

1. Gita is a software engineer. She has a task that needs to be done in the next two hours, and knows she could do it by hand in that amount of time. She also knows she could write code to automate it in less than 30 minutes. The code itself will take some additional time to run. The script involves 8 tasks. Each must be done one after the other. Because of how the data is being processed, the time to perform two adjacent tasks is half as long as it takes to do the next task. The first task will take 2 seconds. The fourth will take 20 seconds.
- (a) Write a system of equations describing the length of each task.

- (b) She plugs this into her computer and gets the matrix

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 20 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 56 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 152 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 416 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1136 \end{pmatrix}$$

Given that it takes no more than 30 minutes to write the script. Will Gita be able to complete the task in under two hours? Explain.

2. (courtesy A. Raymond)

- (a) Use **Gauss-Jordan elimination** to find the general solution for the following system of linear equations:

$$\begin{aligned} & z_2 + 3z_3 - z_4 = 0 \\ -z_1 - z_2 - z_3 + z_4 &= 0 \\ -2z_1 - 4z_2 + 4z_3 - 2z_4 &= 0 \end{aligned}$$

- (b) Give an example of a solution to the previous system of linear equations.
- (c) The points $(1, 0, 3)$, $(1, 1, 1)$, and $(-2, -1, 2)$ lie on a unique plane $a_1x_1 + a_2x_2 + a_3x_3 = b$. **Using your previous answers**, find an equation for this plane. (Hint: think about the relationship between the previous system and the one you would need to solve in this question.)

3. Consider the following linear system,

$$\begin{aligned}y_1 - 3y_2 + 5y_3 &= b \\2y_1 + 4y_2 &= 10 \\3y_1 + 7y_2 + ay_3 &= 3b.\end{aligned}$$

We see that the system involves two unknown constants, a and b . You enter this into a computer and find that the system in echelon form is:

$$\left[\begin{array}{ccc|c} 1 & -3 & 5 & b \\ 0 & 1 & -1 & 1 - \frac{b}{5} \\ 0 & 0 & \frac{a+1}{16} & \frac{b}{5} - 1 \end{array} \right].$$

- (a) For what values of a and b does the system have no solution?
- (b) For what values does the system have infinitely many solutions?
- (c) Give an example of a pair of values a and b for which the system has exactly one solution, and find that solution.