic growth (at the neural level) that really identify the secret architectures of the "knowing I". We really have the possibility to enter the unexplored territories of epistemic complexity.

5.2 Meaning Clarification and the "Thinking I"

Merleau Ponty, as is well known, is in line with Brentano and Koffka in considering the phenomenal *Umwelt* as 'already there', perception consisting precisely in detaching (*dégager*) the nucleus of this 'already there'. The distinctive nature of the *gestalt* is not as something alive in itself, independently of the subject which has to "insert" into it its relationship with the world; nor, however, is it constructed by the subject. It is not absolute, since experience shows that it can be made to vary, yet nor is it purely related to the Self, since it provides an *Umwelt* which is objective (transcendent). In this sense, perception does not constitute a simple act of synthesis.

According to this viewpoint, Quine too considers, for instance, predication as something more than mere conjunction (the mere synthesis, Brentano would have said, of a subject-notion and a predicate-notion), not least since it ultimately coincides with an act of perception and, as we have seen before, with the actual constitution of specific perceptual structures.[9] When we say 'the apples are red', this for Quine means that the apples are immersed in red. Predication, indeed, finds its basis on a far more complex act than simple conjunction- composition.

It should, however, be underlined that when in his later work Quine gives an illustration of the kind, he is quite consciously and carefully re-examining not only some of Brentano's original ideas on the thematic of perception, but also a number of basic assumptions behind Husserl's idea of relations between perception and thought. Can colour be grasped, Husserl asked, independently of the surface supporting it? Quite clearly not: it is impossible to separate colour from space. If we allow our imagination to vary the object-colour and we try to annul the predicate relative to the extension, we inevitably annul even the possibility of the object-colour in itself, and reach an awareness of impossibility. This is what essence reveals: and it is precisely the procedure of variation which introduces us to the perception of essence. The object's *eidos* is constituted by the invariant which remains unchanged throughout all the variations effected.

In Husserl's opinion, together with perception it is necessary to conceive of acts based on sensory perceptions in parallel with the movements of categorial foundation taking place at the intentional level.[10] These acts offer a precise "fulfilment" to the complex meanings which for us constitute the effective guides to perception. When I observe gold, I see not yellow on the one hand and gold on the other, but 'gold-is-yellow'. 'Gold is yellow' constitutes a fact of perception, i.e. of intuition. The copula, the categorial form par excellence, cannot in itself be 'fulfilled': yet in the perception of the fact that 'gold is yellow', the copula too is a given. The sentence is filled up in its entirety simultaneously with its formation at the categorial level. It is in this sense that intuition itself takes on a form. Categorial intuition, as opposed to sensory or sensible intuition, is simply the evidencing of this formal fact, which characterizes any possible intuition. I do not see-perceive primary visions and their link: I see immersion, Quine would say: I see the whole, and perceive an act of realized synthesis. This, Vailati would add, is the sense in which meanings function as the tracks guiding all possible perception. A categorial form, then, does not exist in and for itself, but is revealed and developed through its embodiments, through the concrete forms showing its necessity, and which unfold it according to specific programs that constitute, simultaneously, themselves as program-performers. It is thus meaning which has the power to produce forms, this constituting the intuition according to its categorial nature.

Category cannot be reduced to grammar because it is not outside the object. According to Husserl,[9] we need to conceive of a type of grammar which is immanent to language, which must necessarily be the grammar of thought, of a thought which reveals itself as language in action, a language that, in its turn, constitutes itself as the Word of Reality, like a linguistic corpus, i.e. a construction articulated linguistically, according to precise grammatical and semantic patterns, which gradually manages to express itself as Reality. In contemporary terms, we could say that Husserl's language in action is characterized by the fact that the origin of meaning within the organization relative to the complex coupled system is merely an emergent property. What is self-organizing is the function together with its meaning, and it is in this sense that, as stated above, meaning for Husserl is able to produce form-functions.

It thus becomes clear how for Husserl form, or articulation, can be considered as precisely that it can only be constituted as object through a formalizing abstraction. Hence the birth of a very specific intuition which can only be the result of a founding act. It is in this founding activity that the ultimate sense of categorial objectivity lies: this is the case, for instance, of mathematical evidence, which relates to the existence of a structure only insofar as it is accessed by an ordered series of operations. As we have just seen in the case of fractal structures, we have to ascertain, for instance, that the limit objects with the self-similarity property truly exist.

Thus, the actual reality of an object is not given by its immediate appearance, but by its foundation, it shows itself as something constituted through a precise act. The innate meaning of an object is that of being itself within an act of intuition. There is a moment, for example, when a circle ceases to be a circle by means of a variation procedure: it is this moment which marks the limits of its essence. Being itself identifies the very idea of intuition. To have an intuition of a sensible or abstract object means possessing it just as it is, within its self-identity, which remains stable in the presence of specific variations at both a real and possible level. The realm of intuition, in this sense, is the realm of possible fulfilments. To have intuition of an object means having it just as it is ['the thing itself']: breaking down the limits of the constraints distinguishing its quiddity. The intuition, for example, of a complex mathematical object means possessing it as itself, according to an identity which remains unaltered through all real or possible variations. An object is a fixed point within a chain-operation, and only through this chain can its meaning reveal itself.

It should be born in mind, however, that a categorial form can only be filled by an act of intuition which is itself categorized, since intuition is not an inert element. In this sense, complex propositions can also be fulfilled, and indeed every aspect in a complete proposition is fulfilled. It is precisely the proposition, in all its complexity, which expresses our act of perception. A correspondence thus exists between the operations of the categorial foundation and the founded intuitions. To each act of categorial intuition a purely significant act will correspond. Where there exists a categorial form which becomes the object of intuition, perceived on an object, the object is presented to our eyes according to a new "way": precisely the way related to the form: we see the table and chair, but we can also see in the background of this perception the connection existing between these two different things, which makes them part of a unique whole. The analysis of the real nature of categorial intuition thus leads Husserl, almost by the hand, to the question of holism. But it is of course this question – the sum of the problems posed by the relationship between thought and its object – which, as we know, constitutes one of Husserl's basic points of affinity with his mentor Brentano. Brentano's slant on questions of this kind pointed the way for Husserl in his own analysis, and proved a blueprint for the development of the different stages of his research. For Brentano consciousness is always consciousness of something, and inextricably linked to the intentional reference. At the eidetic level this means that every object in general is an object for a consciousness. It is thus necessary to describe the way in which we obtain knowledge of the object, and how the object becomes an object for us.

While, then, the question of essences seemed initially to be taking Husserl towards the development of a rigorously logical science, a *mathesis universalis*, the question of intentionality obliges him to analyze the meaning, for the subject, of the concepts used at the level of logical science. An eidetic knowledge had to be radically founded. Husserl thus proceeds along a path already partially mapped out, albeit in some cases only tentatively, by Brentano, gradually tracing an in-depth analysis of the concept of completeness before arriving at a new, more complex concept, that of organicity. Kant's category of totality, Brentano's unity of perception and judgement, and experimental research in the field of *Gestalt* theory thus come together, at least in part, in a synthesis which is new. But further analysis of the concept of organicity then suggests other areas of thematic investigation, in particular at the level of *Experience and Judgement*, principally regarding the concepts of *substratum* and dependence. At the end of his research trajectory, then, Husserl returns to the old Brentanian themes concerning the nature of the judgement, offering new keys of interpretation for the existential propositions of which Brentano had so clearly perceived the first essential theoretical contours.

It is this area of Husserl's thought which interested Quine and Putnam in the 1970s and 1980s, and Tieszen, Carsetti and Petitot in the 1990s. New sciences and conceptual relations enter the arena: e.g. the relationship between Logic and Topology, and, simultaneously, between perceptual forms and topological forms, etc. Here we may distinguish those particular main forces which shape the continuing relevance and originality of that secret line of thought linking together Brentano and the two main streams represented by Husserl's logical analyses, on the one hand, and the experimental research of the *Gestalt* theoreticians, on the other.

Quine's and Putnam's revisitation of the Brentano-Husserl analysis of the relation between perception and judgement was of considerable importance in the development of contemporary philosophy. It was no isolated revisitation, however, Husserl's conception of perception-thought relations constituting a source of inspiration for many other thought-syntheses. Recent years, in particular, have witnessed another rediscovery of the phenomenological tradition of equal importance: that linked to the philosophical and "metaphysical" meditations of the great contemporary logician K. Gödel. Its importance at the present moment is perhaps even more emblematic, in comparison, for instance, with Quine's and Putnam's rediscoveries, with respect to today's revisitation of Husserlian conception. For many aspects, Gödel's rereading constitutes a particularly suitable key to pick the lock, as it were, of the innermost rooms containing Husserl's conception of the relations between perception and thought.

As we have seen in Chap. 4, the departure point of Gödel's analysis is Husserl's distinction between sensory intuition and categorial intuition. Gödel, however, speaks in terms not of categorial intuition but of rational perception, and it is precisely this type of perception which allows for a contact with concepts, and through which we reach mathematical awareness. In his opinion, the conceptual content of mathematical propositions has an objective character. The concepts constitute an objective reality which we can only perceive and describe, but not create. In this sense, this form of rational perception is in some ways comparable with sense-perception. In both cases, according to Gödel, we come up against very precise limits, possible illusions, and a precise form of inexhaustibility.

A very clear example of this inexhaustibility is provided by the unlimited series of new arithmetic axioms that one could add to the given axioms on the basis of the incompleteness theorem: axioms which, in Gödel's opinion, are extremely self-explanatory in that they elucidate only the general content of the concept of set. Gödel's comment on this in 1964 is as follows: 'We possess something like a perception of the objects of set theory'. This perception is a sort of mathematical intuition: a rational perception. But how can the intuition of essence be reached? How is it possible to extend awareness of abstract concepts? Or to understand the relations interconnecting these concepts: i.e. the axioms which represent them?

None of this, in Gödel's opinion, can be done by introducing explicit definitions for concepts or specific demonstrations for axioms, which would necessarily require further abstract concepts and the axioms characterizing them. The correct procedure is, conversely, to clarify the meaning, and this act of clarifying and distinguishing is, for Gödel, the central nucleus of the phenomenological method as delineated by Husserl. The theorems of incompleteness would seem, in effect, to suggest the existence of an intuition of mathematical essences (of a capacity in us to grasp abstract concepts), for which no reductionist explanation is possible. This kind of intuition is required above all for specific mathematical problems, for obtaining proofs of coherence for formal systems, etc. The theorems thus demonstrate clearly that that particular essence constituting "mathematical truth" is something more than a purely syntactical or mechanical concept of provability, while guaranteeing full mathematical rigour.

A rigorous science, in other words, as Husserl maintained, is more than a purely formal science. It also requires a transcendent aspect, and it is at this level that new mathematical axioms gradually come to light, arising not only from formal and deductive procedures. The unlimited series of new arithmetical axioms which present themselves in the form of Gödel's sentences, and which can be added to the already-existing axioms on the basis of the theorems of incompleteness, constitutes, in particular, a classic example of this process of successive revelation-constitution. These new axioms clearly represent precise evidence which is not extrapolable from preceding axioms *via* mere formal deduction. They can thus be used in order to solve previously-undecidable problems. According to Gödel, this is a clearly-defined way of explaining our intuition of an essence. An even more interesting example is provided by the Paris-Harrington theorem, a genuinely-mathematical statement referring only to natural numbers which, however, remains undecidable at the PA level. Its proof requires the use of infinite sets of natural numbers, the theorem providing a sound example of Gödel's concept of the need to ascend to increasingly more elevated levels of complexity to solve lower-level problems. In a number of works between 1951 and 1956, Gödel returns to one of his favorite examples: the unlimited series of axioms of infinity in Set theory. These are not immediately evident, only becoming so in the course of the development of the mathematical construction. To understand the first transfinite axioms it is first necessary to develop the set theory to a very specific level, after which it is possible to proceed to a higher stage of awareness in which it will be possible to "see" the following axiom, and so on.

In Gödel's opinion, this is a very impressive example of the procedure of meaning clarification (as well as of the process of rational perception) Husserl had posited. It is precisely by utilizing our intuition of essence as related to the concept of a "set" that set-theoretic problems in general can be solved. It is also necessary, he goes on, for us constantly to recognize new axioms logically independent of those previously established in order to solve all mathematical-level problems, even within a very limited domain. One case in point is the possible solution of the Continuum problem.

Here Gödel states explicitly that the theorems of incompleteness demonstrate how mental procedures can prove to be substantially more powerful than mechanical same, since the procedures they use are finite but not mechanical, and able to utilize the meaning of terms. This is exactly what happens in the case of the intuition of mathematical essences. In offering us the possibility of understanding the nature of this process of categorial intuition, Phenomenology allows us to avoid both the dangers of Idealism, with its risk of an inevitable drift towards a new metaphysics, and Neopositivism's instant rejection of all possible forms of metaphysics.

While the theorems of incompleteness are not derivable from the doctrine of Phenomenology, they offer a better focus on the irreducible nature of mathematical essences, not least, for instance, through the clarification offered by the concept of 'mechanically-computable function' as analyzed by Turing. Gödel thus finds hidden truths within an epistemological perspective that many may have considered outdated and obsolete. His conceptual instruments, however, belong to an analytical tradition which is not that of Phenomenology. According to the great Austrian logician, Phenomenology must, basically, be considered as a method of research, it consists of a manifold of procedures of meaning clarification and these procedures appear indissolubly connected to specific patterns of selective activities growing up in a continuous way. From an effective point of view, Gödel and Turing actually open the way to the development of non-standard mathematics, to the possibility of building, from within, specific conceptual frameworks that prove immersed in meaning but according to an evolutionary perspective. Hence the need to abandon Neoplatonism and embrace the methodological tools of Phenomenology, but also the need to explore the new territories associated with the processes of emergence of meaning not limiting ourselves to a mere clarification of it.

5.3 Knowledge and Reality: The Role of Conceptual Constructions

With respect to this frame of reference, Reality presents itself as a set of becoming processes characterized by the presence-irradiation of a specific body of meaning and by a dynamic (iterative) *compositio* of generative fluxes having an original character. These processes then gradually articulate through and in a (partially-consistent) unifying development warp with internal fluctuations and integrations of generative patterns. It is exactly this functional, self-organizing and teleonomic warp, in the conditions of "fragmentation" in which it appears and is reflected at the interface level through the unfolding of a specific canalization process, that the "ideal" mind progressively manages to read, reconstruct and "enumerate" with regard to its specific functional aspects and living constraints, ultimately synthesizing and reflecting it into an operating architecture of causal and interrelated (surface) maps. It is the successful enumeration that individuates the mind in action. In this way, it is then possible to identify (and inscribe) a whole, complex *score* which will function as the basis for the reading-reconstruction of the aforementioned functional warp. However, to read-identify-represent the score will necessarily require the contemporary discovery-hearing of the underlying harmony. Only the individual capable of representing and tuning the warp as living harmony, and the score as silent object, will actually be able to depict him/herself as "I" and as subject. This individual will then not only be able to observe objects, but will itself be able to see the observing eye, modeling those objects. The I able to portray itself as such will be able to rediscover the roots of the very act of seeing, positing itself as awareness and as the actual instrument allowing both for the progressive surfacing by forms of the original Source, and the semantic "grafting" as it articulates at the deep level.

It is thus through the progressive metamorphosis and the embodiment of the coder (in accordance with the emergence each time of specific "ideas") that new Nature can begin to speak, and Reality can channel itself (*in primis* as regards the "external" selection), according to its deep dimension, ultimately surfacing and expressing as an activity of synthetic multiplication. It is the face-texture of the effected construction which provides the guidelines for the I's edification; and indeed the original Source which gradually surfaces reflects itself in the constructed work, thereby allowing the effective emergence of an "observer" who finally reveals itself as a cognitive agent able to inspect the Nature around him in accordance with the truth, i.e., we are actually faced with the very multiplication of the cognitive units. The system is thus able to see according to the truth insofar as it constitutes itself as an I and as consciousness, i.e., to the extent that it can "see" (and portray-represent) its own eye which observes things.

This can be better understood if we start from a number of simple considerations. The visual process, as stated above, occurs within a coupled system equipped with self-reflection, in which a precise distinction obtains between vision and thought, although they maintain a constant and indissoluble functional exchange. A coupled system of the kind subsumes the articulation of a series of specific processes: a process of mirroring, a process of reflection, a process of intentional observation, etc. It also subsumes the successive outlining of functions which self-regulate, as well as the progressive construction of increasing forms of real autonomy. That function which self-organizes with its meaning, and which posits itself as emergent, is "experienced" as vision insofar as it manages to establish itself, at the network level, as a tool for the specific edification and integration of biological circuits capable of realizing in themselves a partial but adequate engraving-presentation (on the surface level) of the original *Sinn* (of the deep processes of "scanning" and constrained unification articulating at the level of the system-body of meaning).

Whoever posits himself as a tool-modality for an adequate reading process on behalf of Goddess-Muse (whoever adds, namely, himself to Nature along the inscription of a score) has no choice but to listen to the melody that comes from the echoing of meaning within the Temple of life, the harmony, that is to say, linked to the possession on the part of the truth of his opening eyes. It is the self-organization of himself on behalf of the hero as a biological network (along the embodiment process) which allows the effective realization of this possession as well as the opening up to the petrifaction and the self-recognition in an image, which is the reflection of himself in the face of the Goddess (only the hero that is reflected in his possessed being can, indeed, come to listen to the harmony). Here is the I of a mind in action. It is to the extent that meaning, *via* Arianna, comes to guide and support the observation activity through the embodiment (in accordance with the truth) that original creativity can, in turn, surface to itself thus avoiding the dissipation up to determine the birth of a specific replication process: hence the possibility to achieve the life of a body as real invariance. I can posit, in this sense, myself as an I only by listening to the Other. Indeed, I can play the score which concerns me only if I am able to listen to the melody of the Other which comes to inhabit me and marks the possession of my eyes on behalf of the truth: here we can find the primary root of self-conscious I.

What emerge are circuits of the flesh which, through integration and assimilation, are ultimately reflected in the half-closed eyes of Ariadne, albeit only once the precise meaning of the effected infixions has been understood and displayed. It is by a similar means that the body of the Minotaur is so smooth at the end of the metamorphosis, opening his eyes to the role of the Goddess and achieving invariance by immersing himself within her. Having discovered and manifested the trajectory of the constraints (and sutures) within himself, he is able to offer himself as support in the action of "conception". The Minotaur proffers himself as Ariadne's means of self-reading, thus of presiding over the ongoing embodiment: the means, that is to say, of unwinding the thread within the file. The Goddess-Muse who comes to be reflected in his eyes (the Minotaur finally becoming reflected in her face, as Picasso's famous painting has it) represents in herself the achievement of meaning as meaning dislocated in ambient incompressibility, an achievement which seals the accomplishment of the metamorphosis.

It is Ariadne who carries the light with her, in her hand, and who, as truth in action and Goddess-Muse of meaning, guides and illuminates the steps of the Minotaur, of the original ideal emotion which explodes for thousand cascades and which wishes to possess her. To sharpen creativity and avoid its consumption by the fire of dissipation, the thread must be unraveled and Daphne-Ariadne must arborize as a labyrinth (although open to exploration). Some inner light must oversee the nesting process and thereby avoid the fall into crystallization: the light which is offered the Minotaur as a guide through the labyrinth and as the secret thread relative to the plot of the infixions. It is exactly in the light of this thread that a file can then witness the opening of the Minotaur's eyes. The shepherd and the Nymph: the Nymph becomes Nature and the shepherd an observer, but only insofar as meaning self-organizes according to a precise nesting process. In this way, rather than fragmenting himself in the fire of dissipation, Apollo is obliged to anchor himself to the stone of observation. Apollo has no choice but to become embodied if he is to avoid falling victim to dissolution, but this means entering the realms of invariance by establishing an indissoluble bond with the observing eye. An incarnation for death in anticipation of salvation and the saving of life.

The thread has been unraveled and gathered into its secret luminosity, and shadow can now reveal itself in all its complexity along the wounds of light: the wounds which have turned the originally wild and radiant creature into man and martyr, through the path indicated by the cuts inflicted along the route to cognition: Narcissus-Minotaur (creativity in action) illuminated (and possessed) by new meaning even as it dies (Endymion). On the other hand, and conversely, the original light, the file, will only be able to reveal itself through the spurts which spring up along the path of the shadow: Marsyas, the incisions and the spurting blood. These incisions, together with the spurts, cause the irruption of a new file: hence the outlining of the light of an outpouring shadow, and Marsyas illuminated by new life (albeit in the Other) even within his death – the life of the poet and of chimeras. What this represents is the shadow of light in action: shadow becoming life, and light, in turn, becoming cognition: the infixions lead to the stone as the incisions lead to the gridiron. While Marsyas burns in the fire of the meadows, Narcissus drowns in the waters of meaning. He bears himself like the modality of the unraveling of the thread, like Ariadne's reading of herself, the very elucidation of herself in accordance with the truth proper to the effected observation.

The eyes (and mind) of the Minotaur gradually open in agreement with a phantasmagoria of images and forms (written in the language of mathematics) as the eyes of Ariadne, once again in the house of meaning, narrow in the secret light shining from within themselves. The conceptual instruments of the craftsman are rarified in their turn as the eye of Clio's intellect burns brightly, the Muse having returned to the house of creativity. While The Minotaur succeeds in observing, Marsyas succeeds in thinking (through the programmes and models of the brain); his son is taken to himself by the God as Work in action and Artefact-programme (as a successful simulation machine) supporting inspiration. It is the burning gridiron and the new, self-inscribing cipher which feed the God's new thought and which supports the rediscovered triumph of his light: here we can find the file in action within the thread as well as the origins of the new irruption. Disembodiment takes place when Clio returns to the house of creativity, and the God becomes pure abstraction, pure artifice. At the end of the metamorphosis, the craftsman's brain closes in a chain-link of programmes and a soul may burn and issue forth in the fire of (intellectual) abstraction and of the Work-programme: we are dealing with a simulation which opens to new productivity, the productivity of meaning, as well as to a semantics which becomes generative.

Realizing the labyrinth and revealing its roots are the necessary steps if the Muse is to illuminate the original information with truth, allowing it to constitute itself as semantic information (the Minotaur and St. Sebastian) and as prelude for the new conception. It is in the moment the Muse reads (and plays) herself as a score, in connection with the full emergence of creativity as living melody linked to specific motifs and invariants, that a new conception can take place and the Painter can use his brush to depict the inborn and new harmony. The journey into abstraction begins here, now that it is no longer Ariadne who is present with her thread, presiding over the unraveling of the incarnation, but Clio with her file presiding over the volutes of abstraction. In the case of Ariadne we can inspect a Muse playing herself as a score while Nature unravels herself through the successive addition of different observers. The hero who adds himself as observer endorses the inscription and thus the self-reading of the Goddess-Muse; it is precisely in this sense that the opening of the labyrinth represents the means whereby Ariadne manages to read herself and unravel her own thread. If this reading reveals as successful the labyrinth will finally acquire a human nature. Lastly, the Muse will be fulfilled and will reflect the hero: eternity and invariance joined together but with respect to the ongoing replication.

The hero (Narcissus) realizes in itself as a stone (and as a column of the Temple of life) an engraving concerning a surface representation (by fixed points and symbolic retractions) of the original *Sinn* which affects him. In general, the hieroglyphics and the images at the level of the Temple concern, first of all, the writing of the names of the different heroes through the indications of the performing actions that underlie, at the semantic level, their existence. Narcissus can only see to the extent to which he can engrave his image (through the embodiment) and petrify. Moreover, insofar as he petrifies he is also obliged to drown in the waters of meaning. Only in this way, can he manage to see (and exist) as an I and as a subject. Actually, when I follow as a subject the score related to my own body, enclosing myself in the presentation of this very score, a harmony necessarily comes to inhabit the eyes of my mind (and I can, finally, identify myself through this harmony). In this way, Narcissus positing himself as a stone and as the primary factor of replication, identifies himself with the eye set forever in the stone in accordance with the eternal time of a Destiny (as shown by S. Dalì), the Destiny proper to the invariance of his being. *Esse est percipere* (in the first sense of the *Via*) and coincides with the unfolding of a Destiny. It is the realization, at the surface level, of the body of meaning (the very reading of its image in the mirror) that gives rise to the eternal replication of Narcissus as a flower according to the truth. In this way, the hero stands as an instrument for the operational nesting and the constrained irradiation (on the surface level) of meaning as an acting unification drive. Hence the unfolding of a Nature full of meaning and necessarily linked to the successive "addition" of different observing systems.

The Minotaur who, his metamorphosis completed, succeeds in reflecting himself as man and mortal is no longer a multiplicity of 1,000 emotions, but a mind able to reflect within itself the meaning which concerns it and in which his mind is in turn reflected. Thus Narcissus-Minotaur can offer himself as the source of the possible replication, as a nucleus rooted deeply within life, like a hero who gives life having inscribed in himself life's *Sinn* in accordance with the truth, a hero, in particular, able to offer himself, through engraving, as witness to the achieved inscription of this very *Sinn* within his flesh. The result is a consciousness (adjunction + testimony + observation) which is always consciousness of a body and its sensations, as well as of the ideas which reach embodiment through inscription. Narcissus (as achieved embodiment) deploys (and recognizes himself in) the truth belonging to Nature as invariance (i.e., the truth related to life as invariance). Marsyas, on the other hand, (as achieved abstraction), captures within himself the life of the Work as morphogenesis (i.e., the life related to meaning considered as the very root of morphogenesis). Narcissus observes his reflection in the mirror and sees the face of the Muse, recognizing himself within it and ideally engraving within himself the essentials features of this same face. He immerses himself in his own truth in consonance with the infixions perpetrated, thus succeeding in seeing objectively, although along the construction of himself (and in his offering himself as source and model of replication). What emerges is an incarnate body composed of proteins, which manages to live on the surface and appears as perfectly honed to the specific functionality of its own being (insofar as it operates as replication in action): the functionality of a flower by means of which an Annunciation may come to manifest itself. The inscription and the testimony-consciousness therefore represent the foundation of its being as a cognitive being, a being in life but in accordance with the truth.

In the case of Marsyas, on the other hand, incisions and cipher represent the foundations of his being as an agent of transformation, of his being for truth but according to a specific disembodiment. Government (through observation) and use (through manipulation): I see in that I observe, and inscribe the *Sinn* of the Other within me by offering myself as witness. It is through these means that I incarnate. At Marsyas' level, however, I think in that I palpate and manipulate (the brain palpates the world as a network in action. We are faced with Nikolais' glove in action as well as with a soul burning in the abstraction). On the one hand, truth is the necessary condition for seeing: becoming flesh implies precisely this – the emergence of creativity in truth and its offering itself as semantic creativity (at the level of categorial intuition). On the other hand, thinking is only possible in the new life: in meaning emerging into life, and offering itself as productive meaning (at the level of intuitive categorization). In other words, thinking is only possible within the Work, through the *Via* concerning the operant abstraction and Marsyas' burning in the air. In Marsyas' case it is the Other who becomes a witness within me: the consciousness of the Other which I hear/feel in my inner being and in my progression towards the necessary addition. I add myself to Nature, and am added to the Work to enable another consciousness to be born animated by the pipe which enchants. Consciousness immerses itself in Nature once, and once it exalts itself (and separates) within the Work. The testimony is once within the crystal and once within the smoke, in the volutes of a sound which curl within each other until the beat of a renewed harmony can be heard in the delineation of new morphogenesis.

In this sense, insofar as we execute and establish at the level of the operating mind the correct "enumeration" we, actually, sacrifice to meaning feeding thus the invariance of its very body-system. The resulting picture is of a world characterized by deep invariance and by a constant composition and restructuring of schemes. This composition works at the horizontal and vertical level with a functional and constant internal "thickening" of the involved processing streams in accordance with a precise *bricolage*. We are actually dealing with the building of the Temple of life and with the opening of the eye of Horus, the God capable of inspecting all the details of the correlated universe. The invariance concerning the Temple is not a given, it is to the extent of the ongoing (continuous) adjunction of the different observing systems as well as of the unification work progressively put in place. In this sense, it also appears as necessarily related to the irruption connected to the morphogenesis process.

In the self-organizing emergence of meaning, then, we find, simultaneously, a process of "scanning" and one of stabilization and reduction through fixed points. This process can then gradually recognize itself in the realized emergence as an act of vision concerning the emergence itself. It is therefore not surprising that, as soon as the emergence (and the unfolding by unification) of meaning occurs correctly, vision appears veridical. What this particularly presupposes as an essential component of the process is also the articulated presence of definite capacities of self-reflection and precise replication-mechanisms at the level of perceptual constructions. If, actually, it is obvious that no thought can exist which has not first filtered through the senses, it is equally clear that there can be no effective vision, at the level of the "hero" in action, unless specific elaboration has taken place able to "coagulate" the selective activity. The outline offered by the hero serves first of all to propose possible integration schemes able to support and prime the nesting proper to meaning selection. The hero must constitute itself as the real carrier and the witness of the "enumeration" process as mind in action. At the moment of the realization of the renewed embodiment, new vision by *gestalten* emerges, and the offered outline considered as an independent instrument, must be abandoned because superseded. In this sense, it is true that at the level of the eyes of mind we have visual cognition, and not intellectual reading. Function and meaning articulate together, but in accordance with the development of a process of *adequatio*, and not of autonomous (even if mediated) creation. I will be unable to think of vision during emergence, but will be able to use it, once realized, to determine the rising of further simulation models. Growth, modulation, and successive integration thus exist 'within and among' the channels together with specific differentiation processes. In order to see more and more I have to support a better canalization of the original flow and to feed a more coherent "circumscription" activity at the meaning level. Hence specific Forms will reveal themselves as natural forms through the progressive realization of my embodiment: in order to join meaning and canalize the

Sinn I have to "fix" the emerging flow into the genesis relative to the Form, I have to give life to specific prototypes and I have to recognize myself by identifying previously my role in and through them.

Amodal completion, for instance, emerges in this context as a privileged window opened on a microcosm which is largely articulated according to the fibres proper to the architecture of mind. Objects are identified through the qualities elaborated and calculated along and through the channels. I neither colonizes nor occupy, to use Freeman's words: I offer myself as a stone and I am engraved but in the presence of life. What remains on my petrified flesh, the operative selection, is the inscription by means of cancellations and negative engravings of the deep functional patterns according to which the real processes "pulsate". I become the witness of the truth along the full unfolding of my embodiment.

The non-standard shadow of meaning affects me, albeit in the ongoing light of creativity. Meaning is revealed in the shadow which animates and which beats on the stone (Poussin, The Shepherds of Arcadia) which, however, is not that of simple death, but of the very eternity of meaning (albeit linked in itself to a ritual death): an eternity connected each time to the accomplishment of the new path pursued by the coder-shepherd. This is the shadow which, like Daphne in action, vitalizes creativity and burns within it like truth: the fire which truth carries with it, wrapped in its hand (Bernini, The Truth) – hence the emergence of creativity, albeit in meaning. Meaning's possibility to affect is offered as natural support of conception and therefore of the road to resurrection, in accordance with the reprofiling of a mind-Nymph. Marsyas giving life with his blood feels the God's design as it is incised on his skin, thereby permitting the surfacing of the new coder. Before this, the Minotaur must pass through death, to allow truthful vision and real reflection. His death will later be recounted in relation to the shepherd's path, which begins with the escape from the labyrinth. I exist as a being and as consciousness-vision-testimony when I feel meaning kindling within me like a shadow which burns in truth even as I add myself as an observer. Apollo burns with love and is reflected in Daphne. He cannot, however, possess her without losing his unity, unless in arborization: hence the God who becomes observer: it is now the truth which burns within him while his eyes open as he becomes Nature and mortal, until entirely filled with Daphne's image. He will have no choice but to reflect himself in her as achieved arborization (with half-closed eyes), like the Goddess who has returned to the house of meaning (Goddess in herself yet for him). Now the flame burning within the observer will quicken into life by the route of cyclical replication (which thus reveals itself as the true "presentation" of the new invariance). What appears is Ariadne in action, the Goddess who illuminates and makes self-reading possible: to the extent to which she reads herself, she introduces embodiment and allows the externalizing of light as a factor in observation (and no longer of dissipation): hence the possibility for Apollo to be inhabited by the shadow of meaning and its *Sinn*, the means of possessing Daphne constituting itself as semantic information in action. Apollo, however, is able to recover his inner creativity since Daphne nests within herself, successfully recovering her light there. It is on account of Daphne-Ariadne's reading-generating that Apollo can become an observer, just as the existence of the labyrinth-plot imprisoning the hero allows his eyes to be opened down to the moment he can be reflected in the countenance of Ariadne and recognize himself in her. This is the conquest of meaning (the possession of the hero's eyes on behalf of the truth) which identifies and confers unity and autonomy (in meaning) to the hero. From this proceeds self-awareness, although with Ariadne's help, by means of the hero's identification in the features proper to the Goddess. In the case of Marsyas, on the other hand, it is the synthesis in action operated productively which individuates the genesis of awareness leading from stammering to speech. In a nutshell: the Minotaur is, indeed, the means whereby Ariadne reads herself and thinks through forms, albeit with respect to the (half-closed) eyes of the mind. The shadow burns deep down as truth which gradually possesses the eyes of the observer adding him/herself as a column. Marsyas, on the contrary, provides the means for Clio to conceive of herself, and to see through concepts by activating the eye of the intellect.

Narcissus exists to the extent that he sees, but he may see because is able to inscribe himself through the realization of a specific engraving. This engraving involves the design and the construction of a score as well as the individuation of the corresponding code. It's just as he comes to see in such a way that he may give rise to his replication. The hero just plunging into meaning and drowning in it can find his identity and ensure his replica as a flower. In this sense, it is only the sacrifice of Narcissus that may allow meaning to act as ruler in action, as a harbinger of the new conception. The invariance cannot affirm itself except in relation to the establishment of a holistic drive and the triumph of the unity of mind. The construction is interwoven with cognitive acts and lies at the foundations of life. But the transition to the stone presupposes the ability on behalf of the source and the coder (working from a logical point of view, in the frame of the second-order level) to operate a complete reduction to the first-order level thus arriving to speak the language of the heterogeneous algebras. It is in this way that we can define the theoretical contours of a reflexive model able to permit a more coherent integration and articulation of the channels, laying the foundation for the self-organized synthesis of ever-new neural circuits. Objects, in their quality of being immersed in the real world, then emerge as related to other objects possessing different features, and so on. Through and beyond these interrelations, holistic properties and dimensions then gradually reveal themselves, which I must grasp in order to see the objects with their meaning, if I am to understand the meaning of things. Apples exist not in isolation, but as objects on a table, on a tree: as we have seen in Chap. 4, they are, for instance, in Quine's words, 'immersed in red', a reality I can only grasp by means of a complicated second-order process of analysis and comparison which must thereafter be reduced, through concatenations of horizontal and vertical constraints as well as the determination of specific rules and precise fixed points, to the first-order level. I thus need constant integration of channels and formal instruments to grasp information of the kind, i.e. to assimilate structural and holistic relations and relative ties in an adequate way.

In other words, I will understand the meaning of things only if I am able to give the correct coagulum recipes with a view to being selected so as to grasp and capture not only the superficial aspects of objects in the world, but their mutual relations as they interact in depth, in obedience, for instance, to a specific intensional dimension. Only if I provide the correct coagulum, and select the right languages, will these relations emerge through the trigger operated by the "selective" procedures proper to semantic information. Information about the outside world and the "genealogical" apparatus feeding it is thus extended: hence the need for a guide to the first articulation and growth of the mechanisms of vision, the need for an intended unfolding of a particular thread: the thread of operational meaning, Ariadne's thread, primarily. The rhythm-scanning of the original labyrinth operated by Ariadne together with the operations concerning the "replica" in action allow the eyes of mind to open: herein we can recognize the progressive opening of the eyes of the Minotaur led by the hand through the process of metamorphosis. Ariadne is a lesson in how to "think by forms": how to order and unify (according to a semantic unification and a representation process) generative thoughts and functional patterns in order to see, while the Minotaur represents the far-flung multiplicity of channels, the pure creativity in action that progressively canalizes itself as (and through) a model (on the basis of the full unfolding of *telos*' activities), thus allowing for the emergence of a specific (and concurrent) mental activity. Selected and guided by Ariadne, and beginning mentally to perceive her (at the level of the inner "replica" effected on the basis of the blueprint of the original scanning), he becomes aware of a new process of self-organization articulating within his channels. Besides thinking (by forms) of Ariadne, he will also be able to see an external world to the extent to which he himself has been selected by means of intended "circumscriptions" and filled by intuitions (and he will also be able to see himself as a part of the world-Nature: i.e., as an observer). The self-organization of the channels coincides with the successive stages in his metamorphosis, with his own gradual cognitive development and with his very achievement of a form of effective, intellectual autonomy. It is exactly within the secret paths of this process of metamorphosis that we can ascertain the objective articulation, at a first (and deep) level, of the specific procedures proper to rational perception as well as the actual expression of the autonomy proper to the *telos*.

This last "fresco" constitutes, however, only a tentative outlining of the first sense of the *Via*. As we have seen, we are really dealing with two different and coupled procedures: the real apprehended not only through categorial intuition, through a filling-in process utilizing the intuitions and the construction of the "garland", but also through intuitive categorization, the engraving of the file, the emergence of meaning-laden concepts within the meandering of intuition. Here we can inspect the appearance of a multiple thought-process which emerges from within the space originally belonging to pure virtuality: we are finally faced with a new nucleus of creativity which can emerge only insofar as other nuclei are shedding their old selves as a consequence of their positing as true "inventors". It is in this causing to emerge as well as in the ongoing metamorphosis that the primary nuclei of creativity can find (and recover) the secret cipher of their realization (which thus reveals itself as indissolubly linked to the procedures of intuitive categorization). I see a *Natura naturata* populated by observers and make myself into an observer. I think of a *Forma formata* full of works and posit myself as an inventor and a craftsman, as a simulation process in action. In other words, I think insofar as I make myself into a means for the emergence of thought systems, in that I come to see *via* principles through these systems, to the extent to which they manage to reach true autonomy. I think insofar as I make myself into a means for the autonomous articulation of the Other's thought and its transmission, in that creativity in action comes to "add" myself to the true life just as, through me, another creativity-thought manages to spring. The thought of the Other which manages to reach autonomy thus transits through my work, my dying in the flesh, my very offering to the Other of my incised skin, hence the progressive outlining of specific modules of pure abstraction: this necessarily constitutes a process of "disincarnation" (i.e., my exiting from myself, from my own self and my emergence as disincarnate 'soul' (Guido Reni, the disembodied intellect), as an individual performing itself as pure abstraction).

Forms shape themselves through concepts. They can do this on the basis of the file-engraving in (and through) spatial and temporal determinations (linked to the rhythm-scanning operated by the original form production), i.e. through linkage by "ring-threading" *via* schemata. The schema is the reduction-medium which realizes the correct incisions at the surface (and neural) level and allows the forms to operate conceptually; it permits form-unification on productive bases and by means of conceptual constructions. In actual fact, this is not a case of simple synthesis (to know is not merely to order), but of a coupled self-organization process realized on the basis of a specific project and of a nesting process in action: hence the need of a continuous agreement between the two different selective forces and a constant, harmonious merging of the processes of rational perception and those of intuitive categorization.

It is a similar process of coupled self-organization which allows the inspection and superseding of limits together with the potential invention of new procedures. It is this, then, that allows cognitive activity, and not simple regimentation, to arise. If no link is operative between the two selections (through the method-*Via*), between the universe of pure virtuality and the world of the productive intellect, then there can be no linguistic growth or new cognition. Concepts and attractors therefore come to live and articulate in a dynamic and co-evolutionary context of forms (characterized by their autonomous becoming) which are articulated in accordance with specific file-engraving procedures. At the same time, specific choices and fusions emerge from the form-production process on the basis of precise incision procedures and of the semantic nesting process in action. These choices manage to dictate from my interiority and it is precisely through these choices that the Other can reveal itself, while an "I" manages to reach its achieved form (turning thus into something other than itself): i.e., while a "vision" by principles is progressively identifying itself as "I" and brain in action, as a *Cogito* characterized by an objective (neural) existence. It is this "I" then, which, at the end of the whole trajectory, will, through "his" body ("my" body) and through the gift of his incised skin, be able to posit itself as the real basis of the successive emergence of new generativity. The unfolding of life, as well as of cognitive activity, is thus linked to a continuous process of "opening-up" of depth information as scanned by the rhythm proper to the ever-new (and strange) song of the God.

Thus, what allows the emergence of new concepts *via* symmetry choices and real fusions appears linked to the dialectics between incisions by conceptual schemata, on the one hand, and semantic nesting (with the necessary limit superseding), on the other hand. The ensuing conceptual articulation allows the ongoing form-production process to reflect itself and "think" of itself as an operant synthesis on the basis of the application of specific non-standard procedures which then can gradually manage to define themselves in accordance with a specific reduction process: the ruler finally manages to perceive itself through principles (once the engraving will reach its completion), and "thinks" of itself through simulation models and conceptual schemes. Actually, productive meaning can posit itself as the basis of the inner growth of these schemes, of the very shaping of their autonomy only to the extent that the concepts manage to engrave themselves at the level of form production (thus ensuring that the intuitions can reveal themselves to themselves in the resulting backlit fabric: hence the very possibility of measuring the forms in action). It is in this way that the *telos* can reveal itself in the expression of a surface Artwork acting as a needle that sews (Vermeer, The Lacemaker), an Artwork (a non-standard portrait) that becomes arch, frame, and blend of colours, a blend that can then take on a name (as well as an objective connotation) and articulate itself insofar as it reflects itself as thought in action, as a cipher able to assure the emergence of new possible forms of vision and life.

We no longer have a name that engraves itself (with its Echo) as a hieroglyph, but a conceptual configuration that hovers like light and harmony, which does not simply inscribe itself in the truth but appears able to release a new surge of emotion. We have concepts that affect neural architectures in action allowing an hidden soul to unfold itself but in the Other, in the fire of the new abstraction, A song rises up giving the right directions for the recovery of the reasons of life, but in accordance with a renewed emergence of meaning.

As we have said in Chap. 4, the original "glove" that imposes shape on itself acts as support for the inscription as well as for the outlining of the limitation procedures through the nesting process: linkage operated by the *telos* allows the abstract drawing-frame to emerge (dressed only in a tunic: Poussin, Self-portrait), as linked to that construction of a creative Artwork through which other nuclei will manage to identify and recognize themselves. What is presented, then, is a vision by principles, a process of ongoing abstraction. As we have seen, the file which narrates of itself provides the support for the realization of a specific nesting process allowing a progressive semantic unification at the level of the activity of form-production. Hence a vision in action (at the level of the ruler) which can reflect itself as thought, and can ultimately see by principles according to well-defined unification and abstraction procedures. A new nucleus of individual creativity emerges through which new postulates and axiomatic systems manage to find concrete self-expression; a production of meaningful forms is effected which disincarnates itself in pure abstraction. Ring-threading by *schemata* constitutes the specific modality whereby the file can be engraved in the living forms operating at the level of neural simulation. It allows vision to operate by principles, and to unify semantic production modules on generative lines. This determines a specific embedding which progressively realizes itself on the basis of the Clio guide, thus determining the emergence of new creativity nuclei as a result of the effected abstraction. The concepts are the ongoing, operant modules of unification at the level of form production, and constitute the means whereby the "glove" can shape itself once the neural engraving (Vailati) has reached its completion: hence the necessity of a coupled process of self-organization constantly in action. The concepts define themselves through the glove just as the intuitions define themselves through the file: fixed points of disimprisonment and imprisonment. Body's intelligence articulates and self-enlightens through concepts, just as rational perception articulates through intuitions. The unity of thought is multiplied and the multiplicity of pure virtuality is unified. Here we are faced with a mind observing outside itself the multiplicity of living processes in action within the unity of an unique space and a brain which, on the contrary, senses within itself an acting unification drive within the frame of the multiple distinction of different temporal successions. The *telos* acts as link among concepts (the final product of the engraving of the file through incisions) and intuitions (the final product of the imprisonment of the thread of meaning in the categorial apparatus). Where linkage exists there also exists the synthesis of the intuitions through rational tools (namely, unification): life is able to recognize itself as *Natura naturata* at a cognitive level. There exists, on the other hand, a multiplication of concepts by intuitions: meaning is thus able to "think of itself" as *Forma formata*. The linkage is given by the awareness in action, by the individuation each time of the correct point of encounter between vision and thought: depth which exhibits its inner extension at the surface level, and surface which opens to multiple levels of profundity. Here we can find some first hints in order to individuate the ultimate roots of the articulated (and targeted) process of knowledge construction.

Notes

1. See for more information: Carsetti A. (2004), "The embodied meaning" in *Seeing, Thinking and Knowing*, (A. Carsetti ed.), Dordrecht, Kluwer A. P, 307–331.
2. Cf. Varela F. (1975), "A calculus for self-reference" *International Journal of General Systems*, 2:5–24. Kauffman L.H. (2005), "Eigenform", *Kybernetes*, 34: 129–150.
3. Cf. Petitot J. (2008), *Neurogéometrie de la vision,* Paris, Les Editions de l'Ecole Polytechnique, 397.
4. Cf. Scott D. (1980), "Relating theories of the lambda calculus" in *To H.B. Curry: Essays on Combinatory Logic, Lambda Calculus and Formalism*, (P. Seldin & R. Hindley eds.), New York, Academic Press, 403–450.
5. Cf. Kripke S. (1975), "Outline of a theory of truth", *Journal of Philosophy*, 19: 690–716.
6. Cf. von Foerster H. (1981), "Objects: tokens for (eigen-) behaviors", in *Observing Systems, The Systems, Inquiry Series*, Salinas, Intersystems Publications, 274–285; Kauffman L.H. (2003), "Eigenforms – Objects as tokens for eigenbehaviours", *Cybernetics and Human Knowing,* 10(3–4): 73–89.

7. Cf. Hutchinson J. (1981), "Fractals and self-similarity", *Indiana University Journal of Mathematics*, 30: 713–747.
8. Cf. Carsetti A. (2010), "Eigenforms, natural self-organization and morphogenesis", *La Nuova Critica,* 55–56: 75–99; Carsetti A. (2011), "The emergence of meaning at the co-evolutionary level: An epistemological approach". doi:10.1016/j.amc.2011.08.039.
9. Cf. Reeb G. (1979), *L'analyse non-standard, vieille de soixante ans?*, Strasbourg, I.R.M.A.
10. Cf. in particular: Husserl E. (1964), *Erfahrung und Urteil*, London, Allen and Unwin.

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