Sparse and Low-Rank Decomposition

- Optimize over communication, computation and storage resources in mobile big data systems

\[ \mathcal{P}_1: \text{minimize } \mathcal{L}(X) + \lambda R(X) \]

subject to \( \text{rank}(X) = r. \)

- \( \mathcal{L} \) is a smooth convex loss function; \( R \) is a non-smooth (possibly non-convex) sparsity inducing function

We will focus on the family that the conventional convex relaxation approaches (e.g., nuclear-norm relaxation) are inapplicable!

Example I: Topological Interference Alignment

- Topological interference alignment problem in partially connected K-user interference network [1]

\[ \mathcal{P}_2: \text{minimize } \text{rank}(X) \]

subject to \( X_0 = 1, \forall i = 1, \ldots, K, \]

\( X_0 = 0, \forall i \neq j, (i, j) \in V. \)

- \( V \) is the index set of connected transceiver pairs

The well-known nuclear norm relaxation approach fails: always return identity matrix.

Key conclusion: \( \text{DoF} = 1 \)

Example II: Network Topology Control

- Network topology control problem in the partially connected K-user interference network [2]

\[ \mathcal{P}_3: \text{minimize } \|X\|_0 \]

subject to \( X_0 = 1, \forall i = 1, \ldots, K, \]

\( \text{rank}(X) = r. \)

- \( \|X\|_0 \): side information for network connectivity, storage and computation load

The widely used mixed \( \ell_1 \)-norm and nuclear norm convex penalty relaxation approach is inapplicable!

Example III: User Admission Control

- User admission control problem in the communication, caching and distributed computing networks [3]

\[ \mathcal{P}_4: \text{maximize } \|\text{diag}(X)\|_0 \]

subject to \( X_0 = 0, \forall i \neq j, (i, j) \in V, \]

\( \text{rank}(X) = r. \)

A simple \( \ell_1 \)-norm relaxation approach yields the objective unbounded and non-convex

Smoothing Regularized Sparsity Inducing Functions

- To find a good sparsity pattern, we propose a smoothed regularized version of the problems \( \mathcal{P}_1, \mathcal{P}_2 \) and \( \mathcal{P}_3 \) as a smooth fixed-rank optimization problem \( \mathcal{P}_5 \)

For problem \( \mathcal{P}_1 \), we have \( \mathcal{L}(X) = \sum_i (X_i - 1)^2 + \sum_{i,j \in V} X_{ij}^2 \) and \( \lambda = 0. \)

- Riemannian pursuit: alternatively perform fixed-rank optimization and rank updating

For problem \( \mathcal{P}_2 \), we have \( \mathcal{L}(X) = \sum_i (X_i - 1)^2, R(X) = \|X\|_1, \) and \( \lambda \geq 0. \)

For problem \( \mathcal{P}_3 \), we have \( \mathcal{L}(X) = \sum_{i,j \in V} X_{ij}, R(X) = \|\text{diag}(X)\|_2^2 - \|\text{diag}(X)\|_1, \) and \( \lambda, \rho \geq 0. \)

\( \|\text{diag}(X)\|_2^2 \) serves the purpose of bounding the overall objective function.

Algorithm: Matrix Manifold Optimization

- Solve fixed-rank problem \( \mathcal{P}_5 \) by Riemannian optimization

- Generalize Euclidean gradient (Hessian) to Riemannian gradient (Hessian)

\[ \nabla_A f(X^{(k)}) = P^{(k)}(\nabla f(X^{(k)})) \]

\[ X^{(k+1)} = R_{X^{(k)}}(-\alpha^{(k)} \nabla_A f(X^{(k)})) \]

Experiment Results

- Performance of algorithms on problems \( \mathcal{P}_2 \) and \( \mathcal{P}_3 \)

References