Evaluating Neoclassical Growth Theory

Harvard Economics 1011B
Professor Gabriel Chodorow-Reich
Spring 2020

Overview of growth and income differences

- Kaldor facts.
- Solow model.
  - Growth from capital accumulation and exogenous technology.
- Neoclassical growth model.
  - Growth from equilibrium capital accumulation and exogenous technology.
  - Efficiency result.
- **Confronting neoclassical growth theory with evidence.**
- Other and deeper theories of cross-country growth differences.
- Growth over time.
- Cross-country welfare differences beyond GDP.

Outline

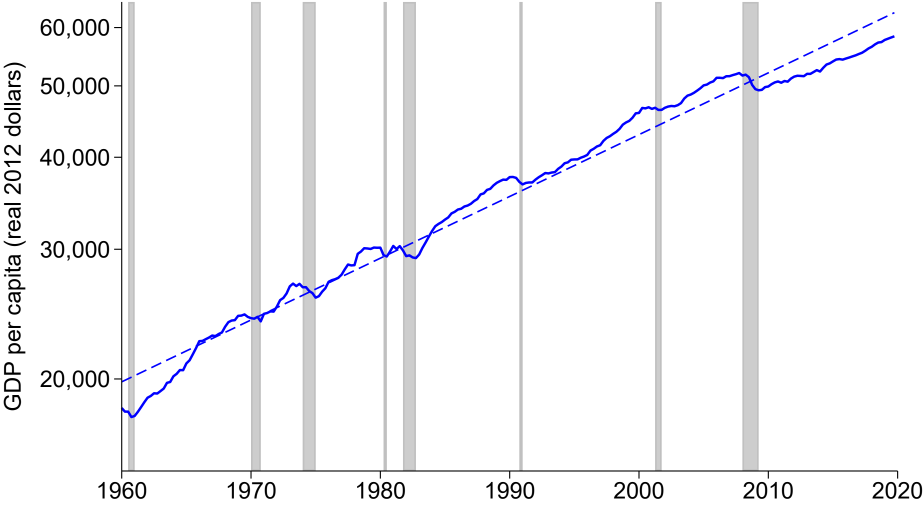
1. Revisiting stylized facts
2. Explaining cross-country differences

Outline

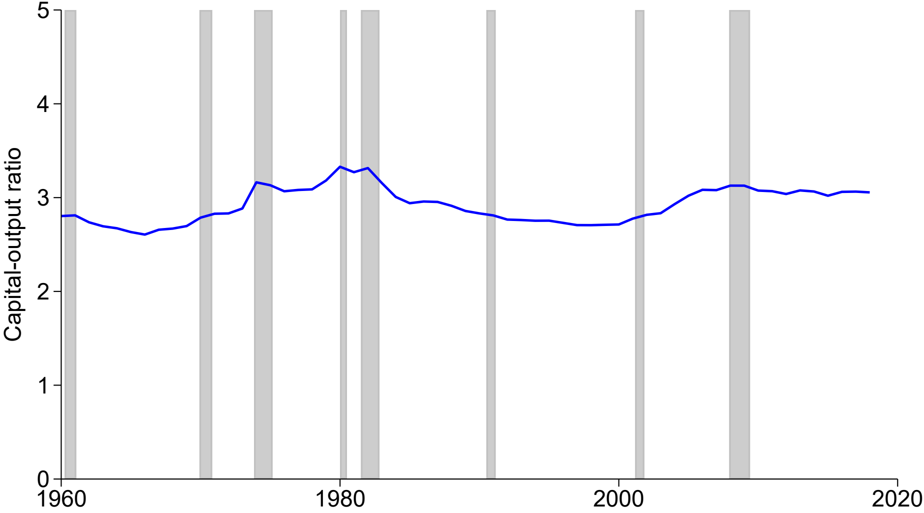
Stylized facts

1. Constant long-run growth rate: long-run growth rate is g. ✓
2. Constant capital output ratio: $K_{bgp}/Y_{bgp} = k_{bgp}/y_{bgp} = s/(n+g+\delta)$ (Solow) + $s_{bgp} = \delta\alpha/(\beta^{-1}-1+\delta)$ (neoclassical). ✓
3. Labor share constant: $F(K,AL) = K^{\alpha}(AL)^{1-\alpha} \Rightarrow w = F_L = (1-\alpha)K^{\alpha}A^{1-\alpha}L^{-\alpha} \Rightarrow wL/Y = 1-\alpha$. ✓ (but labor share declining recently...)
4. Constant real interest rate: $r^K_{bgp} = F_{K_{bgp}}$. ✓ (but level off and secular stagnation...)

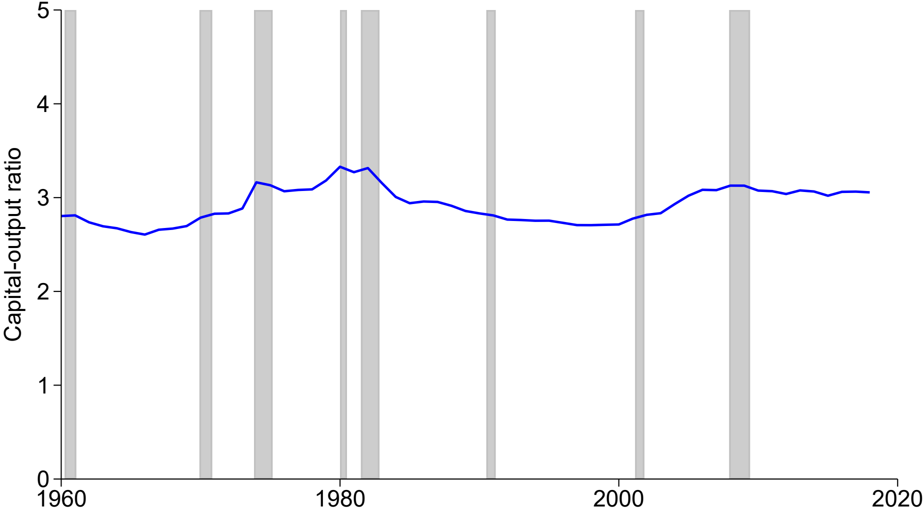
STABLE RATE OF GROWTH



MAGNITUDE: CAPITAL-OUTPUT RATIO

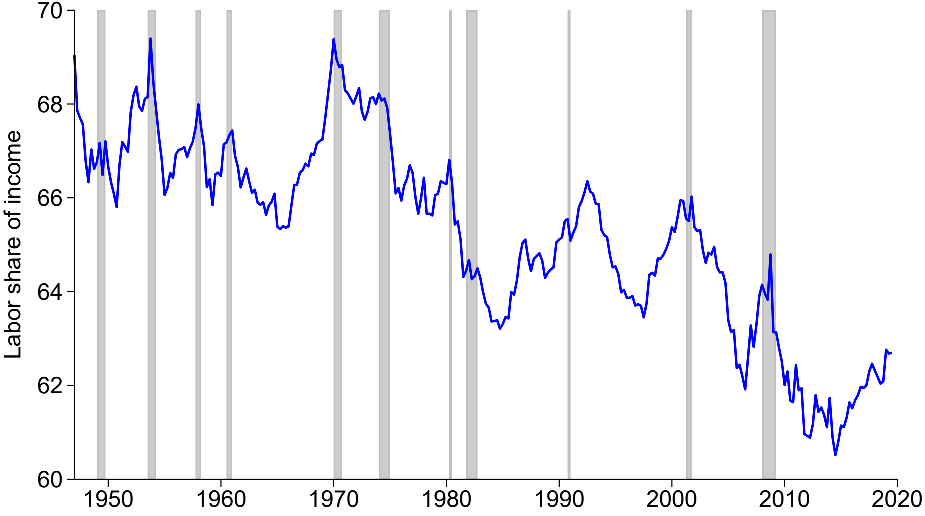


Magnitude: capital-output ratio

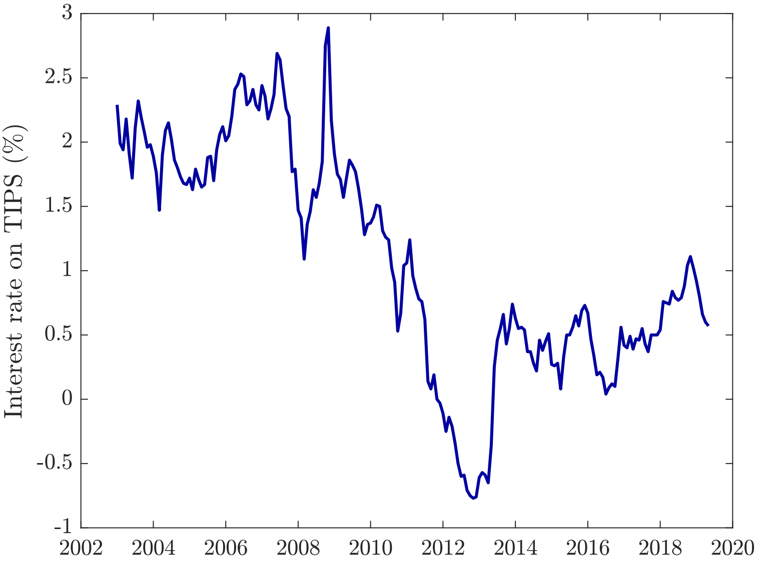


Calibration: $s = 0.2, \delta = 0.04, n = 0.01, g = 0.015 \Rightarrow \frac{K}{Y} = \frac{k}{y} = \frac{s}{n+g+\delta} = 3.25.$

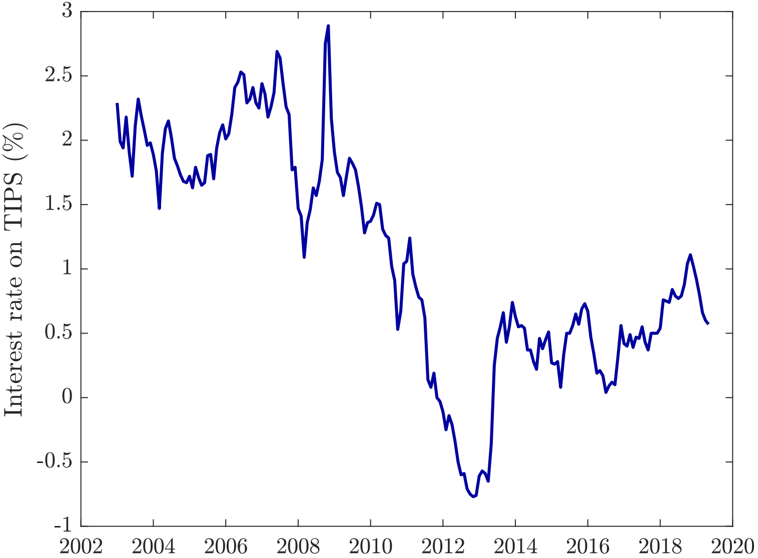
Stable labor share



MAGNITUDE: REAL INTEREST RATE



Magnitude: real interest rate



Calibration: $r = F_K - \delta = \alpha k^{\alpha-1} - \delta = \frac{\alpha y}{k} - \delta = \frac{0.35}{3.25} - 0.04 = 6.8\%.$

Outline

1. Revisiting stylized facts
2. Explaining cross-country differences

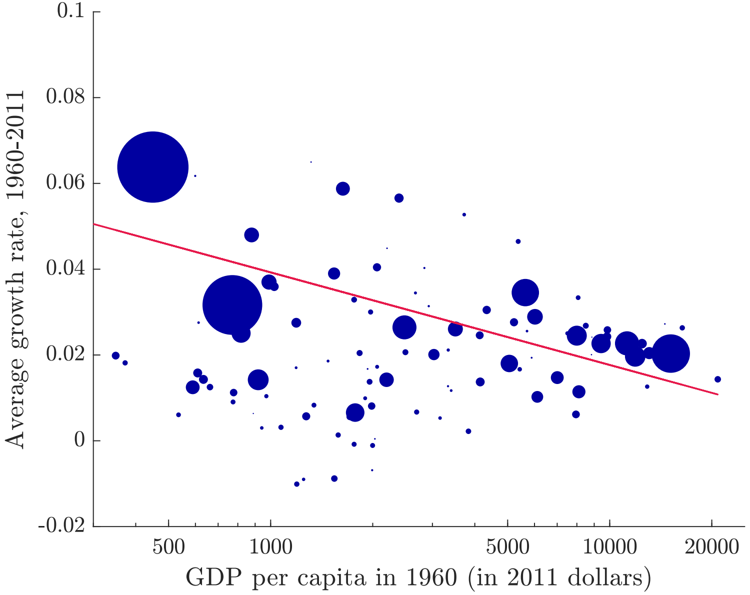
HYPOTHESES

The Solow/neoclassical framework offers three possibilities for explaining cross-country income differences:

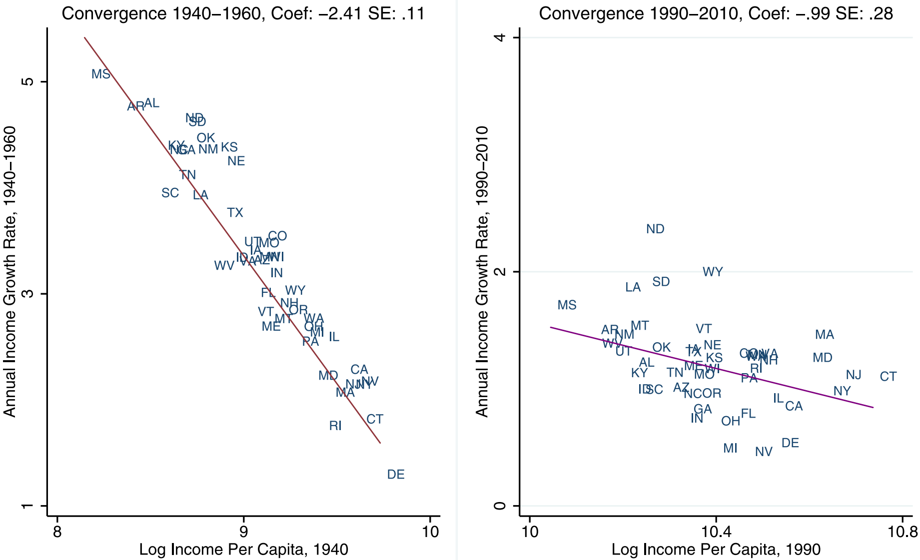
1. Some countries are not at their BGP level.
2. Different BGP saving rates, perhaps due to different levels of impatience.
3. Level differences in A across countries.

(1)$\Rightarrow$unconditional convergence (why?) (2)$\Rightarrow$conditional convergence (why?) (1) and (2)$\Rightarrow$cross-country differences in output explained by differences in capital stocks.

Off BGP: unconditional convergence



Off BGP: conditional convergence



Source: Ganong and Shoag (2017). Why Has Regional Income Convergence in U.S. Declined?

Different capital stocks: direct measurement

- Production function with human capital:

$$\text{Country } i: \qquad Y_i = K_i^{\alpha}(A_i H_i)^{1-\alpha},$$

$$\text{where:} \qquad H_i = e^{\phi(E_i)} L_i.$$

Why model this way?

Different capital stocks: direct measurement

- Production function with human capital:

$$\text{Country } i: \qquad Y_i = K_i^{\alpha}(A_i H_i)^{1-\alpha},$$

$$\text{where:} \qquad H_i = e^{\phi(E_i)} L_i.$$

 Why model this way?

- Measurable expression:

$$\text{Competitive wage:} \qquad w_i = \frac{\partial Y_i}{\partial L_i} = (1-\alpha) K_i^{\alpha} \left(A_i e^{\phi(E_i)}\right)^{1-\alpha} L_i^{-\alpha},$$

$$\text{so:} \quad \frac{\partial \ln w_i}{\partial E_i} = (1-\alpha)\phi'(E_i).$$

Different capital stocks: direct measurement

- Production function with human capital:

$$\text{Country } i: \qquad Y_i = K_i^{\alpha}(A_i H_i)^{1-\alpha},$$

$$\text{where:} \qquad H_i = e^{\phi(E_i)} L_i.$$

  Why model this way?

- Measurable expression:

$$\text{Competitive wage:} \qquad w_i = \frac{\partial Y_i}{\partial L_i} = (1-\alpha) K_i^{\alpha} \left(A_i e^{\phi(E_i)} \right)^{1-\alpha} L_i^{-\alpha},$$

$$\text{so:} \quad \frac{\partial \ln w_i}{\partial E_i} = (1-\alpha)\phi'(E_i).$$

- Some algebra:

$$\left(\frac{K_i}{Y_i}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{H_i}{L_i}\right) A_i = \left(\frac{K_i}{K_i^{\alpha}(A_i H_i)^{1-\alpha}}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{H_i}{L_i}\right) A_i = \frac{K_i^{\alpha}(A_i H_i)^{1-\alpha}}{L_i}$$

$$= \frac{Y_i}{L_i}.$$

Direct measurement: calibration

$$\frac{Y_i}{L_i} = \left(\frac{K_i}{Y_i}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{H_i}{L_i}\right) A_i.$$

- Solow:

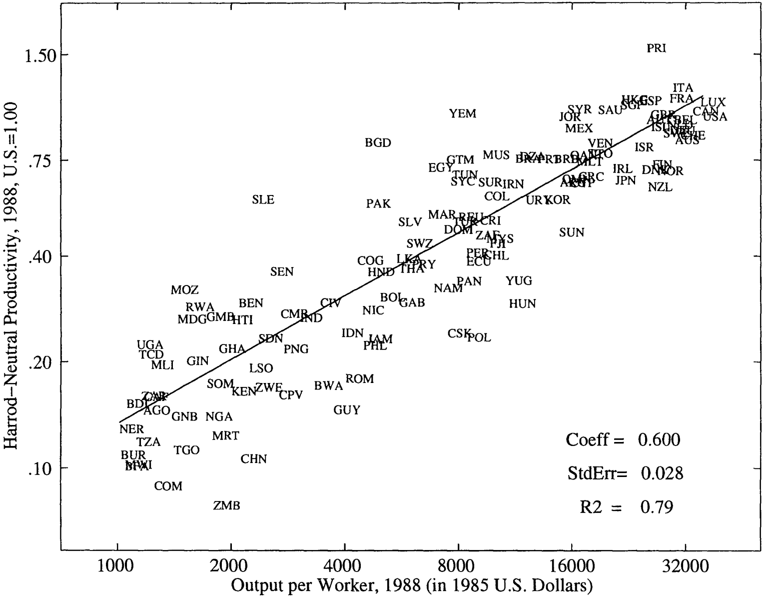
$$\frac{K_{bgp}}{Y_{bgp}} = \frac{k_{bgp}}{y_{bgp}} = \frac{\left(\frac{s}{n+g+\delta}\right)^{\frac{1}{1-\alpha}}}{\left(\frac{s}{n+g+\delta}\right)^{\frac{\alpha}{1-\alpha}}} = \frac{s}{n+g+\delta}.$$

- Investment rates (and hence capital-output ratios) differ by up to $3\times$.
- $\alpha \approx 1/3 \Rightarrow \frac{\alpha}{1-\alpha} \approx 1/2$.
- Therefore, physical capital differences can account for a factor of about $3^{1/2} = 1.73$ in differences in output per worker across countries.
- 8 additional years of school in richest relative to poorest and return of 9% per year $\Rightarrow$ factor of $2\times$.
- But output per worker differs by factor of 30.

Direct measurement (Hall and Jones, QJE 1999)

Country	Y/L	Contribution from $(K/Y)^{\alpha/(1-\alpha)}$	H/L	A
United States	1.000	1.000	1.000	1.000
Canada	0.941	1.002	0.908	1.034
Italy	0.834	1.063	0.650	1.207
West Germany	0.818	1.118	0.802	0.912
France	0.818	1.091	0.666	1.126
United Kingdom	0.727	0.891	0.808	1.011
Hong Kong	0.608	0.741	0.735	1.115
Singapore	0.606	1.031	0.545	1.078
Japan	0.587	1.119	0.797	0.658
Mexico	0.433	0.868	0.538	0.926
Argentina	0.418	0.953	0.676	0.648
U.S.S.R.	0.417	1.231	0.724	0.468
India	0.086	0.709	0.454	0.267
China	0.060	0.891	0.632	0.106
Kenya	0.056	0.747	0.457	0.165
Zaire	0.033	0.499	0.408	0.160
Average, 127 countries:	0.296	0.853	0.565	0.516
Standard deviation:	0.268	0.234	0.168	0.325
Correlation with Y/L (logs)	1.000	0.624	0.798	0.889
Correlation with A (logs)	0.889	0.248	0.522	1.000

Direct measurement (Hall and Jones, QJE 1999)



Different capital stocks: interest rates

- Maybe capital stocks are mis-measured.
- Suppose countries i and j have the same $A = 1$ and α. Then:

Interest rate: $$r_i + \delta = r_i^K = F_{K,i} = \alpha \left(\frac{K_i}{A_i L_i} \right)^{\alpha - 1}.$$

Relative rates: $$\frac{r_i + \delta}{r_j + \delta} = \frac{r_i^K}{r_j^K} = \left(\frac{k_i}{k_j} \right)^{\alpha - 1} = \left(\frac{y_i}{y_j} \right)^{\frac{\alpha - 1}{\alpha}}.$$

- For $\alpha = 1/3$, to explain relative output per worker of $x = y_i / y_j$, require relative return on capital of x^{-2}.
- For example, output per worker in Mexico 30% of U.S. Would require relative return on capital $0.3^{-2} = 11\times$ higher in Mexico.
- No evidence of massive capital flows to poor countries to take advantage of these return differentials.

Required interest rate differentials

Country	$x = \frac{y}{y_{US}}$	$\frac{k}{k_{US}} = x^{\frac{1}{\alpha}}$	$r = r_{US}^{K} x^{\frac{\alpha-1}{\alpha}} - \delta$
Switzerland	1.02	1.05	7.0%
USA	1	1	7.4%
Portugal	0.47	0.12	41%
Mexico	0.30	0.031	104%
China	0.25	0.019	146%
India	0.089	0.001	1,002%
Ethiopia	0.026	0.00003	9,819%

Conclusion

- Neoclassical model matches basic Kaldor facts.
- Cannot quantitatively explain cross-country income differences with differences in capital stocks.
- In accounting sense, A important. Models so far have nothing deep to say about A.