1. What is $\int_0^{1/4} \frac{\tan(\pi x)}{[\ln(\cos(\pi x))+1]^5} dx$? *Give an exact answer.*

   Let $u = \ln(\cos(\pi x)) + 1$. The integral becomes

$$\frac{1}{\pi}\int_1^{\ln(\sqrt{2}/2)+1} u^{-5} du = \frac{1}{4\pi}\left[(\ln(\sqrt{2}/2)+1)^{-4} - 1\right].$$

2. Approximate $\int_1^5 \sqrt{x+1}\, dx$ using a righthand Riemann sum and $n = 4$ rectangles. Is this an over or underestimate?

$$1(\sqrt{3}+\sqrt{4}+\sqrt{5}+\sqrt{6})$$

   overestimate

3. Let A be the area of the region between the curves $y = \frac{1}{\sqrt{2}}$, $y = x^{1/4}$ and $y = x^2$.

   (a) Find A by integrating with respect to x. *Hint: Draw a graph - you will need two integrals.*

$$\int_0^{1/4} x^{1/4} - x^2 dx + \int_{1/4}^{1/2^{1/4}} \frac{1}{\sqrt{2}} - x^2 dx.$$

   (b) Find A by integrating with respect to y. *Hint: You only need one integral this time.*

$$\int_0^{1/\sqrt{2}} y^4 - \sqrt{y} dy.$$

4. For $x > 0$ find the value of x which minimizes the function $f(x) = \int_{2x}^{3x} \ln t dt$.

   The derivative is $f'(x) = 3\ln(3x) - 2\ln(2x)$. To minimize we solve $f'(x) = 0$.

$$\begin{aligned} 0 &= 3\ln(3x) - 2\ln(2x) \\ 0 &= \ln(27x^3) - \ln(4x^2) \\ 0 &= \ln\left(\frac{27x^3}{4x^2}\right) \\ 0 &= \ln\left(\frac{27}{4}x\right) \end{aligned}$$

   This means that $\frac{27}{4}x = 1$ and so $\boxed{x = \frac{4}{27}}$.

5. Let B be the region between the curve $y = \sqrt{x-1}$, the x-axis and the vertical line $x = 5$.

   (a) Set up an integral expressing the volume obtained from rotating B about the x-axis.

   $$\int_1^5 \pi(x-1)dx.$$

   (b) Set up the integral with respect to y for the volume of the solid obtained by rotating B about the y-axis.

   $$\int_0^2 \pi[5^2 - (y^2+1)^2]dy.$$

   (c) Set up the integral with respect to x for the volume of the solid obtained by rotating B about the y-axis.

   $$\int_1^5 2\pi x\sqrt{x-1}dx.$$

6. Compute the following integrals:

   (a) $\displaystyle\int_0^{2\pi} |\sin t - \cos t|\, dt$

   $$-\int_0^{\pi/4} \sin t - \cos t dt + \int_{\pi/4}^{5\pi/4} \sin t - \cos t dt - \int_{5\pi/4}^{2\pi} \sin t - \cos t dt$$

   (b) $\displaystyle\int \frac{1}{(1+\sqrt{x})^3}\, dx$

   Let $u = 1 + \sqrt{x}$ this means $du = \frac{1}{\sqrt{x}}dx$ and so $dx = 2\sqrt{x}du$. Making this substitution gives

   $$2\int \frac{\sqrt{x}}{u^3}du.$$

   To take care of the $\sqrt{x}$ we notice that $u - 1 = \sqrt{x}$. This integral becomes

   $$2\int \frac{u-1}{u^3}du = 2\int u^{-2} - u^{-3}du, \qquad \text{you can solve the integral from here}$$