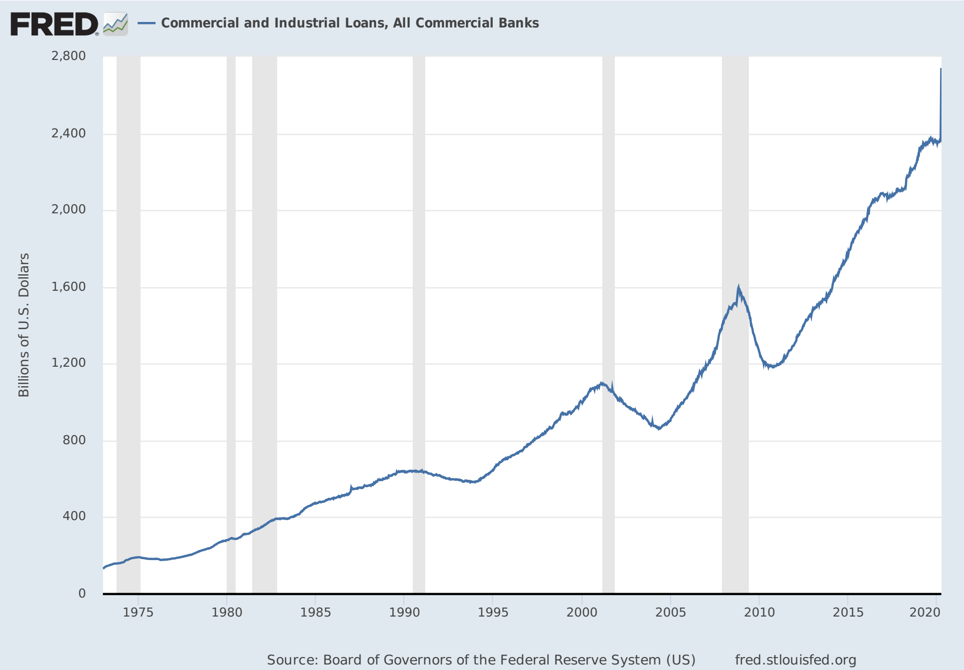
FRED
Commercial and Industrial Loans, All Commercial Banks
Billions of U.S. Dollars
2,800
2,400
2,000
1,600
1,200
800
400
0
1975
1980
1985
1990
1995
2000
2005
2010
2015
2020
Source: Board of Governors of the Federal Reserve System (US)
fred.stlouisfed.org

New Keynesian Model

Harvard Economics 1011B
Professor Gabriel Chodorow-Reich
Spring 2020

Outline

Outline

1. Overview
2. Mankiw and Weinzerl (BPEA 2011), flexible prices
3. Mankiw and Weinzerl (BPEA 2011), sticky prices

New Keynesian versus old Keynesian

- IS-MP style models was dominant into the 1970s. It's the model most economists carry around in their head.
- Drawbacks of IS-MP:
    1. The behavioral equations do not have the forward-looking component of optimization decisions.
    2. The time horizon is funny. Assumption of sluggish or exogenous inflation okay in short run but not in long run.
    3. The time horizon is funny. Short term interest rate versus long term interest rate.
- New Keynesian models have the benefit of fully specified optimizing behavior by private agents but at the cost of substantial stylization. You will see even a model with a number of simplifying assumptions will involve substantial algebra to solve.

Outline

Overview

- There are a lot of derivations in what follows. But much of it we have already worked through.
- I am loosely following Mankiw, Greg and Matthew Weinzierl (2011). Brookings Papers on Economic Activity. `http://www.brookings.edu/~/media/projects/bpea/spring-2011/2011a_bpea_mankiw.pdf`.
- The slides are also comprehensive. You should go through them on your own. Understand the general approach, the derivation methods, and the intuition.
- Model has same structure as the core of the Dynamic Stochastic General Equilibrium (DSGE) models widely used at central banks.

Model overview

- There are two time periods, $t = 1, 2$.
  - 2 periods minimum necessary to have intertemporal component. Can extend to infinite horizon.
- 3 types of agents: household, firm, central bank.
- Household will make consumption/saving decision similar to consumption lecture.
- Firm will make investment decision similar to investment lecture.
- We will derive first order conditions, budget constraints, and market clearing conditions and define an equilibrium in which all of these conditions hold simultaneously.
- First with flexible prices, then with sticky prices.

Household

- A household consumes C_1, C_2. Household receives income from profits of a firm, $P_1\Pi_1, P_2\Pi_2$. Let $P_1 W = P_1\Pi_1 + P_2\Pi_2/(1+i_{1,2})$ denote present value of household's wealth.
- Household solves:

$$\max_{C_1, C_2} \frac{C_1^{1-\frac{1}{\sigma}} - 1}{1-\frac{1}{\sigma}} + \beta \frac{C_2^{1-\frac{1}{\sigma}} - 1}{1-\frac{1}{\sigma}} \tag{1}$$

s.t.

$$P_1 C_1 + \frac{P_2 C_2}{1+i_{1,2}} = P_1 W, \tag{2}$$

$$P_t C_t \leq M_t, \ t = 1, 2. \tag{3}$$

- The maximization eq. (1) and the budget constraint eq. (2) should look familiar from our analysis of household consumption.
- Equation (3) is a *cash in advance (CIA)* constraint. It says total nominal spending must be less than money holdings.

CIA

- Timing:

  1. Enter period with bond wealth or debt and any cash balance. Receive or make interest payments.
  2. Choose holdings of nominal bond to carry over to next period. So in period 1, your portfolio consists of money M_1 and bonds B_1. Recall that $B_1 < 0$ means you are borrowing.
  3. Consume in period 1 subject to the constraint $P_1 C_1 \leq M_1$.
  4. Carry over any left over cash balance into next period.

- Lemma: if $i_{t,t+1} > 0$, then agent will carry over no cash into $t+1$. That is, CIA constraint binds with equality.
- Intuition: cash offers nominal interest rate of zero and is strictly dominated by bond if $i > 0$.
- We will assume the case $i_{t,t+1} > 0$ for the rest of this lecture.

Household FOC

- Euler equation:

$$\frac{u'(C_1)}{P_1} = (1 + i_{1,2})\beta \frac{u'(C_2)}{P_2} \tag{4}$$
$$\Leftrightarrow \frac{C_1^{-\frac{1}{\sigma}}}{P_1} = (1 + i_{1,2})\beta \frac{C_2^{-\frac{1}{\sigma}}}{P_2} \tag{5}$$
$$\Leftrightarrow C_1^{-\frac{1}{\sigma}} = \beta(1 + r_{1,2})C_2^{-\frac{1}{\sigma}}. \tag{6}$$

  Last line uses definition of real interest rate and inflation: $1 + r_{1,2} = (1 + i_{1,2}) / \left(\frac{P_2}{P_1}\right)$.

- CIA binds:

$$P_1 C_1 = M_1, \; P_2 C_2 = M_2. \tag{7}$$

- $i_{1,2}, M_2$ are exogenous policy variables and will be colored green.

Household consumption

- Solve Euler equation for C_2:

$$C_2 = C_1 \left(\beta(1 + r_{1,2})\right)^{\sigma}.$$

Household consumption

- Solve Euler equation for C_2:

$$C_2 = C_1 \left(\beta(1 + r_{1,2})\right)^{\sigma}.$$

- Substitute into budget constraint $P_1 C_1 + P_2 C_2/(1 + i_{1,2}) = P_1 W$:

$$P_1 C_1 + \frac{P_2 C_1 \left(\beta(1 + r_{1,2})\right)^{\sigma}}{1 + i_{1,2}} = P_1 W.$$

HOUSEHOLD CONSUMPTION

- Solve Euler equation for C_2:

$$C_2 = C_1 \left(\beta(1 + r_{1,2})\right)^{\sigma}.$$

- Substitute into budget constraint $P_1 C_1 + P_2 C_2/(1 + i_{1,2}) = P_1 W$:

$$P_1 C_1 + \frac{P_2 C_1 \left(\beta(1 + r_{1,2})\right)^{\sigma}}{1 + i_{1,2}} = P_1 W.$$

- Divide through by P_1 and rearrange:

$$W = C_1 + \frac{C_1 \left(\beta(1 + r_{1,2})\right)^{\sigma}}{1 + r_{1,2}} = \left[1 + \beta^{\sigma}(1 + r)^{\sigma - 1}\right] C_1$$

$$\Rightarrow C_1 = \frac{1}{1 + \beta^{\sigma}(1 + r_{1,2})^{\sigma - 1}} W, \tag{8}$$

$$C_2 = \frac{\left(\beta(1 + r_{1,2})\right)^{\sigma}}{1 + \beta^{\sigma}(1 + r_{1,2})^{\sigma - 1}} W. \tag{9}$$

HOUSEHOLD CONSUMPTION

- Solve Euler equation for C_2:

$$C_2 = C_1 \left(\beta(1+r_{1,2})\right)^{\sigma}.$$

- Substitute into budget constraint $P_1 C_1 + P_2 C_2/(1+i_{1,2}) = P_1 W$:

$$P_1 C_1 + \frac{P_2 C_1 \left(\beta(1+r_{1,2})\right)^{\sigma}}{1+i_{1,2}} = P_1 W.$$

- Divide through by P_1 and rearrange:

$$W = C_1 + \frac{C_1 \left(\beta(1+r_{1,2})\right)^{\sigma}}{1+r_{1,2}} = \left[1+\beta^{\sigma}(1+r)^{\sigma-1}\right] C_1$$

$$\Rightarrow C_1 = \frac{1}{1+\beta^{\sigma}(1+r_{1,2})^{\sigma-1}} W, \tag{8}$$

$$C_2 = \frac{\left(\beta(1+r_{1,2})\right)^{\sigma}}{1+\beta^{\sigma}(1+r_{1,2})^{\sigma-1}} W. \tag{9}$$

- Same consumption function we derived in consumption lecture.

Firm

- Firms have production technology $Y_t = A_t K_t$.
- K_1 is given. $K_2 = I_1$ is chosen in period 1. Note assumption of full depreciation.
- Firm solves:

$$\max_{K_2} P_1 \underbrace{(Y_1 - K_2)}_{\Pi_1} + \frac{1}{1+i_{1,2}} P_2 \underbrace{A_2 K_2}_{\Pi_2}. \tag{10}$$

- Note 1: $Y_1 = A_1 K_1$ is pre-determined.
- Note 2: Firm discounts nominal profits using nominal interest rate.
- A_1, K_1, A_2 are exogenous real variables and will be colored red.

Firm FOC

- FOC (K_2):

$$
\begin{aligned}
P_1 &= \frac{P_2 A_2}{1 + i_{1,2}} \\
\Rightarrow \frac{1 + i_{1,2}}{P_2 / P_1} &= A_2 \\
\Rightarrow 1 + r_{1,2} &= A_2. \qquad (11)
\end{aligned}
$$

- Result: gross real interest rate equal to real marginal product of capital in period 2.

- Note: $r_{1,2} < 0$ if $A_2 < 1$.

PROFITS

- Substitute eq. (11) into firm profits:

$$
\begin{aligned}
P_1 W &= P_1(Y_1 - K_2) + \frac{P_2 A_2 K_2}{1 + i_{1,2}} \\
\Rightarrow W &= \Pi_1 + \Pi_2/(1 + r_{1,2}) \\
&= Y_1 - K_2 + \frac{A_2 K_2}{1 + r_{1,2}} \\
&= Y_1 - K_2 + \frac{(1 + r_{1,2}) K_2}{1 + r_{1,2}} \\
&= Y_1. \qquad (12)
\end{aligned}
$$

- Intuition: MRT=ART=$1 + r_{1,2}$ $\Rightarrow$zero economic profits in period 2.

Central bank

- The central bank sets the nominal interest rate in period 1 $i_{1,2}$ and buys or sells bonds to satisfy the household's choice of holding bonds and money given the nominal interest rate.
- The central bank sets the money supply M_2 in period 2.
- Why different instruments in each period? It doesn't make sense to have the central bank set an interest rate $i_{2,3}$ because this is only a two period model.
- In period 1, central bank sets i and M adjusts. We could equivalently assume central bank chooses M and i adjusts. (Proof in a few slides.)

Equilibrium

- Definition: given initial condition $Y_1 = A_1 K_1$, and central bank policies $i_{1,2}, M_2$, an equilibrium in this economy is a set of real variables $\{C_1, C_2, Y_1, Y_2, I_1\}$, and prices $\{P_1, P_2, r_{1,2}\}$ such that:

  1. The household satisfies the consumption functions eqs. (8) and (9).
  2. The firm satisfies its FOC eq. (11) and the value of the firm W satisfies eq. (12).
  3. The *market clearing* conditions $C_1 + I_1 = Y_1 = A_1 K_1, C_2 = Y_2 = A_2 K_2$ hold.

- The exogenous variables are $A_1, K_1, A_2, i_{1,2}, M_2$.
- The next slide uses eqs. (8), (9), (11) and (12) and other definitions to characterize the equilibrium values.

EQUILIBRIUM

$$1 + r_{1,2} = A_2,$$

Equilibrium

$$
\begin{aligned}
1 + r_{1,2} &= A_2, \\
Y_1 &= A_1 K_1,
\end{aligned}
$$

Equilibrium

$$
\begin{aligned}
1 + r_{1,2} &= A_2, \\
Y_1 &= A_1 K_1, \\
C_1 = \frac{1}{1 + \beta^{\sigma}(1 + r_{1,2})^{\sigma - 1}} W &= \frac{1}{1 + \beta^{\sigma} A_2{}^{\sigma - 1}} Y_1,
\end{aligned}
$$

EQUILIBRIUM

$$
\begin{aligned}
1 + r_{1,2} &= A_2, \\
Y_1 &= A_1 K_1, \\
C_1 = \frac{1}{1 + \beta^{\sigma} (1 + r_{1,2})^{\sigma - 1}} W &= \frac{1}{1 + \beta^{\sigma} A_2{}^{\sigma - 1}} Y_1, \\
C_2 = \frac{(\beta (1 + r_{1,2}))^{\sigma}}{1 + \beta^{\sigma} (1 + r_{1,2})^{\sigma - 1}} W &= \frac{(\beta A_2)^{\sigma}}{1 + \beta^{\sigma} A_2{}^{\sigma - 1}} Y_1,
\end{aligned}
$$

EQUILIBRIUM

$$
\begin{aligned}
1 + r_{1,2} &= A_2, \\
Y_1 &= A_1 K_1, \\
C_1 = \frac{1}{1 + \beta^{\sigma}(1 + r_{1,2})^{\sigma-1}} W &= \frac{1}{1 + \beta^{\sigma} A_2{}^{\sigma-1}} Y_1, \\
C_2 = \frac{(\beta(1 + r_{1,2}))^{\sigma}}{1 + \beta^{\sigma}(1 + r_{1,2})^{\sigma-1}} W &= \frac{(\beta A_2)^{\sigma}}{1 + \beta^{\sigma} A_2{}^{\sigma-1}} Y_1, \\
I_1 = Y_1 - C_1 &= \left[1 - \frac{1}{1 + \beta^{\sigma} A_2{}^{\sigma-1}}\right] Y_1,
\end{aligned}
$$

EQUILIBRIUM

$$
\begin{aligned}
1+r_{1,2} &= A_2, \\
Y_1 &= A_1 K_1, \\
C_1 = \frac{1}{1+\beta^{\sigma}(1+r_{1,2})^{\sigma-1}} W &= \frac{1}{1+\beta^{\sigma} A_2{}^{\sigma-1}} Y_1, \\
C_2 = \frac{(\beta(1+r_{1,2}))^{\sigma}}{1+\beta^{\sigma}(1+r_{1,2})^{\sigma-1}} W &= \frac{(\beta A_2)^{\sigma}}{1+\beta^{\sigma} A_2{}^{\sigma-1}} Y_1, \\
I_1 = Y_1 - C_1 &= \left[1 - \frac{1}{1+\beta^{\sigma} A_2{}^{\sigma-1}}\right] Y_1, \\
Y_2 = A_2 I_1 &= A_2 \left[1 - \frac{1}{1+\beta^{\sigma} A_2{}^{\sigma-1}}\right] Y_1,
\end{aligned}
$$

EQUILIBRIUM

$$
\begin{aligned}
1 + r_{1,2} &= A_2, \\
Y_1 &= A_1 K_1, \\
C_1 = \frac{1}{1+\beta^\sigma (1+r_{1,2})^{\sigma-1}} W &= \frac{1}{1+\beta^\sigma {A_2}^{\sigma-1}} Y_1, \\
C_2 = \frac{(\beta(1+r_{1,2}))^\sigma}{1+\beta^\sigma (1+r_{1,2})^{\sigma-1}} W &= \frac{(\beta A_2)^\sigma}{1+\beta^\sigma {A_2}^{\sigma-1}} Y_1, \\
I_1 = Y_1 - C_1 &= \left[1 - \frac{1}{1+\beta^\sigma {A_2}^{\sigma-1}}\right] Y_1, \\
Y_2 = A_2 I_1 &= A_2 \left[1 - \frac{1}{1+\beta^\sigma {A_2}^{\sigma-1}}\right] Y_1, \\
M_1 = \text{next slide} &= \frac{1}{\beta(1+i_{1,2})} (\beta A_2)^{1-\sigma} M_2,
\end{aligned}
$$

Equilibrium

$$
\begin{aligned}
1 + r_{1,2} &= A_2, \\
Y_1 &= A_1 K_1, \\
C_1 = \frac{1}{1+\beta^{\sigma}(1+r_{1,2})^{\sigma-1}} W &= \frac{1}{1+\beta^{\sigma} A_2{}^{\sigma-1}} Y_1, \\
C_2 = \frac{(\beta(1+r_{1,2}))^{\sigma}}{1+\beta^{\sigma}(1+r_{1,2})^{\sigma-1}} W &= \frac{(\beta A_2)^{\sigma}}{1+\beta^{\sigma} A_2{}^{\sigma-1}} Y_1, \\
I_1 = Y_1 - C_1 &= \left[1 - \frac{1}{1+\beta^{\sigma} A_2{}^{\sigma-1}}\right] Y_1, \\
Y_2 = A_2 I_1 &= A_2 \left[1 - \frac{1}{1+\beta^{\sigma} A_2{}^{\sigma-1}}\right] Y_1, \\
M_1 = \text{next slide} &= \frac{1}{\beta(1+i_{1,2})} (\beta A_2)^{1-\sigma} M_2, \\
P_1 = \frac{M_1}{C_1} &= \left[\frac{1}{1+\beta^{\sigma} A_2{}^{\sigma-1}} A_1 K_1\right]^{-1} \frac{1}{\beta(1+i_{1,2})} (\beta A_2)^{1-\sigma} M_2,
\end{aligned}
$$

EQUILIBRIUM

$$
\begin{aligned}
1 + r_{1,2} &= A_2, \\
Y_1 &= A_1 K_1, \\
C_1 = \frac{1}{1+\beta^\sigma (1+r_{1,2})^{\sigma-1}} W &= \frac{1}{1+\beta^\sigma A_2{}^{\sigma-1}} Y_1, \\
C_2 = \frac{(\beta(1+r_{1,2}))^\sigma}{1+\beta^\sigma (1+r_{1,2})^{\sigma-1}} W &= \frac{(\beta A_2)^\sigma}{1+\beta^\sigma A_2{}^{\sigma-1}} Y_1, \\
I_1 = Y_1 - C_1 &= \left[1 - \frac{1}{1+\beta^\sigma A_2{}^{\sigma-1}}\right] Y_1, \\
Y_2 = A_2 I_1 &= A_2 \left[1 - \frac{1}{1+\beta^\sigma A_2{}^{\sigma-1}}\right] Y_1, \\
M_1 = \text{next slide} &= \frac{1}{\beta(1+i_{1,2})} (\beta A_2)^{1-\sigma} M_2, \\
P_1 = \frac{M_1}{C_1} &= \left[\frac{1}{1+\beta^\sigma A_2{}^{\sigma-1}} A_1 K_1\right]^{-1} \frac{1}{\beta(1+i_{1,2})} (\beta A_2)^{1-\sigma} M_2, \\
P_2 = \frac{M_2}{C_2} &= \left[\frac{(\beta A_2)^\sigma}{1+\beta^\sigma A_2{}^{\sigma-1}} A_1 K_1\right]^{-1} M_2.
\end{aligned}
$$

Derivation of M_1

Euler eq. (line 1), multiply LHS by C_1/C_1, RHS by C_2/C_2, and rearrange (line 2), substitute using $M_1 = P_1 C_1, M_2 = P_2 C_2$ (line 3), substitute using expression for C_1/C_2 from previous slide (line 4), simplify (line 5):

$$
\begin{aligned}
\frac{C_1^{-\frac{1}{\sigma}}}{P_1} &= (1+i_{1,2})\beta \frac{C_2^{-\frac{1}{\sigma}}}{P_2} && \text{(Euler eq.)} \\
\Rightarrow P_1 C_1 &= \frac{1}{\beta(1+i_{1,2})}\left(\frac{C_1}{C_2}\right)^{1-\frac{1}{\sigma}} P_2 C_2 && \text{(algebra)} \\
\Rightarrow M_1 &= \frac{1}{\beta(1+i_{1,2})}\left(\frac{C_1}{C_2}\right)^{1-\frac{1}{\sigma}} M_2 && \text{(substitute } M_1, M_2\text{)} \\
&= \frac{1}{\beta(1+i_{1,2})}\left((\beta A_2)^{-\sigma}\right)^{1-\frac{1}{\sigma}} M_2 && \text{(substitute } C_1/C_2\text{)} \\
&= \frac{1}{\beta(1+i_{1,2})}(\beta A_2)^{1-\sigma} M_2.
\end{aligned}
$$

Note one-to-one correspondence between M_1 and $i_{1,2}$. Choosing $i_{1,2}$ and letting M_1 adjust is same as choosing M_1 and letting $i_{1,2}$ adjust.

MONETARY NEUTRALITY I

- Suppose the central bank raises M_2. What changes?

- M_2 does not enter expressions for C_1, C_2, I_1, Y_1, Y_2. So no changes in real variables.

- The price rises immediately in period 1:

$$\frac{\partial P_1}{\partial M_2} = \left[\frac{1}{1+\beta^{\sigma} {A_2}^{\sigma-1}} A_1 K_1\right]^{-1} \frac{1}{\beta(1+i_{1,2})} (\beta A_2)^{1-\sigma}.$$

- Result: changes in *future* monetary policy affect *current* variables. This should remind you of forward guidance.

Monetary neutrality II

- Suppose the central bank lowers $i_{1,2}$. What changes?
- $i_{1,2}$ does not enter expressions for C_1, C_2, I_1, Y_1, Y_2. So no changes in real variables.
- $i_{1,2}$ does not enter the expression for P_2. The future price level is unchanged.
- The price rises immediately in period 1. Easiest to see by computing inflation from the Fisher relationship:

$$1 + \pi_{1,2} = \frac{P_2}{P_1} = \frac{1 + i_{1,2}}{1 + r_{1,2}} = \frac{1 + i_{1,2}}{A_2}.$$

- Result: decrease in $i_{1,2}$ requires lower inflation to make the Fisher relationship hold, which requires an increase in P_1.

Change in A_2

- Suppose A_2 falls.
- Using previous slide, either the nominal interest rate must fall or inflation must rise.
- Preview of next class: if $i_{1,2}$ cannot fall further because of the zero lower bound, then the economy "needs" inflation.

Summary

- This is example of fully microfounded model.

- *Monetary neutrality*: Changes in the central bank's policies $i_{1,2}$ or M_2 have no effect on real variables. That is, $C_1, C_2, Y_1, Y_{2,1}$ are all independent of $i_{1,2}$ and M_2. Instead, changes in monetary variables affect prices and inflation.

- Key to monetary neutrality is price flexibility.

- Extrapolate from 2 period model. Prices are flexible in the long run, so money is neutral in the long run.

Outline

Overview

- We now introduce price stickiness.

- In fact, we introduce an extreme form of price stickiness: P_1 is pre-determined.

- In reality, some firms adjust their prices each period and others don't. Assumption of P_1 completely fixed will allow us to see implications of rigid prices without keeping track of many firms each with a different price.

- Loose interpretation: firms set prices in a period 0 based on expectations about conditions in period 1, and actual conditions may differ from those expectations.

- This single assumption will break monetary neutrality.

(DIS)EQUILIBRIUM

- In the flexible price equilibrium, P_1 depends on the policy variables $i_{1,2}, M_2$ and the technology A_2.
- With P_1 pre-determined, market clearing may fail for one or more markets.
- Why? Markets clear by prices adjusting.
- Claim: the equality $Y_1 = C_1 + I_1 = A_1 K_1$ is replaced by the inequality $Y_1 \leq A_1 K_1$.
- Strict inequality is a recession: actual output is below what the economy could produce.
- Intuition: with flexible prices, Y_1 is exogenous, and P_1 is endogenous. Now P_1 is exogenous, and so Y_1 is endogenous.
- There is a second case we will ignore where P_1 is "too low," and demand exceeds supply in period 1. More complicated models rule out this case by making producers monopolists or by making P_1 downward nominally rigid.

Sticky price equilibrium

- Expressions that do not change:

$$1 + r_{1,2} = A_2.$$

- Expressions that change:

$$
\begin{aligned}
P_1 &= P_1, \\
P_2 = \frac{1+i_{1,2}}{1+r_{1,2}} P_1 &= \frac{1+i_{1,2}}{A_2} P_1, \\
C_2 = \frac{M_2}{P_2} &= \frac{A_2}{(1+i_{1,2})P_1} M_2, \\
C_1 = \left(\beta(1+r_{1,2})\right)^{-\sigma} C_2 &= \frac{\beta^{-\sigma} {A_2}^{1-\sigma}}{(1+i_{1,2})P_1} M_2, \\
I_1 = \frac{C_2}{A_2} &= \frac{1}{(1+i_{1,2})P_1} M_2, \\
Y_1 = C_1 + I_1 &= \frac{1+\beta^{-\sigma} {A_2}^{1-\sigma}}{(1+i_{1,2})P_1} M_2, \\
Y_2 = A_2 I_1 &= \frac{A_2}{(1+i_{1,2})P_1} M_2.
\end{aligned}
$$

Monetary non-neutrality

- All real variables C_1, C_2, I_1, Y_1, Y_2 depend multiplicatively and positively on the stance of monetary policy $\mathscr{M}$:

$$\mathscr{M} = \frac{M_2}{(1+i_{1,2})P_1}.$$

- Central bank can increase $\mathscr{M}$ by raising M_2 or by lowering $i_{1,2}$.
- Thus two tools to raise current output: lower current nominal interest rate, or raise future money supply. This should remind you of our discussion of the term structure and forward guidance.
- Central bank can produce "full employment" in period 1 by setting $\mathscr{M}$ such that following holds with equality:

$$C_1 + I_1 = \mathscr{M}\left(1 + \beta^{-\sigma} {A_2}^{1-\sigma}\right) = Y_1 \leq A_1 K_1.$$

- Stabilization: to ensure full employment, central bank should react to shocks to current and future technology, and preferences.
- Next class: what happens if M_2 is fixed and the i required to restore the flexible price allocation is less than zero?

Welfare

- Compare consumptions under flexible and sticky prices:

	C_1	C_2
Flexible prices	$\frac{1}{1+\beta^{\sigma}A_2{}^{\sigma-1}}A_1K_1$	$\frac{(\beta A_2)^{\sigma}}{1+\beta^{\sigma}A_2{}^{\sigma-1}}A_1K_1$
Sticky prices	$\frac{\beta^{-\sigma}A_2{}^{1-\sigma}}{(1+i_{1,2})P_1}M_2$	$\frac{A_2}{(1+i_{1,2})P_1}M_2$

- Use inequality concerning full employment from previous slide:

$$C_1^{\text{sticky}} = \left(\beta^{-\sigma}A_2{}^{1-\sigma}\right)\mathscr{M} \leq \left(\beta^{-\sigma}A_2{}^{1-\sigma}\right)\frac{A_1K_1}{1+\beta^{-\sigma}A_2{}^{1-\sigma}}$$

$$= \frac{1}{1+\beta^{\sigma}A_2{}^{\sigma-1}}A_1K_1 = C_1^{\text{flexible}}.$$

- You will verify same inequality for C_2 on the problem set.
- Implication: consumer welfare $u(C_1)+\beta u(C_2)$ maximized at flexible price allocation.
- Intuition: welfare under constrained maximization is weakly less than welfare under unconstrained maximization. In this case, welfare is weakly higher under flexible prices.
- Implication: policy should aim to replicate flexible price allocation.

Technical comment

- Monetary policy transmission mechanism in this model is different from what we emphasized with IS-MP.
- In IS-MP, monetary policy worked by lowering the real interest rate, which makes spending today relatively cheap, which stimulates spending today.
- In this model, the real interest rate is outside the central bank's control and pinned down by A_2.
- Instead, monetary policy stimulates output through a wealth effect. Easiest to see by thinking about raising M_2.
- Both intertemporal substitution and wealth effect channels plausible in reality.

DSGE generalization

- Modern central banks rely heavily on Dynamic Stochastic General Equilibrium (DSGE) models for forecasting and policy analysis.
- The model we just studied is dynamic (multiple periods) and general equilibrium.
- Stochastic comes from specifying distributions of shocks (e.g. to future technology) and agents' expectations.

Other features of DSGE models

- Infinite horizon ($t = 1, 2, 3, \ldots$).
- Endogenous labor supply.
- Frictional unemployment.
- Capital adjustment costs.
- Inflation dynamics from monopolistic firms solving multiperiod pricing problem.
- Limits on household and firm borrowing.
- More complicated preferences.
- More shocks: financial frictions, markup shocks, etc.
- Cost of complications: can no longer write endogenous variables as explicit functions of parameters and exogenous variables. Instead approximate policy functions on computer...

Outline