ELB

Harvard Economics 1011B
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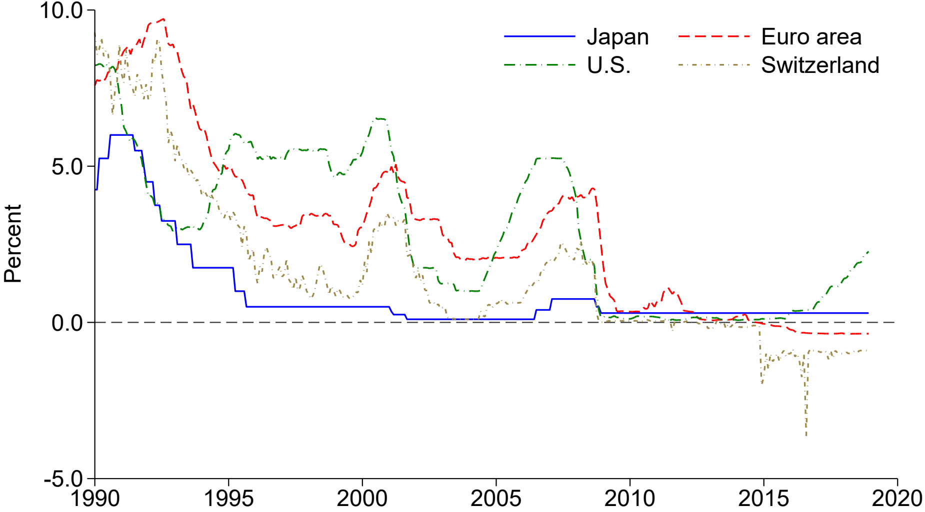
Outline

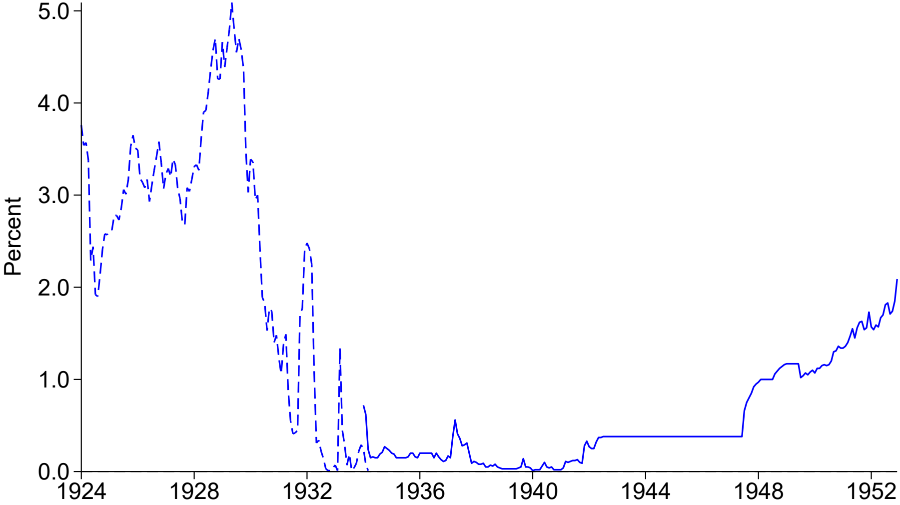
1. What is the effective lower bound?
2. Krugman (BPEA 1998) flexible price
3. Krugman (BPEA 1998) sticky price
4. Adding uncertainty

Outline

Short term interest rates around the world



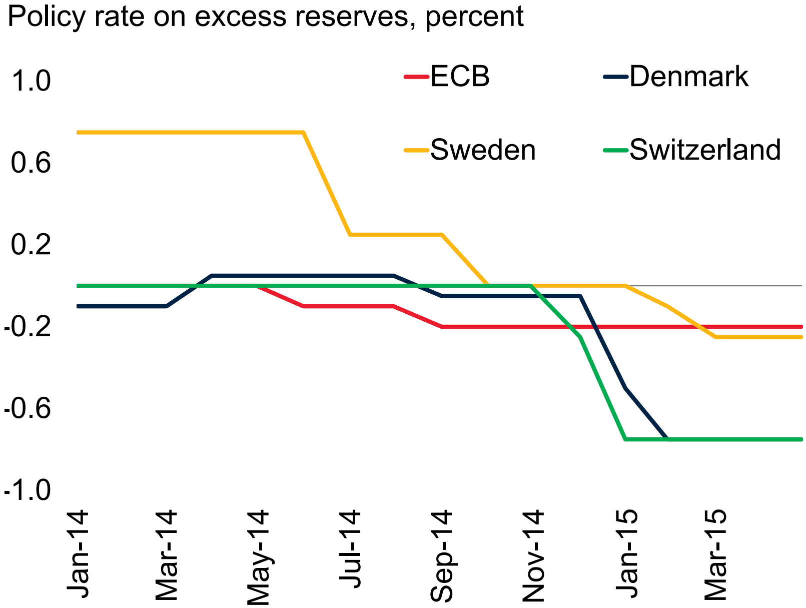
U.S. TBill rate during Great Depression



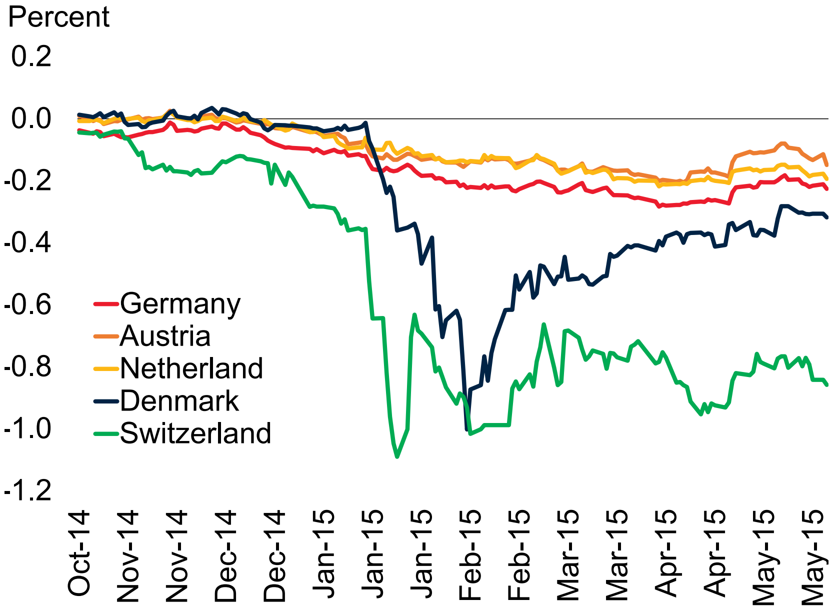
ZLB IN THEORY

- Investment/liquidity choice: hold bond or currency.
- Bond pays interest $i_{t,t+1}$. Currency pays interest of zero.
- If $i_{t,t+1} < 0$, then hold currency instead.
- When $i_{t,t+1} = 0$, money and bonds are perfect substitutes.

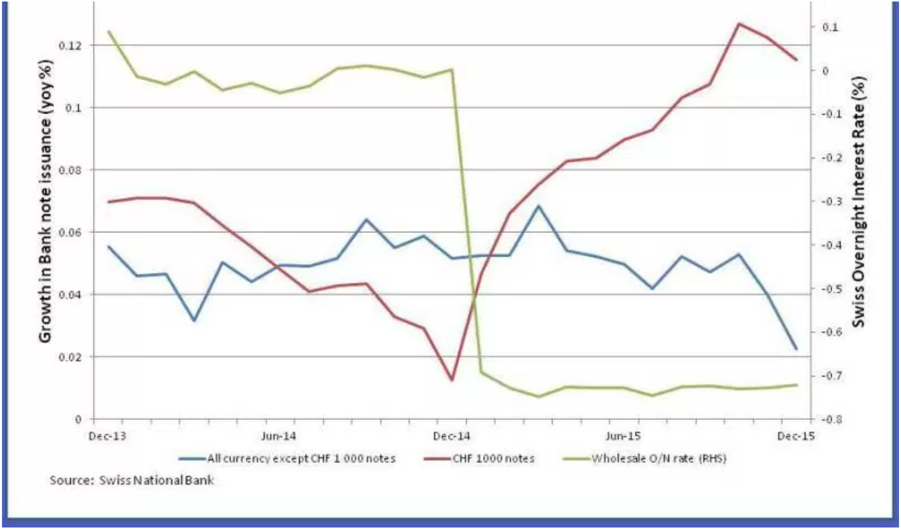
Crossing the ZLB – policy rates



Crossing the ZLB – 2 year bond rates



CROSSING THE ZLB – SWISS CASH DEMAND



ELB IN PRACTICE

- Regulatory concerns important: money market mutual funds.
- Storage costs: holding currency requires safe, fire-proofing, etc.
  - Gold has storage cost of $\approx 0.2\%$ per year, but takes up less space and is fire resistant. Crude oil has storage cost of $1-10\%$ per year.
  - 1000 CHF note versus 500 euro note versus 100 dollar bill.
- Transaction demand: modern economy operates by large movements of cash electronically.
  - Easy to impose negative interest rate on electronic deposits.
  - Credit card fees $\approx 1-3\%$ per year.
- Payment flexibility: people can prepay taxes, etc.
- We'll assume 0 for convenience, but actual lower bound appears to be slightly below 0.

Aside: eliminate 100 dollar bill?

Value of currency in circulation, in billions of dollars as of December 31 of each year

	$1	$2	$5	$10	$20	$50	$100	$500 to $10,000	TOTAL
2018	$12.4	$2.5	$15.3	$20.1	$188.5	$89.2	$1,343.5	$0.3	$1,671.9
2017	$12.1	$2.4	$14.8	$19.6	$183.8	$86.4	$1,251.7	$0.3	$1,571.1
2016	$11.7	$2.3	$14.2	$19.2	$177.2	$83.5	$1,154.8	$0.3	$1,463.4
2015	$11.4	$2.3	$13.7	$19.0	$171.3	$79.8	$1,082.2	$0.3	$1,380.0
2014	$11.0	$2.2	$13.1	$18.9	$162.2	$76.9	$1,014.5	$0.3	$1,299.1
2013	$10.6	$2.1	$12.7	$18.5	$155.0	$74.5	$924.7	$0.3	$1,198.3
2012	$10.3	$2.0	$12.2	$17.7	$148.9	$72.5	$863.1	$0.3	$1,127.1
2011	$10.0	$1.9	$11.8	$17.2	$141.1	$69.6	$782.6	$0.3	$1,034.5
2010	$9.7	$1.8	$11.5	$16.6	$130.6	$66.9	$704.6	$0.3	$942.0
2009	$9.6	$1.7	$11.2	$16.2	$127.5	$65.3	$656.4	$0.3	$888.3
2008	$9.5	$1.7	$11.0	$16.3	$125.1	$64.7	$625.0	$0.3	$853.2
2007	$9.3	$1.6	$10.8	$16.2	$121.8	$63.0	$569.3	$0.3	$792.2
2006	$9.0	$1.5	$10.5	$16.0	$119.2	$62.8	$564.1	$0.3	$783.5
2005	$8.8	$1.5	$10.3	$15.5	$115.4	$62.1	$545.0	$0.3	$758.8
2004	$8.3	$1.4	$9.8	$15.1	$107.6	$60.6	$516.7	$0.3	$719.9
2003	$8.2	$1.3	$9.7	$15.1	$107.8	$59.9	$487.8	$0.3	$690.2
2002	$8.0	$1.3	$9.4	$14.9	$103.7	$58.5	$458.7	$0.3	$654.8
2001	$7.8	$1.3	$9.2	$14.7	$100.9	$57.0	$421.1	$0.3	$612.3
2000	$7.7	$1.2	$8.9	$14.5	$98.6	$55.0	$377.7	$0.3	$563.9
1999	$7.5	$1.2	$9.0	$16.2	$116.1	$64.7	$386.2	$0.3	$601.2
1998	$7.0	$1.2	$8.0	$14.3	$90.9	$50.5	$320.1	$0.3	$492.2

Includes Federal Reserve notes, U.S. notes, and currency no longer issued

Outline

Overview

- Model and notation very similar to last lecture.
- Agents: household, firm, central bank.
- Infinite horizion: $t = 1, 2, 3, 4, \ldots$, but all action will occur in period 1.
- Household owns firm and chooses consumption to maximize utility.
- Central bank sets $M_1, M_2, M_3, \ldots$.
- ZLB constraint: $i_{1,t} \geq 0$.
- *Endowment economy*: firm exogenously produces Y_t in period t. Sometimes referred to as a *Lucas tree* model.
- Assumption: $Y_2 = Y_3 = \ldots = Y^*, M_2 = M_3 = \ldots = M^*$.
- Procedure: define maximization problems, take first order conditions, define equilibrium.

Household's problem

$$\max_{\{C_t\}} \sum_{t=1}^{\infty} \beta^{t-1} \frac{C_t^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}}$$

s.t.

$$\sum_{t=1}^{\infty} \frac{P_t C_t}{1 + i_{1,t}} = \sum_{t=1}^{\infty} \frac{P_t Y_t}{1 + i_{1,t}}, \tag{1}$$

$$P_t C_t \leq M_t, \quad \forall t. \tag{2}$$

Comments:

1. Value of firm is present value of stream of dividends $P_t Y_t$.
2. Equation (2) is a cash in advance constraint. See previous lecture.
3. $i_{1,t}$ is nominal interest rate between period 1 and t, with $i_{1,1} = 0$.

Household's FOC and market clearing

- Euler equation. For any $h > 0$:

$$\beta^h \left(\frac{C_{t+h}}{C_t}\right)^{-\frac{1}{\sigma}} = \frac{1+i_{1,t}}{1+i_{1,t+h}} \frac{P_{t+h}}{P_t} = \frac{1}{1+r_{t,t+h}}.$$

- This is generalized Euler equation. Easy to see for $t = 1, h = 1$:

$$\beta(1+r_{1,2}) C_2^{-\frac{1}{\sigma}} = C_1^{-\frac{1}{\sigma}}.$$

- Market clearing:

$$C_t = Y_t \;\; \forall t.$$

- CIA:

$$\forall t: \quad i_{t,t+1} > 0 \Longrightarrow P_t C_t = M_t.$$

- *LM* curve: $i_{1,t} = \max\left\{\frac{M_t}{\beta M_1}\left(\frac{C_1}{C_t}\right)^{1-\frac{1}{\sigma}} - 1, 0\right\}$.

LM CURVE

- Derive relationship between money and interest rate as in previous lecture:

$$
\begin{aligned}
\frac{C_1^{-\frac{1}{\sigma}}}{P_1} &= (1+i_{1,t})\beta^{t-1}\frac{C_t^{-\frac{1}{\sigma}}}{P_t} \\
\Rightarrow 1+i_{1,t} &= \beta^{1-t}\frac{P_t C_t}{P_1 C_1}\left(\frac{C_1}{C_t}\right)^{1-\frac{1}{\sigma}} \\
\Rightarrow i_{1,t} &= \begin{cases} \beta^{1-t}\frac{M_t}{M_1}\left(\frac{C_1}{C_t}\right)^{1-\frac{1}{\sigma}} - 1, & \beta^{1-t}\frac{M_t}{M_1}\left(\frac{C_1}{C_t}\right)^{1-\frac{1}{\sigma}} > 1 \\ 0, & \text{otherwise} \end{cases}.
\end{aligned}
$$

- *LM curve*: $i_{1,t} = \max\left\{\beta^{1-t}\frac{M_t}{M_1}\left(\frac{C_1}{C_t}\right)^{1-\frac{1}{\sigma}} - 1, 0\right\}$.

Firm's problem

- Firm's problem is trivial because produced output "grows on a tree."

EQUILIBRIUM

- Given output process $Y_1, Y_2, \ldots$ and central bank policies $M_1, M_2, \ldots,$, equilibrium is set of quantities $C_1, C_2, \ldots$, prices $P_1, P_2, \ldots$ and interest rates $i_{1,1}, i_{1,2}, i_{1,3}, \ldots$ such that:

1. Household satisfies its Euler equations.
2. Household satisfies its money demand LM curves.
3. Household satisfies the cash in advance constraint.
4. Market clearing holds: $C_t = Y_t \quad \forall t$.

Solution $t = 2, 3, \ldots$

- Claim. For $t > 1$:

$$
\begin{aligned}
C_t = Y_t &= Y^*, &\quad (3)\\
1 + i_{t,t+1} = 1 + i^* &= \beta^{-1}, &\quad (4)\\
P_t = \frac{M_t}{C_t} &= \frac{M^*}{Y^*}. &\quad (5)
\end{aligned}
$$

- Proof:

1. Market clearing is immediately satisfied by eq. (3).
2. Equation (5) implies $P_t = P_{t+h}$ for $t > 1, h \geq 0$, which implies by the Fisher relationship $i_{t,t+1} = r_{t,t+1}$. Then because $C_t = C_{t+1}$, the Euler equation gives $1 + r_{t,t+1} = \beta^{-1}$.
3. $r_{t,t+1} > 0 \Rightarrow i_{t,t+1} > 0 \Rightarrow$ CIA binds $\Rightarrow P_t C_t = M_t$.

Solution $t = 1$

- $P_1, i_{1,2}$ are endogenous period 1 variables.
- Equilibrium conditions:

$$\frac{C_1^{-\frac{1}{\sigma}}}{P_1} = \beta(1+i_{1,2})\frac{C^{*-\frac{1}{\sigma}}}{P^*},$$

$$i_{1,2} = \max\left\{\beta^{-1}\frac{M^*}{M_1}\left(\frac{C_1}{C^*}\right)^{1-\frac{1}{\sigma}} - 1, 0\right\}.$$

- Substitute using market clearing conditions $C_1 = Y_1, C^* = Y^*$ and rewrite slightly:

$$\frac{P^*}{P_1} = \beta(1+i_{1,2})\left(\frac{Y^*}{Y_1}\right)^{-\frac{1}{\sigma}},$$

$$i_{1,2} = \max\left\{\beta^{-1}\frac{M^*}{M_1}\left(\frac{Y_1}{Y^*}\right)^{1-\frac{1}{\sigma}} - 1, 0\right\}.$$

Experiments

- ZLB doesn't bind: $M_1 \uparrow \Rightarrow i_{1,2} \downarrow \Rightarrow P_1 \uparrow$. Increasing money supply generates higher period 1 price level. Why? Holding M_2 fixed, higher M_1 lowers money *growth*, which reduces inflation.

- ZLB binds: $M_1 \uparrow$ has no further effect on any other variables. Central bank raises M_1 by buying government bonds from public. When $i_{1,2} = 0$, this is exchange of one zero interest rate asset for another.

- What makes ZLB bind? Real interest rate is:

$$1 + r_{1,2} = \beta^{-1} \left(\frac{Y^*}{Y_1} \right)^{\frac{1}{\sigma}}.$$

Real interest rate is lower if β is larger (household is more patient and wants to save more) or growth rate of output is lower (household wants to save more to smooth consumption).

Outline

Overview

- As in Mankiw and Weinzierl, simplest assumption is P_1 is fixed and output is demand determined.

- Formally, P_1 now exogenous, $C_1 = Y_1$ endogenous.

- Two equation system in endogenous variables $C_1, i_{1,2}$:

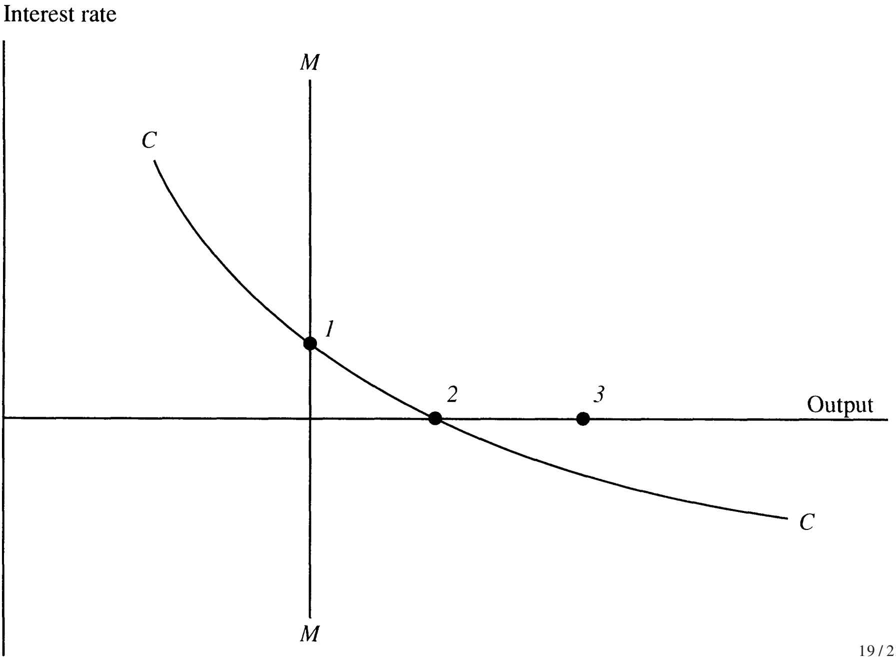
$$C_1 = \beta^{-\sigma} \left(\frac{1 + i_{1,2}}{1 + \pi_{1,2}} \right)^{-\sigma} Y^*,$$

$$i_{1,2} = \max \left\{ \beta^{-1} \frac{M^*}{M_1} \left(\frac{C_1}{Y^*} \right)^{1 - \frac{1}{\sigma}} - 1, 0 \right\},$$

  where $\pi_{1,2} = P^*/P_1 - 1$ is the inflation rate.

- Key difference relative to Mankiw and Weinzierl: assume $Y_2 = Y^*$ fixed, so $1 + r_{1,2}$ is endogenous.

Figure 2. Relationships between Output and the Interest Rate



Economics

- If potential output in period 1 is at point 3, economy needs negative nominal interest rate to achieve full employment.
- Assumption of P_1 fixed also fixes rate of inflation (or deflation) $\pi_{1,2}$.
- By zero lower bound and Fisher relationship, minimum possible real interest rate is $r_{1,2}^{\text{min}} \approx -\pi_{1,2}$.
- If $r_{1,2}^{\text{full emp.}} < r_{1,2}^{\text{min}}$, then economy is in liquidity trap with under employment, and period 1 central bank policy $(M_1, i_{1,2})$ is powerless to help.

Policy responses

1. Forward guidance: reduce long term interest rate. Equivalent in model to raising M^*. Commit to act irresponsibly.

2. Raise inflation target. Equivalent in model to raising M^*. Commit to act irresponsibly.

3. Devalue exchange rate: either $NX \uparrow$ or inflation $\uparrow$.

4. Fiscal policy: see IS-MP lecture on effectiveness of fiscal policy when nominal interest rate is held fixed.

5. Structural reform: Recall $1 + r_{1,2} = \beta^{-1} \left(\frac{Y^*}{Y_1} \right)^{\frac{1}{\sigma}}$. Raise full employment real interest rate by raising Y^*.

Outline

Sticky price model with uncertain Y^*

- Suppose $Y^* \in \{Y^L, Y^H\}$ with probability $p, 1-p$:

$$C_1 = \beta^{-\sigma}\left(\frac{1+i_{1,2}}{1+\pi_{1,2}}\right)^{-\sigma}\left[(1-p)\,Y^H + pY^L\right],$$

$$i_{1,2} = \max\left\{\beta^{-1}(1+\pi_{1,2})\,C_1^{-\frac{1}{\sigma}}\left[(1-p)\left(Y^H\right)^{\frac{1}{\sigma}} + p\left(Y^L\right)^{\frac{1}{\sigma}}\right] - 1, 0\right\}.$$

- Note: assumes inflation-targeting central bank.
- If $i_{1,2} = 0$, then $p\uparrow \Rightarrow C_1\downarrow$. Wealth effect through Euler equation.
- If $i_{1,2} > 0$, then $p\uparrow$ affects $i_{1,2}$. If $\sigma < 1$ (enough risk aversion), $i_{1,2}\downarrow$ (precautionary savings).

Add commitments

- Suppose you can increase consumption in the future by spending today. Should you?
- Same problem as firm deciding whether to invest today or next period.
- Same *real option* to waiting to spend.
- Intuition: who today is planning a vacation for August?
- Further reduces C_1 (and I_1 if modeled) at given real interest rate.
- I expect this will be drag on recovery once restrictions are lifted.