1. Textbook problems 4, 38, 41, 50.

Solution:

4. Ans. (a) The probability duck i is hit by hunter j is $0.6/12 = 1/20$. The probability the first duck is killed is $1-(19/20)^6$, so the expected number killed is $12 \cdot [1-(19/20)^6] = 3.178$, (b) $6 \cdot (0.6) = 3.6$

38. Ans. (a) The probablity a Poisson with mean 0.5 is ≥ 3 is 0.0144. (b) The mean is $4000(0.0144) = 57.6$. The standard deviation is $\sqrt{4000(0.0144)(0.9856)} = 7.5$. ≥ 70 corresponds to $(69.5 - 57.6)/7.5 = 1.586$ so the normal approximation is $P(\chi \geq 1.586) = 0.0564$.

41. CORRECT ANSWER IS 1 minus what is written!

Ans. The mean of one bet is $-.1$. The second moment is 11.5 so the variance is $11.5 - (.1)^2 = 11.49$ and the standard deviation is 3.389. The mean of 100 bets is -10. The standard deviation is $3.389 \cdot \sqrt{100} = 33.89$. ≥ 0 corresponds to $(-0.5+10)/33.89 = 0.28$ so the normal approximation is $P(\chi \geq 0.28) = 0.3897$

50. Ans. $0.1014 \pm 2\sqrt{(.1)(.9)/2,809} = [0.1004, 0.01024]$

2. Alice and Bob are sharing french fries by simultaneously flipping coins. Alice's coin has probability 1/2 of heads. She eats two fries each time she flips a head. Bob's coin has probability 1/10 of landing heads. He eats five fries each time his coin lands heads. Let X be the number of flips until Alice eats 100 fries, and let Y be the number of flips until Bob does.

(a) Find the mean and variance of X and Y.

Solution: Notice we can write $X = \sum_{i=1}^{50} X_i$ with $X_i = \text{geometric}(1/2)$ and $Y = \sum_{i=1}^{20} Y_i$ with $Y_i = \text{geometric}(1/5)$. It follows that

$$EX = 100, \quad EY = 200.$$

$$\text{var}(X) = 50\,\text{var}(X_1) = \frac{1-\frac{1}{2}}{(1/2)^2} = 100.$$

$$\text{var}(Y) = 20\frac{1-\frac{1}{10}}{(1/10)^2} = 1800.$$

(b) Use normal approximation to estimate $P(\{X < 120\} \cap \{Y > 120\})$.

Solution: Let's assume that $X = N(100, 10)$ so $P(X < 120) = .975$. We can set $Y = N(200, \sqrt{2250}) \approx N(200, 42)$. So $P(Y > 120) = .97$. The intersection has probability

$$.975 * .97 = .945$$

(c) Use the previous part to estimate $P(X < Y)$. Is your answer an over or underestimate? Explain how you could improve the estimate.

Solution: The answer from the previous part is an estimate on this event. It is an underestimate though, because $\{X < 120\} \cap \{Y > 120\} \subseteq \{X < Y\}$. This could be improved by considering the event $E_k = \{X < k\} \cap \{Y > k\}$ and finding k that maximizes $P(E_k)$. We just figured out the case $k = 120$.

3. You are walking towards your favorite tree and start out one mile away. You move closer by first rolling a six-sided die to obtain the number D_i. You then cover a $1 - \frac{D_i}{10}$ fraction of the distance remaining between you and the tree. So if you roll a 3 on your first roll you would move to be .3 miles from the tree. If you then roll a 6 you would move to $.3 * .6$ miles away from the tree, and so on.

   (a) Write an expression for your distance, S_n, to the tree after n steps.

   Solution: $S_n = (D_1/10) \cdots (D_n/10)$.

   (b) What is ES_n?

   Solution: $ES_n = (3.5/10)^n = .35^n$

   (c) What is $E \log(D_1/10)$?

   Solution: $\frac{1}{6} \sum_{k=1}^{6} \ln(k/10)$

   (d) Explain why the law of large numbers guarantees $\ln S_n \approx nE \log(D_1/10)$.

   Solution: $\ln S_n = \sum_{i=1}^{n} \ln(D_i/10)$. This is a sum of iid random variables, so if we divide by n it ought to converge to $E \ln D_1$.

   (e) What does $e^{\ln S_n}$ converge to?

   Solution: $e^{(n/6)\sum \ln(k/10)} = ((.1. * 2. * 3 * .4. * 5 * .6)^{1/6})^n = .29^n$.

   (f) There is a conflict between (b) and (e). Why are they different, and which answer actually predicts how close you will be to the tree after n steps?

   Solution: Unlikely events make the expected value larger than what the most likely behavior will be. The answer to (e) is more accurate.

Answers to Homework 3

5. 5.03
6. The right answer when $n = 100$ and $m = 10$ is 90.226. I don't want to give away the general formula.
10. $(3.5)^2$, 79.965
16. 0, no
20. .1111, .0013 (w/o histogram) .0021 (w/ histogram correction) You should use a histogram correction.
22. .309, .027
26. 0.177
31. .1056
34. .2702
37. 157
46. .359
49. $.04 \pm 2\sqrt{(.04)(.96)/625}$ so $[.024, .056]$.
59. ≥ 23