1. Anna and Burt are moving at constant speeds along straight lines in the xy-plane. They both start moving at the same time. Anna starts from the origin and reaches the point $(4,3)$ in 1 second. Burt starts from the point $(1,2)$ and reaches the point $(5,8)$ in 2 seconds.

   (a) Give Anna's parametric equations of motion.

   $(x(t), y(t)) = (4t, 3t)$

   (b) Give Burt's parametric equations of motion.

   $(x(t), y(t)) = (2t+1, 3t+2)$

   (c) Determine the time when Anna and Burt will be closest together.

   The distance formula tells us that the distance squared between Anna and Burt is given by

   $$D(t) = (4t - (2t+1))^2 + (3t - (3t+2))^2 = 4t^2 - 4t + 5.$$

   This is a parabola which has a minimum at its vertex $\frac{-(-4)}{2\cdot 4} = \frac{1}{2}$.

2. Consider two circles of radius 1. Circle A is centered at the origin, and circle B is centered at the point $(1,0)$. Let P be the point of intersection in the positive quadrant. Find the equation for the tangent line to circle A at P and find the equation of the tangent line to circle B at P.

   Circle A has equation $x^2+y^2=1$ and circle B has equation $(x-1)^2+y^2=1$. Using algebra we can find that they intersect at the point $(\frac{1}{2}, \frac{\sqrt{3}}{2})$. We then know that the radial line of circle A to P has slope $\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$ and therefore the tangent line to circle A at P has slope $-\frac{1}{\sqrt{3}}$, and then has equation

   $$y - \frac{\sqrt{3}}{2} = -\frac{1}{\sqrt{3}}(x - \frac{1}{2}).$$

   Similarly the radial line for circle B through P has slope $\frac{\frac{\sqrt{3}}{2}-0}{\frac{1}{2}-1} = -\sqrt{3}$. It follows that the tangent line to circle B at P has equation

   $$y - \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}(x - \frac{1}{2}).$$

3. (a) A 12 cm rod is attached to a wheel of radius 4 cm and allowed to slide along the x-axis. The wheel turns counterclockwise at 3 revolutions per minute. The rod starts laying flat on the x axis. Write the x-coordinate $Q(t)$ at time t.

$Q(t) = 4\cos(6\pi t) + \sqrt{12^2 - (4\sin(6\pi t))^2}$.

(b) Find the values of x so that: $e^{\ln\left(\ln\left(\frac{e^x}{e^{x^2}}\right)+2\right)} = 0$

Using properties of exponents and logs we can write the above as

$$\begin{aligned}
2 &= e^{\ln\left(\ln\left(\frac{e^x}{e^{x^2}}\right)+2\right)} \\
&= \ln\left(\frac{e^x}{e^{x^2}}\right) + 2 \\
&= \ln(e^x) - \ln(e^{x^2}) + 2 \\
0 &= x - x^2 \\
&= x(1-x).
\end{aligned}$$

So $x = 0$ and $x = 1$ are the solutions.