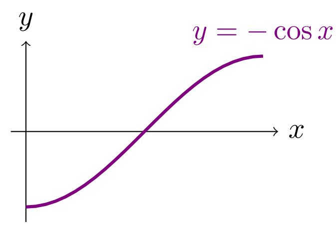
1. A trough shaped like the solid obtained by rotating the curve $y = -\cos x$ from $0 \leq x \leq \pi$ around the y-axis is filled with water. The trough has a spigot 4 meters above its top rim.



   (a) Write an integral for the work done to pump all of the water out of the trough.

   (b) Part of the integral is similar to $\int (\arccos(-y))^2 dy$. Compute this integral.

   (c) The other part is similar to $\int y(\arccos(-y))^2 dy$. Compute this. (DIFFICULT - DO THIS PROBLEM LAST.)

   (d) Put together (a), (b), (c) to find the work done pumping the trough dry. (DITTO)

2. A vat contains 100 liters of yogurt. Pure yogurt flows in at 5 L/min. A bacteria colony is growing inside the vat (assume it is mixed uniformly at all times). Mixed yogurt flows out at 5 L/min. Initially there are 1000 bacteria. Let $P(t)$ represent the number of bacteria at time t. Left alone, the bacteria grow at a rate proportional to P (i.e. kP for some constant k).

   (a) Find an equation for $\frac{dP}{dt}$ in terms of k and t.

   (b) Solve for $P(t)$ in terms of k and t.

   (c) Suppose that left alone the bacteria population doubles every hour. Compute $\lim_{t\to\infty} P(t)$.

   (d) Let k be as in (c). Instead of pure yogurt, say we let bacteria filled yogurt flow in at 5 L/min. What concentration of bacteria flowing in (call it b bacteria/liter) will result in $\lim_{t\to\infty} P(t) = 1000$?

3. Compute the following integrals.

   (a) $\displaystyle\int_1^\infty \frac{1}{t(1+\sqrt{t})^2}\,dt$

   (b) $\displaystyle\int \frac{x^2}{\sqrt{8-2x-x^2}}\,dx.$

4. Given $y(0) = 1$, solve the following differential equation for $y(x)$

$$(1 - 2x)(1 + y^2) = y'.$$

5. Let $F(x) = \int_{-x}^{\sin x} e^{\sqrt{t}} dt$.

   (a) Find $\lim_{h \to 0} \frac{F(h)}{h}$.

   (b) Setup (but DO NOT evaluate) an integral expressing the arc length of F from on $[0, \pi]$.