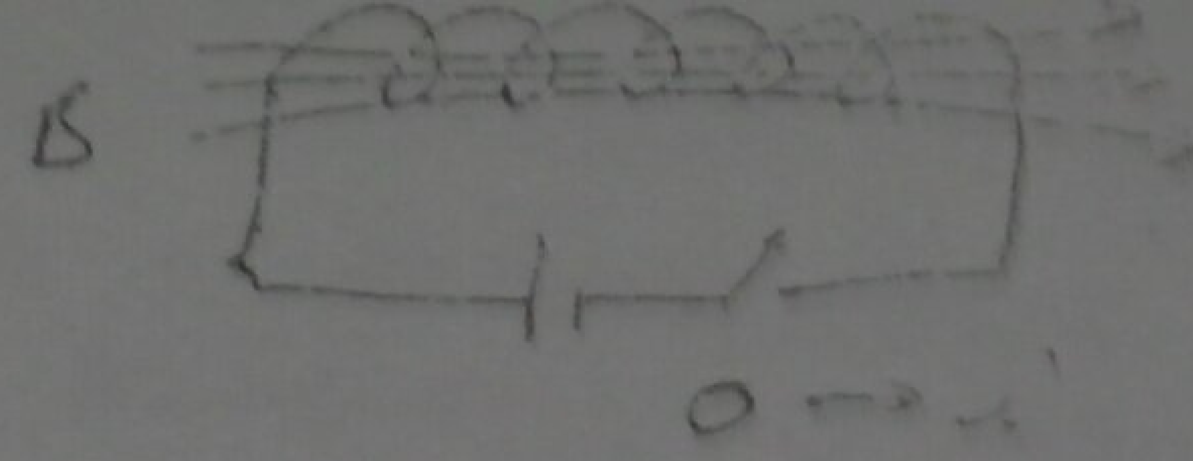


Analysis of Coupled Circuits

11.1 INTRODUCTION

In this chapter, the aim is to analyse magnetically coupled circuits [illegible] the interconnected loops of an electric network through magnetic [illegible]

11.2 SELF INDUCTANCE

When a current changes in a circuit, the magnetic flux linking the same circuit changes (and vice versa) and an e.m.f is induced in the circuit. This induced emf is proportional to the rate of change of current.

i.e.,

$$v = L\frac{di}{dt} \quad ...(11.1)$$

where v = induced voltage

$\frac{di}{dt}$ = rate of change of current

L = constant of proportionality called self-inductance

[The unit of self-inductance is *Henry*]

However, from the concepts of basic Electrical Engineering, the self-inductance is also expressed as

$$L = \frac{N\phi}{i} \quad ...(11.2)$$

$$\begin{bmatrix} N = \text{no. of turns in the circuit} \\ \phi = \text{flux linkage} \end{bmatrix}$$

Substitution of (11.2) in (11.1) yields

$$v = L\frac{d\left(\frac{N\phi}{L}\right)}{dt}$$

or

$$v = L \times \frac{1}{L} \times N\frac{d\phi}{dt} = N\frac{d\phi}{dt} \quad ...(11.3)$$

Comparing (11.1) and (11.3),

$$L\frac{di}{dt} = N\frac{d\phi}{dt}$$

or,

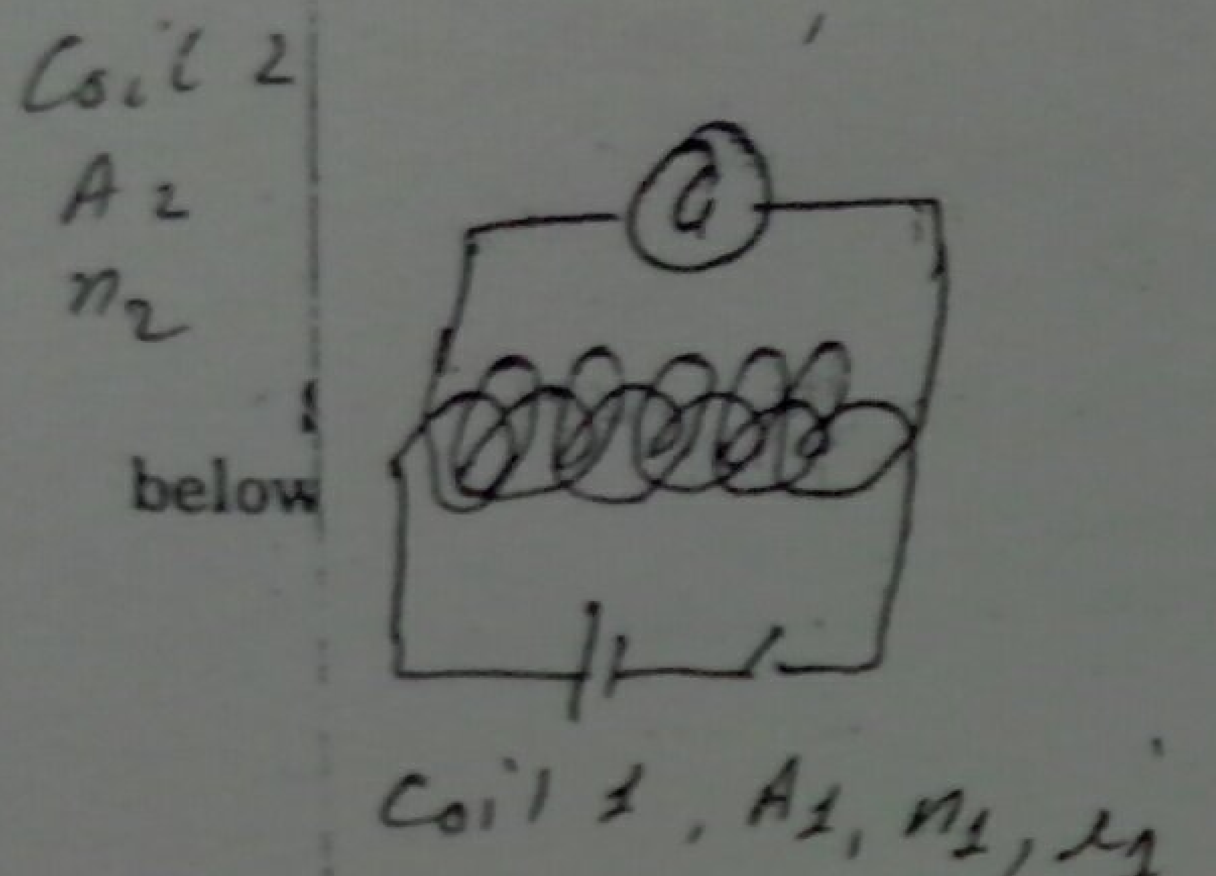
$$L = N\frac{d\phi}{di} \quad ...(11.4)$$

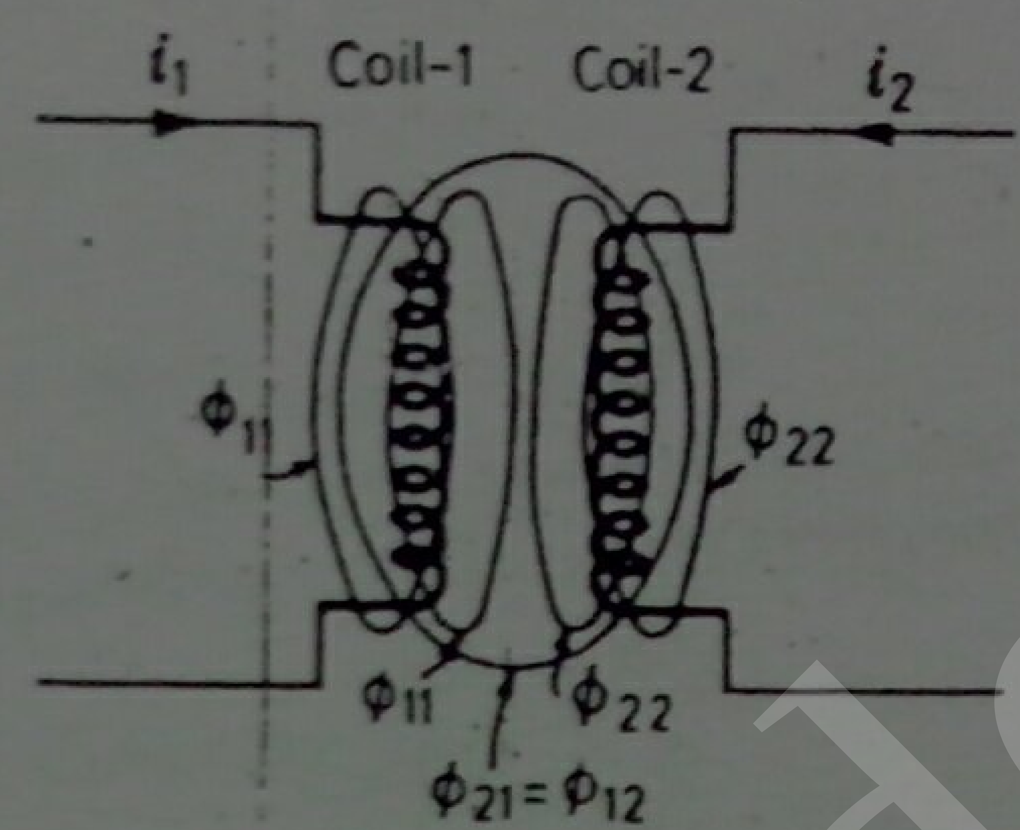
Expression (11.4) represents the magnitude of self-inductance [illegible]

Definition. – If two coils are placed in such a way that change of current in one coil induce emf in other coil, then this phenomenon is called mutual Induction

11.3 MUTUAL INDUCTANCE

In Fig. 11.1 let two coils carry currents i_1 and i_2 [both alternating currents]. Each coil will have leakage flux (ϕ_{11} and ϕ_{22} for coil 1 and coil 2 respectively) as well as mutual flux (ϕ_{21} or ϕ_{12} where, the flux of coil-2 links coil-1 or flux of coil-1 links coil-2).





[N_1 = no. of turns in coil-1
N_2 = no. of turns in coil-2]

Fig. 11.1. Linking of Flux.

$\phi_2 \propto i_1$

$\phi_2 = M i_1$

The induced voltage of coil-2 is given by

$$v_{L_2} = N_2 \frac{d\phi_{12}}{dt} \quad ...(11.5)$$

Again, since ϕ_{12} is related to the current of coil-1 and the induced voltage is proportional to the rate of change of i_1

$$\therefore \quad v_{L_2} = M \frac{di_1}{dt} \quad ...(11.6)$$

[M = constant of proportionality termed as *mutual inductance* between the two coils].

Comparing equations (11.5) and (11.6),

$$M \frac{di_1}{dt} = N_2 \frac{d\phi_{12}}{dt}$$

$$\therefore \quad M = N_2 \frac{d\phi_{12}}{di_1}. \quad ...(11.7)$$

Similarly, $$M = N_1 \frac{d\phi_{21}}{di_2} \quad ...(11.8)$$

When the coils are linked with air as medium, the flux and current are linearly related and the expressions of mutual inductance are modified as

$$M = N_2 \frac{\phi_{12}}{i_1} \quad ...(11.7(A))$$

and $$M = N_1 \frac{\phi_{21}}{i_2} \quad ...(11.8(A))$$

It may be observed at this stage that the mutual inductance is the bilateral property of the linked coils.

11.4 COEFFICIENT OF COUPLING

It is defined as the fraction of total flux that links the coils.

i.e., K, the *coefficient of coupling* $= \dfrac{\phi_{12}}{\phi_1} = \dfrac{\phi_{21}}{\phi_2}$

Since, $\phi_{12} < \phi_1$ and $\phi_{21} < \phi_2$ hence the maximum value of K is unity.

Multiplying equations (11.7(A)) and (11.8(A)), we get

$$M^2 = N_1 N_2 \frac{\phi_{21}\,\phi_{12}}{i_1\, i_2} = N_1 N_2 \frac{K\phi_1 \,.\, K\phi_2}{i_1 \,.\, i_2}$$

$$= K^2 N_1 \frac{\phi_1}{i_1} \,.\, N_2 \frac{\phi_2}{i_2}$$

$$= K^2 L_1 L_2 \qquad \left[\because\ L = \frac{N\phi}{i}\right]$$

$$\therefore \qquad M = K\sqrt{L_1 L_2} \qquad \text{...(11.9)}$$

11.5 SERIES CONNECTION OF COUPLED COILS

Let two coils, of self-inductances L_1 and L_2 are connected in series such that the voltage induced in coil-1 is v_{L_1} and that in coil-2 is v_{L_2} while a current i flows through them. Let M_{12} be the mutual inductance.

$$\therefore \qquad v_{L_1} = L_1 \frac{di}{dt} + M_{12}\frac{di}{dt} = (L_1 + M_{12})\frac{di}{dt} \qquad \text{...(11.10)}$$

and $$v_{L_2} = L_2 \frac{di}{dt} + M_{12}\frac{di}{dt} = (L_2 + M_{12})\frac{di}{dt} \qquad \text{...(11.11)}$$

$$\therefore \text{Net voltage} = v_L = v_{L_1} + v_{L_2} = (L_1 + M_{12})\frac{di}{dt} + (L_2 + M_{12})\frac{di}{dt}$$

$$= \frac{di}{dt}[L_1 + M_{12} + L_2 + M_{12}]$$

or, $$v_L = (L_1 + L_2 + 2M)\frac{di}{dt} \qquad [\text{putting } M_{12} = M]$$

$\therefore$ In case of series connection, as shown in Fig. 11.2, the total inductance

$$L = L_1 + L_2 + 2M \qquad \text{...(11:12)}$$

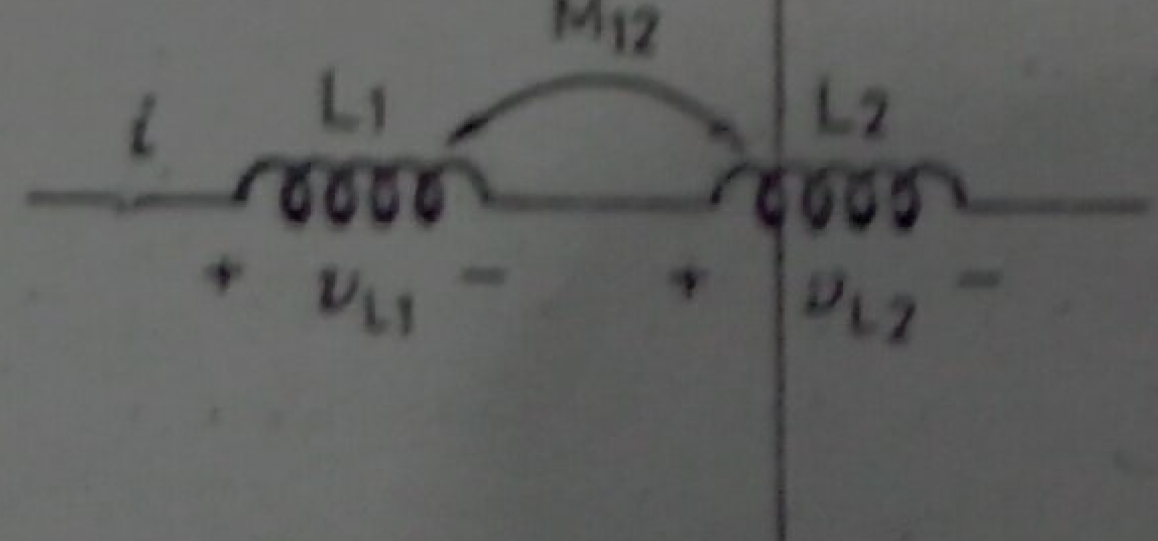


Fig. 11.2. Mutually Coupled coils in Series. [Here the flux of both the coils mutually assist each other]

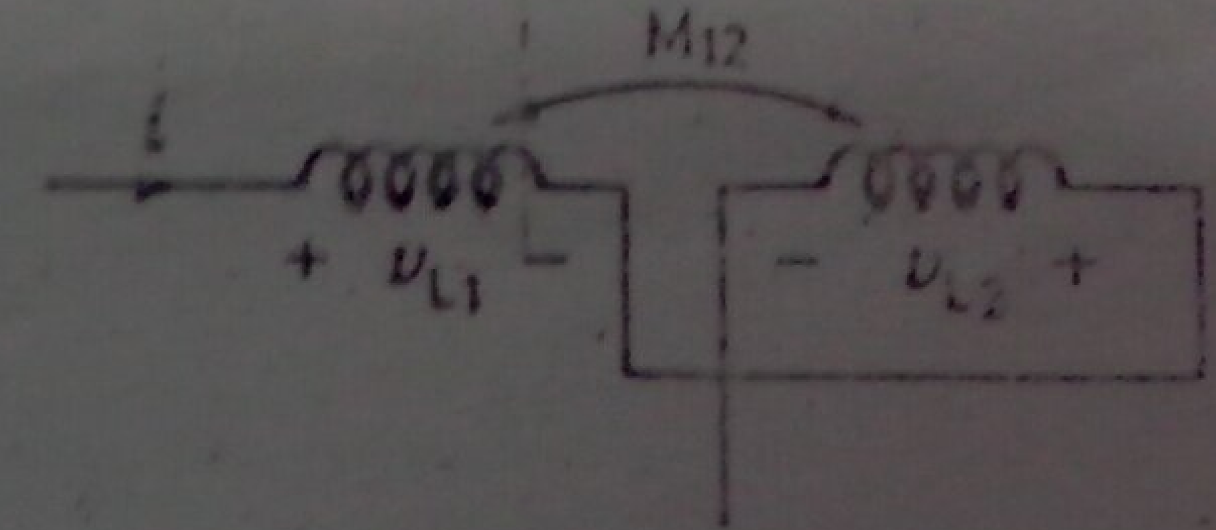


Fig. 11.3. Mutually Coupled coils in Series. [Here, the flux of both the coils mutually oppose each other]

However, in Fig. 11.3, where the coils are still series connected but the flux of both the coils oppose each other,

$$v_{L_1} = (L_1 - M_{12})\frac{di}{dt} \quad \text{and} \quad v_{L_2} = (L_2 - M_{12})\frac{di}{dt},$$

giving

$$v_L = (L_1 + L_2 - 2M_{12})\frac{di}{dt}.$$

The net inductance, here, is thus

$$L = (L_1 + L_2 - 2M) \qquad ...(11.13)$$

11.6 MODELLING OF COUPLED CIRCUITS

In Fig. 11.4, each circuit contains a voltage source and the respective mesh currents being i_1 and i_2, application of KVL yields

$$v_1 = R_1 i_1 + L_1\frac{di_1}{dt} \pm M\frac{di_2}{dt} \qquad ...(11.14)$$

and

$$v_2 = R_2 i_2 + L_2\frac{di_2}{dt} \pm M\frac{di_1}{dt} \qquad ...(11.15)$$

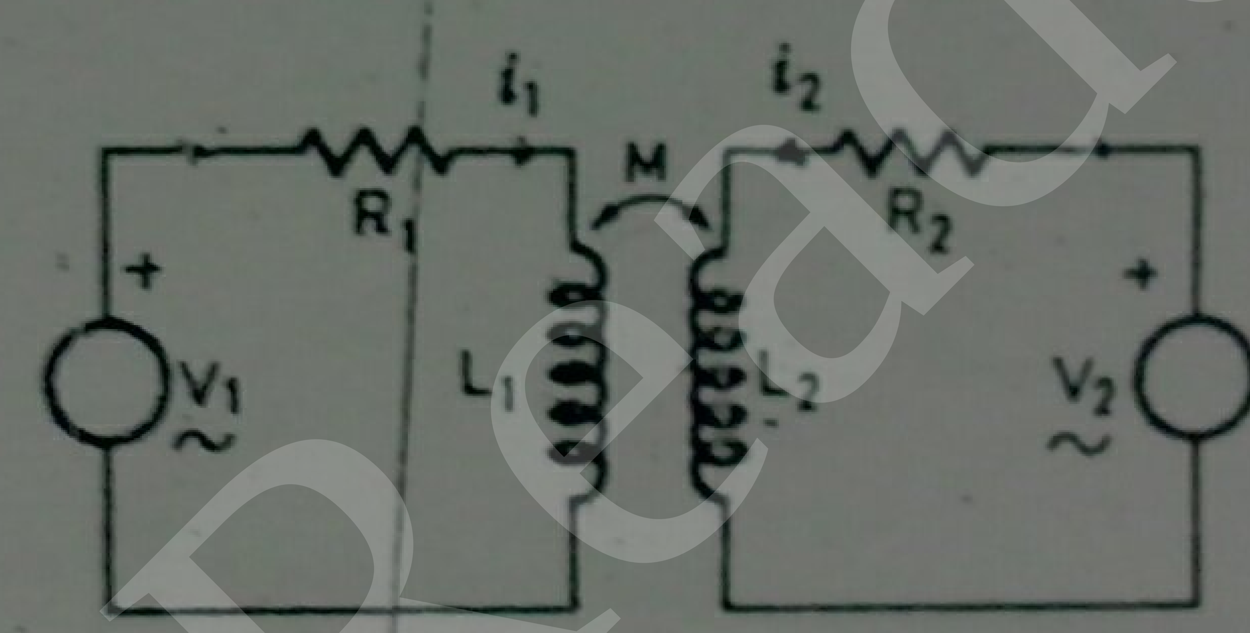


Fig. 11.4. Coupled coils fed by voltage sources.

If the flux of both the coils oppose each other, equations (11.14) and (11.15) modify as

$$v_1 = R_1 i_1 + L_1\frac{di_1}{dt} - M\frac{di_2}{dt} \qquad ...(11.14(A))$$

$$v_2 = R_2 i_2 + L_2\frac{di_2}{dt} - M\frac{di_1}{dt} \qquad ...(11.15(A))$$

Assuming sinusoidal voltage sources, at steady state, the equations (11.14(A)) and (11.15(A)) are further modified as

$$v_1 = I_1 R_1 + j\omega L_1 I_1 - j\omega M I_2$$

$$v_2 = I_2 R_2 + j\omega L_2 I_2 - j\omega M I_1$$

i.e.,

$$v_1 = (R_1 + j\omega L_1) I_1 - j\omega M I_2 \qquad ...(11.14(B))$$

$$v_2 = -j\omega M I_1 + (R_2 + j\omega L_2) I_2 \qquad ...(11.15(B))$$

In matrix form,

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \qquad ...(11.16)$$

where,

$$\left.\begin{matrix} Z_{11} = R_1 + j\omega L_1 & Z_{12} = -j\omega M \\ Z_{21} = -j\omega M & Z_{22} = R_2 + j\omega L_2 \end{matrix}\right\}$$

- On the otherhand if the flux of both coils assists each other, we can write

$$v_1 = R_1 i_1 + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \quad \text{and} \quad v_2 = R_2 i_2 + L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

This time $Z_{12} = Z_{21} = +j\omega M$

11.7 DOT CONVENTION IN COUPLED COILS

To determine the relative polarity of the induced voltage in the coupled coil, the coils are marked with *dots*. On each coil, a dot is placed at the terminals which are instantaneously of the same polarity on the basis of mutual inductance alone. When the currents through each of the mutually coupled coils are going away from the dot or towards the dot, the mutual inductance is +ve while for the case when the current through the coil is leaving the dot for one coil and entering the other, the mutual inductance is –ve (Fig. 11.5 (*a*) and (*b*)).

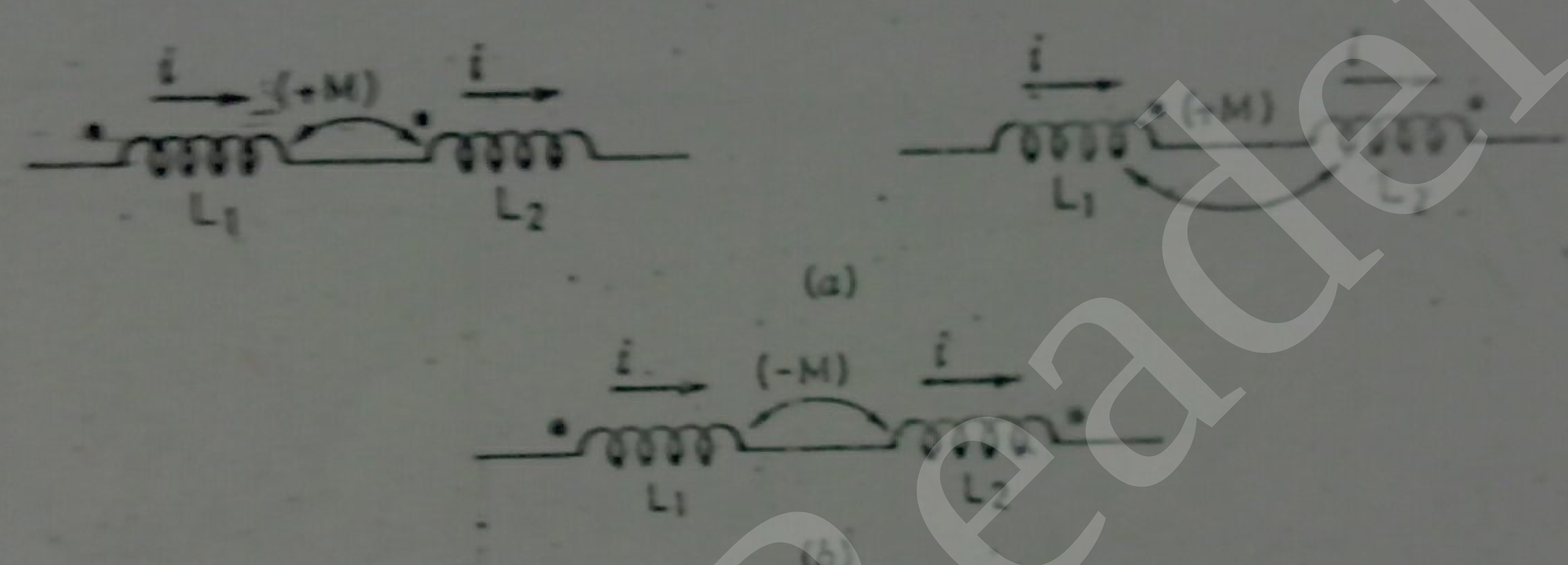
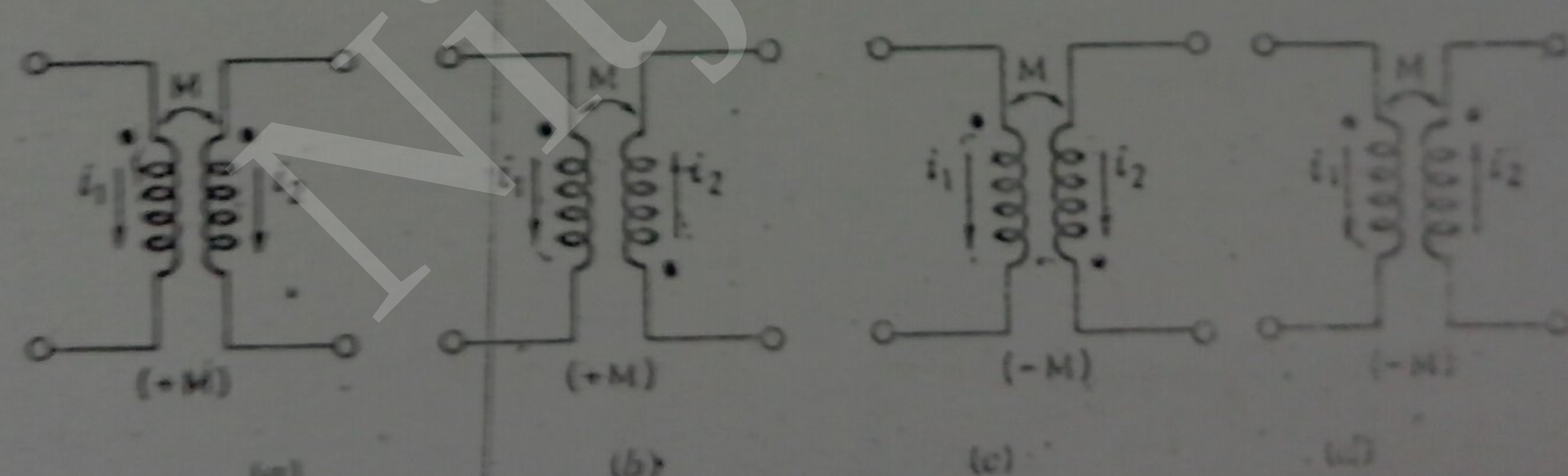


Fig. 11.5. +ve and -ve dot convention for series connected mutually coupled coils.

The dot convention of few possibilities of mutually coupled transformer coils are indicated below (Fig. 11.6).

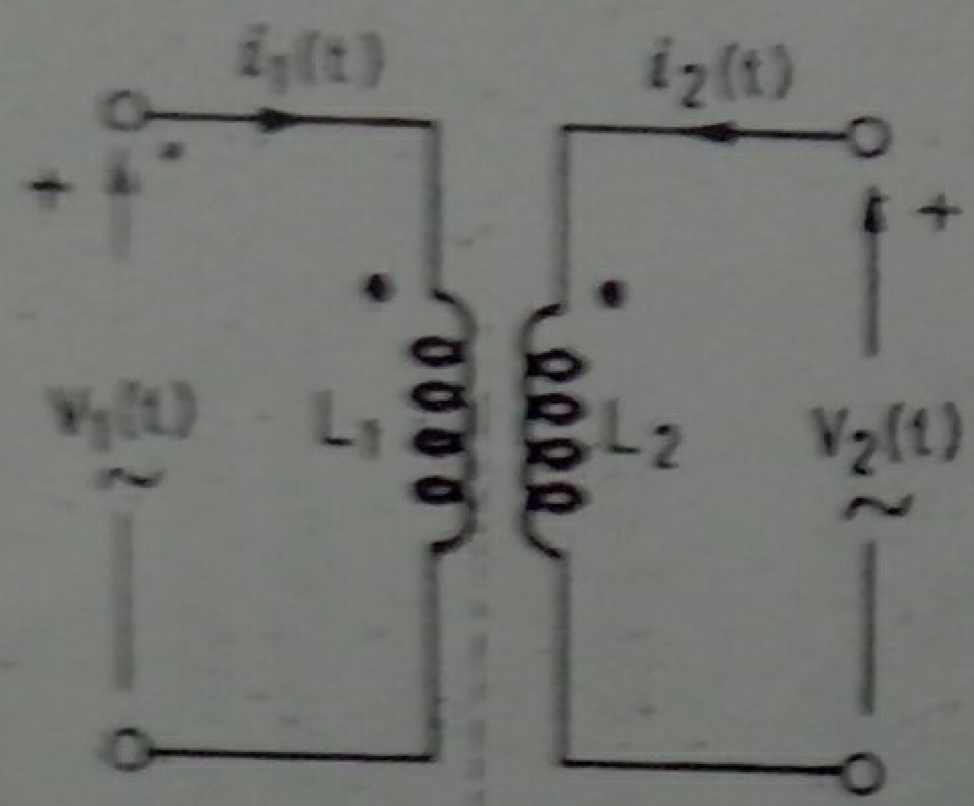


[Physically (-*M*) means that the windings of the coil are wound in reverse way from one another]

Fig. 11.6. Dot convention of transformer coils.

11.8 ELECTRICAL EQUIVALENTS OF MAGNETICALLY COUPLED CIRCUITS

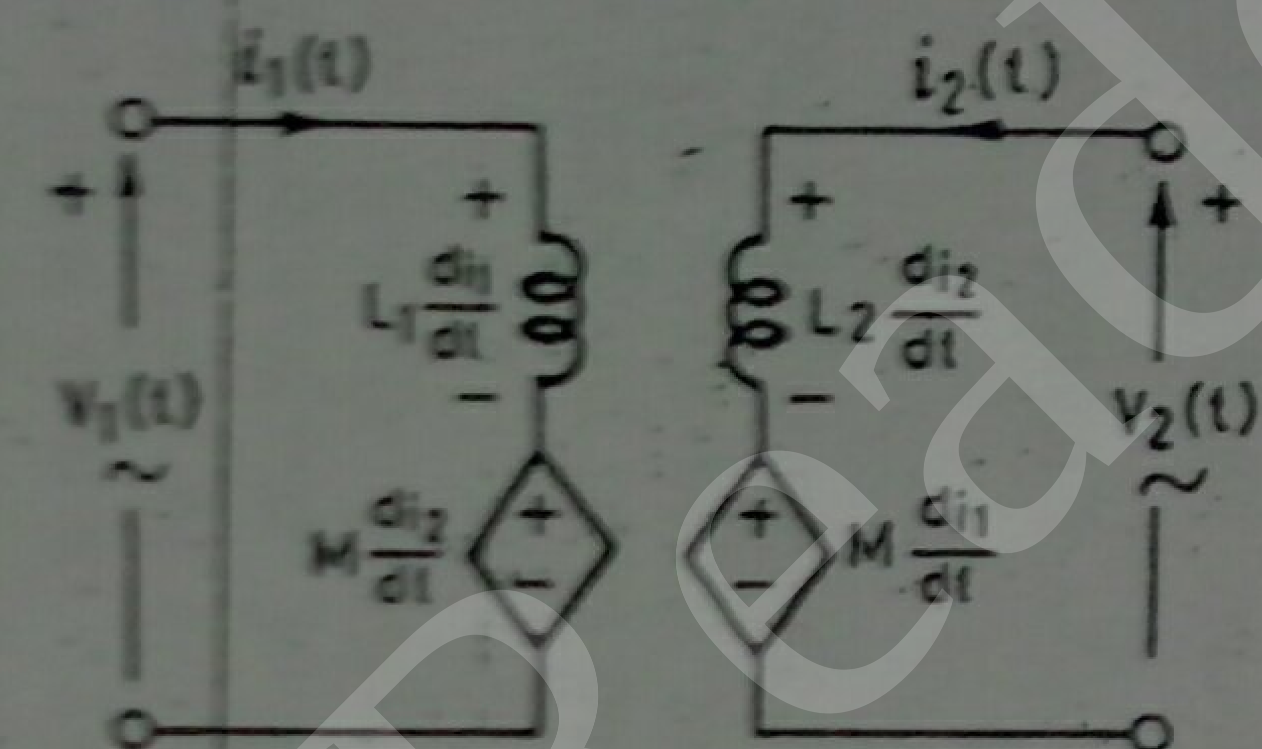
Fig. 11.7 represents circuit in time domain. In electrical equivalent representation of this circuit [Fig. 11.8 (*a*)], the mutually induced voltages may be shown as controlled voltage sources (*M* is +ve for both the loops as



$$
\begin{cases}
v_1(t) = L_1 \dfrac{di_1(t)}{dt} + M \dfrac{di_2(t)}{dt} \\
v_2(t) = L_2 \dfrac{di_2(t)}{dt} + M \dfrac{di_1(t)}{dt}
\end{cases}
$$

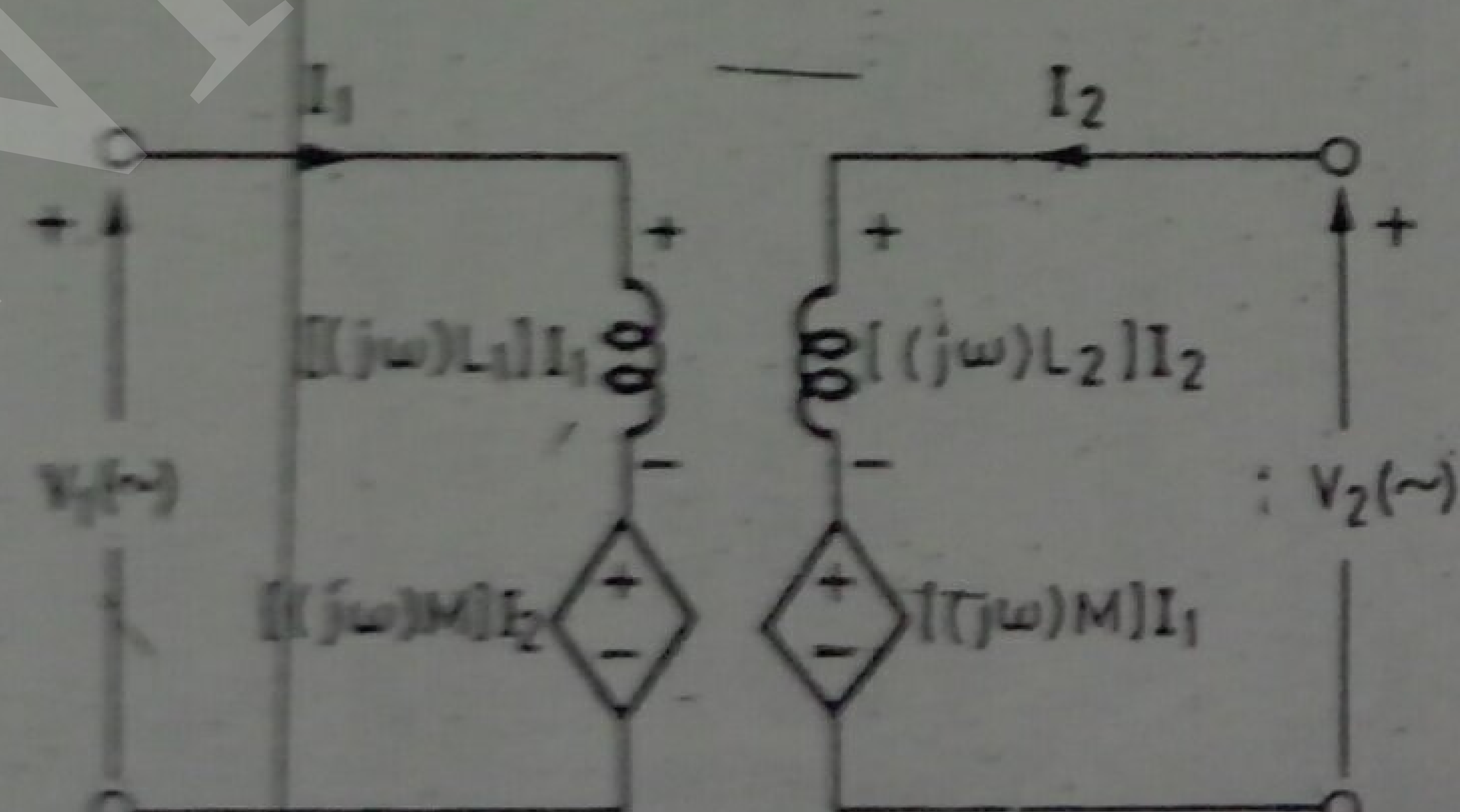
Fig. 11.7. Coupled circuit and the instantaneous mesh equations.

in both the coils the currents enter the dots). In the frequency domain equivalent circuit, '$j\omega$' term is to be included with the induced voltages (Ref. Fig. 11.8 (*b*)). Table 11.1 represents different combinations of couplings and their equivalent circuits.



[$L_1 \dfrac{di_1}{dt}$ and $L_2 \dfrac{di_2}{dt}$ are the drops in the self-inductances and $M \dfrac{di_1}{dt}$ as well as $M \dfrac{di_2}{dt}$ are the drops due to mutual inductance].

(*a*) Time domain representation of coupled circuits.

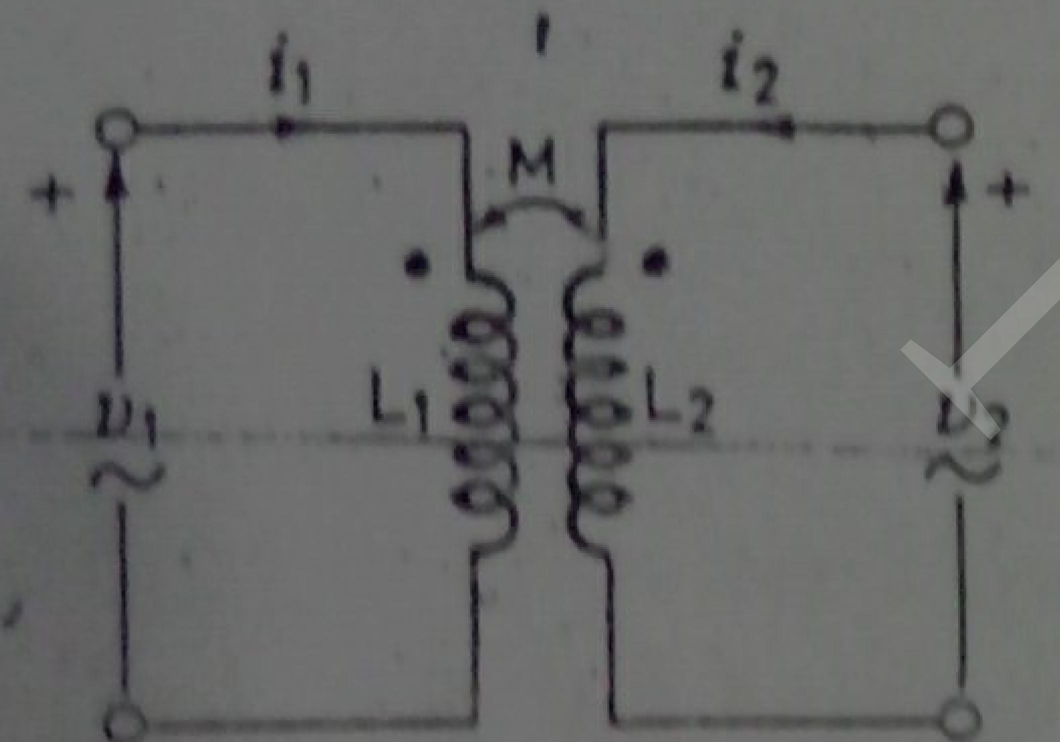
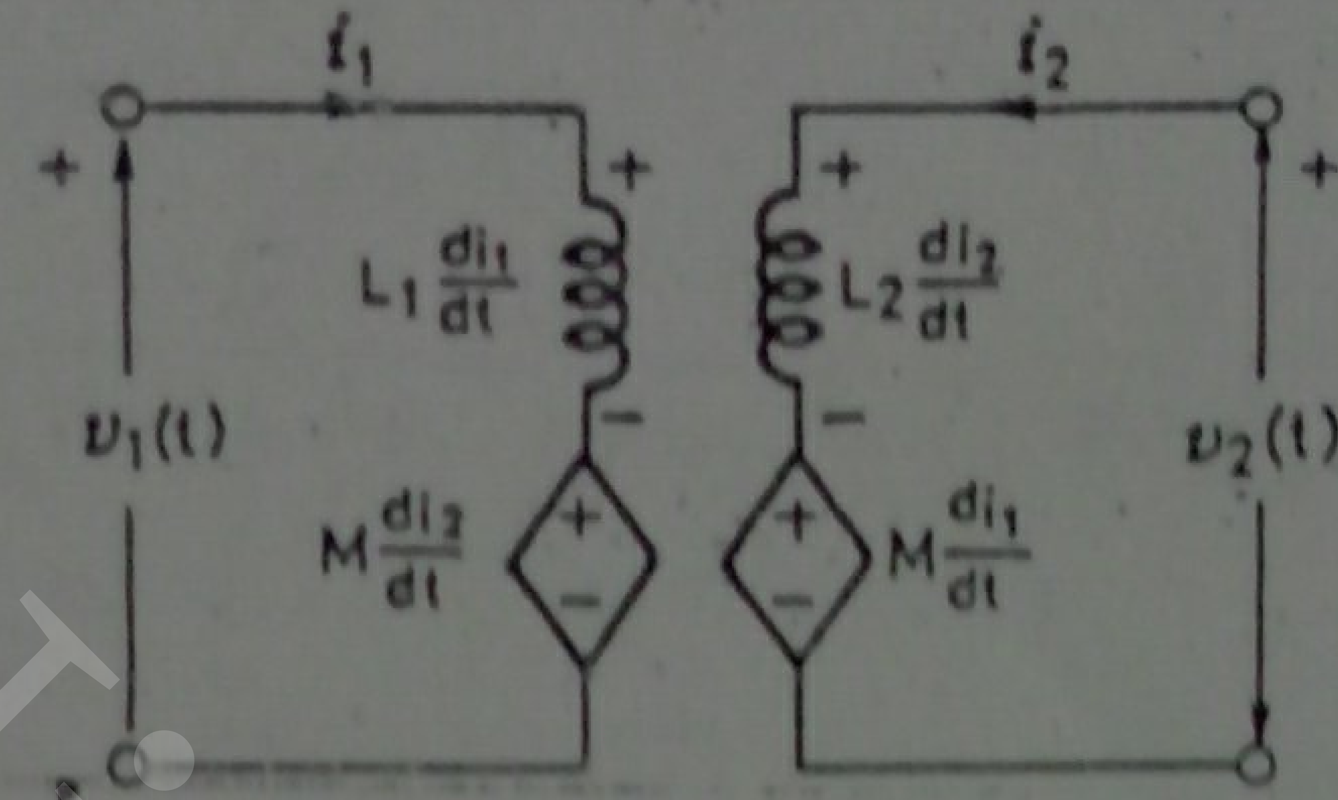


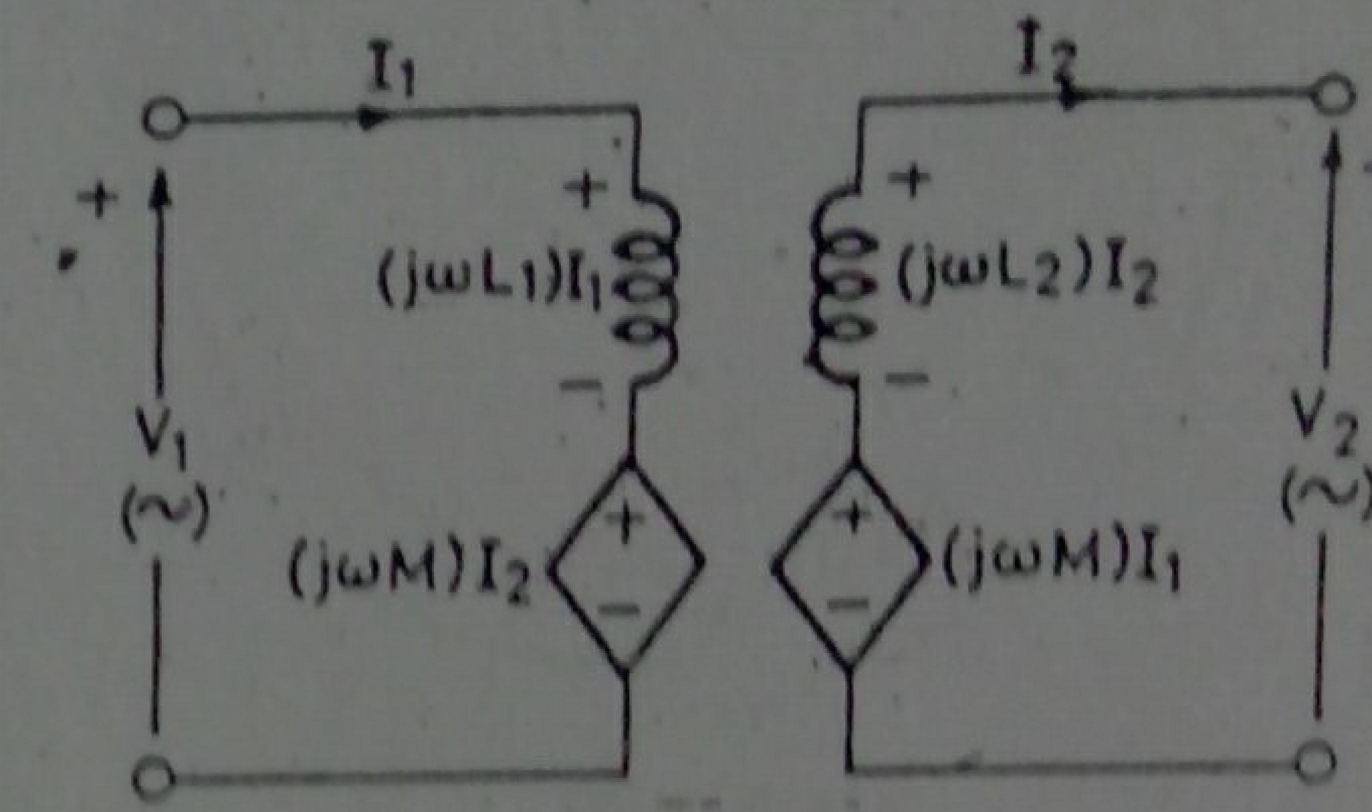
[Here, $V_1 = [(j\omega) L_1] I_1 + [(j\omega) M] I_2$
$V_2 = [(j\omega) M] I_1 + [(j\omega) L_2] I_2$].

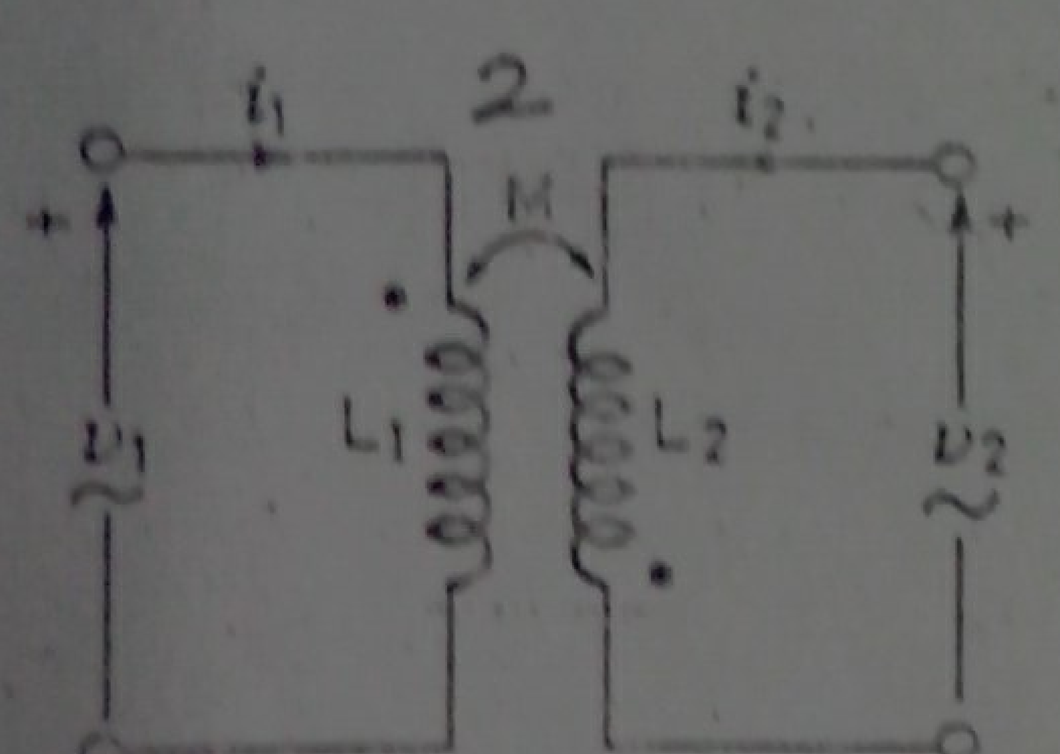
(*b*) Frequency-domain representation of coupled circuit.

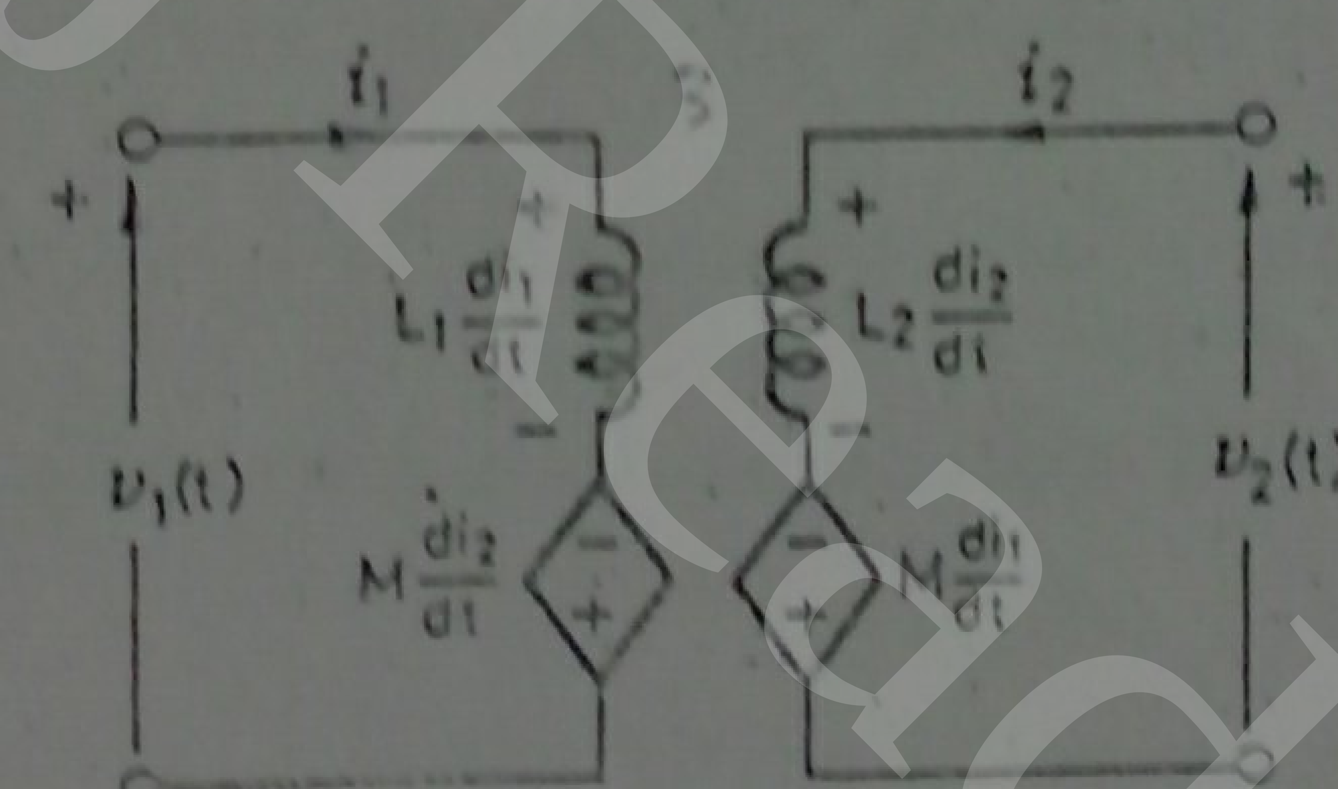
Fig. 11.8. Representation of coupled circuits in equivalent electrical form.

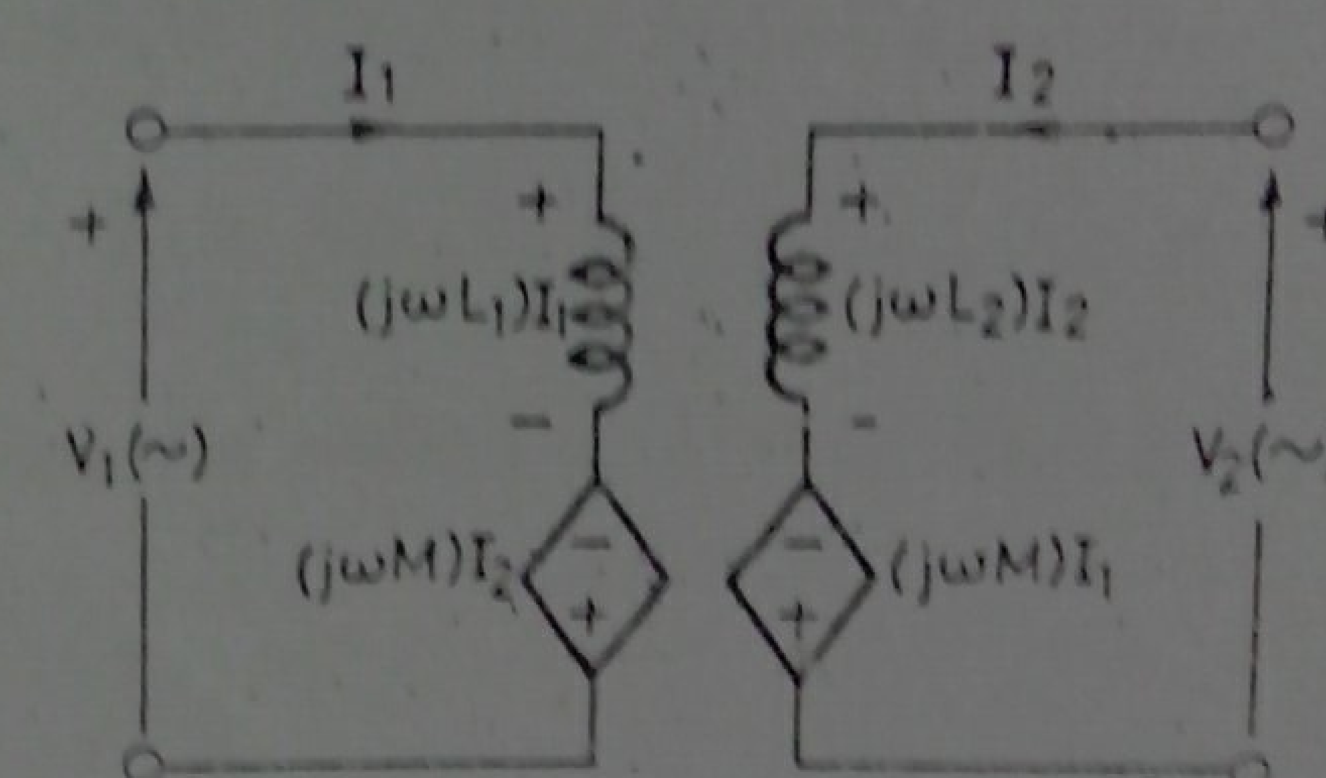
TABLE 11.1.

<table>
<tr><th>Different Combination of Coupling</th><th>Time Domain Equivalent Circuit</th><th>Frequency Domain Equivalent Circuit</th></tr>
<tr>
<td>[In both the coils, the currents enter the dot end and hence M term would be +ve in both the loops]

</td>
<td>

$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$
$v_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$</td>
<td>

$V_1 = j\omega L_1 \,.\, I_1 + j\omega M \,.\, I_2$
$V_2 = j\omega M I_1 + j\omega L_2 \,.\, I_2.$</td>
</tr>
<tr>
<td>

[In left loop, the current enters the dot end while in right loop, the current comes out of the dot end. Hence M terms will be –ve]</td>
<td>

$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$
$v_2 = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$</td>
<td>

$V_1 = (j\omega L_1) I_1 - (j\omega M) I_2$
$V_2 = -(j\omega M) I_1 + (j\omega L_2) I_2$</td>
</tr>
</table>

(Table Contd.)

6-22. Definitions Concerning Magnetic Circuit

1. **Magnetomotive force (m.m.f.).** It drives or tends to drive flux through a magnetic circuit and corresponds to electromotive force (e.m.f.) in an electric circuit.

M.M.F. is equal to the work done in joules in carrying a unit magnetic pole once through the entire magnetic circuit. It is measured in ampere-turns.

In fact, as p.d. between any two points is measured by the work done in carrying a unit charge from one point to another, similarly, m.m.f. between two points is measured by the work done in joules in carrying a unit magnetic pole from one point to another.

2. **Ampere-turns (AT).** It is the unit of magnetomotive force (m.m.f.) and is given by the product of number of turns of a magnetic circuit and the current in amperes in those turns.

3. **Reluctance.** It is the name given to that property of a material which opposes the creation of magnetic flux in it. It, in fact, measures the opposition offered to the passage of magnetic flux through a material and is analogous to resistance in an electric circuit even in form. Its unit is AT/Wb.*

$$\text{reluctance} = \frac{l}{\mu A} = \frac{l}{\mu_0 \mu_r A}\,; \quad \text{resistance} = \rho\frac{l}{A} = \frac{l}{\sigma A}$$

In other words, the reluctance of a magnetic circuit is the number of amp-turns required per weber of magnetic flux in the circuit. Since 1 AT/Wb = 1/henry, the unit of reluctance is "reciprocal henry."

4. **Permeance.** It is reciprocal of reluctance and implies the ease or readiness with which magnetic flux is developed. It is analogous to conductance in electric circuits. It is measured in terms of Wb/AT or henry.

5. **Reluctivity.** It is specific reluctance and corresponds to resistivity which is specific resistance'.

6-23. Composite Magnetic Circuit

In Fig. 6-26 is shown a composite magnetic circuit consisting of three different magnetic materials of different permeabilities and lengths and one air gap ($\mu_r = 1$). Each path will have its own reluctance. The total reluctance is the sum of individual reluctances as they are joined in series.

$$\therefore \quad \text{total reluctance} = \Sigma\frac{l}{\mu_0 \mu_r A}$$

$$= \frac{l_1}{\mu_0 \mu_{r1} A_1} + \frac{l_2}{\mu_0 \mu_{r2} A_2} + \frac{l_3}{\mu_0 \mu_{r3} A_3} + \frac{l_a}{\mu_0 A_g}$$

$$\therefore \quad \text{flux } \Phi = \frac{\text{m.m.f.}}{\Sigma \dfrac{l}{\mu_0 \mu_r A}}$$

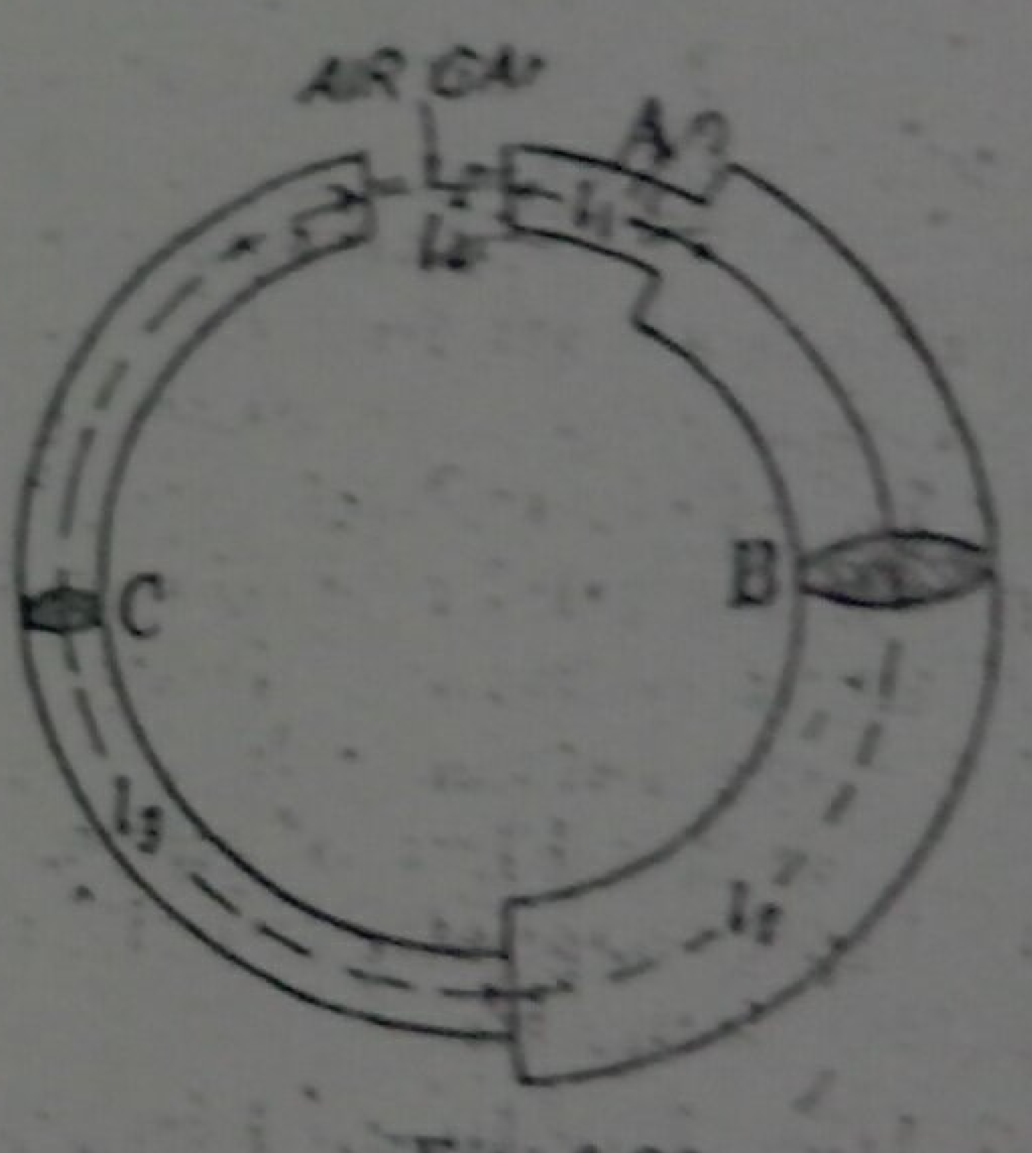


Fig. 6.26

6-24. How to Find Ampere-turns ?

It has been shown in Art. 6-15 that

$H = NI/l$ AT/m or $NI = H \times l$ $\therefore$ ampere-turns AT $= H \times l$

Hence, following procedure should be adopted for calculating the total ampere-turns of a composite magnetic path.

(i) Find H for each portion of the composite circuit. For air, $H = B/\mu_0$, otherwise $H = B/\mu_0\mu_r$.

(ii) Find ampere-turns for each path separately by using the relation AT $= H \times l$