LAKSHYA JEE

LAKSHYA KO HAR HAAL ME PAANA HAI

Relations & Functions

Lecture: 02

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Today's Goal: :

Introduction of Relations: ✓

Types of Relations: ✓

Introductions of Relations:

we can say that

A is father of B
A is wife of B
L_1 is parallel to L_2
} examples of relations

$2x+3y=5$ } eqⁿ in x & y represents relation in x & y

$x=1 \Rightarrow y=1$

$x=2 \Rightarrow y=\frac{1}{3}$

$\underbrace{\text{sum of } x \,\&\, y = 4}$

Domain, Co-domain & Range of Relations:

$R : \mathbb{Z} \to \mathbb{Z}$

$R = \{(a, b) \mid a^2 + 3b^2 = 28 ;\ a, b \in \mathbb{Z}\}$

Domain of a Relation:

The set of all first elements of the ordered pairs in a relation R form a set A to a set B is called the domain of the relation R.

find Domain & Range

$a^2 + 3b^2 = 28$

If $a = \pm 1 \Rightarrow b = \pm 3$

Range of a Relation:

The set of all second elements in a relation R from a set A to a set B is called the range of the relation R. The whole set B is called the codomain of the relation R. Note that range is subset of codomain.

If $a = \pm 2 \Rightarrow b$ Not integer

If $a = \pm 3 \Rightarrow b$ not integer

If $a = \pm 4 \Rightarrow b = \pm 2$

$a = \pm 5 \Rightarrow b = \pm 1$

Domain $= \{1, -1, 4, -4, 5, -5\}$

Range $= \{3, -3, 2, -2, 1, -1\}$

Problems based on Domain & Range:

Let A = $\{4, 5, 7\}$ and B = $\{2, 4, 6\}$ be two sets and let a relation R be a relation from A to B is defined as $R = \{(x, y) : x < y, x \in A, y \in B\}$, then the difference between the sum of elements of domain and range of R is-

(a) 2 (b) 3 (c) 4 (d) 5.

By Tuesday, Answer = 3

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Types of Relations:

1. **Empty / Void Relation** $\longrightarrow$ Number of null relation = 1

Let A be a set. Then ϕ subset of $\underbrace{A \times A}$ and so it is a relation on A. This relation is called the void or empty relation on A.

eg: $A = \{1, 3, 5\}$, $B = \{2, 4, 6\}$ } $R : A \to B$ is defined as

$R = \{(a, b) \mid |a-b| \text{ is even}, a \in A, b \in B\} = \phi$ (Null relation)

2. **Universal Relation** $\to$ $\boxed{\text{no of univer. relation} = 1}$

Let A be a set. Then $A \times A \subseteq A \times A$ and so it is a relation on A. This relation is called the universal relation on A.

$A = \{1, 2\}$, $B = \{3\}$ $\Rightarrow R = \{(1, 3), (2, 3)\} = A \times B$

Note :

The void and the universal relations on a set A are respectively the smallest and the largest relations of A.

Types of Relations:

3. **Identity Relation:**

Let A be a set. Then the relation $I_A = \{(a, a) : a \text{ is member of } A\}$ on A is called the identity relation on A.

In other words, a relation I_A on A is called the identity relation if every element of A is related to itself only.

$A = \{1, 2, 4\}$

$A \times A = \{(1,1), (1,2), (1,4), \ldots\}$

$R = \{(1,1), (2,2), (4,4)\}$ is Identity relation → no. of Identity relation $= 1$

$R = \{(1,1), (2,2), (4,4), (1,4)\}$ → Not identity relation

Types of Relations:

$n(A) = n$

no. of reflexive relations from A to A

$$\boxed{2^{n^2-n}}$$

No need to derive

4. **Reflexive Relation:**

A relation R on set A is said to be reflexive if every element of A is related to itself.

$A = \{1, 2, 4\}$

$A \times A$

$R_1 = \{(1,1), (2,2), (1,2)\}$ ✗

$R_2 = \{(1,1), (2,2), (4,4)\}$ ✓

$R_3 = \{(1,1), (2,2), (4,4), (4,2)\}$ ✓

5. **Symmetric Relation:**

A relation R on a set A is said to be a symmetric relation iff $(a, b) \in R \Rightarrow (b, a) \in R$ for all $a, b \in A$ *

i.e. $aRb \Rightarrow bRa$ for all $a, b \in A$

$A = \{1, 2, 4\}$

$R: A \to A$

$R_1 = \{(1,1), (1,2), (2,1)\}$ ✓

$R_2 = \{(1,1)\}$ ✓

$R_3 = \{(1,1), (2,4)\}$ ✗

$R_4 = \{(1,2), (2,4)\}$ ✗

No. of symmetric relations $= \frac{n(n+1)}{2}$

Types of Relations:

6. **Transitive Relation:**

Let A be any set. A relation R on A is said to be a transitive relation iff $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$ for all $a, b, c \in A$

i.e. aRb & $bRc \Rightarrow aRc$ for all $a, b, c \in A$.

If aRb
and bRc
then aRc $\Rightarrow$ then transitive

If aRb
and bRc
then $a \not R c$ $\Rightarrow$ not transitive

If $a \not R b$ and $b \not R c$
i.e. isolated relation is transitive relation by default.

$A = \{1, 2, 4\}$

$\underline{R : A \to A}$ ✓

~~R_1~~ $= \{(1,1), (2,4)\}$ as of isolated

~~R_2~~ $= \{(1,1)\} \Longrightarrow \{(1,1), (1,1), (1,1)\}$ wrong set

~~R_3~~ $= \{(1,2)\}$

$R_4 = \{(1,2), (2,4), (4,1)\}$ not transitive

$(1,4)$

A is wife of B

~~Transitive~~

(1, 1)

(1, 1)

(1, 1)

A is father of B

A is father of B

B is father of C

but A is not father of C

not transitive

Thank You Lakshyians