Lecture plan

1. Asset pricing
2. Investment
3. Money, inflation, monetary policy
4. IS-MP model
5. New Keynesian model
6. Zero lower bound
7. International macroeconomics
8. Financial system and runs
9. The European Crisis
10. The Great Recession
11. The COVID-19 Recession

Asset Pricing

Harvard Economics 1011B
Professor Gabriel Chodorow-Reich
Spring 2020

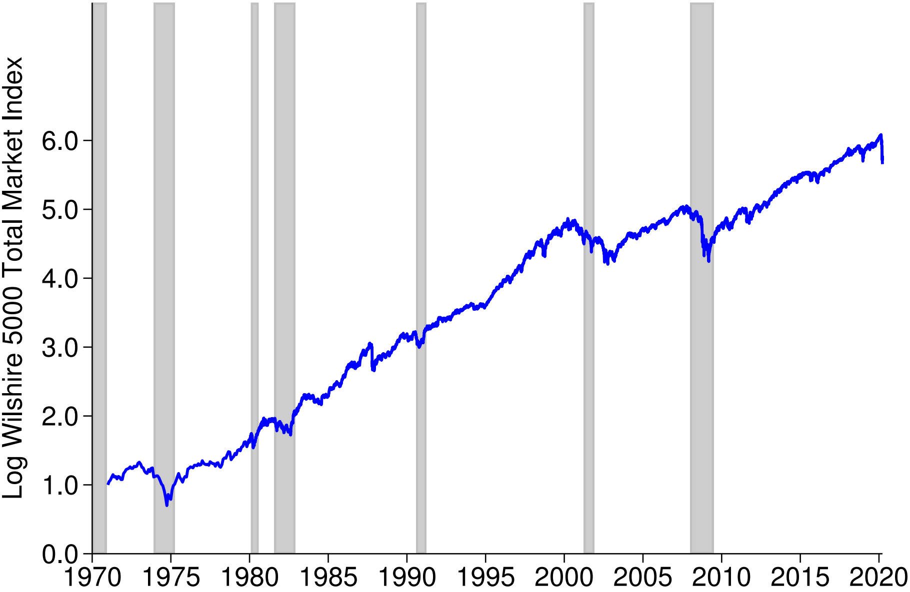
Outline

1. Overview
2. Background probability theory
3. Asset pricing with risk neutrality
4. Asset pricing with risk aversion
5. Interest rates and arbitrage

Outline

1. Overview
2. Background probability theory
3. Asset pricing with risk neutrality
4. Asset pricing with risk aversion
5. Interest rates and arbitrage

WILSHIRE 5000



Why study asset pricing?

- Key transmission channel for monetary policy.
- Keynes's "animal spirits" as driver of economic fluctuations.
- Consumption determinant ("stock market wealth effect").
- Next lecture: investment responds to stock prices.
- Many recessions begin with asset price decline:
  1. Black Tuesday, 1929 crash, and Great Depression.
  2. Dot Com bubble and 2001 recession.
  3. House prices, stock prices, and 2007-2009 Great Recession.
  4. 2020 COVID-19 recession.
- But not all asset price declines presage a recession:
  - Black Monday, 1987 crash.
  - Paul Samuelson: "The stock market has predicted nine of the last five recessions."

Outline

1 Overview

2 Background probability theory

3 Asset pricing with risk neutrality

4 Asset pricing with risk aversion

5 Interest rates and arbitrage

Probability measure

- This is formalizing something we know from everyday life.
- Consider an uncertain occurance, such as what will be the level of the S&P 500 index at market close today.
- An *event* is some subset of the possible outcomes, such as "the S&P 500 will close at 2000 (after rounding)."
- We can partition all possible outcomes into a set of events. For example, 0,1,2,3,...,2000,2001,2002,...,2999, 3000 and above.
- We sometimes refer to an event as a "state of the world."
- The probability of event j is the likelihood of event j occurring. Denote it $\pi(j)$.
- Two important properties:
  1. For all j, $\pi(j) \geq 0$.
  2. $\sum_j \pi(j) = 1$.

Expectation operator

- Let $x(j)$ denote the realized value in state j.
- Expectation operator E is the inner product of the probabilities and the realized values:
$$E[x] = \sum_j \pi(j)\, x(j)\,.$$
- The E operator is a summation.
    - Expectation of a sum is the sum of the expectations.
    - We can pass a derivative through it.
    - These properties continue to apply when event space is continuous.
- Sometimes we want to be explicit about the time horizon and information set. Let $\pi_{t+h|t}(j)$ denote the probability of event j in period $t+h$ conditional on information known at time t.
- Conditional expectation: $E_t[x_{t+h}] = \sum_j \pi_{t+h|t}(j) x_{t+h}(j)$.

Outline

1. Overview
2. Background probability theory
3. Asset pricing with risk neutrality
4. Asset pricing with risk aversion
5. Interest rates and arbitrage

Setup

- Asset pays *stochastic* dividend each period. Stochastic means varying over time and uncertain.
- Assumption: there is a safe asset yielding a return $1+r$ each period.
- Risk neutrality means \$1 in expectation in period $t+h$ is worth $\$\frac{1}{(1+r)^h}$ today.
    - Intuition: Take $\frac{1}{(1+r)^h}$ and invest it at return $1+r$ each period, and in period $t+h$ you will have \$1.
    - Same discounting as without uncertainty in consumption lecture. Risk neutrality$\approx$agent ignores uncertainty.
- Let $D_t(j)$ denote the dividend if state j is realized.

Recursive representation of price

- *Ex dividend price* if you planned to hold asset for one period and then sell it:

$$P_t = \frac{E_t[D_{t+1} + P_{t+1}]}{1+r}.$$

- Solve forward:

$$\begin{aligned}
P_t &= \frac{1}{1+r} E_t[D_{t+1}] + \frac{1}{1+r} E_t[P_{t+1}] \\
&= \frac{1}{1+r} E_t[D_{t+1}] + \frac{1}{1+r} E_t\left[\frac{1}{1+r} E_{t+1}[D_{t+2}] + \frac{1}{1+r} E_{t+2}[P_{t+2}]\right]
\end{aligned}$$

$$\ldots = \sum_{h=1}^{T} \frac{E_t\left[D_{t+h}\right]}{(1+r)^h} + \frac{E_t\left[P_{t+T}\right]}{(1+r)^T}.$$

We have used the property $E_t\left[E_{t+h}[X_{t+h}]\right] = E_t[X_{t+h}]$.

Buy-and-hold formula

- Price if you plan to hold asset forever:

$$P_t = \sum_{j=1}^{J} \pi_{t+1|t}(j) \frac{D_{t+1}(j)}{(1+r)} + \sum_{j=1}^{J} \pi_{t+2|t}(j) \frac{D_{t+2}(j)}{(1+r)^2} + \ldots +$$

$$= E_t \left[\sum_{h=1}^{\infty} \frac{D_{t+h}}{(1+r)^h} \right] = \sum_{h=1}^{\infty} \frac{E_t[D_{t+h}]}{(1+r)^h}.$$

- Buy-and-hold price is discounted sum of future expected dividends.
- Equal to recursive price if $\lim_{T\to\infty} E_t\left[P_{t+T}\right]/(1+r)^T = 0$ ("no bubbles condition").
- Theoretically useful but operationally impossible: requires constructing an expectation of D for every period in the future.

Pricing formula, more structure

- Suppose D_t evolves as:

$$D_{t+1} = (1+g)D_t + \varepsilon_{t+1}.$$

  - ε_t is a *random variable* that may take a different value each period. It is unknown ahead of time, with $E_t \varepsilon_{t+1} = 0$.
  - This is example of a *stochastic process*.
  - Assumption: $r > g$.
- What is $E_t [D_{t+h}]$?

Pricing formula, more structure

- Suppose D_t evolves as:

$$D_{t+1} = (1+g)D_t + \varepsilon_{t+1}.$$

  - ε_t is a *random variable* that may take a different value each period. It is unknown ahead of time, with $E_t\varepsilon_{t+1} = 0$.
  - This is example of a *stochastic process*.
  - Assumption: $r > g$.
- What is $E_t\left[D_{t+h}\right]$?

$$\begin{aligned}
E_t[D_{t+h}] &= E_t\left[(1+g)D_{t+h-1} + \varepsilon_{t+h}\right] \\
&= E_t\left[(1+g)\left((1+g)D_{t+h-2} + \varepsilon_{t+h-1}\right) + \varepsilon_{t+h}\right]\dots \\
&= E_t\left[(1+g)^h D_t + \sum_{j=1}^{h}(1+g)^{h-j}\varepsilon_{t+j}\right] \\
&= (1+g)^h D_t.
\end{aligned}$$

  - Last line follows because D_t known at time t and $E_t[\varepsilon_{t+h}] = 0\ \forall h > 0$.

Gordon growth formula

- From two slides ago:

$$P_t = \sum_{h=1}^{\infty} \frac{E_t[D_{t+h}]}{(1+r)^h}.$$

- From previous slide,
  $D_{t+1} = (1+g)D_t + \varepsilon_{t+1} \Rightarrow E_t[D_{t+h}] = (1+g)^h D_t$.
- Substitute:

- Gordon growth formula: $P_t = \frac{E_t[D_{t+1}]}{r-g}$.

Gordon growth formula

- From two slides ago:

$$P_t = \sum_{h=1}^{\infty} \frac{E_t[D_{t+h}]}{(1+r)^h}.$$

- From previous slide,
$D_{t+1} = (1+g)D_t + \varepsilon_{t+1} \Rightarrow E_t[D_{t+h}] = (1+g)^h D_t$.
- Substitute:

$$\begin{aligned} P_t &= \sum_{h=1}^{\infty} \frac{(1+g)^h D_t}{(1+r)^h} = \sum_{h=1}^{\infty} \left(\frac{1+g}{1+r} \right)^h D_t \\ &= \sum_{h=0}^{\infty} \left(\frac{1+g}{1+r} \right)^h D_t - D_t = \frac{D_t}{1 - \frac{1+g}{1+r}} - D_t \\ &= \frac{(1+g)D_t}{r-g} = \frac{E_t[D_{t+1}]}{r-g}. \end{aligned}$$

- Gordon growth formula: $P_t = \frac{E_t[D_{t+1}]}{r-g}$.

Excess volatility

- Gordon growth formula: $P_t = \frac{E_t[D_{t+1}]}{r-g}$.

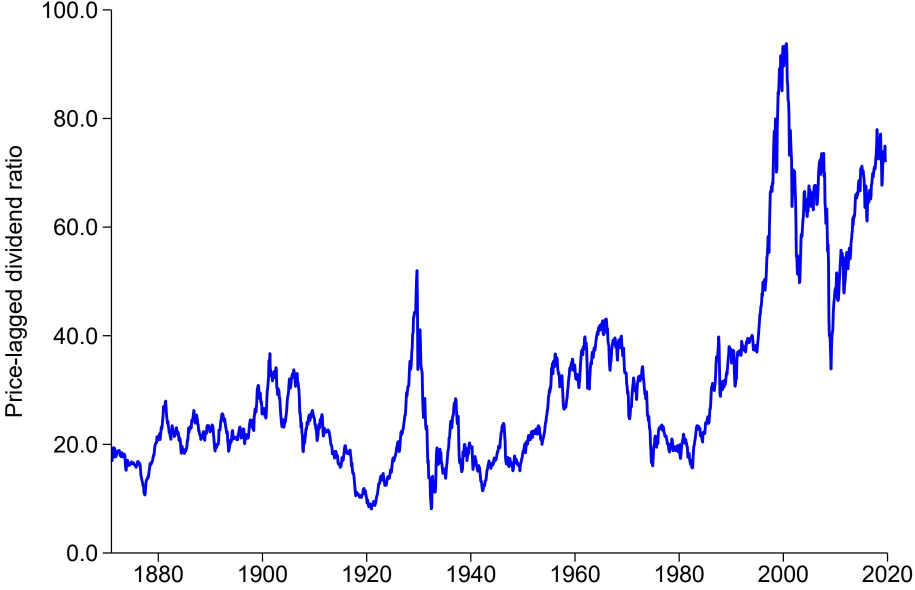
- Rewrite formula:

$$\frac{P_t}{E_t[D_{t+1}]} = \frac{1}{r-g}.$$

- Formula predicts price-dividend ratio fluctuates because r or g fluctuates.

- Assume "adaptive" expectations: $E_t[D_{t+1}]$ is recent realizations of D_t.

SHILLER P/D RATIO



Excess volatility

- Gordon growth formula: $P_t = \frac{E_t[D_{t+1}]}{r-g}$.

- Rewrite formula:

$$\frac{P_t}{E_t[D_{t+1}]} = \frac{1}{r-g}.$$

- Price-earnings ratio fluctuates because r or g fluctuates.

- Does volatility in P/D reflect volatile dividend growth (g volatile) or volatile discounting of future cash flows (r volatile)?

- Campbell and Shiller (JF 1988): "dividend-price ratios are too volatile to be accounted for by news about future dividends."

OUTLINE

1. OVERVIEW
2. BACKGROUND PROBABILITY THEORY
3. ASSET PRICING WITH RISK NEUTRALITY
4. ASSET PRICING WITH RISK AVERSION
5. INTEREST RATES AND ARBITRAGE

Euler equation with uncertainty, setup

- Income and asset returns depend on state of the world.

Euler equation with uncertainty, setup

- Income and asset returns depend on state of the world.
- Utility depends on consumption: $u = u(c_t(j))$.
- Consider investor deciding how much to hold of asset. Let ξ denote this quantity, and y_t exogenous income. Two period model:

Euler equation with uncertainty, setup

- Income and asset returns depend on state of the world.
- Utility depends on consumption: $u = u(c_t(j))$.
- Consider investor deciding how much to hold of asset. Let ξ denote this quantity, and y_t exogenous income. Two period model:

$$\max_{\xi} u(c_t) + \beta \sum_{j} \pi(j)\, u(c_{t+1}(j)) = \max_{\xi} u(c_t) + \beta E_t\left[u(c_{t+1})\right] \text{ s.t.}$$

$$c_t = y_t - P_t \xi,$$

$$c_{t+1}(j) = y_{t+1}(j) + \xi\left(D_{t+1}(j) + P_{t+1}(j)\right).$$

Euler equation with uncertainty

$$\max_{\xi} u(c_t) + \beta E_t [u(c_{t+1})] \text{ s.t.}$$

$$c_t = y_t - P_t \xi,$$
$$c_{t+1}(j) = y_{t+1}(j) + \xi (D_{t+1}(j) + P_{t+1}(j)).$$

- Substitute constraints into objective:

$$\max_{\xi} u(y_t - \xi P_t) + \beta E_t [u(y_{t+1}(j) + \xi (D_{t+1} + P_{t+1}))].$$

- FOC:

$$P_t u'(c_t) = \beta E_t \left[u'(c_{t+1}) (D_{t+1} + P_{t+1}) \right].$$

- Interpret: Buy small amount ε more of asset today, lowering consumption today by εP_t, which lowers utility today by $\varepsilon P_t u'(c_t)$. Dividend plus resale value increases consumption tomorrow by $D_{t+1}(j) + P_{t+1}(j)$, which increases utilty tomorrow by $u'(c_{t+1}(j))(D_{t+1}(j) + P_{t+1}(j))$. Utility tomorrow gets discounted back to utility today by factor β.

Stochastic discount factor

- Rearrange FOC:

$$P_t = E_t\left[\beta\frac{u'(c_{t+1})}{u'(c_t)}(D_{t+1}+P_{t+1})\right]$$
$$= \sum_j \pi(j)\left[\beta\frac{u'(c_{t+1}(j))}{u'(c_t)}(D_{t+1}(j)+P_{t+1}(j))\right].$$

- Compare to risk neutral recursive formula:

$$P_t = E_t\left[\frac{1}{1+r}(D_{t+1}+P_{t+1})\right] = \sum_j \pi(j)\left[\frac{1}{1+r}(D_{t+1}(j)+P_{t+1}(j))\right].$$

- Risk neutral pricing: total payoff $D_{t+1}(j)+P_{t+1}(j)$ discounted at same rate $1+r$ in every state of the world j, and price is expectation of discounted sum.
- Risk aversion: payoff in state j discounted by $\beta\frac{u'(c_{t+1}(j))}{u'(c_t)}$.
- $u''(c)<0 \Longrightarrow$ payoffs in states with lower consumption over-weighted relative payoffs in states with higher consumption.

Intuition

$$\frac{1}{1+r} \text{ versus } \beta \frac{u'(c_{t+1}(j))}{u'(c_t)}.$$

- Does correlation of stock market and your consumption matter?
- If value of marginal dollar depends on the state of the world, then yes.
- Warren Buffet: "Be fearful when others are greedy and greedy when others are fearful."
- Campbell-Shiller: stock volatility because discounting volatility.
- Risk averse discounting: discount rate might vary because of path of consumption, correlation between consumption and asset returns, or utility function shifting to disliking risk more.
- Modern asset pricing is about distinguishing which of these factors is important or something else.
- Institutional asset pricing: interpret $u'(c)$ as value of marginal dollar to financial traders (hedge funds, investment banks, etc.) rather than as consumption.

Why has the stock market been falling?

1. Expectations of future dividends revised down because of COVID-19.
2. States of the world where dividends fall a lot are also states of the world where people are laid off, have health emergencies, etc. and have high marginal value of a dollar.
3. Financial institutions are concerned about disruptions in financial system and want to convert as much to cash as possible.

Outline

1 Overview

2 Background probability theory

3 Asset pricing with risk neutrality

4 Asset pricing with risk aversion

5 Interest rates and arbitrage

Overview

- A stock pays dividends D_{t+1} and has resale value P_{t+1}. The issuer of the stock does not guarantee the value of D_{t+1} or P_{t+1}.
- A pure discount nominal bond is a promise to pay a fixed sum (the face value) at a date in the future.
- Example: once every four weeks the U.S. Treasury issues a new 52 week Treasury bill which promises to pay \$100 in exactly 52 weeks.
- Example: most Mondays the U.S. Treasury issues a new 13 week Treasury bill.
- What would you pay to buy a 52 week Treasury bill?

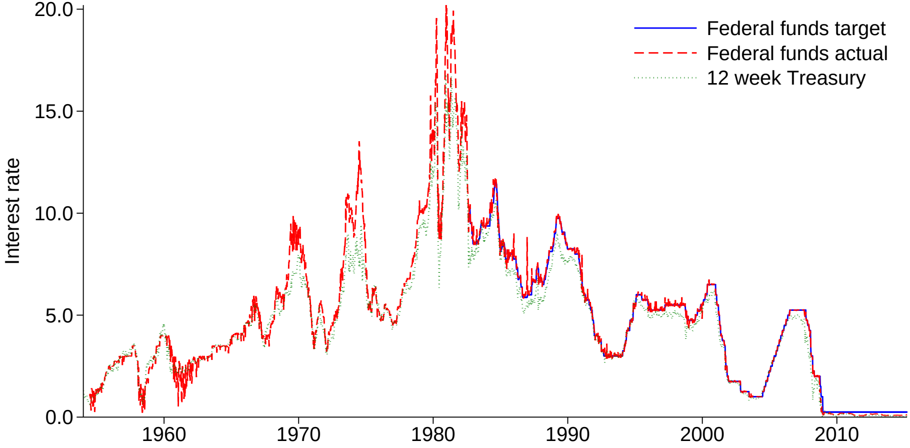
Bond prices and interest rates

- Suppose you pay P_t^{52} for 52 week Treasury bill.
- Compute return to holding bond:

$$R_{t,t+52} = \frac{100}{P_t^{52}}.$$

- Define nominal interest rate as $i_{t,t+52} = R_{t,t+52} - 1$.
- Approximation if $P_t^{52} \approx 100$:
$i_{t,t+52} = \frac{100 - P_t^{52}}{P_t^{52}} = \frac{100 - P_t^{52}}{100} \frac{100}{P_t^{52}} \approx \frac{100 - P_t^{52}}{100} = 1 - .01 P_t^{52}.$
- Example: bond costs \$99, interest rate is $100/99 - 1 = 0.010101\ldots$.
- Example: bond costs \$98, interest rate is $100/98 - 1 = 0.020408\ldots$.
- Result: bond prices move inversely with interest rates.

Preview: monetary policy



- For many purposes, we can just assume the Fed picks the 4 week or 13 week Treasury bill rate.

Term structure

- What is return on a 52 week Treasury bill?
- Consider alternative investment: buy 13 week bill and obtain return $R_{t,t+13}$, in 13 weeks invest proceeds in another 13 week bill, in another 13 weeks invest proceeds in another 13 week bill...
- Expected return:

$$
\begin{aligned}
t+13 :& R_{t,t+13} \\
t+26 :& E_t[R_{t,t+13} R_{t+13,t+26}] \\
t+39 :& E_t[R_{t,t+13} R_{t+13,t+26} R_{t+26,t+39}] \\
t+52 :& E_t[R_{t,t+13} R_{t+13,t+26} R_{t+26,t+39} R_{t+39,t+52}].
\end{aligned}
$$

- This strategy is risky, because future interest rates are uncertain.
- Risk neutrality means indifferent between this strategy and investing directly in 52 week bill:

$$R_{t,t+52} = E_t[R_{t,t+13} R_{t+13,t+26} R_{t+26,t+39} R_{t+39,t+52}].$$

Expectations hypothesis of term structure

- Useful fact: if x, y are small, then $(1+x)(1+y) \approx 1+x+y$.
- Using $R_{t,t+j} = 1 + i_{t,t+j}$ and the above fact:

$$i_{t,t+52} \approx E_t[i_{t,t+13} + i_{t+13,t+26} + i_{t+26,t+39} + i_{t+39,t+52}].$$

- This is the expectations hypothesis of the term structure. It states that a long-term interest rate is equal to the sum of expected short term interest rates over the contract horizon.
- Corollary: if the Fed can control short term interest rates, it can also affect long term interest rates through expectations of future short term interest rates.
- As above, if investors are not risk neutral, then they will demand a risk premium for holding the risky asset.

Forward guidance

March 2015 FOMC statement:
"the Committee judges that an increase in the target range for the federal funds rate remains unlikely at the April FOMC meeting. The Committee anticipates that it will be appropriate to raise the target range for the federal funds rate when it has seen further improvement in the labor market and is reasonably confident that inflation will move back to its 2 percent objective over the medium term... When the Committee decides to begin to remove policy accommodation, it will take a balanced approach consistent with its longer-run goals of maximum employment and inflation of 2 percent. The Committee currently anticipates that, even after employment and inflation are near mandate-consistent levels, economic conditions may, for some time, warrant keeping the target federal funds rate below levels the Committee views as normal in the longer run."

Janet Yellen on normalizing monetary policy

March 2015: "what matters for financial conditions and the broader economy is the entire expected path of short-term interest rates and not the precise timing of the first rate increase. The spending and investment decisions the FOMC seeks to influence depend primarily on expectations of policy well into the future, as embedded in longer-term interest rates and other asset prices. More important than the timing of the Committee's initial policy move will be the strategy the Committee deploys in adjusting the federal funds rate over time, in response to economic developments, to achieve its dual mandate. Market participants' perceptions of that reaction function and the implications for the likely longer-run trajectory of short-term interest rates will influence the borrowing costs faced by households and businesses, including the rates on corporate bonds, auto loans, and home mortgages."

Outline

1. Overview
2. Background probability theory
3. Asset pricing with risk neutrality
4. Asset pricing with risk aversion
5. Interest rates and arbitrage