The Politico-Economic Dynamics of China's Growth

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Abstract

China's rapid growth has been driven by policy reforms and political changes that significantly reduce market frictions. Policy reforms are determined by the government with its own politico-economic considerations. This paper makes the first step to embed the politico-economic considerations in a macro model, to endogenously study government policies, market frictions, and growth. In the model, an elite runs the government and maximizes its own income, facing a political constraint: getting enough supporters. It provides a high enough income for state sector workers to buy their support. It also controls capital allocations in the state and the private sectors to balance between keeping enough supporters and extracting more tax income from the private sector. These policies generate an initial decline of labor and capital market frictions but keep the frictions persistent in the long-run, which are harmful to the growth. The calibrated model can quantitatively account for salient aspects of China's development since 1998. (JEL: E22 E24 O41 O43 P16)

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1 Introduction

China has been growing at a stellar rate for about 40 years. During this era, a sequence of policy reforms have been taken to reduce market frictions. The reduction of frictions has resulted in the rapid economic growth. For example, in late 1997, the 15th Congress of the Communist Party of China officially acknowledged the private sector as an important pillar of the economy, and this reform led to a rapid state to private transition. Song et al. (2011) show that the labor and capital reallocations to the more efficient private sector have contributed significantly to the rapid growth since 1998. It is generally acknowledged that policy reforms are crucial for the great transformation of China (see Chow (2015)) and they are determined by the government with its own politico-economic considerations (see Shirk (1993) for the political logic of economic reforms). However, as of today, there is no theory that embeds the politico-economic considerations into a macro model. This paper offers a first step in this direction by explicitly modeling the objective function of the government and its political constraint: the government is run by the elite who maximizes its own income, subject to the constraint that enough workers support it. Then the policy reforms and the dynamics of market frictions can be endogenously studied. Having a microfoundation for policies and frictions is useful for understanding the dynamics of frictions and growth in the past and necessary for predicting the future growth. Otherwise, the government policies and the dynamics of frictions have to be assumed exogenously, usually following the trend before. This assumption can be largely counterfactual. For example, the recent trends of labor and capital wedges and allocations significantly deviate from the trends before 2008. Hsieh and Song (2015) document that after 2007, the capital wedge between state and private firms stopped declining. Storesletten and Zilibotti (2014) show that the reallocation of labor to private firms has stopped since 2008. A model that can endogenously generate these changes in frictions is necessary for predicting the future development.

How does the government set policies to satisfy the political constraint - getting enough supporters? First, it provides state sector workers a high income to buy their support, using both wage and transfer. This policy creates labor market distortions. For example, Ge and Yang (2014) find that China's state sector workers have been enjoying a wage premium between 20% and 30%, and this labor market wedge has been persistent and evening increasing. State sector workers, not surprisingly, are found to be more supportive of the current regime and less supportive of democracy. Chen and Lu (2011) study survey data of Chinese individuals and show that the state sector workers and the middle-class are less supportive of democratic values, including multi-party competition, freedom of

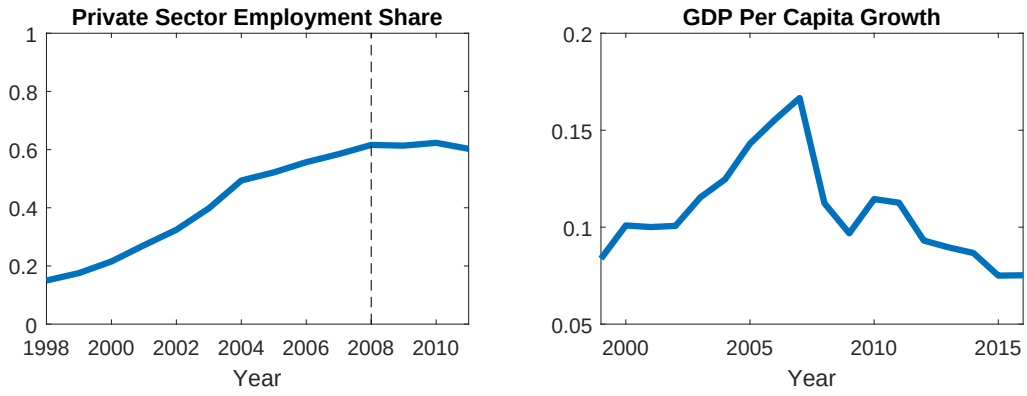

Figure 1: Private sector employment share in manufacturing and growth of GDP per capita PPP adjust. Source: Song et al. (2011) and The World Bank.

demonstrations, etc. Second, the government balances the capital allocations in state and private firms to maintain enough workers in the state sector. As shown in Storesletten and Zilibotti (2014) and reproduced here in Figure 1, China's private sector employment share in manufacturing stopped increasing when it reached about 60% in 2008, after ten years of rapid growth. In other words, the state sector employment has stopped declining and have stayed around 40% since 2008. How does the state maintain this significant fraction of workers, while state firms are less productive than private firms? It is through the capital allocation in favor of state firms. State firms get more financial resources, for example, bank loans, so they can keep investing and hiring a large fraction of workers. Brandt and Zhu (2010) document that the state sector's investment share stays around 60% though their employment share has been declining from 1998 to 2007.

In this paper, I embed the government's political constraint in a two-sector growth model and study how the politico-economic considerations shape the policy reforms and market frictions and how policies and frictions affect the growth. In the model, a political elite runs the government and can extract surplus from state firms and tax the private sector.[1] However, it faces a political constraint, that is, support from a sufficient number of workers. The government can use the following policy tools to maintain the support and maximize the elite's income: in the labor market, setting the wage and the transfer to state sector workers, and in the capital market, controlling capital allocations to state firms and private firms. First, the government sets the income of state sector workers sufficiently high - as high as the income level that they expect in the democracy - so that state sector workers prefer to support the current regime instead of democracy. To increase the income of state sector workers, the government sets the wage and the transfer, balancing the costs:

[1]The elite and the government can be used interchangeably in this context, except later when we discuss democracy.

labor distortion from regulating the wage and the direct cost of using the transfer. When the transfer from the government is high, the wage burden to state firms is low, so state firms tend to hire more workers than the efficient level. The redundant labor generates the labor wedge: a lower labor productivity in the state sector than in the private sector. Second, the government controls capital allocations in the state and the private sectors to keep enough workers in the state sector.[2] When the private sector is small and its capital level is low, maintaining enough political support is not costly, because the number of workers in the state sector is large and workers' expected wage in the democracy is not high. When private sector capital grows, a trade-off for the government emerges: a larger private sector contributes more tax, but it also requires a higher cost of maintaining the support. If the private sector capital is so high that the state employment share declines to the critical level for sustaining the regime, the government needs to invest enough in the state sector, to maintain enough workers there. For an even higher level of private sector capital, the government prefers to restrain the size of the private sector, for example, by reducing loans to private firms.

Because the government capital market policy changes when the private sector grows, the economy's growth pattern also changes accordingly. It develops along a three-stage transition. The first is *rapid growth*, during which the private sector grows fast, capital and labor are reallocated to the private sector, and the productivity gain from the reallocation contributes to the output growth. As privatization continues and the state employment share declines to the critical level, the economy enters the second stage, *state capitalism*. In this stage, the government over-invests in the state sector to keep the state employment share sufficiently large. This stage features the stop of privatization and the large state investment. Though the private sector relative size stops growing, its absolute size of capital can still grow, while the state sector capital grows proportionally. The economy still grows fast, partly due to the growth of capital. As the private sector capital keeps growing, the economy enters the third stage, and there are two possible outcomes. The first is the *middle-income trap*. The government chooses to maintain the existing political system and the necessary frictions. It creates barriers for the private sector capital growth, for example by tightening lendings to private firms. The private sector capital growth slows down and the economy stops growing before reaching the efficient income level. This happens if the cost of sustaining the regime is low, e.g., if costs of investment and transfer are low. The other possible outcome is *sustained growth*. As the private sector capital grows, it becomes too costly to continue investing in the less efficient state sector and therefore the government chooses to change the political system towards democracy.

[2] In addition to the capital allocation, the transfer is also used to help hiring enough state sector workers.

Frictions in the labor and the capital markets disappear, and the economy keeps growing until the income reaches a high level.

The model is calibrated to China's manufacturing sector and can account for salient aspects of China's growth experience since 1998, especially the following three: the state to private transition, the dynamics of labor and capital market wedges between state and private firms, and the output growth. The first stage - *rapid growth* - corresponds to Chinese economy between 1998 and 2007, during which the private sector employment share increases rapidly, labor and capital productivities of state firms relative to private firms increase, and output grows fast. See the growth of private sector employment share and the growth of GDP per capita in Figure 1. As documented in Storesletten and Zilibotti (2014), the private sector employment share in the manufacturing sector increases from 15% to 80%. The model in this paper can generate this fast labor reallocation from state to private firms, with the help of the fast increase of private sector capital, which benefits from capital reallocation from state firms and the capital accumulation of private firms. Second, the labor productivity and the capital productivity of state firms relative to private firms both increase. Hsieh and Song (2015) document an increase of labor productivity of state firms relative to private firms from 60% to 75%, and an increase of relative capital productivity from 34% to 46%. Using the TFP growth of state firms relative to private firms implied by Hsieh and Song (2015) as the input, the model can successfully account for this trend of labor and capital wedges. Third, the model generates a high output growth rate of above 10%, which is consistent with the high GDP per capita growth in China during this period.

The *state capitalism* stage starts when the employment share of the state sector reached the critical level in 2008. State employment share stays around 40% afterward. The model generates this stagnation of labor reallocation because the political constraint requires sufficiently many state sector workers. The model also generates an continuous increase of labor productivity of state firms relative to private firms, and a much slower increase of capital productivity of state firms, qualitatively consistent with the findings in Hsieh and Song (2015). The capital productivity grows slowly for two reasons: the over-investment in the state sector to maintain supporters and the decline of transfer relative to income. As the income increases, the cost of using transfer increases, and the relative size of transfer declines. This leads to an increase in wage, labor productivity and capital-labor ratio in the state sector, and consequently the slow growth of the capital productivity. Finally, the growth rate decreases but the large investment keeps it from dropping too much, consistent with the discussion in Zilibotti (2017). The output growth rate is still above 5% in 2017 and stays above 4% until 2027.

The model also provides predictions for future dynamics of wedges, economic growth, and potential development paths. The third stage in the calibrated economy is the *middle-income trap*. The model predicts that the labor and capital market frictions will persist in the future. Frictions harm the future economic growth, which slows down and stops before the income converges to its potential high level. The growth slows down because the government reduces lendings to private firms and the capital market frictions increase. The frictions persist because the government chooses to sustain the current regime instead of democratization. There is another possible development path: *sustained growth*. This path emerges if costs of investment and transfer are much larger than in the benchmark calibration. Then the elite may find sustaining the regime too costly and may choose to democratize. The frictions are reduced and the growth continues until the income converges to a high level.

This paper is related to three strands of literature. The first is the literature on China's economic growth given labor and capital market frictions. This paper contributes by providing the microfoundation for the capital and labor market frictions and studying their endogenous evolution. Brandt and Zhu (2010) use a three sector growth model with labor and capital wedges between state and private firms to account for China's growth. They identify private sector TFP growth as the main contributor. Song et al. (2011) construct a two-sector growth model to study China's growth with the transition from the state to the private sector, given the financial constraints on private firms. Cheremukhin et al. (2015) study China's structural changes and the evolution of wedges in the labor and the capital markets and their contribution to growth from 1953. This paper offers explanations for the dynamics of the frictions in the past, and suggests that these frictions will be persistent within the current political institutions, so they will be harmful to China's future growth. Second, this paper contributes to the study of the interaction between political development and economic development. One side of the interaction is that political institutions affect economic development. For example, Acemoglu (2008) studies the economic performance in oligarchic societies, in comparison with democratic societies. He shows that an oligarchic society may achieve higher efficiency at first because producers, being the elite, can protect their properties better and invest more than in a democracy. However, in the long-run, the elite blocks new entrepreneurs from entering, and the economic growth becomes slower. This paper's analysis on the growth in the oligarchy is in line with the spirit of Acemoglu (2008), but with an important difference in the implications on efficiency. The higher short-run growth in an oligarchy in this paper is not because of higher efficiency but precisely because of its inefficiency: labor and capital market distortions. In the beginning, the growth is fast because the initial output level

is low given the labor market distortions, and in the second stage, the economy grows fast because of over-investment in the state sector. The faster growth in the oligarchic does not necessarily imply the larger output. Even if the output is high, it does not imply a high income for the majority of the population: the workers' income in the oligarchy is always lower than in the democracy. The mechanism generating a slower long-run growth in the oligarchy is similar to Acemoglu (2008): in the later stage, the elite creates barriers for entrepreneurs. The other side of the relation, i.e., how economic development affects political development, is also studied in this paper. Economic progress may lead to political development, but the political progress only happens under the right conditions. This is related to but different from the modernization theory started by Lipset (1959). The third strand is the literature on the middle-income trap. There are many important factors determining of the middle-income trap, such as the income threshold, labor costs, population dynamics, etc. This paper contributes to the literature by studying one of them - the government policies. Why do some countries successfully adopt good policies to rapidly grow out of poverty but then suddenly fail to implement the right policies that can help them to coverage to rich countries? This paper offers a theoretical explanation: the interest of the elite aligns with growth in the beginning but not in the later stage. It also studies the right government policies that help the middle-income countries to grow further and the conditions that can induce the government implementing these right policies.

The rest of the paper is organized as follows. Section 2 presents the model. In Section 3, the model is calibrated to the manufacturing sector in China. The quantitative model accounts for the time series in China's recent development and offers predictions for the future development. Extensions of the model, for example the alternative development path, are also studied. Section 4 concludes. Proofs and more details of the model are in the Online Appendix.

2 The Model

This section presents a two-sector dynamic general equilibrium growth model. The two sectors are the state (S) sector and the private (P) sector. There are a continuum of state firms and private firms. They are standard neoclassical firms: they produce the same final goods using capital and labor, and they maximize profits taking prices as given. There are three groups of infinitely many agents: the elite (e), private entrepreneurs (p) and workers (w). The population size of workers is normalized to 1, while the sizes of the elite and the entrepreneurs are infinitesimal - denoted by a number ε. The elite supplies capital to state

firms, and entrepreneurs supplies capital to private firms. They receive incomes from capital returns, consume and save. Workers supply labor and receive wages. A worker may work in an S firm or a P firm. For a group of agents, we can use a representative agent to describe their behavior, e.g., "the S firm", "the P firm", "the elite", "the entrepreneur", etc.[3]

The political economy is the following. The economy starts in an oligarchic regime, where the elite controls the government but faces a political constraint: it needs sufficiently many workers to support the regime. If the number of supporters is large enough, the regime survives; otherwise it collapses, and democratization occurs. Each worker decides whether to support the regime, comparing her current income with her expected income in democracy. To maintain enough supporters, the government can strategically influence incomes of workers using the following policies: setting the state sector wage, giving transfers to state sector workers, and changing capital allocations in the two sectors.

2.1 Preferences, Technology and Markets

Agents live for infinite periods. The lifetime utility of the elite is the following:

$$U = \sum_{t=0}^{\infty} \beta^t \log(c_{et}).$$

The elite discounts the future at the rate β and the rate of relative risk aversion equals one. Same for the entrepreneur. The elite and the entrepreneur receive incomes from the capital that they supply to the state (S) sector and the private (P) sector, respectively. After the government redistribution, they consume and save the rest for the next period. The savings will affect their capital supply and capital income in the next period.[4] Each worker supplies one unit of labor to a firm. Workers live hand-to-mouth and are myopic: they consume all of the current period income and care only about the current period income.

The S firm and the P firm are different in two aspects: access to the capital market and productivities. First, the S firm rents capital in the S sector, which is supplied by the elite, and the P firm can only rent capital in the P sector, supplied by the entrepreneur. They can hire from the same pool of workers.[5] Second, the S firm is less productive than the P firm. The technologies of the S and the P firms are described by the following Cobb-Douglas

[3] We don't use "the worker" to describe workers because there are state and private workers, and they don't act collectively in their political decisions.

[4] Details of government redistribution and the capital market will be discussed in subsection 2.3 below.

[5] In the equilibrium, all workers prefer to work in the S sector because the income in the S sector is higher, and then a fraction of them are randomly drawn into the S firm given its labor demand.

production functions:

$$\begin{aligned} Y_{St} &= z_{St} K_{St}^{\alpha} L_{St}^{1-\alpha}, \\ Y_{Pt} &= z_{Pt} K_{Pt}^{\alpha} L_{Pt}^{1-\alpha}, \end{aligned}$$

where z_{jt}, K_{jt}, and L_{jt}, $j \in \{S, P\}$ stand for TFP, capital and labor in sector j at time t respectively, and $z_{St} < z_{Pt}$. Capital depreciates at the rate δ. Firms are neoclassical. Given their profit maximization problems, wages and capital returns are equal to the marginal productivities, as follows:

$$w_{St} = (1-\alpha) z_{St} K_{St}^{\alpha} L_{St}^{-\alpha}, \tag{1}$$

$$w_{Pt} = (1-\alpha) z_{Pt} K_{Pt}^{\alpha} L_{Pt}^{-\alpha}. \tag{2}$$

Similarly, the gross returns of capital (before depreciation and tax) are:

$$\begin{aligned} r_{St} &= \alpha z_{St} K_{St}^{\alpha-1} L_{St}^{-\alpha}, \\ r_{Pt} &= \alpha z_{Pt} K_{Pt}^{\alpha-1} L_{Pt}^{-\alpha}. \end{aligned} \tag{3}$$

In the financial market, there is a representative competitive bank. It has access to the international bond market where the interest rate r is exogenously given. The gross rate of return is denoted by $R = 1 + r$. So the bank serves as an intermediary which allows domestic agents to save and borrow at this interest rate. The elite can borrow from the bank, and invest the bank loan and her asset in the S firm. There is no constraint on how much the elite can borrow. The entrepreneur can also borrow from the bank, but she faces a credit constraint: the bank loan cannot exceed $\eta_t - 1$ fraction of her asset. In other words, the maximal ratio of capital over asset is η_t:

$$K_{Pt} \leq \eta_t a_{pt},$$

where a_{pt} and K_{Pt} represent the entrepreneur asset and the P firm capital, respectively. η_t may vary over time, depending on the government's policy, which will be described below in subsection 2.3.

2.2 The Political Constraint

The economy starts in an oligarchy. The representative elite runs the government and sets policies to maximize her utility. However, she faces the following political constraint: she

needs to get political support from sufficiently many workers. Otherwise, the revolution occurs, and the regime switches to democracy. Here we can take the incomes of the elite and workers in the democracy as given, and they are determined by the equilibrium in the democracy which will be studied in subsection 2.7. In each period, each S sector worker decides whether to support the oligarchic regime or not. If the number of supporters in the S sector is larger than a critical level $\underline{L}$, the regime survives in this period. In the equilibrium of the calibrated model, the income of P sector workers is lower than the income of the S sector workers, and they always prefer democracy, so it is without loss of generality to consider the large enough amount of supporters from the S sector as the sufficient and necessary condition for sustaining the oligarchy.[6] Let us denote a worker's expected income in the democracy as y_{wt}^D. We will see later that the worker's income in the democracy includes the wage and the tax from the elite and the entrepreneur. Now the political constraint is equivalent to the following two economic constraints: (1) a high enough S sector worker income:

$$y_{wSt} = w_{St} + T_t \geq y_{wt}^D, \tag{4}$$

where y_{wSt} stands for the S sector worker's income in period t, which includes the wage paid by the S firm w_{St} and the transfer from the government T_t; and (2) a large enough S sector employment share:

$$L_{St} \geq \underline{L}. \tag{5}$$

2.3 The Government

In the beginning of each period, the government controlled by the elite can choose to democratize voluntarily or to stay in the oligarchy. If the first is chosen, the regime switches to democracy forever. If the government chooses the latter, it sets the following policies: the S sector wage, the transfer to S sector workers, the capital in the S sector, and the credit constraint of the P firm.[7] Moreover, the government taxes P sector workers and the entrepreneur at an exogenous rate $\tau > 0$.[8] I assume that the tax is imposed on the gross

[6] This political process can be formalized as a global game based on Morris and Shin (2000). Details of the game are in the Online Appendix.

[7] In the oligarchy, the elite and the government are equivalent and can be used interchangeably. Here it is the elite who decides the capital in the S sector. It chooses the loan from the bank and the total investment in the S sector. For simplicity, we can say that the government chooses the capital in the S sector.

[8] The tax rate can be endogenized. I also consider the extension that the government can choose the tax rate on the entrepreneur given an upper bound $\bar{\tau}$, which is exogenously determined by the state capacity described in Besley and Persson (2009). Then the government can potentially choose $\tau < \bar{\tau}$. In the calibrated model, introducing this does not change the results, because along the transition the government always sets

return on the entrepreneur's investment in the P firm, i.e., $r_{Pt}K_{Pt}$, so that the tax income from P sector workers and the entrepreneur together simply equals τ fraction of the P sector output. The government budget includes the profit from the S firm, the tax income from the P sector, minus the transfer to S workers. The surplus of the government budget is claimed by the elite.

In the model, the variable T_t is the transfer to S sector workers. The direct cost of using the transfer is a convex function: $b_t T_t^2$. b_t can start from a low level representing a small direct cost of using the transfer. There is also an indirect cost of the transfer: it affects the wage paid by the S firm and the labor demand and productivity, as we will see later. This parameter will be calibrated to match the labor productivity of state firms in the quantitative model. The transfer is a government policy that increases the income of state sector workers without affecting the labor cost of state firms. In reality, there are also other policies with the same effects on workers' income and wage. In this sense, this variable T_t can also be used to generally represent other policies that increase state sector workers' income in oligarchy relative to the income in democracy, as long as their costs are paid by the government but not state firms. There are three types of them. First, the cash compensation to state workers paid by the government. It includes direct cash transfer in the narrow sense of the variable T_t, and also labor subsidies which are paid by the government. The latter increases the worker's final income but does not increase state firms' labor cost. Second, non-cash benefits, including housing, education and tax benefits that state sector workers enjoy. For example, giving state sector workers better access to public schools increases the support of state sector workers if they believe that this benefit will disappear if the regime changes. The third includes political propaganda and policies that affect workers' expectations on their income in the oligarchy and their income in the democracy. For example, if a political campaign convinces workers that the current government is not corrupted and very efficient or that democratization leads to a large uncertainty and a large cost, it increases political support to the current government. In the model, all these types of policies can be represented by the variable T_t, because they work exactly in the same way as the cash transfer: they increase the income of state sector workers, and they are paid by the government but not state firms, so they do not affect the labor cost of state firms or directly distort the labor demand of state firms. The direct cost of some of these policies can be low. For example, giving state sector workers better access to existing public schools is almost free for the government. More details on how to formally show the equivalence of these three groups of policies are in the

$\tau = \bar{\tau}$, while the case $\tau < \bar{\tau}$ only happens when P sector is extremely small: smaller than the size in 1998 when the calibrated economy begins.

Online Appendix. Moreover, if some policies and factors increase the workers' utility, e.g., non-monetary benefits of democracy, and their effects are approximately additive to their income, they can also be captured by this variable T_t. Because the cost and the size of T_t are endogenously calibrated, the interpretation does not affect the model predictions.

Notice that the government can set S sector capital without being constrained. It can borrow as much as it wishes from the international bond market to invest in the S sector at the interest rate r. The government can also influence the P sector capital , but with some limitation: it can set the P firm financial constraint η_t in a bounded region $\left[\underline{\eta}, \bar{\eta}\right]$. The interpretation of choosing η_t is that the government can create barriers to entrepreneurs' access to the financial market, or give administrative instructions to state banks on how much loan is allocated to entrepreneurs (see Brandt and Zhu (2000)). $\underline{\eta}$ represents the leverage of the private firm given the highest level of restriction on the financing of the P firm. [9] $\bar{\eta}$ represents the natural level of leverage if the government does not restrict private sector borrowing at all. [10] The setting on the financial market is similar to Song et al. (2011), while an important difference is that here state sector capital and private sector credit constraint are endogenously determined by the government. Then the model can endogenously account for the dynamics of the capital market friction and can be used to study the future trend of the capital market friction.

2.4 Timing of Events

Since the government is run by the elite, its actions are equivalent to and described as the actions of the elite. Then the events in each period are the following:

1. In the beginning of period t, the elite decides whether to voluntarily democratize or not. If so, the political system switches to democracy forever; if not, the following events occur.

2. Capital allocation: the elite sets K_{St} and η_t, and then the entrepreneur chooses K_{Pt}.

3. The elite sets the S sector wage w_{St} and the transfer to S sector workers T_t.

4. The S firm and the P firm hire workers. Workers are randomly selected to the S firm.

5. S sector workers decide whether to support the regime. If there are not enough supporters, the political system switches to democracy. If there are enough supporters, the oligarchy survives.

[9] For example, if the most stringent policy that the government can set is to let the bank lend nothing to the entrepreneur, then the entrepreneur has to finance her investment using her own savings. This implies $\underline{\eta} = 1$.

[10] In this case, there is still a natural limit on the lending to private entrepreneurs, because of the following moral hazard problem: if a private entrepreneur gets a loan that is too large compared to her asset, she may choose to default on paying back the loan.

6. Firms produce. Labor and capital incomes are distributed. Taxes are collected and the transfer is made.

7. The elite and the entrepreneur consume and save. The economy enters the next period.

Notice that the game in oligarchy may end in either step 1 or 5 after democratization. If so, the exit payoffs of the elite and the workers are determined by the equilibrium in democracy.

2.5 The Equilibrium in Oligarchy

Let us first look at the entrepreneur's problem:

$$\max_{\{K_{Pt},a_{pt}\}} \sum_{t=0}^{\infty} \beta^t \log c_{pt} \tag{6}$$

$$\begin{aligned} \text{s.t. } c_{pt} &= Ra_{pt} + y_{pt} - a_{p,t+1}, \\ y_{pt} &= \left(\left(1-\tau\right) r_{Pt} - \delta\right) K_{Pt} - rK_{Pt}, \\ K_{Pt} &\leq \eta_t a_{pt}, \end{aligned} \tag{7}$$

where the entrepreneur takes the price sequence r_{Pt} as given. This implies that each entrepreneur is infinitesimal and her choice does not affect aggregate variables such as the return to capital and the political system. The decision of K_{Pt} is simple: if r_{Pt} is large enough and the net return of capital is larger than the cost of borrowing, i.e.,$\left(1-\tau\right) r_{Pt} - \delta > r$, she wants to borrow as much as possible, i.e., $K_{Pt} = \eta_t a_{pt}$. If the net return of capital equals the international interest rate, she is indifferent between investing in the P firm and saving in the bank, and the capital is determined by the international interest rate. If the net return of capital is lower, she does not borrow but saves all her asset in the bank. The last case does not happen in the equilibrium. The saving decision of the entrepreneur is also simple: given the log-utility, she saves a fraction β of her total resource $Ra_{pt} + y_{pt}$ for the next period: $a_{pt+1} = \beta\left(Ra_{pt} + y_{pt}\right)$. Moll (2014) solves a similar problem. The solution to this problem formally proved in the Online Appendix.

The elite's maximization problem is the core of the equilibrium. We can define this problem recursively. It contains two stages. First, the elite chooses the political system and solves

$$W\left(a_e, a_p, t\right) = \max\left\{W^O\left(a_e, a_p, t\right), W^D\left(a_e, a_p, t\right), W^R\left(a_e, a_p, t\right)\right\}, \tag{8}$$

where W is the value function representing the elite's lifetime utility, W^O, W^D and W^R are

the value functions conditional on choosing to sustain oligarchy, to voluntarily democratize, and to stay in oligarchy without getting enough support and resulting in the revolution. W^D and W^R are determined by the equilibrium in democracy discussed in subsection 2.7. There we show that W^D simply equals the elite's lifetime utility given asset a_e and its rate of return r, and W^R is similar except that the first-period income can be different given the already determined K_S and K_P. Notice that t is also a state variable because model parameters, including TFPs, change over time. If the elite chooses W^O, then she stays in power and sets government policies η, K_S, w_S and T subject to the political constraint: maintaining enough supporters, or equivalently, two economic constraints: paying S sector workers high enough income and keeping enough S sector workers. Moreover, she decides on consumption and saving to maximize the lifetime utility. In the case that oligarchy is sustained, the recursive problem is:

$$\begin{aligned} W^O\left(a_e, a_p, t\right) &= \max_{\eta, K_S, w_S, T, c_e, a_e'} \log c_e + \beta W\left(a_e', a_p', t+1\right) \\ \text{s.t. } w_S + T &\geq y_w^D, \\ L_S &\geq \underline{L}, \\ a_e' &= R a_e + y_e - c_e, \\ a_p' &= \beta\left(R a_p + y_p\right). \end{aligned} \tag{9}$$

Variables y_w^D, L_S, y_e and y_p are functions of the policies η, K_S, w_S and T. Given these policies, the entrepreneur rationally expects the return to P sector capital and accordingly chooses K_P subject to $K_P \leq \eta a_p$. Then given K_S and K_P, workers calculate their expected income if democratization occurs, i.e., y_w^D. Given w_S and K_S, the S firm demands labor L_S according to marginal labor productivity equation 1. Then the labor, wage and capital returns in the private sector are also determined, by $L_P = 1 - L_S$, equation 2 and 3. Given the equilibrium allocations and prices, the incomes of the entrepreneur and the elite - y_p and y_e - can also be calculated, according to equation 7 and

$$y_e = \alpha K_S^{\alpha} L_S^{1-\alpha} - (r+\delta) K_S + \tau z_P K_P^{\alpha} L_P^{1-\alpha} - b T^2.$$

Notice that z_P and b are time-varying and can be considered as functions of t. We can see that the elite's current period income contains three components: (1) the profit of the S firm: $\alpha K_S^{\alpha} L_S^{1-\alpha} - (r+\delta) K_S$; (2) the tax income from the private sector: $\tau z_P(t) K_P^{\alpha} L_P^{1-\alpha}$; and (3) the cost of using transfer: bT^2.

A crucial feature of this problem is that the elite can borrow without being constrained. In other words, she can choose K_S and other variables, independent of a_e to maximize her

income. Even if a_e is low, the elite can still choose the same sequence of policies as the case that a_e is high to achieve the same level of lifetime income except for the initial difference in Ra_e. Then a_e only matters for consumption smoothing. Therefore, the representative elite's problem can be separated into two sub-problems. First, maximization of lifetime income independent of a_e by choosing government policies. Second, maximization of lifetime utility by choosing a_e to smooth consumption. The formal proof is in the Online Appendix. The first sub-problem can be written as:

$$\begin{aligned} V\left(a_p,t\right) &= \max\left\{V^O\left(a_p,t\right),V^D\left(a_p,t\right),V^R\left(a_p,t\right)\right\}, \\ V^O\left(a_p,t\right) &= \max_{\eta,K_S,w_S,T} y_e+\frac{1}{R}V\left(a_p',t+1\right) \\ \text{s.t. } w_S+T &\geq y_w^D, \\ L_S &\geq \underline{L}, \\ a_p' &= \beta\left(Ra_p+y_p\right), \end{aligned} \tag{10}$$

where V is the value function representing the discounted lifetime income of the elite excluding the return to its initial asset Ra_e, and V^O, V^D, V^R stand for the value functions in the cases of choosing sustaining oligarchy, voluntary democratization and revolution. The second subproblem is simply spending the lifetime income to maximize the lifetime utility:

$$\begin{aligned} W\left(a_e,a_p,t\right) = \max_{\{c_{es}\}_{s=t}^{\infty}} &\sum_{s=t}^{\infty}\log c_{es} \\ \text{s.t. } &\sum_{s=t}^{\infty}\frac{c_{es}}{R^s}=Ra_{et}+V\left(a_p,t\right). \end{aligned}$$

where $V\left(a_p,t\right)$ is the maximized lifetime income from the first subproblem.

2.6 Analytical Properties of the Equilibrium

Here we highlight two important analytical properties, including how the S sector labor and capital are determined and the associated labor and capital wedges. We also provide intuitions for some other crucial properties.

Let us first focus on how the S sector labor is determined, given the capital. From the first economic constraint - high enough state sector income - represented by equation 4, we can see that the S sector wage is determined by the expected income in democracy and

the transfer:

$$w_{St} \geq y_{wt}^{D} - T_t.$$

If the above equation holds with equality, then the S firm's marginal productivity of labor, captured by equation 1, determines the S sector demand for labor given capital and wage:

$$\begin{aligned} L_{St} &= \left((1-\alpha) z_{St}\right)^{\frac{1}{\alpha}} w_{St}^{-\frac{1}{\alpha}} K_{St} \\ &= \left((1-\alpha) z_{St}\right)^{\frac{1}{\alpha}} \left(y_{wt}^{D} - T_t\right)^{-\frac{1}{\alpha}} K_{St}. \end{aligned} \tag{11}$$

This equation shows that the larger transfer implies the lower wage, and then the larger labor demand of the S firm. Intuitively, when the government directly pays a large transfer to the S sector worker, the labor cost is low for S firm, and it may be willing to hire redundant labor. This result can be formally stated as the following:

Theorem 1. *If $T_t \geq \frac{\tau\alpha}{1-\alpha} w_{St}$, then $L_{St} \geq L_{St}^{*}$, and $\frac{Y_{St}}{L_{St}} \leq \frac{Y_{Pt}}{L_{Pt}}$.*

If the transfer is larger than a certain fraction of the wage, there is redundant labor in the S sector. The S sector labor is larger than the efficient level L_{St}^{*} given the S and P sector capital, and the S sector labor productivity is lower than the P sector.

The proof is in the Online Appendix.[11] This result states that if the equilibrium transfer is large enough, the labor wedge - a lower S sector labor productivity - exists. The optimal level of transfer chosen by the government depends first on the cost of using transfer, b_t. If b_t is low enough, the condition in Theorem 1 is satisfied, and the S sector labor productivity is low. Then as we will see in the quantitative model, choosing b_t helps to calibrate the dynamics of the labor wedge. Notice that b_t is not the only factor that determines T_t. When the government chooses T_t, it also considers its dynamic impacts on other variables, including y_{wt}, K_{St}, K_{Pt}, etc. For example, a larger T_t implies a higher L_{St} and a smaller L_{Pt}, which slows down the private sector growth, and if the government wants the P sector to grow faster, it may optimally choose a not too large T_t, even if b_t is very low.

As we see from equation 11, L_{St} depends on K_{St}, conditional on other variables for example T_t. A large K_{St} implies a large L_{St}, and it helps to satisfy the second economic constraint - the large enough state sector employment share - represented by equation 5.

[11] The reason for getting this simple expression for the lower bound of T_t is that the expected income in the democracy is also a function of the S and P sector capital, as we will see in subsection 2.7. It includes the wage in democracy and the tax income on the elite and the entrepreneur. The tax income is transferred to w workers and it equals $\frac{\tau\alpha}{1-\alpha}$ fraction of the wage. So if the transfer in the oligarchy is larger than the transfer in democracy, the S sector wage is lower than the efficient wage, implying redundant labor and lower labor productivity.

A large T_t also implies a large L_{St}, but given T_t in a certain region, a large enough K_S is necessary for keeping L_{St} greater than $\underline{L}$. This result can be formally stated as follows.

Theorem 2. *Given any μ, if $T_t \leq \mu w_{St}$, then $L_{St} \geq \underline{L}$ and $T_t + w_{St} \geq y_{wt}^D$ imply $K_{St} \geq \kappa K_{Pt}$, where $\kappa = \frac{\left(1+\frac{\tau\alpha}{1-\alpha}\right)^{\frac{1}{\alpha}} L}{(1+\mu)^{\frac{1}{\alpha}} - \left(1+\frac{\tau\alpha}{1-\alpha}\right)^{\frac{1}{\alpha}} L} \left(\frac{z_{Pt}}{z_{St}}\right)^{\frac{1}{\alpha}}$.*

If the transfer as a fraction of wage is bounded above, then to sustain oligarchy, a large enough K_{St} relative to K_{Pt} is necessary.

The detail of the proof is in the Online Appendix. The idea is that given the transfer as a fraction of the wage below an upper bound, to achieve the high enough income for S workers, the S sector wage cannot be too low. Given the S sector wage, keeping enough workers in the S sector implies investing enough capital in the S sector relative to the P sector. This minimal S sector capital requirement can generate over-investment compared to the case without the binding constraint on state employment, and a capital wedge between S and P firms. First, in an efficient allocation, a higher P sector capital implies a lower S sector capital, as discussed in Song et al. (2011), because with more capital, P firms can hire more workers, and then the S firms hire fewer workers and need less capital. In this model, as the P sector capital grows, the political constraint requires that the S sector capital grow proportionally, and this can generate the over-investment in the S sector. Second, if κ is not too low, then given a lower TFP in the S sector, the large enough S sector capital implies the capital market wedge: a lower S sector capital productivity compared to the P sector.

The above two Theorems together imply that if the transfer is bounded above and below, then the political constraint generates labor and capital wedges between S and P firms. As we will see in the quantitative exercise, this is indeed the case: the equilibrium transfer varies in a bounded region. So there are labor and capital wedges. How the wedges evolve can be studied in details with the aid of the quantitative model in Section 3.

2.7 The Equilibrium in the Democracy

Here we explain the equilibrium in the democracy and show how the incomes of workers and the elite are determined. The key difference from oligarchy is that in the democracy the government is run by the representative worker because the workers' dominating population size guarantees that they win the majority voting. The government then taxes the elite and the entrepreneur at the exogenous rate $\tau > 0$. The economy is simply a

competitive equilibrium given taxes.[12] The economy is similar to the two-sector growth model in the competitive equilibrium in Song et al. (2011).

The competitive labor market implies that wages in the S and the P sector are the same, determined by the marginal productivity of the labor.

$$w_t^D = (1-\alpha)\, z_{St}\, (K_{St})^{\alpha} \left(L_{St}^D\right)^{-\alpha} = (1-\alpha)\, z_{Pt}\, (K_{Pt})^{\alpha} \left(L_{Pt}^D\right)^{-\alpha},$$

where L_{St}^D and L_{Pt}^D represent the labor allocation in the democracy. The representative worker's income includes the wage and the transfer. The transfer equals the tax collected from the elite and the entrepreneurs. So we have:

$$\begin{aligned} y_{wt}^D &= w_t^D + \tau\left(\alpha z_{St}\, (K_{St})^{\alpha} \left(L_{St}^D\right)^{1-\alpha} + z_{Pt} K_{Pt}^{\alpha} \left(L_{Pt}^D\right)^{1-\alpha}\right) \\ &= \left(1 + \frac{\tau\alpha}{1-\alpha}\right) w_t^D, \end{aligned} \tag{12}$$

given that $L_{St}^D + L_{Pt}^D = 1$. Remember that a worker's final income in the democracy - y_w^D, given the capital allocation, is the reference point for her political decision in the oligarchy, and it includes the wage w_t^D and the transfer $T_t^D = \frac{\tau\alpha}{1-\alpha} w_t^D$. If in the oligarchy the transfer to S worker is greater or equal to $\frac{\tau\alpha}{1-\alpha}$ fraction of the S worker wage w_{St}, then the S sector worker wage w_{St} is lower or equal to w_t^D, because $w_{St} + T_t = y_{wt}^D = w_t^D + T_t^D$. This explains why in Theorem 1, $\frac{\tau\alpha}{1-\alpha}$ shows up as the lower bound of transfer in oligarchy as a share of wage to generate a low S sector wage and labor productivity.

The capital market in the democracy is also competitive. Given that the elite can borrow from the international market at the interest rate r, in the case that the S sector exists, then the net rate of return for the elite's investment in S firms is also r. The other case is that the S sector does not exist because the P sector capital is large enough. Then given the large P sector capital and wage, S sector capital return is lower than r, so the S sector capital is 0 and only P firms produce. In this case, the elite can save her asset in the bank and receives the return at the rate r. In both cases, the elite's income simply equals the return from her asset at the rate r. This implies that the elite's income in democracy, except for the return to her asset, is 0, i.e., $V^D \equiv 0$. The elite's income in the case of revolution - V^R - is similar but slightly different. In this case, the elite does not choose to democratize at the beginning of the period but chooses to stay in the oligarchy. She sets the capital allocation but does not get enough support. Then revolution happens and brings in democracy. Because the elite has set the capital allocation, her income in

[12] The distortion in the democracy is the dynamic distortion of capital taxes. There are no static distortions for example in the labor market.

this period is determined by the existing capital allocation. After this period, the elite's income is determined by the capital allocation in democracy and equals the return from her asset at the rate r. So $V^R = ((1-\tau) r_S - \delta - r) K_S$. In the quantitative model, revolution is always dominated by sustaining oligarchy or voluntary democratization, because when the P sector capital is small, the elite prefers to sustain oligarchy: $V^O > V^R$, and when the P sector capital is large enough that the elite wants to democratize, it prefers to do it in the beginning of the period: $V^D \geq V^R$. We can safely ignore V^R.

The entrepreneur faces the credit constraint, so potentially the rate of returns to her asset and the P sector capital can be higher than r. In this case, the entrepreneur wants to borrow more but her supply of capital is constrained by $K_{Pt} \leq \eta_t a_{pt}$. The government prefers a higher η_t which implies a higher K_{Pt}, and a higher w_t, so it always sets $\eta_t = \bar{\eta}$.

The dynamic equilibrium in the democracy is similar to Song et al. (2011). The P sector grows as the entrepreneur accumulates more and more asset. Eventually, all workers move to the P sector. More details are in the Online Appendix.

3 Quantitative Analysis

In this section, I calibrate the model to the Chinese economy and show that the quantitative model can deliver a successful account of China's growth experience since 1998, including the private sector growth, the labor and capital market frictions, and the economic growth.[13] More specifically, I calibrate the model to the manufacturing sector in China. Differential TFP growths in the state and the private sectors are inputs of the model, and other key parameters including the discount factor of the entrepreneur and the cost of transfer are chosen to match the growth of the private sector employment share, and the state sector labor productivity relative to the private sector. Then the calibrated model is used to predict the future output growth, the dynamics of frictions and the development paths.

3.1 Calibration

The model is calibrated to the Chinese manufacturing sector, following Song et al. (2011). The production function is Cobb-Douglas with the capital share $\alpha = 0.5$ (Bai et al. (2006)). The annual depreciation rate is set as $\delta = 0.1$ (Song et al. (2011)). The world interest rate is $r = 5\%$. The leverage of private firms in China is 24%, according to Li et al. (2008), and

[13] In late 1997, the 15th Congress of the Communist Party of China officially endorsed the role of private firms, and from 1998 on state firm privatization starts.

this implies $\bar{\eta} = 1.24$. $\underline{\eta}$ is set to 1, implying no lending to the P firm at all.

The TFP of the S firm relative to the P firm is set to match the labor and capital productivity gaps between the state and the private firms estimated by Hsieh and Song (2015). They report in Table 5 that the state firm labor productivity is 60% of the private firm labor productivity in 1998 and 75% in 2007. They also report the numbers for capital productivity: 35% and 47% respectively. In the neoclassical framework, the Cobb-Douglas production function implies that the TFP gap can be backed out using the labor and capital productivity gaps, as follows:

$$\frac{z_{St}}{z_{Pt}} = \left(\frac{Y_{St}/L_{St}}{Y_{Pt}/L_{Pt}}\right)^{1-\alpha} \left(\frac{Y_{St}/K_{St}}{Y_{Pt}/K_{Pt}}\right)^{\alpha},$$

where Y_{St}/L_{St} stands for the labor productivity in the S sector, etc. This equation implies that S firm TFP is 45% of P firm TFP in 1998 and 58% in 2007. [14] The annual growth rate of the P firm TFP between 1998 and 2007 is 3%, which is the average private sector TFP growth estimated by Brandt and Zhu (2010). This TFP growth rate is assumed to continue until 2030, gradually decline to 2% in 2040 and stay at 2% after that. The TFP gaps between the S and P sector in 1998 and 2007 discussed above imply that the S sector TFP grows at 4% between 1998 and 2007. Afterward, the S firm TFP growth rate is assumed to linearly decline until it converges to the P firm TFP growth rate in 2017. This implies that today the S firm TFP is 68% of the P firm TFP and the gap stays at this level after 2017.

The discount factor of the entrepreneur is set to $\beta = 0.825$ to match the growth of the private sector employment share estimated by Storesletten and Zilibotti (2014). They find that the private sector employment share increases from 15% in 1998 to a bit more than 60% in 2008 and stays around that level afterward. The average of the private employment share from 2008 to 2012 is 61.5%, so we set $\underline{L} = 38.5\%$. The initial period of the model economy corresponds to the year 1998, so the employment share in 1998 is also used to pin down the initial asset of the entrepreneur. The tax rate is set to $\tau = 20\%$. This implies that the final income of the S sector worker is on average about 20% higher than the income of the P sector worker between 1998 and 2007. The cost parameter of using transfer b is chosen to calibrate the labor productivity of S firms relative to P firms because a higher cost leads to less transfer, a higher S sector wage and a higher S firm labor productivity. Setting b to 0.28 in 1998 matches the initial labor wedge, and letting b grow by 0.04 annually matches the labor wedge in 2007. I assume that b keeps growing until 2040,

[14] These numbers are lower than the TFP gaps estimated using the monopolistic competition framework in Hsieh and Song (2015) - 55% and 75% but are broadly consistent with the estimation by Brandt and Zhu (2010) who also use the neoclassical framework - 45% and 50%. This paper uses the TFP gaps backed out using the neoclassical framework to make the model internally consistent.

Figure 2: The Solution of the Equilibrium at the Steady State

which is the year that all parameters including the TFP growths reach their steady-state levels.

3.2 The Solution

In this subsection, I illustrate the solution of the calibrated model. In the long-run, the key variables - including asset, capital, and income - grow at the same rate of the TFP growth - 2%, so here I normalize all these variables using this growth rate so that the normalized variables converge to a steady state in the long-run. Using the normalized variables, Figure 2 shows the solution of the elite's recursive problem at the steady state, i.e., how policies and equilibrium variables depend on the state variable - the entrepreneur's asset a_p. The key feature is that government policies respond to a_p differently, depending on whether a_p is small or large. First, when a_P is small, a larger a_P implies a larger K_P, a smaller K_S, and a smaller L_S. This relation is similar to the one in the competitive equilibrium in the democracy: when the P sector is larger and employs more workers, the S sector shrinks and uses less labor and capital. However, when a_p is large enough - larger than 0.8 in this case, L_S and K_S do not decrease as a_p increases. L_S stays at the critical level $\underline{L}$, because of the political constraint: keeping enough workers in the S sector. This is because given T and the corresponding w_S, keeping $\underline{L}$ workers in the S sector implies that K_S needs to be large enough relative to K_P, as stated in Theorem 2. Second, if a_p is larger than the threshold that makes the elite prefer a smaller P sector - 1.2 in this case, the elite chooses to lower the leverage of the P firm η to keep K_P not too large. The reason is that now a

larger K_P requires a larger K_S, and the marginal return from K_P, i.e., the tax income from the P sector, is dominated by the marginal cost of K_S, i.e., the interest cost minus the return of the investment. The elite adjusts η to keep K_P around her preferred level. However, there is a lower bound for η, so when a_p is large enough, the elite cannot keep K_P at her preferred level anymore. Then K_P increases with a_p again. So when a_p and K_P are too large, the elite's income becomes low. The elite's lifetime income is maximized when a_p is at a medium level, as the fourth panel of Figure 2 shows. Notice that the transfer generally follows the pattern of K_S, as shown in the fifth panel: when a_p and L_P are relatively small, T does not increase much when a_p increases; but when a_p is large enough, T increases as a_p increases. It is also because when a_p and K_P are large, buying support becomes costly but also necessary.

Above is the solution at the steady state. In fact, in all other periods, the policy functions are qualitatively similar though there are some quantitative differences in parameters and policy functions. For example, in the initial periods, S firm TFP and the cost of using transfer are lower than the steady-state levels. These differences affect the choice of K_S, w_S and T. The elite prefers to use more transfer and less S sector capital. Still, the key properties discussed above, i.e., how K_S, η, and T respond to a_p conditional on the size of a_p, are still similar.

To sum up, the government policies depend on the entrepreneur asset. If the entrepreneur asset is small enough such that the S sector employment share is above the critical level, the S sector capital and labor decrease as the P sector capital increases. If the entrepreneur asset is large enough, the S sector employment share stays at the critical level, and the S sector capital and transfer increase with the entrepreneur asset and P sector capital. If the entrepreneur asset reaches a further high level, the leverage of the P firm is reduced by the government to keep the private sector capital from growing larger.

3.3 The Dynamics

Given the solutions for all periods, we can simulate the economy starting from an initially small P sector, whose employment share equal 15% as documented by Storesletten and Zilibotti (2014). The dynamics from 1998 on are illustrated in Figure 3 and 4. The dashed lines are the data, while the solid lines are the time series generated by the model economy. As we can see from Figure 3, the targeted moments - state sector employment share from 1998 to 2012, state firm labor productivity as a percentage of the private firm labor productivity in 1998 and 2007, and state firm capital productivity - are matched well.[15]

[15]The dashed line in the first panel of Figure 3 represents the state sector employment share in each year between 1998 and 2012, calculated by Storesletten and Zilibotti (2014). The dashed line in the second plan

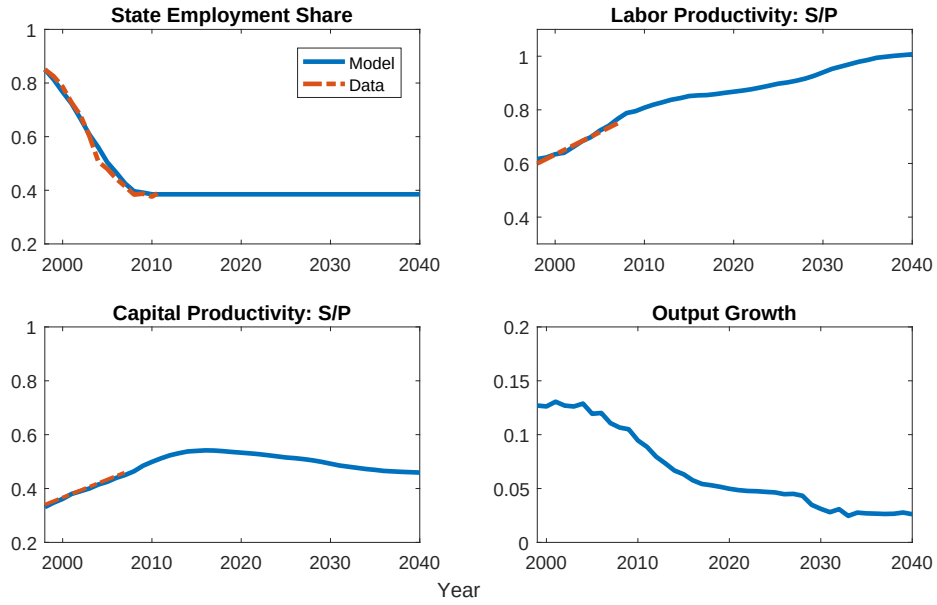

Figure 3: The Dynamics of the Economy

The untargeted moments, including the output growth and the labor productivity gaps after 2008, are also generally consistent with the observations. For example, the output growth stays high above 10% until 2008, slowly goes down, and in 2017 it is still above 5%. [16]

From 1998 to 2008, the state sector employment share declines from 85% to 38.5%. Capital and labor in the private sector grow very fast (see Figure 4). This is the first stage of the development - *rapid growth*. In this stage, the main driving force is the private sector growth, in terms of both the relative size (employment share) and the absolute size (capital level). The output growth stays above 10%. The growth of the private sector TFP also contributes significantly to the output growth. Though the state sector TFP grows faster, the relative size of the state sector declines, so the contribution of the private sector TFP to the output growth becomes relatively more important. The labor productivity of state firms relative to private firms grows from 60% to 75%, while the capital productivity grows from 35% to 47%, as documented in Hsieh and Song (2015). Both labor and capital productivities of state firms grow following the growth of the state sector TFP. In addition to TFP growth, which increases both labor and capital productivities, the transfer also affects labor and capital productivities, differently: it directly reduces wage and labor

shows the labor productivity of state firms relative to the productivity of private firms in 1998 and 2007, documented in Hsieh and Song (2015). They do not report the labor productivities in the years between 1998 and 2007, so the line simply connects these two years. It is similar for the capital productivity in panel 3.

[16] The only exception is the output growth shown in Figure 3. It is not normalized by the 2% long-run growth, so it corresponds to the nominal output growth.

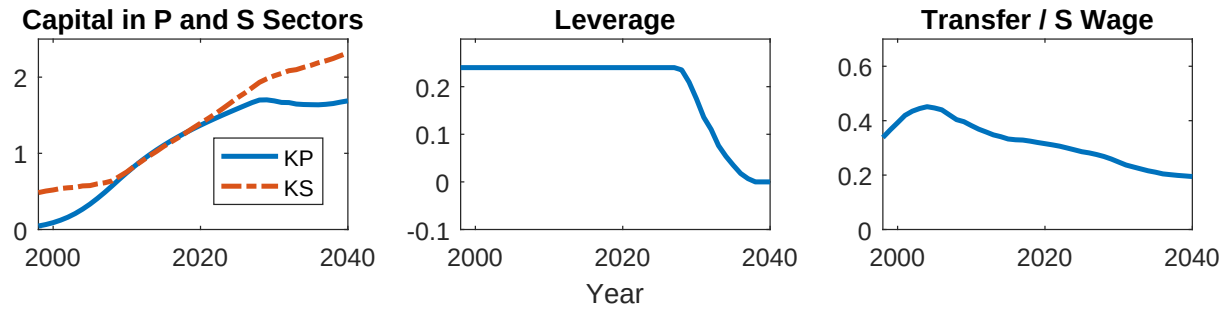

Figure 4: The Dynamics of The Economy: More Variables

productivity of S firms and indirectly increases the capital productivity through the lower capital-labor ratio. In this stage, the size of transfer relative to wage does not change much: it stays around 40%, as we can see from the third panel of Figure 4, so the S firm labor and capital productivities grow as fast as the S firm TFP.[17] Why is the transfer relatively stable in this stage? There are two counter-forces affecting the dynamics of the transfer. First, the cost of transfer increases over time because both the cost parameter and the income level grow. This creates incentives for the government to reduce the transfer as a share of income. Second, given the initially small private sector, the government prefers an initially low level of transfer to help the private sector to grow. It is because more transfer implies more redundant labor in the state sector and reduces the private sector labor and output. As the private sector grows over time, the government has a lower incentive to increase the size of the private sector, so it worries less about the redundant labor in the state sector and may want to increase the transfer. The two forces together generate a relatively stable level of transfer going slightly up and then down in this stage. So the dynamics of the labor wedge is mainly determined by the dynamics of TFPs, but not much by the change of the distortion of transfer. So is the capital wedge. In later stages, when the ratio of transfer to wage declines, growths of the labor and the capital productivities start to differ.

After 2008, the state employment stops declining and stays around 38.5%. This implies that the economy enters a different stage of development: *state capitalism*. In this stage - from 2008 to 2027 - the private sector capital keeps growing, but at a slower speed. The state sector capital increases proportionally to the private sector capital, to maintain the state sector employment. The private sector capital growth slows down because the employment does not increase anymore and the wage is pushed up by the large investment in the state sector. The private sector growth is still an important direct driving force

[17] Brandt and Zhu (2010) document that non-wage compensations are about 50% of the wage in the early 1990s and about 25% in 2000s. The transfer in the model is broadly in the same range but slightly larger. This is reasonable considering that the transfer in the model represents not only the non-wage transfer but more factors including the impact of political propaganda, etc.

of growth, but now the private sector does not grow in terms of its relative share in the economy although its capital level still grows. As the private sector capital grows, the state sector capital also grows proportionally, and this over-investment helps to keep the aggregate growth rate from declining too fast. From 2008 to 2017, the output growth declines but it still stays above a reasonably high level of 5%. Another important feature in this stage is that the growth of labor productivity in the state sector differs from that of the capital productivity. Because the transfer as a share of income decreases when the cost of transfer grows as the income and the parameter b grow over time. When the transfer from the government is lower, the wage paid by state firms is higher, and state firms hire less redundant labor. This is one driving force for the growth of the labor productivity in the state sector, in addition to the TFP growth. The reduction of the redundant labor in the state sector also implies a higher capital-labor ratio and a lower capital productivity. The over-investment in the state sector is the other reason for the low capital productivity. So as we can see from Figure 3, the model generates a continuous growth of the state firm labor productivity. In comparison, the capital productivity does not increase much and stays around 50%. This prediction is qualitatively consistent with the finding in Hsieh and Song (2015). They show that from 2007 to 2012, the labor productivity of state firms relative to that of private firms keeps growing but at a slower rate than before, while the capital productivity stops increasing. [18]

As the entrepreneur asset and the private sector capital keep growing, the tax income from the private sector can be dominated by the cost of maintaining the regime. Then the elite chooses to directly slow down the growth of the private sector capital by reducing the lending to private firms. In the model, this happens in 2028, and then the private sector capital stops growing. The economy enters the third stage: *middle-income trap*. The credit constraint on private firms become even tighter and private firms find it even harder to grow. The long-run private sector size is smaller than the efficient level in the democracy, for two reasons. First, private firms' loans and capital are reduced by the government. Second, the private sector employment is lower than in democracy due to the political constraint. Eventually, the output growth slows down and then stays the steady-state level - 2%, before the income reaches the level in the democracy.

In summary, the model generates a three-stage transition. The first two stages are

[18] There are some qualitative differences from the findings in Hsieh and Song (2015): growths of labor and capital productivities after 2007 in this paper are faster than in their paper. The main reason is that the TFP growths after 2007 in this paper are faster. The quite slow growths of productivities after 2007 in Hsieh and Song (2015) imply a large drop of TFP growth rate after 2007 to about one-fourth of the rate before in the neoclassic framework. In this paper, the state sector TFP growth is assumed to decline smoothly, so the changes in state-sector productivity growths are not that dramatic.

consistent with China 's growth experience since 1998. The model matches the fast privatization which started in 1998 and stopped in 2008. It can also account for the dynamics of the labor and the capital market frictions: the declining labor productivity gap between state firms and private firms, and a larger capital productivity gap which initially went down but recently stopped decreasing. It also generates reasonably output growth until today. The model offers the following explanation for the dynamics of frictions and the growth in the recent years: the political constraint - buying support from state sector workers using wage and transfer - creates the redundant labor and the over-investment in the state sector, but the private sector still grows and contributes to the rapid growth. Finally, the model predicts that frictions will be persistent and the private sector capital growth will slow down. The rapid growth in the current regime is not sustainable compared to in the democracy.

3.4 Sustained Growth

In the benchmark calibration, the capital market frictions persist, because it helps the elite to sustain the regime and maximize her income. This harms the private sector growth and the economic growth. The alternative case is that the elite chooses to take political reforms, including liberalizing the financial market and democratization, and then the private sector can better access the financial market and grow further. This does not happen in the benchmark model because the cost of sustaining the regime is not so high: the government can borrow from the international market and invest in the state sector at a low interest rate, and the cost of using transfer is not large. If the costs are large enough, the elite chooses to democratize. [19]

To study the consequence of democratization, here I consider the case that when the economy enters the third stage, instead of choosing to restrict the private sector growth, the government chooses to democratize and remove all the labor and capital market frictions. [20] Figure 5 shows the dynamics after democratization in 2028 in solid lines, and in comparison, the dynamics in the benchmark model in dashed lines. As we can see, after democratization, the labor and capital market frictions are removed. First, the leverage

[19] For example, if the world interest rate r becomes much higher or the economy is a closed economy and the elite cannot borrow from the international market, then investing in the state sector is more costly and the elite may give up the oligarchic regime. The other case is that the cost of T becomes very large. Given the general interpretation of T, including the value of political propaganda and the expected utility in democracy, this can happen if the economic growth also brings in higher education, more demand for equal rights, and better organized civil society, then it becomes more and more difficult for the elite to buy the support. This is the case that modernization theory (see Lipset (1959)) considers.

[20] This can happen if the cost of transfer becomes large enough, e.g., the cost becomes $\underline{b} + b_t T_t^2$, and $\underline{b}$ is sufficiently large.

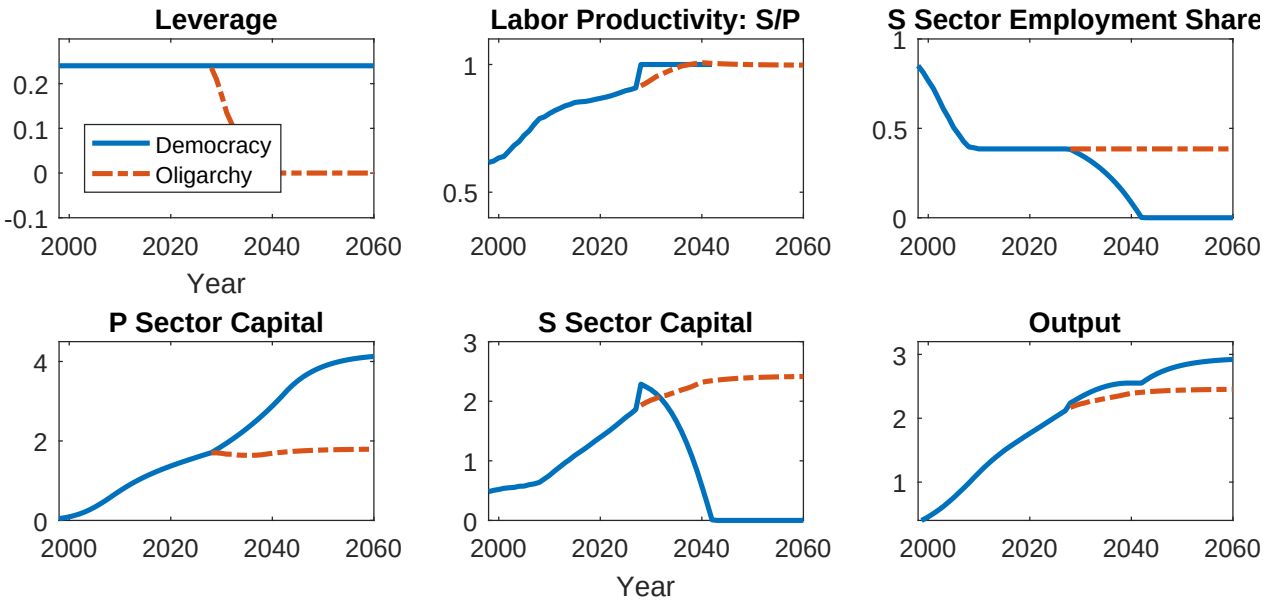

Figure 5: Dynamics After Democratization

of private firms stays at the highest level. Second, the state sector labor productivity immediately converges to the private sector labor productivity. Third, the employment share in the state sector is allowed to decline again. As we can see from panel 4 and 5, the private sector capital grows faster than in the benchmark, and the state sector capital initially goes up but then quickly decreases to 0. The private sector grows faster because now lendings to private firms and the employment are not restricted by the government. The state sector capital initially goes up because the capital level is determined by the competitive market but not the monopolistic government, and later it decreases as the private sector grows. As shown in panel 6, the output after democratization grows faster than in the benchmark, especially in two periods: right after democratization, the growth speeds up because of the faster growth of the private sector; and then right after all the state firms exit the market, which happens around 2042, the growth speeds up again. The second period of faster growth is because the efficient capital in the economy grows faster when there is no decrease of state sector capital anymore. Afterwards, only the private sector capital accumulation drives the growth. Eventually, the output in democracy converges to a higher level than in oligarchy because of the removal of capital and labor market frictions.

3.5 Discussions

In this subsection, we discuss more on the model's mechanics, extensions and implications. I first discuss the core mechanism that generates the rapid growth by analyzing the

efficiency and the redistribution in the oligarchy. Then I discuss the consequences of an extension of the model by assuming that the economy is closed. Finally, I study the implications of the model in the middle-income trap and the middle-class activism.

Efficiency v.s. Redistribution In this model, the growth in oligarchy is faster in the beginning but then slower in the long-run, compared to the growth in democracy. This result and the intuition behind are similar to Acemoglu (2008): in the beginning, the elite's interest is in line with growth and the government policies are growth-enhancing, but in the long-run, the elite's interest conflicts with growth and the policies are set to restrict growth. However, the mechanics and the implications on efficiency and redistribution are different. In Acemoglu (2008), an oligarchic society produces more than a democratic society in the beginning because it achieves higher efficiency given that the elite - the major producers - can protect their properties better than in democracy. In this paper, the initial rapid growth is because of the inefficiencies - the labor and the capital market distortions. First, the initial rapid growth is because of the initial low output level resulted by the labor market distortion. The initial output level is lower than in democracy because of the redundant labor in the state sector, and given the initially low output level, the growth can be fast, as the redundant labor gradually declines. However, the labor market distortion still exists and the output level is generally lower than in democracy. In this sense, the output efficiency in oligarchy is lower, though the growth is faster. Figure 6 compares the dynamics in democracy starting from the first period (blue lines) with the dynamics in the benchmark economy in oligarchy (red dashed lines). We can see that the labor productivity of state firms always equals the productivity of private firms (panel 1). This higher efficiency in the democracy implies that the initial output level is higher than in the oligarchy (panel 5), while the output growth is lower (panel 6 shows that the growth in the democracy is around 10% from 1998 to 2008 and then drops to about 4% in 2017). Second, the output growth in the state capitalism stage is also fast, driven by the inefficient over-investment in the state sector. In this stage, the output level in the oligarchy can catch up with the output level in the democracy. We can see from Figure 6 that the state sector capital grows fast in oligarchy and it catches up with the level in democracy around 2028 (panel 4), which implies that the output level in oligarchy catches up with the level in democracy quickly and gets very close in 2028 (panel 5). [21]However,

[21]In alternative calibrations, e.g., that the state sector TFP is higher, the output in the oligarchy can be higher than the counterpart in the democracy in the state capitalism stage. Also notice that the capital level is lower in oligarchy in the beginning. This also contributes to the initial low output in oligarchy. There are two reasons for the initially low level of state sector capital. First, the oligarchic government is the monopolizer of the state sector capital so initially it prefers to set the state sector capital at the low

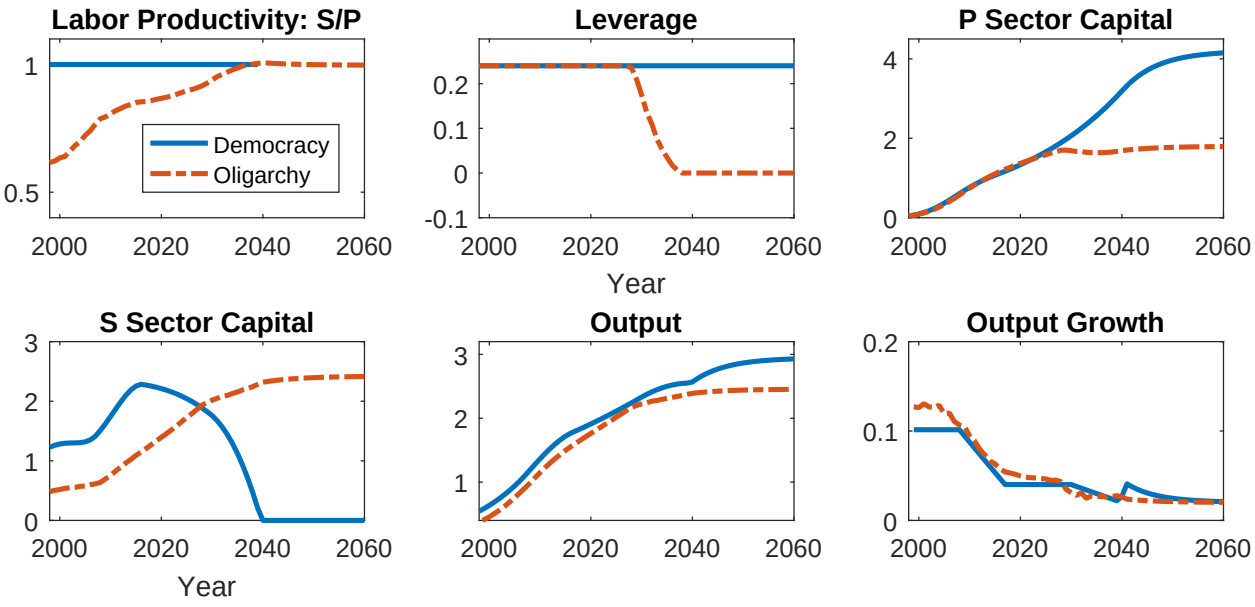

Figure 6: Dynamics in Democracy

this high output is mainly because of the very high level of state sector capital, but not because of the efficiency of the capital and the labor. If we measure efficiency by only the output level, we can state that the efficiency in oligarchy can be close to or even higher than in democracy, because it is efficient in generating fast capital accumulation. However, if we measure the efficiency by the productivity of resources, for example, the capital, the efficiency in oligarchy is still lower.

The efficiency can also be measured using the incomes and the welfare of the agents.[22] Then how the output is redistributed matters for the aggregate welfare and the efficiency. The large capital in the state sector aims for sustaining the regime, so it generates a higher income for the elite. However, we know that state sector workers expect to get the same level of income in the democracy; and private sector workers get more in the democracy. The population size of workers dominates that of the elite, so the aggregate welfare in the oligarchy is lower because it results in lower incomes for the majorities. In other words, the welfare and the efficiency in the oligarchy is lower than in the democracy because of the larger inequality.

monopolistic level for higher return to capital. Second, the inefficient labor allocation implies a lower return to capital so there is a lower incentive for investment. However, when the private sector employment share reaches the critical level, the government has to set the state sector capital high enough to maintain enough supporters. Afterwards, the state sector capital in the oligarchy exceeds the level in the democracy.

[22] An economy can obtain a higher output but a lower efficiency level. Consider, for example, a neoclassic growth model with a capital subsidy.

The Closed Economy We can assume that the elite cannot borrow from the international market and can only use the domestic savings to invest in the state sector. Let us first consider the case that the elite does not manipulate the domestic savings and the corresponding interest rate. Then the elite has to use her own saving to finance the state sector investment, and the elite's asset becomes a state variable of the equilibrium. As the entrepreneur asset grows and the economy enters the state capitalism stage, the elite has to increase her saving to maintain enough capital in the state sector. In the case that the time discount of the elite is high, the elite's saving can be low in the open economy but in the closed economy, the elite is forced to save more and consume less. The cost of maintaining oligarchy becomes larger. Two possible cases may emerge: one is that the elite finds it too costly to maintain oligarchy and chooses to democratize; the other is that the elite still chooses to sustain the oligarchy and it adopts even harsher policies to restrict the private sector growth. This has an interesting implication: international sanctions on non-democratic countries, which forbid the governments from accessing the international market, may create two opposite results. Some governments give up or fail in maintaining the regime, for example, the Soviet Union. In some other countries for example Iran, the governments respond by tightening their controls over the economies and then the societies become even less democratic. This is the opposite of what the sanctions aim to achieve. The different consequences of sanctions are consistent with the findings by Grauvogel and von Soest (2013) and von Soest and Wahman (2015).

If we consider the case that the government can influence the domestic savings and the interest rate, then the government wants to increase the savings in the domestic banking system and reduce the corresponding interest rate. Compulsory savings for example through the pension system and regulated low interest rate for savings in the bank can help to increase the control over the politico-economic system and reduce the cost of maintaining the regime. If we consider an half-closed economy where the population have access to the international financial market but under the regulation of the government, then the capital control and the recent tightening capital outflow in China also help the government to keep more financial resources under its control and to maintain the regime.

The Middle-Income Trap In the model, the government policies generate a rapid growth in the beginning, but then become detrimental to the growth, when the elite's interest conflicts with the private sector growth. This explains why some governments adopt the right policies to achieve rapid growths out of the poverty but then implement the "wrong" policies that stop the convergence to rich countries. The negative impacts of government policies can be more detrimental than what was described above. In the benchmark

calibration, the restriction that the government can put on private firms is moderate: at the maximal, it can set the lending to private firms at zero. This reduces their capital by 24%. It leads to a lower long-run output but not that far from the level in the democracy. In reality, the government may implement much harsher policies to restrict the private firms and these policies may harm the growth more. The government can directly confiscate the capital of some private firms. Acemoglu and Robinson (2012) discuss such an example in the steel industry in China. The government can also forbid private firms from entering or operating in certain industries, which they may have invested in before. This policy has two consequences. First, the investment of private firms can be become even lower than the entrepreneur asset. This happens when some entrepreneurs cannot invest in the industries as they did before. Second, profits and the TFP of private firms are reduced when private firms are excluded from some profitable industries. These policies can be modeled as allowing the government to set $\eta_t < 1$ or reduce z_{Pt}. [23] If these are allowed, the long-run output in oligarchy can even go lower. In the benchmark calibration for China, if $\eta_t < 1$ is allowed, the government reduces η_t in the long-run to 0.9, and this reduces the long-run output by another 10%. In many other middle-income countries with different conditions, the problem can be much larger. The governments may be more afraid of the private sector growth, for example if the productivities of state firms or elite-controlled firms are lower than in the benchmark calibration, then they implement even harsher policies and harm the long-run output much more. Those countries can stop growing when their incomes are still far away from those of rich countries, and they may stop converging with large income gaps between them and the U.S.

The Middle-Class Activism An important political implication of this model is that the middle-class do not necessarily support democracy. What we learn from the European history and also from the modernization theory is that as the middle-class receive higher incomes and better education, their demand for democracy grows over time and eventually leads to democratization. A similar mechanism also exists in this paper: as the income increases, the cost of buying support goes up and this puts more pressure on the oligarchic government. However, this paper shows that the growing demand for democracy and democratization do not necessarily happen, because of two counter-forces. First, the

[23] In this model, the implications of these two policies are similar, for the following reason: the P firm output is $z_{Pt}\left(\eta_t a_{pt}\right)^{\alpha} L_{Pt}^{1-\alpha} = \left(z_{Pt}^{\frac{1}{\alpha}} \eta_t a_{pt}\right)^{\alpha} L_{Pt}^{1-\alpha}$, and decreasing z_{Pt} by α percent is equivalent to decrease η by 1 percent in terms of their impacts on output. They are only different in their impact on the cost of the P firm, because reducing η reduces the borrowing of the P firm and the related cost, while reducing z_{Pt} does not. However, given the relatively low interest rate for borrowing compared to the high rate of return to P firm capital, this difference is small.

government can respond to the demand of democracy from the middle-class by providing them state sector jobs and paying them high incomes. Then the middle-class, being afraid of losing their advantages in the existing regime, become even less supportive of democratization compared to the lower-class. Notice that in this case a large fraction of the middle-class are actually created by the state. This force is especially strong in countries with historically large state sectors, for example in pre-communist countries where many people rely on the state but not the market to organize important things in life such as education, health and housing. In addition to the findings for China mentioned above, Rosenfeld (2017) find the similar pattern for the Russian middle-class. He reveals that the middle-class from the state sector were less likely to mobilize against electoral fraud, and argues that potential coalitions in support of democratization are weakened by middle-class growth in state-dependent sectors. Second, even if the demand for democracy from the middle-class is large and grows over time, it is not always the case that this demand can put enough pressure on the government so that it gives up the non-democratic regime. This paper shows the possibility that the government, in a very long period at least, can maintain the regime even though the income and the size of the middle-class have been growing.

4 Conclusion

This paper proposes a political-economic theory to understand China's recent development and to predict China's future economic and political transition. The political constraint - maintaining enough supporters in the state sector - creates the labor and capital market frictions and results in a three-stage transition. The first two stages are *rapid growth* and *state capitalism*, which are consistent with several salient aspects of China's development, including the growth of the private sector, persistent labor and capital market wedges, and rapid output growth. In the future, China is likely to enter a *middle-income trap* with persistent capital market frictions, given the current conditions, especially the economically and politically powerful state. To switch to the other development path that leads to the *sustained growth*, economic and political reforms are necessary, though such reforms may challenge the current regime.

Even though the focus of this paper is in China, it is also useful for understanding the development of many other emerging countries and some developed countries with similar patterns. The key political constraint in the theory also exists in some other countries such as Kuwait where the political elite need to gain support from public workers, and Korea before the 1980s, when politicians need support from citizens from industries tightly

connected to the government. In Korea before 1980, the large conglomerates (chaebol) were granted privileged access to low-cost credit and the employment share of small and medium enterprises (SME) had stagnated. The difference is that after 1980, democratic movements and financial reforms happened together, and the employment share of SMEs increased from 50% to 68% in 1990 and continued in the early 1990s after democracy was consolidated. This political and economic development path is consistent with the second case in the theory: the *sustained growth*. [24]

Moreover, the theory is also useful to think on an important question in development: should other developing countries apply the "China model", i.e., the combination of authoritarian politics and state-guided capitalism, to promote economic growth? Some suggestions in favor of adopting this model are based on its recent success, but the long-run implications should be carefully examined and distinguished from the short-run. This paper provides a quantitative framework to evaluate the economic and political consequences of the "China model", which may become the "Korea model" under the right conditions.

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5 Appendix (Available Online)

First, I provide the proofs of Theorems. Then I provide more details on the model, including the general interpretation of the transfer, the equilibrium in democracy, the microfoundation of the political constraint, and the definition of the recursive equilibrium in oligarchy.

5.1 Proof of Theorem 1

First we express y_{wt}^D as a function of K_{St} and K_{Pt}:

$$y_{wt}^D = \left(1+\frac{\tau\alpha}{1-\alpha}\right)(1-\alpha)\left(z_{St}K_{St}+z_{Pt}K_{Pt}\right)^{\alpha}.$$

Then L_{St} can be expressed as a function of y_{wt}^D, T_t, K_{St} and K_{Pt}, while L_{St}^D as a function of K_{St} and K_{Pt}:

$$\begin{aligned}
L_{St} &= \left((1-\alpha)z_{St}\right)^{\frac{1}{\alpha}} w_{St}^{-\frac{1}{\alpha}} K_{St} \\
&= \left((1-\alpha)z_{St}\right)^{\frac{1}{\alpha}} \left(y_{wt}^D - T_t\right)^{-\frac{1}{\alpha}} K_{St}, \\
L_{St}^D &= \frac{z_{St}^{\frac{1}{\alpha}} K_{St}}{z_{St}^{\frac{1}{\alpha}} K_{St} + z_{Pt}^{\frac{1}{\alpha}} K_{Pt}}.
\end{aligned}$$

Denote $T_t = dw_{St}$, we can then compare L_{St} and L_{St}^D and check under which conditions for d, we get $L_{St} \geq L_{St}^D$:

$$\begin{aligned}
(1-\alpha)^{\frac{1}{\alpha}}\left(\frac{1}{1+d}\left(1+\frac{\tau\alpha}{1-\alpha}\right)(1-\alpha)\right)^{-\frac{1}{\alpha}} \frac{z_{St}^{\frac{1}{\alpha}} K_{St}}{z_{St}^{\frac{1}{\alpha}} K_{St} + z_{Pt}^{\frac{1}{\alpha}} K_{Pt}} &\geq \frac{z_{St}^{\frac{1}{\alpha}} K_{St}}{z_{St}^{\frac{1}{\alpha}} K_{St} + z_{Pt}^{\frac{1}{\alpha}} K_{Pt}}, \\
\left(\frac{1}{1+d}\left(1+\frac{\tau\alpha}{1-\alpha}\right)\right)^{-\frac{1}{\alpha}} &\geq 1, \\
1+d &\geq 1+\frac{\tau\alpha}{1-\alpha}, \\
d &\geq \frac{\tau\alpha}{1-\alpha}.
\end{aligned}$$

5.2 Proof of Theorem 2

Given μ as an upper bound of T_t as a fraction of w_{St}, the constraint $T_t + w_{St} \geq y_{wt}^D$ implies that w_{St} can not be too low:

$$w_{St} \geq \frac{y_{wt}^D}{1+\mu}.$$

Then given the not too low w_{St}, the constraint $L_{St} \geq \underline{L}$ implies a not too small K_{St}:

$$\begin{aligned}
\left(\frac{1}{1+\mu}\left(1+\frac{\tau\alpha}{1-\alpha}\right)\right)^{-\frac{1}{\alpha}} \frac{z_{St}^{\frac{1}{\alpha}} K_{St}}{z_{St}^{\frac{1}{\alpha}} K_{St} + z_{Pt}^{\frac{1}{\alpha}} K_{Pt}} &\geq L, \\
\frac{K_{St}}{K_{St} + (z_{Pt}/z_{St})^{\frac{1}{\alpha}} K_{Pt}} &\geq \left(\frac{1}{1+\mu}\left(1+\frac{\tau\alpha}{1-\alpha}\right)\right)^{\frac{1}{\alpha}} L, \\
\frac{K_{St}}{(z_{Pt}/z_{St})^{\frac{1}{\alpha}} K_{Pt}} &\geq \frac{\left(\frac{1}{1+\mu}\left(1+\frac{\tau\alpha}{1-\alpha}\right)\right)^{\frac{1}{\alpha}} L}{1-\left(\frac{1}{1+\mu}\left(1+\frac{\tau\alpha}{1-\alpha}\right)\right)^{\frac{1}{\alpha}} L}, \\
K_{St} &\geq \frac{\left(1+\frac{\tau\alpha}{1-\alpha}\right)^{\frac{1}{\alpha}} L}{(1+\mu)^{\frac{1}{\alpha}} - \left(1+\frac{\tau\alpha}{1-\alpha}\right)^{\frac{1}{\alpha}} L} \left(\frac{z_{Pt}}{z_{St}}\right)^{\frac{1}{\alpha}} K_{Pt}, \\
&= \kappa K_{Pt}.
\end{aligned}$$

5.3 General Interpretation of Transfer

In the model, the transfer T is the government policy that increases the state sector workers' income but not the wage. The state firms' cost of using labor is not affected by T but the state sector workers income is increased. In the main text, we claim that three groups of policies have the same effects on the workers' income (or utility) and and they can also be represented by T. They include cash compensation, non-cash benefits and some political propaganda and policies. Here we formally illustrate how these policies affect the equilibrium and why they are equivalent to transfer.

First, the cash compensation to state workers paid by the government, including direct cash transfer and also labor subsidies paid by the government. The former is the narrow interpretation of variable T and is denoted as T^{1c}. The latter is denoted as T^{1s}. Second, non-cash benefits, including housing, education and tax benefits that state sector workers enjoy. We can model them as increasing the state sector workers' final income by T^2. Third, political propaganda and policies that affect workers' expected income in oligarchy

if they support the regime and their income in democracy. Let us denote the policies that increase workers' expected income in oligarchy if they support the regime, e.g., the reward of loyalty, as T^{3+}, policies that decrease their expected income if they do not support the regime, e.g., cost of revolution, as T^{3-}, and policies that decrease their expected income in democracy, e.g., cost of the transition to democracy, as T^D. Then a state sector worker supports the regime if

$$w_S + T^{1c} + T^{1s} + T^2 + T^{3+} \geq y_w^D - T^{3-} - T^D. \tag{13}$$

We can then use T to denote $T^{1c} + T^{1s} + T^2 + T^{3+} + T^{3-} + T^D$ and rearrange the constraint as

$$w_S + T \geq y_w^D.$$

Obviously, factors that do not affect the real income but the expected utilities can also be represented by T, if their impacts on utilities are approximately additive to the income. This can include other non-monetary benefits of democracy such as more social rights, and the propaganda that affects workers' expected income in democracy without affecting the real income.

5.4 The Equilibrium in Democracy

The equilibrium is a decentralized competitive equilibrium given taxes, similar to Song et al. (2011). The competitive labor market implies that wages are the same in the S and the P sectors. The competitive capital market in S sector and the elite's infinity borrowing capacity imply that the return of capital to the elite is equal to the interest rate r. This pins down the rate of return to S sector capital and the S sector capital labor ratio. The credit constraint of the entrepreneur and the higher productivity of P firms imply that P sector capital return is higher than in the S sector but the entrepreneur's capital supply is limited by the credit constraint. Given a large enough time preference parameter β and the corresponding high enough saving rate, the entrepreneur keeps accumulating asset and the P sector keeps growing until all workers move to the P sector while S firms no longer produce. Below are more details.

First, in democracy, if the S firm still produces, the rate of return to S firm capital has to be r. If it is greater than r, each elite member wants to supply infinite capital and S sector capital becomes positive infinity; if it is lower than r, the elite does not want to get any loan or supply any capital to S sector. In other words, competition of S sector capital supply implies that the net rate of return to the elite, denoted as ρ_{et}^D, equals the marginal

cost:

$$\rho_{et}^{D} = \left(1-\tau^{D}\right)\alpha z_{St}^{\alpha} K_{St}^{\alpha-1}\left(L_{St}^{D}\right)^{1-\alpha} - \delta = r.$$

This determines S sector capital labor ratio and wage:

$$\begin{aligned}\frac{K_{St}}{L_{St}^{D}} &= \left(\frac{r+\delta}{\left(1-\tau^{D}\right)\alpha z_{st}^{\alpha}}\right)^{\frac{1}{\alpha-1}} \Rightarrow \\ w_{t}^{D} &= (1-\alpha)\left(\frac{z_{St}K_{St}}{L_{St}^{D}}\right)^{\alpha}.\end{aligned}$$

The wage pins down the private sector labor, given capital:

$$\begin{aligned}w_{t}^{D} &= (1-\alpha)\left(\frac{K_{Pt}}{L_{Pt}^{D}}\right)^{\alpha} \Rightarrow \\ L_{Pt}^{D} &= \left(\frac{w_{t}^{D}}{1-\alpha}\right)^{-\frac{1}{\alpha}} K_{Pt}. \end{aligned} \tag{14}$$

Moreover, the elite in democracy gets no transfer, so she only replies on asset return and her income from other sources is simply 0.

The entrepreneur's problem is maximizing lifetime utility by optimally choose each period's capital supply and savings to the next period, taking capital returns and credit constraint as given.

$$\begin{aligned}\max_{\{K_{Pt},a_{pt+1},\}_{t=0}^{\infty}} & \sum_{t=0}^{\infty}\beta^{t}\log c_{pt} \\ \text{s.t. } K_{Pt} &\leq \eta_{t}a_{pt}, \\ y_{pt} &= \rho_{pt}K_{Pt} - rK_{Pt} \\ a_{pt+1} &= Ra_{pt} + y_{pt} - c_{pt},\end{aligned}$$

where ρ_{pt} is the net return of capital to the entrepreneur in democracy. A similar entrepreneur's problem is solved in Moll (2014), and the solution of the problem is simple: the entrepreneur maximizes each period's income and saves a constant fraction of income to the next period, given the properties of the log utility. We solve this sequential problem below.

Suppose that the sequence $\left\{K_{Pt}^{*}, a_{pt+1}^{*}\right\}_{t=0}^{\infty}$ is the optimal solution to the sequential problem. First, K_{Pt}^{*} must maximize y_{pt}, given a_{pt+1}^{*}. Otherwise $\exists t'$ and a feasible $\hat{K}_{Pt'}$

such that $\rho_{pt'}\hat{K}_{Pt'} - r\hat{K}_{Pt'} > \rho_{pt'}K^*_{Pt'} - rK^*_{Pt'}$, we can simply construct a new sequence of $\left\{K^*_{Pt}, a^*_{pt+1}\right\}$ by replacing $K^*_{Pt'}$ by $\hat{K}_{Pt'}$ while keeping all other $K^*_{Pt}$for all $t \neq t'$ and all other a^*_{pt+1}. The new sequence is feasible and implies $c_{pt'} > c^*_{pt'}$ and $\forall t \neq t', c_{pt} = c^*_{pt}$. The lifetime utility of the new sequence is higher. Second, the optimal K^*_{Pt} to maximize y_{pt} given a^*_{pt} is simple:

$$K^*_{Pt}\begin{cases} = \eta_t a^*_{pt} & \text{if } \rho_{pt} > r, \\ \in \left[0, \eta_t a^*_{pt}\right] & \text{if } \rho_{pt} = r, \\ = 0 & \text{if } \rho_{pt} < r. \end{cases}$$

In the calibrated model, the first case happens, because a_{pt} is never that large. Then $K^*_{Pt} = \eta_t a^*_{pt}$ and $y^*_{pt} = \left(\rho_{pt} - r\right)\eta a^*_{pt}$. Adding ra_{pt}, we get the total income, which all come from the return of asset: $y^{tot}_{pt} = Ra^*_{pt} + y^*_{pt} = \left(R + \left(\rho_{pt} - r\right)\eta_t\right)a^*_{pt}$. So $\rho^{tot}_{pt} = R + \left(\rho_{pt} - r\right)\eta_t$ can be considered as the total return to the entrepreneur's saving and the entrepreneur lives on the asset return.[25] Finally, the only problem left is the choice of a_{pt+1}, taking the total return to asset ρ^{tot}_{pt} as given:

$$\max_{\{a_{pt+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \log c_{pt}$$
$$\text{s.t. } a_{pt+1} = \rho^{tot}_{pt} a_{pt} - c_{pt}.$$

Given the log-utility, the substitution effect of the return to saving exactly cancels the wealth effect, and in each period, the agent saves β fraction of the total resource to the next period, i.e., $a^*_{pt+1} = \beta\rho^{tot}_{pt} a^*_{pt}$, no matter how high or low ρ^{tot}_{pt+1} is. If $\beta\rho^{tot}_{pt} > 1$, then a_{pt} keeps growing until $\beta\rho^{tot}_{pt} = 1$. Meanwhile, L^D_P keeps growing according to (14), and L^D_S declines gradually. When a_{pt} reaches $1/\bar{\eta}\left(w^D/1-\alpha\right)^{\frac{1}{\alpha}}$, L^D_P reaches 1, and S sector becomes 0. Afterwards, the entrepreneur keeps accumulating asset, and the economy behaves like a neoclassic growth model.

5.5 Microfoundation of the Political Constraint and the Equilibrium in Oligarchy

In the main text, to keep the model simple and to focus on the quantitative exercise, the political constraint - enough supporters from the state sector - is given as a feature of the

[25] If we consider the other two cases $\rho_{pt} \leq r$, it is not difficult: then the total return to the entrepreneur's asset is simply r.

oligarchic regime. It can be rigorously modeled as a political game of all workers and the elite. Below, I extend the model in the main text to provide the microfoundation for the political constraint.

The government faces the political constraint: it needs support from sufficiently many workers to keep the regime stable. In each period, each worker i, decides whether to support the oligarchic regime ($m_i = 1$) or not ($m_i = 0$). The aggregate mass of supportive workers is $M_w = \int_0^1 m_i di$. If it is larger than a crucial threshold $\underline{L}$, the regime survives this period, otherwise democratization occurs.[26]

The political decision of a worker is made after she gets employed - either by the S firm or the P firm - and before she receives her wage and final income. The expected final income of a worker from sector $j \in \{S, P\}$ in oligarchy is denoted as y_{wj}, and after democratization y^D_{wj}. They are endogenously determined by economic factors, which will be explained in this section. [27] So the payoffs can be summarized as in Table 1. Obviously, for the myopic worker i in sector S, one (weakly) dominating pure strategy is to support oligarchy if and only if the expected income is higher than in democracy, i.e., $m_i = 1$ if $y_{wS} \geq y^D_{wS}$ and $m_i = 0$ if $y_{wS} < y^D_S$. Similar for P sector workers. This strategy expresses the voters' sincere preferences. Without loss of generality, I assume that workers use this strategy, similar to the sincere voting assumption in the literature. [28]This political game is a simple global game of regime switching, in the spirit of Morris and Shin (2000). The game here is simple because there is no heterogeneous information, no cost of being against the regime, and no punishment for the supporters of the regime after the regime collapses.

Table 1: Payoffs of Workers

	$M_w \geq \underline{L}$	$M_w < \underline{L}$
$j = S$	y_{wS}	y^D_{wS}
$j = P$	y_{wP}	y^D_{wP}

[26]The setting that only the support from workers counts is without loss of generality, for two reasons. First, the population size of the workers are much larger than the other two groups, so they should count. Second, the elite and the entrepreneur's welfare is generally higher in oligarchy in all most all cases, so the model is robust to whether considering their political support or not.

In the extreme case that the elite holds dominating political power, and needs little support from workers - e.g., it uses mostly military force to control the citizens - $\underline{L}$ can be close to 0. In the case that the elite needs to win a majority vote, $\underline{L}$ can be 50% if the voting system is fair, or smaller than 50% if the voting system is manipulated in favor of the elite. A regime with a voting system may not necessarily be democracy. It can still be an oligarchy, and the government serves the interests of the elite.

[27]Workers are ex-ante identical, so there is no need for subscript i to denote worker i.

[28]Of course, given that there are a continuum of workers, worker i knows that her action does not affect the aggregate political outcome and feels indifferent about what she does. There are other dominating strategies and equilibria with pure or mixed strategies. However, if there are finite workers and there is some small probability that worker i's choice can be pivotal, then it is wise to follow the sincere strategy described above.

Given this microfounded game, we can describe the equilibrium in greater details in below. Variables in the model of oligarchy can be classified into three groups. First, the exogenous variables. They include: firm productivities (z_S and z_P), minimal number of supporters needed ($\underline{L}$), world interest rate (r), tax rate (τ), the upper and the low bound of the credit constraint ($\underline{\eta}$ and $\bar{\eta}$). The exit payoffs, e.g., the worker's expected income after democratization: y_w^D, are also taken as exogenous variables in the model of oligarchy, though these variables are endogenously generated by the model of democracy. The second group of variables includes those endogenously chosen by the agents which influence the political outcomes. They include the elite's decision on whether to stay in oligarchy or to voluntarily democratize, credit constraint (η), P sector capital (K_S), the S sector wage (w_s) and transfer to the S sector workers (T); and each worker's decision on whether to support the regime (m_i) and its aggregate (M_w). Notice that even if the elite chooses to stay in oligarchy here, the political system is still not decided yet, because later the elite may set policies that attract enough support and sustain oligarchy, or it can set the policies that lead to revolution and democratization. So in fact, the elite can choose three types of political outcome, i.e., $M_e \in \{O, D, R\}$ where O stands for sustaining oligarchy, D for voluntary democratization, and R for revolution. The third group of variables include economic choices by individuals: consumption, saving, etc. For example, private entrepreneur asset (a_p) and the elite's asset (a_e) and the corresponding aggregate variables A_p and A_e. An additional variable is the type of the political system: $M \in \{O, D\}$. Of course, in oligarchy, M always takes value O, but it evolves and may turn to D, in two cases: either $M_e = D$, or $M_e = R$ which means that the elite chooses to stay in oligarchy but does not get enough support - $M_w < \underline{L}$. Then the game in oligarchy ends and the equilibrium in democracy starts.

In the main text, we study the entrepreneur's problem using its sequential form. Here, we write the entrepreneur's problem recursively to help us to define the recursive equilibrium. The entrepreneur, at the beginning of each period, solves the following problem: [29]

$$
\begin{aligned}
W_p\left(a_p; A_e, A_p\right) = \max_{k_p, a_p'} & \log c_{pi} + \beta W_p^{M'}\left(a_p'; A_e', A_p'\right) \qquad (15)\\
\text{s.t. } & c_p = R a_p + y_p - a_p', \\
& y_p = \left(\left(1-\tau\right) r_P - \delta\right) k_P - r k_P, \\
& k_P \leq \eta a_p,
\end{aligned}
$$

[29] The details on how to write the entrepreneur's problem in this recursive form is discussed after the equilibrium is defined.

where k_P and a_p are variables chosen by the representative entrepreneur as an individual, while A_p is the aggregate variable for all entrepreneurs which the representative entrepreneur takes as given. Also, the entrepreneur takes other aggregate variables as given and rationally expect them determined by the state variables by equilibrium functions and laws of motion. These variables including current period price and policy r_P and η, and future aggregate variables: the next period political system $M' \in \{D, O\}$, elite asset A'_e and aggregate entrepreneur asset A'_p. We can denote the equilibrium functions for r_P and η simply as $r_P\left(A_e, A_p\right)$ and $\eta\left(A_e, A_p\right)$. Future variables are expect to follow laws of motion: $M' = \mathcal{L}_M\left(A_e, A_p\right), A'_e = \mathcal{L}_{Ae}\left(A_e, A_p\right), A'_p = \mathcal{L}_{Ap}\left(A_e, A_p\right)$, where $\mathcal{L}_X$ denotes the law of motion of variable X. Because all entrepreneurs are identical, in the equilibrium, $A_p = a_p$. Still, in the recursive form, it is necessary to also use a_p, to avoid the case that an individual entrepreneur believes her choice of a_p can affect the aggregate variables.

Finally, the recursive equilibrium in oligarchy can be defined as follows.[30]

Definition 1. The recursive equilibrium in oligarchy consists of

sets of prices $w_P\left(A_e, A_p\right), r_P\left(A_e, A_p\right)$,

laws of motion $\mathcal{L}_M\left(A_e, A_p\right), \mathcal{L}_{Ae}\left(A_e, A_p\right), \mathcal{L}_{Ap}\left(A_e, A_p\right)$,

value functions $W\left(A_e, A_p\right), W^O\left(A_e, A_p\right), W_p^O\left(a_p; A_e, A_p\right)$,

decision rules $M_e\left(A_e, A_p\right)$, $\eta\left(A_e, A_p\right), K_S\left(A_e, A_p\right), w_S\left(A_e, A_p\right), A'_e\left(A_e, A_p\right), k_P\left(a_p; A_e, A_p\right)$, $a'_p\left(a_p; A_e, A_p\right)$, $m_i\left(w_S, K_S, K_P; A_e, A_p\right)$ and

aggregate variables $K_P\left(A_e, A_p\right), L_S\left(A_e, A_p\right), L_P\left(A_e, A_p\right), M\left(A_e, A_p\right), M_w\left(A_e, A_p\right)$, such that

(a) Given laws of motion, the value function $\left\{W, W^O\right\}$ and decision rules $\{\eta, K_S, w_S, a'_e\}$ solve the elite's problem (8)(9); the value function W_p^O and decision rules $\left\{k_P, a'_p\right\}$ solve the entrepreneur's problem (15); and the decision rule m_i solves workers' problems.

(b) Given $\{w_S, K_S\}$, L_S satisfies the S firm's optimization problem - equation (1). Given $\{w_P, r_P\}$, K_P and L_P satisfy the P firm's optimization problem - equations (2) and (3).

(c) The laws of motion $\left\{\mathcal{L}_M, \mathcal{L}_{Ae}, \mathcal{L}_{Ap}\right\}$are consistent with the equilibrium outcomes $\left\{M, A'_e, A'_p\right\}$. Aggregate variables $\left\{K_P, A'_p, M, M_w\right\}$ are consistent with individual's choices $\left\{k_P, a'_p\right\}$.

Next, we can formally show how to write the entrepreneur's problem and the elite's problem in the current recursive form. the sequential formation of the entrepreneur's problem is as in equation (6). If the entrepreneur is asked to plan the future choices, at the beginning of period 0 given A_e and A_p, the recursive formation simply becomes equation

[30] Notice that here when defining the equilibrium in oligarchy, the equilibrium outcomes in democracy, e.g., $W^D\left(A_e, A_p\right), W^R\left(A_e, A_p\right), W_p^D\left(a_p; A_e, A_p\right)$, can be considered as exogenously given.

(15). In the sequential form, the entrepreneur rationally expects $\{\eta_t, r_{Pt}\}_{t=0}^{\infty}$; while in the recursive form, the entrepreneur expects the current period (η, r_P) using the current period state variables $\left(A_e, A_p\right)$ and predicts the evolvement of future (η, r_P) using the laws of motion of state variables. This is standard, as in classic neoclassic growth models and the household's problem.

It is worth mentioning that in this simple recursive form the problem is as if the entrepreneur chooses k_P and a_p at the beginning of each period using her predictions of η and r_P. This simplification is fine for two reasons: first, we only look at solution given all other agents behave as in the equilibrium; and second, an entrepreneur is infinitesimal and does not thinks that her behavior can affect the aggregate variables and the equilibrium. This recursive form is more restrictive compared to the sequential form which allows for the solution given any sequence of $\{\eta_t, r_{Pt}\}_{t=0}^{\infty}$ which can be off-equilibrium. However, this restriction is standard, like in the recursive formation of the classic neoclassic growth model.

The elite's problem is formed with the similar logic. We write down the recursive problem and the value functions - equation (8) and (9) - as if the elite chooses $\{M_e, \eta, K_S, w_S, A_e'\}$ at the beginning of each period, i.e., in step 1 of timing of events, instead of choosing M_e in step 1, η and K_S in step 2, w_S in step 3 and A_e' in step 8. Similar to the entrepreneur's problem, the elite expects other agents reacting optimally according to the equilibrium solution. This problem is a bit more complicated than the entrepreneur's problem, because the elite takes into account that its choice affects other agents' behaviors and the equilibrium outcome. For example, the elite does not take the entrepreneur's capital supply K_P, the worker's income in democracy y_w^D, entrepreneur income y_p and entrepreneur saving A_p' as given, but understands that they react to choices of (η, K_S, w_S), as we can see in the constraints of problem (9). Though the elite is not infinitesimal, it is the only one: all other agents - entrepreneurs, workers, and firms - are infinitesimal and take aggregate variables as given. So it is correct to form the problem as if the elite chooses all variables at the same time while internalizing the reactions of other agents, given that we are only interested in the equilibrium solution. [31] To show the equivalence, we can write the recursive problem reflecting the elite's choices in each step using more value functions. In step 1, the elite chooses whether to sustain oligarchy, to democratize or to create revolution. Value functions are the same as in equation (8). Next, let us only consider the case when $M_e = O$. In step 2, the elite chooses K_S and η, expecting that afterwards entrepreneurs choose k_P

[31] If there are two larger enough players making decisions sequentially and understanding that their individual choices affect the equilibrium outcome, usually the problem can not be formed as if one makes all choices simultaneously.

which shapes K_P accordingly:

$$W^O\left(A_e, A_p\right) = \max_{\eta, K_S} \beta W^3\left(K_S, K_P; A'_e, A'_p\right)$$
$$\text{s.t. } \frac{K_S}{K_P} \geq \frac{\underline{L}}{z\left(\nu - \underline{L}\right)},$$

where K_P is determined by the entrepreneur's optimal choice of k_P given η, K_S and the expected r_P, denoted as a function $K_P\left(\eta, K_S, A_p\right)$ and is simply $K_P = \eta A_p$ throughout this paper. The constraint is the necessary condition of the capital allocation in the case that oligarchy survives. It is discussed in the equilibrium given capital. In step 3, the elite chooses w_S:

$$W^3\left(K_S, K_P; A_e, A_p\right) = \max_{w_S} W^8\left(y_e, y_p; A_e, A_p\right)$$
$$\text{s.t. } w_S \geq y_w^D\left(K_S, K_P\right),$$
$$L_S\left(w_S, K_S\right) \geq \underline{L}.$$

In the first constraint, function $y_w^D\left(.\right)$ is determined by the equilibrium in democracy, as in equation (12); in the second constraint, function $L_S\left(.\right)$ comes from the FOC of the S firm, as in equation (1). Then, y_e and y_p can be computed using equation (7) and the corresponding equation for y_e, and in the next step, the elite makes decisions conditional on them. In step 7, the elite chooses A'_e:

$$W^7\left(y_e, y_p; A_e, A_p\right) = \max_{A'_e} \log c_e + \beta W\left(A'_e, A'_p\right)$$
$$\text{s.t. } A'_e = RA_e + y_e - C_e,$$
$$A'_p = \beta\left(RA_p + y_p\right).$$

Compared to the simple Bellman equation in the main text, i.e., (9), using more value functions allow for more complete characterization of the problem sequentially, which allows for analyzing the elite's decisions given off-equilibrium variables. For example, W^3 can be used to study the elite's optimal decision if K_P deviates from the equilibrium, i.e., $K_P \neq \eta A_p$; and W^7 can still be solved if y_p is different from the equilibrium level given state variables and the elite's choices. However, if we only care about the equilibrium solution and substitute the equilibrium optimal choices of K_P, y_p, L_S, then we can merge W^8, W^3 into W^O and get the expression of W^O as in the main text, i.e., equation (9). Moreover, given that all other agents are infinitesimal, they have not incentives to deviate from the optimal choice of k_P, y_P, L_S, etc..

We can also write down the Bellman equation for W^R:

$$\begin{aligned} W^R\left(A_e, A_p\right) = & \max_{\eta, K_S, w_S, C_e, A_e'} \log c_e + \beta W^D\left(A_e', A_p'\right) \\ & \text{s.t. } w_S < y_w^D\left(\eta, K_S, w_S, A_p\right) \text{ or } L_S\left(w_S, K_S\right) < \underline{L}, \\ & A_e' = RA_e + y_e^D\left(K_S, \eta, A_e\right) - C_e, \\ & A_p' = \beta\left(RA_p + y_p^D\left(K_S, \eta, A_p\right)\right). \end{aligned}$$

The first constraint means that if the elite chooses to go for policies that lead to revolution, then either the minimal wage constraint or minimal supporter constraint is violated. Then the incomes of the elite and the entrepreneur are determined by the equilibrium in democracy. Notice that in the function of y_p^D, there is no w_S but η. This is because that if in step 3 the elite chooses a w_S that does not buy enough supporters, then the wage is determined by the equilibrium in democracy; while the private sector capital has already been determined by η in step 2, which is before step 3 and before the revolution. In other words, when the revolution happens, the economy enters the equilibrium of democracy given capital allocation in this period, as stated in step 5. One can also decompose this Bellman equation as if the elite chooses variables sequentially, but it will be equivalent.

As we claimed in the main text, the elite's problem can be simplified into two subproblem: maximization of life-time income and then maximization of the utility. The main idea is that because the elite faces no borrowing constraint, its current period asset A_e does not constraint its choices on all other variables, in the current period and in all future periods. So the elite can always choose the same sequence of policies which gives the highest income net of asset return, which is independent of the asset choices.

More formally, the proof is the following. Denote the lifetime utility achieved by solving the two sub-problems - first maximizing lifetime income and then maximizing lifetime utility - as U. Remember that the solution to the original one complete problem gives lifetime utility W. In this proof, we use the sequential form of the dynamic problem, which is equivalent to but simpler than directly looking at the recursive form.

First, $U \leq W$. Let us look at the two subproblems. The solution to the first subproblem achieving V can be denoted as $\left\{\hat{M}_t, \hat{w}_{St}, \hat{K}_{St}, \hat{\eta}_t\right\}_{t=0}^{\infty}$ in the sequential form. The corresponding consumption and saving decisions obtaining U are denoted as $\left\{\hat{C}_{et}, \hat{A}_{et+1}\right\}_{t=0}^{\infty}$. Combine them together, the choice $\left\{\hat{M}_t, \hat{w}_{St}, \hat{K}_{St}, \hat{\eta}_t, \hat{C}_{et}, \hat{A}_{et+1}\right\}_{t=0}^{\infty}$ achieving U is a feasible choice of the original problem, given that in every period the choice set for $M_t, w_{St}, K_{St}, \eta_t$ is independent of A_{et}. So the optimal solution for the original problem should be at least as good as this candidate choice, i.e., $W \geq U$.

Second, $U \geq W$. Denote the choice that solves the original problem and achieves W with stars, as $\left\{M_t^*, w_{St}^*, K_{St}^*, \eta_t^*, C_{et}^*, A_{et}^*\right\}_{t=0}^{\infty}$. Let us compare $\left\{M_t^*, w_{St}^*, K_{St}^*, \eta_t^*\right\}_{t=0}^{\infty}$ with the solution of the first subproblem achieving V: $\left\{\hat{M}_t, \hat{w}_{St}, \hat{K}_{St}, \hat{\eta}_t,\right\}$. Obviously, $\hat{V} = \sum \hat{y}_{et}/R^t \geq \sum y_{et}^{\star}/R^t = V^*$, given that $\left\{M_t^*, w_{St}^*, K_{St}^*, \eta_t^*\right\}_{t=0}^{\infty}$ is also a candidate solution to the first subproblem, again because of the independence of policies from the elite asset. This implies that the lifetime income from solving the first subproblem is as high as the solution of the original problem. Then, in the second sub-problem, choosing $\hat{C}_{e0} = C_{e0}^* + \hat{V} - V^*$, $\left\{\hat{C}_{et}, \hat{A}_{et}\right\}_{t=1}^{\infty} = \{C_{et}^*, A_{et}^*\}_{t=1}^{\infty}$ gives at least as high lifetime utility as $\left\{M_t^*, w_{St}^*, K_{St}^*, \eta_t^*, C_{et}^*, A_{et}^*\right\}_{t=0}^{\infty}$. In other words, consuming the potential extra lifetime income in the first period and following the same strategy of the solution to the original problem can do as good as the optimal solution to the original problem.

Combing these two results, we have $U = W$, i.e., Solving the original lifetime utility maximization problem is the same as solving the two sub-problems.