Labor Supply and Unemployment

Harvard Economics 1011B
Professor Gabriel Chodorow-Reich
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Outline

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Simple model

- For thinking about changes in hours worked across decades or countries, natural to think about frictionless environment.
- Worker has 1 unit of time to allocate to work or leisure:

$$\begin{aligned} & \max U(c,\ell) = u(c) + v(\ell), \\ \text{s.t.} \quad & c \leq (1-\tau)(1-\ell)w + T. \end{aligned}$$

  - c: Consumption.
  - ℓ: Leisure.
  - τ: Tax rate.
  - w: Wage.
- In equilbrium $T = \tau(1-\ell)w$ (balanced budget), but worker does not internalize this and instead treats T as exogenous transfer.

Optimum

- First order conditions with Lagrange multiplier λ:

$$\begin{aligned} c: \quad & u'(c) = \lambda, \\ \ell: \quad & v'(\ell) = \lambda(1-\tau)w. \end{aligned}$$

- Combine:

$$v'(\ell) = (1-\tau)wu'(c).$$

- Interpret: must be indifferent between devoting last (infinitesimal) unit of time to leisure, increasing utility by $v'(\ell)$, or to work, earning after tax wage $(1-\tau)w$ and increasing utility by $(1-\tau)wu'(c)$.

Income and substitution effects

- Across decades, technological process generates growth of w.
- Likewise, w and τ vary across countries.
- What are the predictions for differences in labor supply?
- Functional form assumption:

$$u(c) + v(\ell) = \frac{c^{1-1/\sigma} - 1}{1 - 1/\sigma} - \frac{\theta\varepsilon}{1+\varepsilon}(1-\ell)^{1+1/\varepsilon}.$$

- Solve for $1-\ell$ and differentiate:

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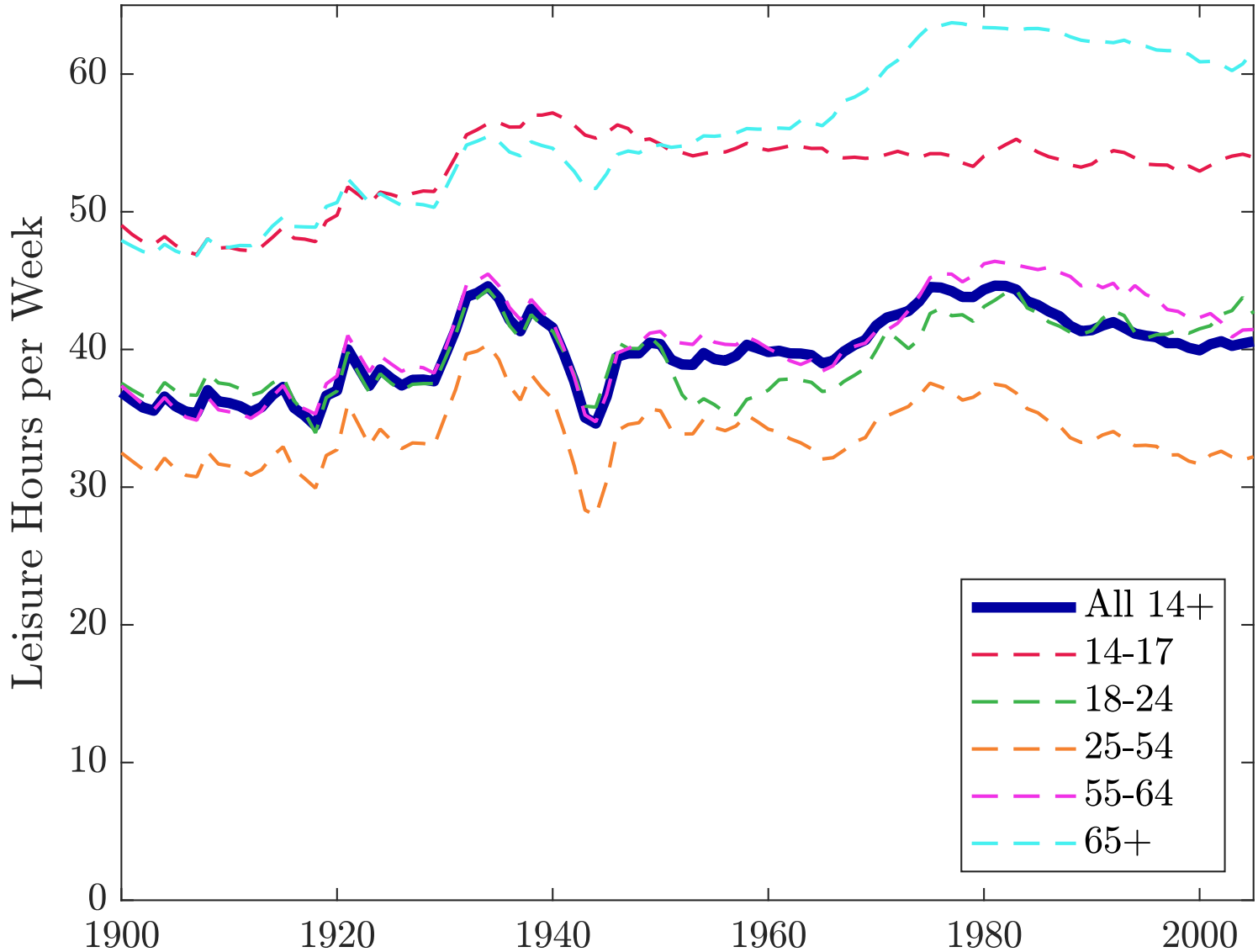
\text{Solve:} \quad & 1-\ell=B(1-\tau)\,w^{\frac{1-1/\sigma}{1/\varepsilon+1/\sigma}},\\
\text{Differentiate:} \quad & \frac{d\ln(1-\ell)}{d\ln w}=\frac{1-1/\sigma}{1/\varepsilon+1/\sigma}.
\end{aligned}$$

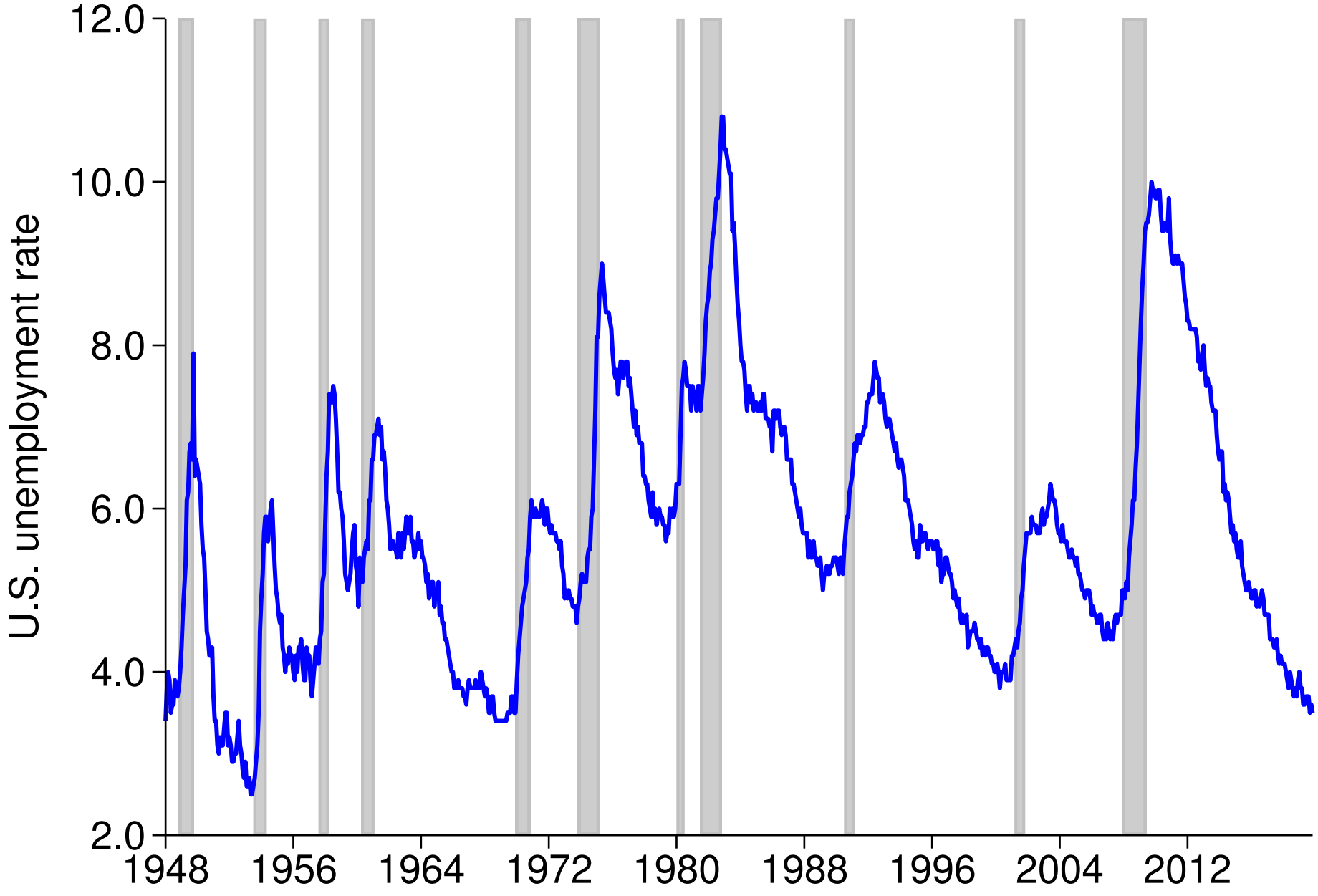
Intuition

- Higher wage encourages substitution toward work.
- Higher wage means higher wealth, encouraging more consumption and more leisure.
- Whether substitution or income effect dominates depends on σ, just as in consumption model.
- With $\sigma = 1$ (log utility), substitution and income effects cancel each other and labor supply remains unchanged.
- In contrast, taxes reduce labor supply because only the substitution margin applies (think of taxes used to finance consumption, e.g. Medicare).
- Different from partial derivative taken in *Beyond GDP* lecture because incorporates response of consumption to permanent change in w. (Unless $\sigma \to \infty$. Why?)

What do data say?



Outline


U.S. unemployment rate
12.0
10.0
8.0
6.0
4.0
2.0
1948
1956
1964
1972
1980
1988
1996
2004
2012

Outline

Theoretical concepts

- Hours worked: input into production.
- Hours paid: includes vacation, sick, bad weather, etc.
- Non-employment: no hours worked or paid.
- Non-employed individuals characterized by unit square in search effort and reservation wage.
  - Transition to employment increasing in search effort, decreasing in reservation wage.
- Underemployment: Positive hours worked or paid but would prefer more hours worked.

Measurement: official unemployment rate

- History: developed during the 1930s at the WPA and the Census Bureau.
- Employed: People are classified as employed if they did any work at all as paid employees during the reference week; worked in their own business, profession, or on their own farm; or worked without pay at least 15 hours in a family business or farm. People are also counted as employed if they were temporarily absent from their jobs because of illness, bad weather, vacation, labor-management disputes, or personal reasons.
- Unemployed: People are classified as unemployed if they meet all of the following criteria: they had no employment during the reference week; they were available for work at that time; and they made specific efforts to find employment sometime during the 4-week period ending with the reference week. Persons laid off from a job and expecting recall need not be looking for work to be counted as unemployed.
- Unemployment rate: The civilian labor force is the sum of employed and unemployed persons. The unemployment rate is the number unemployed as a percent of the labor force.
- Survey reference week: calendar week that contains the 12th day of the month.

OTHER MEASURES

HOUSEHOLD DATA

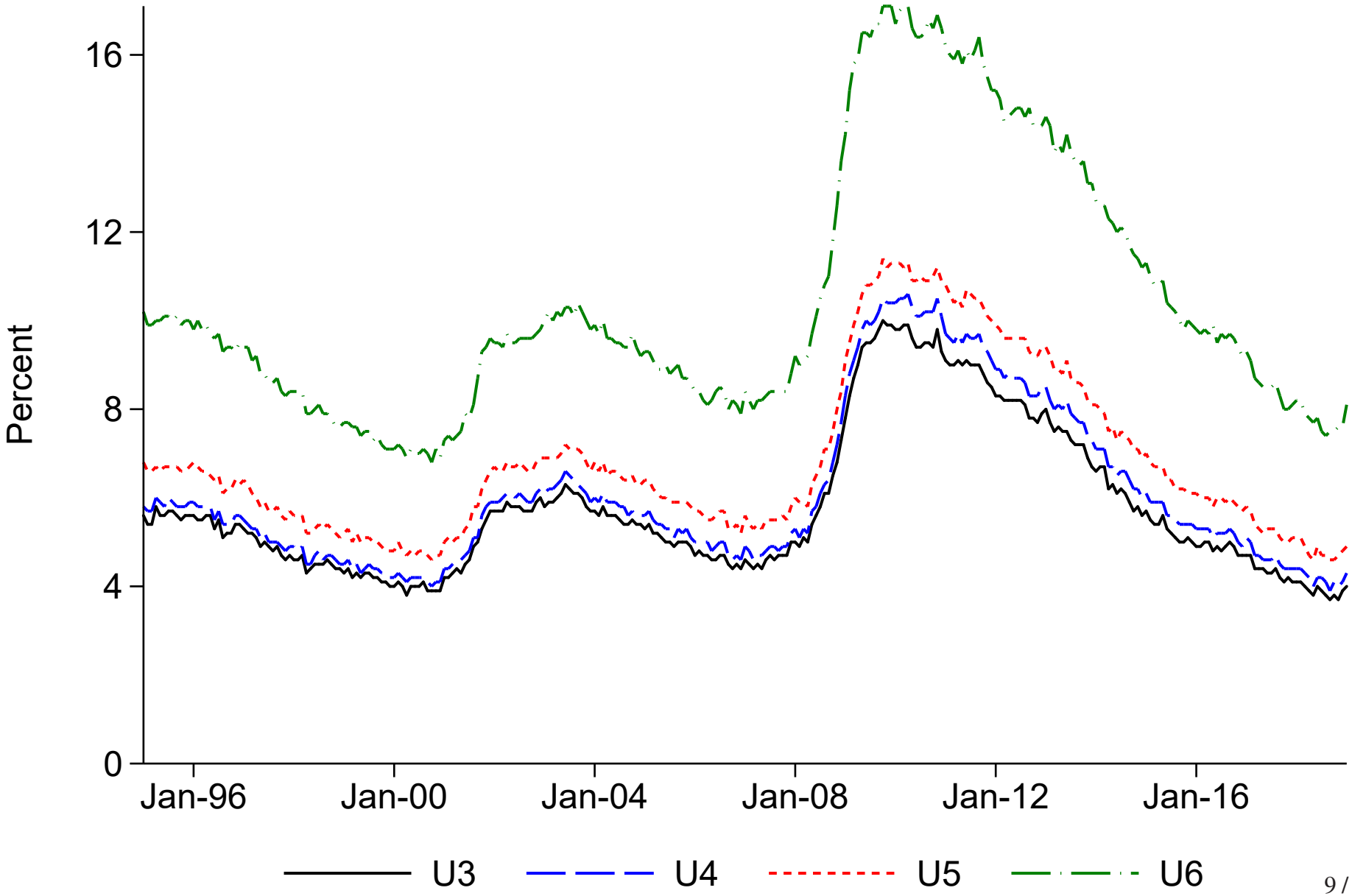
Table A-15. Alternative measures of labor underutilization

[Percent]

Measure	Not seasonally adjusted			Seasonally adjusted					
	Aug. 2015	July 2016	Aug. 2016	Aug. 2015	Apr. 2016	May 2016	June 2016	July 2016	Aug. 2016
U-1 Persons unemployed 15 weeks or longer, as a percent of the civilian labor force	2.1	1.9	1.8	2.2	2.1	1.9	2.0	2.0	1.9
U-2 Job losers and persons who completed temporary jobs, as a percent of the civilian labor force	2.5	2.4	2.4	2.6	2.4	2.3	2.4	2.3	2.4
U-3 Total unemployed, as a percent of the civilian labor force (official unemployment rate)	5.2	5.1	5.0	5.1	5.0	4.7	4.9	4.9	4.9
U-4 Total unemployed plus discouraged workers, as a percent of the civilian labor force plus discouraged workers	5.6	5.5	5.3	5.5	5.3	5.0	5.2	5.2	5.3
U-5 Total unemployed, plus discouraged workers, plus all other persons marginally attached to the labor force, as a percent of the civilian labor force plus all persons marginally attached to the labor force	6.3	6.3	6.0	6.2	6.0	5.7	6.0	6.0	5.9
U-6 Total unemployed, plus all persons marginally attached to the labor force, plus total employed part time for economic reasons, as a percent of the civilian labor force plus all persons marginally attached to the labor force	10.3	10.1	9.7	10.3	9.7	9.7	9.6	9.7	9.7

NOTE: Persons marginally attached to the labor force are those who currently are neither working nor looking for work but indicate that they want and are available for a job and have looked for work sometime in the past 12 months. Discouraged workers, a subset of the marginally attached, have given a job-market related reason for not currently looking for work. Persons employed part time for economic reasons are those who want and are available for full-time work but have had to settle for a part-time schedule. Updated population controls are introduced annually with the release of January data.

MEASURES OF UNDER-EMPLOYMENT

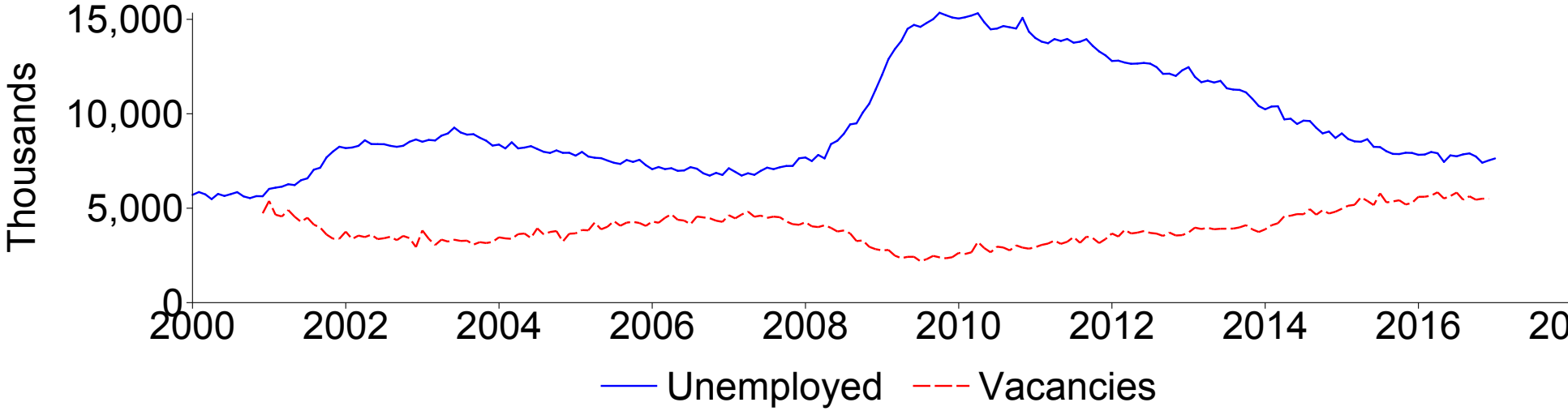


Making definition operational: CPS

- Roughly 60,000 households per month.
- Rotation group structure: household in sample for four months, out for eight months, in for four months.
- Since 1994, reference-dependent survey questionnaire.
- Geographic stratefied sampling procedure:
  - U.S. divided into sets of contiguous counties (PSUs).
  - Large PSUs in sample w.p. 1.
  - Smaller PSUs grouped into strata and one PSU per strata in sample each decade.
  - Within PSU, clusters of geographically adjacent addresses drawn so that entering addresses replace geographically close exiting addresses.
- Separate from "Establishment survey" (CES) which obtains payroll from 147,000 firms (634,000 establishments) each month. CES is larger sample (and near universe after benchmarking to administrative UI records) but no information on activity of non-employed.

Outline

PRIMA FACIE EVIDENCE OF MATCHING FRICTIONS



- Median unemployment duration: 5-25 weeks.
- Average vacancy duration: 1 month.

Large gross flows

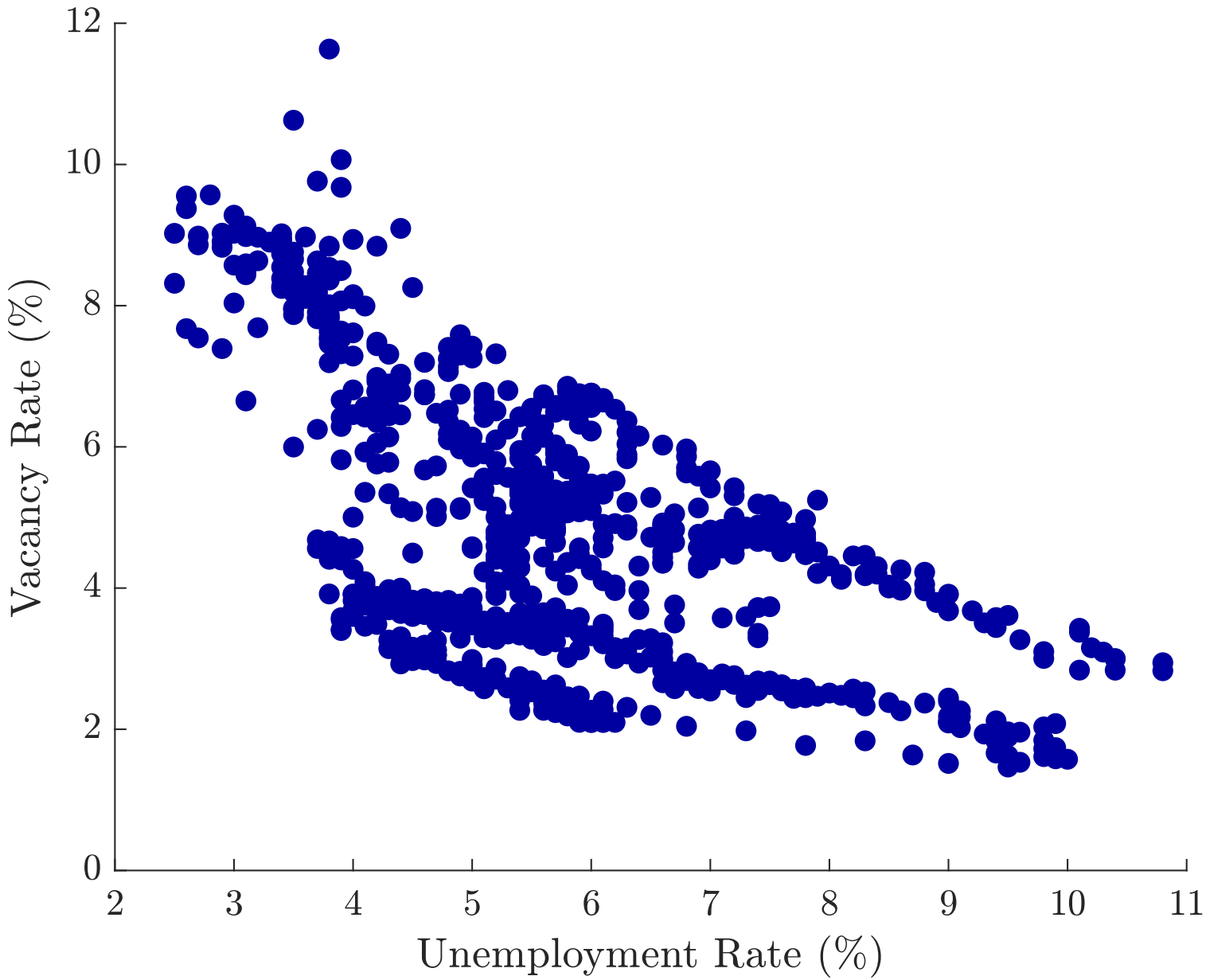
Share of working-age population making labor force status transition:

	E_t	U_t	N_t	All
E_{t-1}	59.29	0.80	1.64	61.73
U_{t-1}	0.91	2.04	0.85	3.80
N_{t-1}	1.53	0.86	32.09	34.48
All	61.73	3.70	34.58	100.01

Fallick and Fleischman, http://www.federalreserve.gov/pubs/feds/2004/200434/200434abs.html.

- Flows from N to E *larger* than flows from U to E.
- Transitions into and out of labor force historically viewed as acyclical.
- Transition *hazard* much larger for U than N.

Beveridge curve



Outline

Goal

Deliver model that:

1. Allows for unemployed workers and vacancies in equilibrium.
2. Generates changes in unemployment in response to shocks.

We will analyze the *search and matching model*.

Ingredients

- All agents either work (employed) or search (unemployed).
- Unemployed workers of mass u search for jobs with fixed effort.
- Firms post v vacancies at cost χ per vacancy to recruit workers.
- Define market tightness $\theta = v/u$.
- Matching function: new matches $m = u^{\alpha} v^{1-\alpha}$.
- Job-finding rate depends on relative number of searchers and vacancies: $f = m/u = \theta^{1-\alpha}$.
- Job-filling rate depends on relative number of searchers and vacancies: $q = m/v = \theta^{-\alpha}$.
- Employed workers produce p each period and receive wage w.
- Unemployed workers produce b (home production).
- Employed workers separate at exogenous rate s (for simplicity).
- Discount future with discount factor β.

Asset pricing preview: value functions

- To analyze model, we will use economics of value functions.
- Simplest case: asset pays dividend d_j each period.
- Recursive representation of time t price: $V_t = d_t + \beta V_{t+1}$.
- Iterate forward:
  $V_t = d_t + \beta (d_{t+1} + \beta V_{t+2}) = \ldots = \lim_{T \to \infty} \sum_{j=0}^{T} \beta^j d_{t+j} + \beta^{T+1} V_{T+1}$.

Asset pricing preview: value functions

- Now consider asset that pays dividend d_L or $d_H > d_L$ each period.
- If d_L, pays d_H next period with probability π_{LH}. If d_H, pays d_L next period with probability d_{HL}.
- Formally, first order Markov transition matrix across states:

$$T = \begin{array}{c} \\ L \\ H \end{array} \begin{array}{c} \begin{array}{cc} L & H \end{array} \\ \begin{pmatrix} 1-\pi_{LH} & \pi_{LH} \\ \pi_{HL} & 1-\pi_{HL} \end{pmatrix} \end{array}$$

- Define P_L and P_H as price of asset if in *state of the world* L or H:

$$V_L = d_L + \beta \left[(1-\pi_{LH}) V_L + \pi_{LH} V_H \right],$$
$$V_H = d_H + \beta \left[\pi_{HL} V_L + (1-\pi_{HL}) V_H \right].$$

Value functions in search model

- Let W_t be the value of having a job and U_t value of unemployment. If employed, receive:
  - Wage w_t.
  - With probability $1-s_t$, remain employed into $t+1$.
  - With probability s_t, separate into unemployment.

  Continuation value is: $\beta\left[(1-s_t)\,W_{t+1}+s_t\,U_{t+1}\right]$.
- We will simplify by assuming constant parameters and $\beta \to 1$: $W = w + [(1-s)\,W + sU]$.
- Similarly, value of being unemployed is: $U = b + [fW + (1-f)\,U]$.
- Value to firm of having worker is: $J = (p-w) + [(1-s)\,J + sV]$.
- Value of vacancy is: $-\chi + [qJ + (1-q)\,V]$.
- Technical note: value functions undefined (all $=\infty$) at $\beta = 1$. But well-defined in limit.

Summary

$$\text{Unemployment:} \quad U = b + [fW + (1-f)U], \tag{1}$$

$$\text{Employment:} \quad W = w + [(1-s)W + sU], \tag{2}$$

$$\text{Filled job:} \quad J = (p-w) + [(1-s)J + sV], \tag{3}$$

$$\text{Vacancy:} \quad V = -\chi + [qJ + (1-q)V], \tag{4}$$

$$\text{Free entry:} \quad V = 0, \tag{5}$$

$$\text{Job finding:} \quad f = \theta^{1-\alpha}, \tag{6}$$

$$\text{Job filling:} \quad q = \theta^{-\alpha}. \tag{7}$$

This is 7 equations in the 7 unknowns U, W, J, V, f, q, θ, given parameters b, s, p, w, α.

Solving the model

$$
\begin{aligned}
\text{(3) and (5):} \quad & J = (p-w)/s, \\
\text{(4) and (5):} \quad & q = \chi/J \\
\text{Combine:} \quad & = \frac{\chi s}{p-w}, \\
+(7): \quad & \theta = \left(\frac{\chi s}{p-w}\right)^{-\frac{1}{\alpha}}, \\
+(6): \quad & f = \left(\frac{\chi s}{p-w}\right)^{\frac{\alpha-1}{\alpha}}, \\
\text{(1) and (2):} \quad & W - U = (w-b) + (1-s-f)(W-U) \\
& = \frac{w-b}{s+f}.
\end{aligned}
$$

Unemployment

- So far, determined values and transition probabilities.
- Law of motion for unemployment: $u_{t+1} = (1 - f_t)\, u_t + s_t\, (1 - u_t)$. Unemployed in $t+1$ are previous unemployed who didn't find jobs or previous employed who separated.
- In our aggregate steady state $f_t = f, s_t = s, u_t = u$. Solve: $u = s/(s+f)$.
- So having determined f, we also determined u.
- We have a model of equilibrium unemployment and vacancies.

Unemployment volatility

- We solved the model assuming time-invariant parameters.
- To consider volatility, we will take a short-cut and compare across steady-states.
- This turns out to be okay in this context because labor market converges very quickly to steady state.
- Formally, comparative static of u w.r.t. p, i.e. what happens to unemployment when the value of what a worker produces changes.
- Allow for wage to change endogenously in response to p.
- From previous slide, we know that u changes if f changes:

$$f = \left(\frac{p - w}{\chi s} \right)^{\frac{1-\alpha}{\alpha}}.$$

Comparative static

$$\frac{d\ln f}{d\ln p} = \left(\frac{1-\alpha}{\alpha}\right)\frac{d\ln(p-w)}{d\ln p} \propto \frac{1}{p-w}\frac{d(p-w)}{d\ln p} = \frac{p-w\left(\frac{d\ln w}{d\ln p}\right)}{p-w}.$$

- If wages increase one-for-one with p, $dw/dp = 1$, no extra incentive for firms to hire and no change in unemployment: $w \times d\ln w/d\ln p = w \times p/w \times dw/dp = p \Rightarrow d\ln f/d\ln p = 0$.
- If wages are sticky, $d\ln w/d\ln p = 0$, then job-finding increases by $p/(p-w)$. Referred to as *fundamental surplus*.
- Closer is w to p, the larger is the percent increase in firm profits with p increases.
- We have a model of volatile unemployment provided wages are sticky enough.

Outline

Coverage

- Everything in lectures except material in lecture 1 and the details of the search and matching model just covered.
- Nothing in readings not also covered in lecture.
- With some prompting, you should be able to set up the objective function and constraint, take first order conditions, and re-arrange to obtain equilibrium conditions.
- You should be able to state and interpret key equilibrium conditions, such as the consumption Euler equation or the relationship between the interest rate and the marginal product of capital.
- You should know the main economic results of the models.
- You should know the main empirical facts we discussed.

What to expect

- 2-3 True/false/uncertain with explanation required.
  - Example: *The balanced growth path level of the capital stock in the neoclassical growth model is equal to the golden rule capital stock. Therefore, the neoclassical growth model achieves the social optimum.*
  - Answer: *The first statement is incorrect; the capital stock in the neoclassical growth model is below the golden rule level if the time preference parameter* $\beta < 1$*. Nonetheless, we proved that the decentralized, market equilibrium of the neoclassical model coincides with the planner's solution and achieves the social optimum. The discrepancy arises because the golden rule capital stock maximizes steady-state consumption, but the implied interest rate would make the household prefer consuming more today if* $\beta < 1$*.*

What to expect

- 2-3 short answer questions.
  - Example: *Explain why interest rate differentials across countries lead to the conclusion that cross-country income differences are largely due to differences in A.*
  - We have the equilibrium condition $r_i + \delta_i = F_{K,i}$, where r_i is the interest rate in country i, δ is the rate of depreciation, and $F_{K,i}$ is the marginal product of capital. Depreciation rates do not differ too much across countries. For reasonable values of the capital elasticity α, the marginal products of capital cannot differ too much either if A does not differ. Therefore, differences in capital cannot explain too much of the cross-country differences in output per worker.

What to expect

- 1-2 long-form questions as in the problem sets.

- For additional practice see Kurlat problems 4.1-4.4, 5.3-5.9, 6.1-6.8, 9.4-9.6, 9.9, 9.11.