SHAPE COULOMBIZATION

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Abstract

Canonical shape analysis is a popular method in deformable shape matching, trying to bring the shape into a canonical form that undoes its non-rigid deformations, thus reducing the problem of non-rigid matching into a rigid one.

As a result, the shape canonization process replaces the original shape by its stretched-thermostat variant.

Influenced by natural phenomena, we propose to perform such a stretching by the simulation of electrostatic repulsion among the vertices of the shape.

Our approach [5]

The repulsion forces can be written as

\[ F \propto \frac{1}{||x_i - x_j||^2} \]

where \( d_{ij} \) is an intrinsic metric.

\[ \text{Coulomb energy: } \mathcal{E}(X) = - \nabla F \]

We propose to solve our problem using alternating minimization:

- step(s) of unconstrained minimization:
  \( X^{(t)} = X^{(t-1)} - \epsilon \nabla \mathcal{E}(X^{(t-1)}) \)

- projection on metric constraints:
  \( X^{(t)} = \text{proj}(X^{(t)}) \)

If the metric constraints are imposed exactly, such a canonical representation is isometric (no metric distortion). However, since closed polyhedral surfaces are known to be rigid, it is necessary to relax the metric constraints.

Handling topological noise

Due to the local nature of the topological noise, we consider an \( L^1 \) violation of the constraints

\[ \sum_{(i,j) \in E} |d_{ij} - ||x_i - x_j|||, \]

in order to exploit the sparsity-inducing properties of the \( L^1 \) norm. In [4], the authors show that the new problem can be solved by a simple re-weighting of the previous fixed-point iteration.

Related works

Elad and Kimmel [3] proposed to perform the canonization by measuring geodesic distances on the shape and embedding them into a Euclidean space by means of multidimensional scaling (MDS):

\[ \min_{X = \{x_1, ..., x_n\}} \sum_{i,j=1}^n (d_{ij} - ||x_i - x_j||)^2. \]

where \( d_{ij} \) is an intrinsic metric.

If the embedding is isometric, then intrinsic similarity between original shapes = extrinsic similarity between canonical forms:

- distortion is data dependent
- sensitivity to topological noise

Main drawbacks:

- \( \mu \) stands for the valence of the vertex \( x_i \).

Distortion control

There are several measures to control the distortion:

- precision-recall curves

Retrieval results

Shapes from TOSCA dataset.

References