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Park, Inverse Park  
and  
Clarke, Inverse Clarke  
Transformations MSS Software  
Implementation  
User Guide



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# Introduction

The behavior of three-phase machines is usually described by their voltage and current equations. The coefficients of the differential equations that describe their behavior are time varying (except when the rotor is stationary). The mathematical modeling of such a system tends to be complex since the flux linkages, induced voltages, and currents change continuously as the electric circuit is in relative motion. For such a complex electrical machine analysis, mathematical transformations are often used to decouple variables and to solve equations involving time varying quantities by referring all variables to a common frame of reference.

Among the various transformation methods available, the well known are:

- Clarke Transformation
- Park Transformation

## Clarke Transformation

This transformation converts balanced three-phase quantities into balanced two-phase quadrature quantities.

## Park Transformation

This transformation converts vectors in balanced two-phase orthogonal stationary system into orthogonal rotating reference frame.

Basically, the three reference frames considered in this implementation are:

1. Three-phase reference frame, in which  $I_a$ ,  $I_b$ , and  $I_c$  are co-planar three-phase quantities at an angle of 120 degrees to each other.
2. Orthogonal stationary reference frame, in which  $I_\alpha$  (along  $\alpha$  axis) and  $I_\beta$  (along  $\beta$  axis) are perpendicular to each other, but in the same plane as the three-phase reference frame.
3. Orthogonal rotating reference frame, in which  $I_d$  is at an angle  $\theta$  (rotation angle) to the  $\alpha$  axis and  $I_q$  is perpendicular to  $I_d$  along the  $q$  axis.

Figure 1 shows the three reference frames.

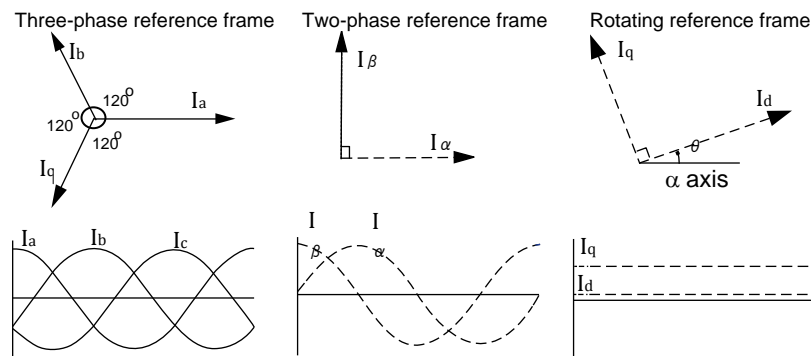
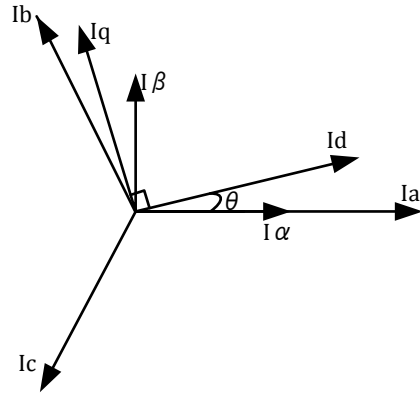


Figure 1 - Reference Frames

The combined representation of the quantities in all reference frames is shown in [Figure 2](#).

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**Figure 2 - Combined Vector Representation**

# Transformations Theory

## Introduction

Clarke and Park transformations are mainly used in vector control architectures related to permanent magnet synchronous machines (PMSM) and asynchronous machines. This section explains the Park, Inverse Park and Clarke, Inverse Clarke transformations.

## Clarke Transformation

The three-phase quantities are translated from the three-phase reference frame to the two-axis orthogonal stationary reference frame using Clarke transformation as shown in Figure 3. The Clarke transformation is expressed by the following equations:

$$I_{\alpha} = \frac{2}{3}(I_a) - \frac{1}{3}(I_b - I_c) \tag{EQ1}$$

$$I_{\beta} = \frac{2}{\sqrt{3}}(I_b - I_c) \tag{EQ2}$$

where,

$I_a$ ,  $I_b$ , and  $I_c$  are three-phase quantities

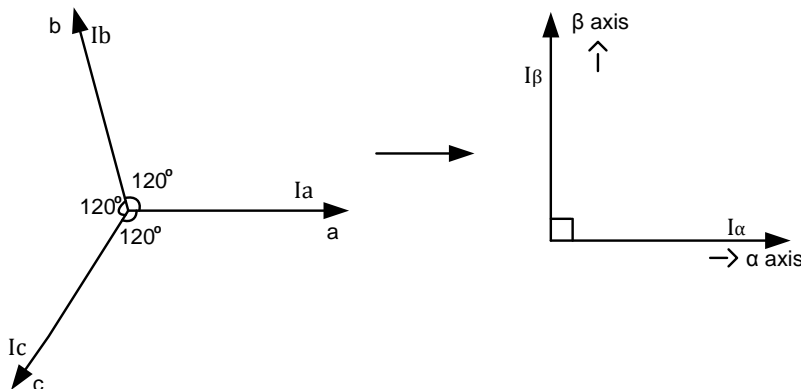
$I_{\alpha}$  and  $I_{\beta}$  are stationary orthogonal reference frame quantities

When  $I_a$  is superposed with  $I_a$  and  $I_a + I_b + I_c$  is zero,  $I_a$ ,  $I_b$ , and  $I_c$  can be transformed to  $I_{\alpha}$  and  $I_{\beta}$  as:

$$I_{\alpha} = I_a \tag{EQ3}$$

$$I_{\beta} = \frac{1}{\sqrt{3}}(I_b - I_c) \tag{EQ4}$$

where  $I_a + I_b + I_c = 0$



**Figure 3 · Clarke Transformation**

## Inverse Clarke Transformation

The transformation from a two-axis orthogonal stationary reference frame to a three-phase stationary reference frame is accomplished using Inverse Clarke transformation as shown in Figure 4. The Inverse Clarke transformation is expressed by the following equations:

$$V_a = V_\alpha \quad \text{EQ5}$$

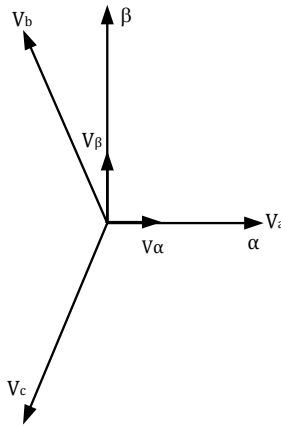
$$V_b = \frac{-V_\alpha + \sqrt{3} * V_\beta}{2} \quad \text{EQ6}$$

$$V_c = \frac{-V_\alpha - \sqrt{3} * V_\beta}{2} \quad \text{EQ7}$$

where,

$V_a, V_b, V_c$  are three-phase quantities

$V_\alpha, V_\beta$  are stationary orthogonal reference frame quantities



**Figure 4 · Inverse Clarke Transformation**

## Park Transformation

The two-axis orthogonal stationary reference frame quantities are transformed into rotating reference frame quantities using Park transformation as shown in Figure 5. The Park transformation is expressed by the following equations:

$$I_d = I_\alpha * \cos(\theta) + I_\beta * \sin(\theta) \quad \text{EQ8}$$

$$I_q = I_\beta * \cos(\theta) - I_\alpha * \sin(\theta) \quad \text{EQ9}$$

where,

$I_d, I_q$  are rotating reference frame quantities

$I_\alpha, I_\beta$  are orthogonal stationary reference frame quantities

$\theta$  is the rotation angle



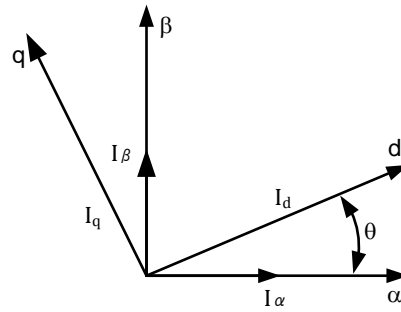


Figure 5 - Park Transformation

## Inverse Park Transformation

The quantities in rotating reference frame are transformed to two-axis orthogonal stationary reference frame using Inverse Park transformation as shown in Figure 6. The Inverse Park transformation is expressed by the following equations:

$$V_{\alpha} = V_d * \cos(\theta) - V_q * \sin(\theta) \tag{EQ10}$$

$$V_{\beta} = V_q * \cos(\theta) + V_d * \sin(\theta) \tag{EQ11}$$

where,

$V_{\alpha}$ ,  $V_{\beta}$  are orthogonal stationary reference frame quantities

$V_d$ ,  $V_q$  are rotating reference frame quantities

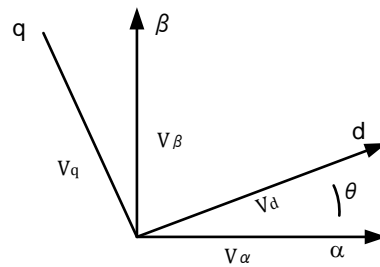


Figure 6 - Inverse Park Transformation



# API Type Definitions

This section describes the type definitions used in MSS software implementation of the Park, Inverse Park and Clarke, Inverse Clarke transformation blocks.

The following are the different data types defined for inputs and outputs.

## Park, Inverse Park and Clarke, Inverse Clarke Transformation Type Definitions

### clarkeinput\_type

Table 1 gives the type definition for the inputs of Clarke transform.

**Table 1 - clarkeinput\_type**

<b>Name</b>	clarkeinput_type	
<b>Type</b>	<pre>typedef struct { int32_t ia; int32_t ib; int32_t ic; }clarkeinput_type;</pre>	
<b>File</b>	Lib.h	
<b>Variable Description</b>	int32_t ia	The phase A current
	int32_t ib	The phase B current
	int32_t ic	The phase C current

### clarkeoutput\_type

Table 2 gives the type definition for the outputs of Clarke transform.

**Table 2 - clarkeoutput\_type**

<b>Name</b>	clarkeoutput_type	
<b>Type</b>	<pre>typedef struct { int32_t ialpha; int32_t ibeta; }clarkeoutput_type;</pre>	
<b>File</b>	Lib.h	
<b>Variable Description</b>	int32_t ialpha	The current component in stationary orthogonal reference frame on alpha axis
	int32_t ibeta	The current component in stationary orthogonal reference frame on beta axis

## invclarkeinput\_type

Table 3 gives the type definition for the inputs of Inverse Clarke transform.

**Table 3 - invclarkeinput\_type**

<b>Name</b>	invclarkeinput_type	
<b>Type</b>	<pre>typedef struct {   int32_t valpha;   int32_t vbeta; }invclarkeinput_type;</pre>	
<b>File</b>	Lib.h	
<b>Variable Description</b>	int32_t valpha	The voltage component in stationary orthogonal reference frame ( $V_\alpha$ )
	int32_t vbeta	The voltage component in stationary orthogonal reference frame ( $V_\beta$ )

## invclarkeoutput\_type

Table 4 gives the type definition for the outputs of Inverse Clarke transform.

**Table 4 - invclarkeoutput\_type**

<b>Name</b>	invclarkeoutput_type	
<b>Type</b>	<pre>typedef struct {   int32_t va;   int32_t vb;   int32_t vc; }invclarkeoutput_type;</pre>	
<b>File</b>	Lib.h	
<b>Variable Description</b>	int32_t va,vb,vc	The Phase Voltages in three phase stationary reference frame

## parkinput\_type

Table 5 gives the type definition for the inputs of Park transform.

**Table 5 - parkinput\_type**

<b>Name</b>	parkinput_type	
<b>Type</b>	<pre>typedef struct {   int32_t ialpha;   int32_t ibeta;   int32_t cos;   int32_t sin; }parkinput_type;</pre>	
<b>File</b>	Lib.h	
<b>Variable Description</b>	int32_t ialpha	The current component in stationary orthogonal reference frame on alpha axis
	int32_t ibeta	The current component in stationary orthogonal reference frame on beta axis

	int32_t cos	Cosine component of electrical angle
	int32_t sin	Sine component of electrical angle

## parkoutput\_type

Table 6 gives the type definition for the outputs of Park transform.

**Table 6 · parkoutput\_type**

<b>Name</b>	parkoutput_type	
<b>Type</b>	<pre>typedef struct { int32_t id; int32_t iq; }parkoutput_type;</pre>	
<b>File</b>	Lib.h	
<b>Variable Description</b>	int32_t id	The direct axis current component in rotor reference frame ( $I_d$ )
	int32_t iq	The quadrature axis current component in rotor reference frame ( $I_q$ )

## invparkinput\_type

Table 7 gives the type definition for the inputs of Inverse Park transform.

**Table 7 · invparkoutput\_type**

<b>Name</b>	invparkinput_type	
<b>Type</b>	<pre>typedef struct { int32_t vd; int32_t vq; int32_t cos; int32_t sin; }invparkinput_type;</pre>	
<b>File</b>	Lib.h	
<b>Variable Description</b>	int32_t vd	The direct axis voltage component in rotor reference frame ( $V_d$ )
	int32_t vq	The quadrature axis voltage component in rotor reference frame ( $V_q$ )
	int32_t cos	Cosine component of electrical angle
	int32_t sin	Sine component of electrical angle

## invparkoutput\_type

Table 8 gives the type definition for the outputs of Inverse Park transform.

**Table 8 - invparkoutput\_type**

<b>Name</b>	invparkoutput_type	
<b>Type</b>	<pre>typedef struct {   int32_t valpha;   int32_t vbeta; }invparkoutput_type;</pre>	
<b>File</b>	Lib.h	
<b>Variable Description</b>	int32_t valpha	The voltage component in stationary orthogonal reference frame ( $V_\alpha$ )
	int32_t vbeta	The voltage component in stationary orthogonal reference frame ( $V_\beta$ )

# API Functions Description

In the current implementation, Clarke transform is used to determine the real ( $i\alpha$ ) and imaginary ( $i\beta$ ) currents from the three phase currents. Inverse Clarke transform is used to determine the three phase voltages in stationary reference frame. Park transform is used for the transformation of real ( $i\alpha$ ) and imaginary ( $i\beta$ ) currents from the stationary to the moving reference frame ( $i_d$ ,  $i_q$ ). Inverse Park transform determines the stationary orthogonal reference frame voltages ( $v\alpha$ ,  $v\beta$ ) from the moving reference frame voltages ( $v_d$ ,  $v_q$ ).

## Park, Inverse Park and Clarke, Inverse Clarke Transformation

This section gives the description of various functions used in the software implementation.

### Clarke\_Lib\_Do

Table 9 describes the Clarke\_Lib\_Do API.

**Table 9 · Specification of API Clarke\_Lib\_Do**

<b>Syntax</b>	void Clarke_Lib_Do(const clarkeinput_type *threephasecurrent_ptr, clarkeoutput_type *twophaserefcurrent_ptr)
<b>Re-entrancy</b>	Re-entrant
<b>Parameters (Inputs)</b>	threephasecurrent_ptr: Pointer to the input structure
<b>Parameters (output)</b>	twophaserefcurrent_ptr: Pointer to the output structure
<b>Return</b>	None
<b>Algorithm Description</b>	This function computes the following equations: $i_{alpha} = i_a$ $i_{beta} = \frac{1}{\sqrt{3}}(i_a + 2 * i_b)$

### InvClarke\_Lib\_Do

Table 10 describes the InvClarke\_Lib\_Do API.

**Table 10 · Specification of API InvClarke\_Lib\_Do**

<b>Syntax</b>	void InvClarke_Lib_Do (const invclarkeinput_type *invclarkeinput_ptr, invclarkeoutput_type *invclarkeoutput_ptr)
<b>Re-entrancy</b>	Re-entrant
<b>Parameters (Inputs)</b>	invclarkeinput_ptr: Pointer to the Inverse Clarke input structure
<b>Parameters (output)</b>	invclarkeoutput_ptr: Pointer to the Inverse Clarke output structure
<b>Return</b>	None
<b>Algorithm Description</b>	This function computes the following equations: $v_a = v_{alpha}$ $v_b = (-v_{alpha} + \sqrt{3} * v_{beta})/2$ $v_c = (-v_{alpha} - \sqrt{3} * v_{beta})/2$

## Park\_Lib\_Do

Table 11 describes the Park\_Lib\_Do API.

**Table 11 - Specification of API Park\_Lib\_Do**

<b>Syntax</b>	void Park_Lib_Do(const parkinput_type *parkinput_ptr, parkoutput_type *parkoutput_ptr)
<b>Re-entrancy</b>	Re-entrant
<b>Parameters (Inputs)</b>	parkinput_ptr: Pointer to the park input structure
<b>Parameters (output)</b>	parkoutput_ptr: Pointer to the park output structure
<b>Return</b>	None
<b>Algorithm Description</b>	This function computes the following equations: $i_d = ialpha * \cos(\theta) + ibeta * \sin(\theta)$ $i_q = ibeta * \cos(\theta) - ialpha * \sin(\theta)$

## InvPark\_Lib\_Do

Table 12 describes the InvPark\_Lib\_Do API.

**Table 12 - Specification of API InvPark\_Lib\_Do**

<b>Syntax</b>	void InvPark_Lib_Do (const invparkinput_type *invparkinput_ptr, invparkoutput_type *invparkoutput_ptr)
<b>Re-entrancy</b>	Re-entrant
<b>Parameters (Inputs)</b>	invparkinput_ptr: Pointer to the Inverse park input structure
<b>Parameters (output)</b>	invparkoutput_ptr: Pointer to the Inverse park output structure
<b>Return</b>	None
<b>Algorithm Description</b>	This function computes the following equations: $valpha = v_d * \cos(\theta) - v_q * \sin(\theta)$ $vbeta = v_q * \cos(\theta) + v_d * \sin(\theta)$



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From the rest of the world, call **650.318.4460**  
Fax, from anywhere in the world **408.643.6913**

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## Technical Support

Visit the Microsemi SoC Products Group Customer Support website for more information and support (<http://www.microsemi.com/soc/support/search/default.aspx>). Many answers available on the searchable web resource include diagrams, illustrations, and links to other resources on website.

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You can browse a variety of technical and non-technical information on the Microsemi SoC Products Group [home page](http://www.microsemi.com/soc/), at <http://www.microsemi.com/soc/>.

## Contacting the Customer Technical Support Center

Highly skilled engineers staff the Technical Support Center. The Technical Support Center can be contacted by email or through the Microsemi SoC Products Group website.

### Email

You can communicate your technical questions to our email address and receive answers back by email, fax, or phone. Also, if you have design problems, you can email your design files to receive assistance. We constantly monitor the email account throughout the day. When sending your request to us, please be sure to include your full name, company name, and your contact information for efficient processing of your request.

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