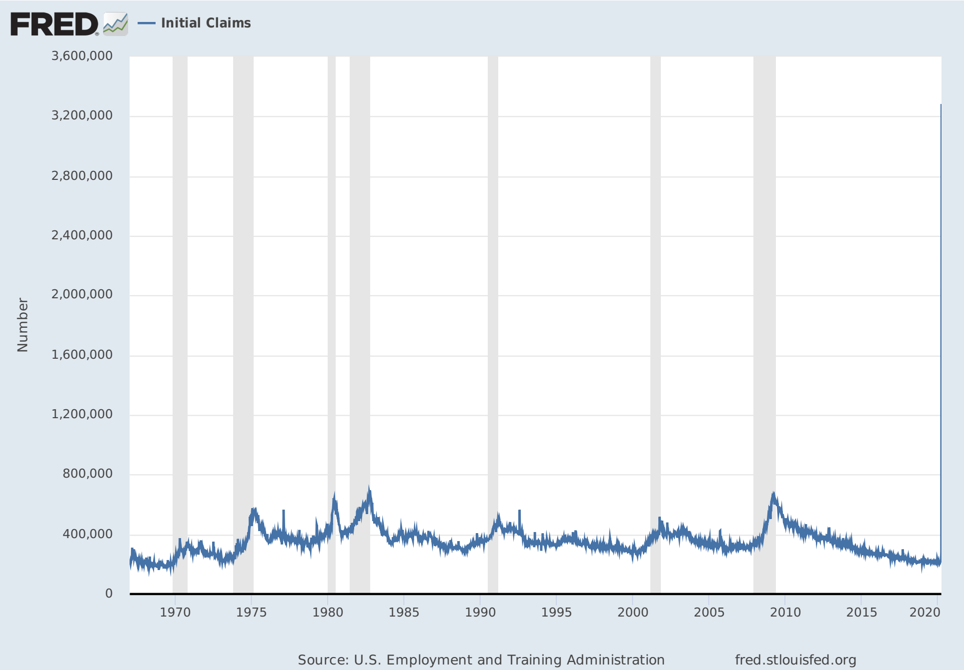

FRED
Initial Claims
3,600,000
3,200,000
2,800,000
2,400,000
2,000,000
1,600,000
1,200,000
800,000
400,000
0
Number
1970
1975
1980
1985
1990
1995
2000
2005
2010
2015
2020
Source: U.S. Employment and Training Administration
fred.stlouisfed.org

SIR-macro model

- Benchmark epidemiological model of COVID-19 is Susceptible-Infected-Recovered (SIR):

$$\begin{aligned} \text{Susceptible:} \quad & \Delta S_t = -\beta_t S_t I_t. \\ \text{Infected:} \quad & \Delta I_t = \beta_t S_t I_t - \gamma I_t. \\ \text{Recovered:} \quad & \Delta R_t = \gamma I_t. \end{aligned}$$

- Population normalized to 1 and recovered includes dead.
- γ is determined by disease and is roughly $1/18$ at daily frequency.
- β_t is "contact rate". Social distancing about reducing β_t.
- β_t responds endogenously to I_t: don't go out if infection rate is high.
- Externality: individuals don't internalize effect of their behavior on β_t.

Principles for policy response

1. Invest money to make this work stoppage as short as possible, neither sparing resources nor shying away from taking risky investments in the development and production of testing, vaccines and new treatments.
2. Provide the necessary support for small and medium businesses to allow them to make their normal credit, rent, and maintenance payments and meet other fixed obligations. Continuity of employment is critical for ensuring that firms and employees can immediately start contributing to rebuilding our economy, once it is safe to suspend the restrictions.
3. Provide the necessary support for individuals whose incomes are affected by work stoppage orders to allow them to make their normal mortgage, credit, and rent payments and meet other inflexible obligations.
4. Assist financial markets in maintaining the productive capacity of large businesses.

From: Economists' Statement on Support for Jobs and Businesses in Response to the Coronavirus Pandemic.

COVID-19 "STIMULUS"

- \$150 billion for hospitals, PPE, testing, etc.
- Direct payments of \$1200 per adult, \$500 per child, phased out for high income households.
- Unemployment Insurance (UI) increase of \$600 per week for four months to roughly \$1000 per week.
- Expansion of eligibility criteria for UI.
- \$367 billion fund for small businesses to maintain payroll or pay rent.
- \$29 billion loans for airlines, \$17 billion loans for companies "critical to maintaining national security.
- \$454 billion in government loans and loan guarantees, mostly to backstop Federal Reserve lending facilities.

Investment

Harvard Economics 1011B
Professor Gabriel Chodorow-Reich
Spring 2020

Outline

Outline

1. Overview
2. User cost model
3. Adjustment costs model
4. Firm investment empirics
5. Investment under uncertainty

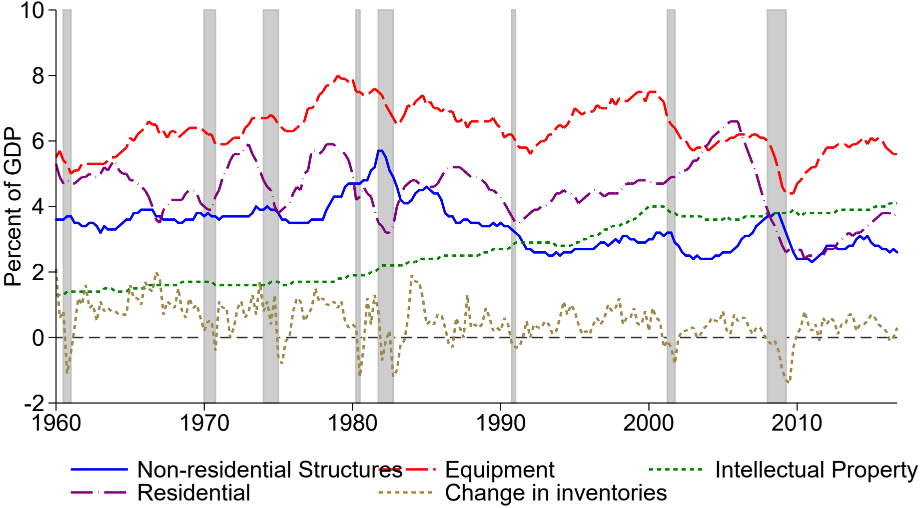
Investment overview

- I is 15% of GDP.
- I supports future consumption (by creating capital).
- I is a volatile component of GDP, so it plays a central role in short-run (i.e., business cycle) fluctuations.
- Dynamics in I provide insight into investor rationality (e.g., animal spirits) and market institutions.
- Many policies are designed to influence I (e.g., investment tax credit).

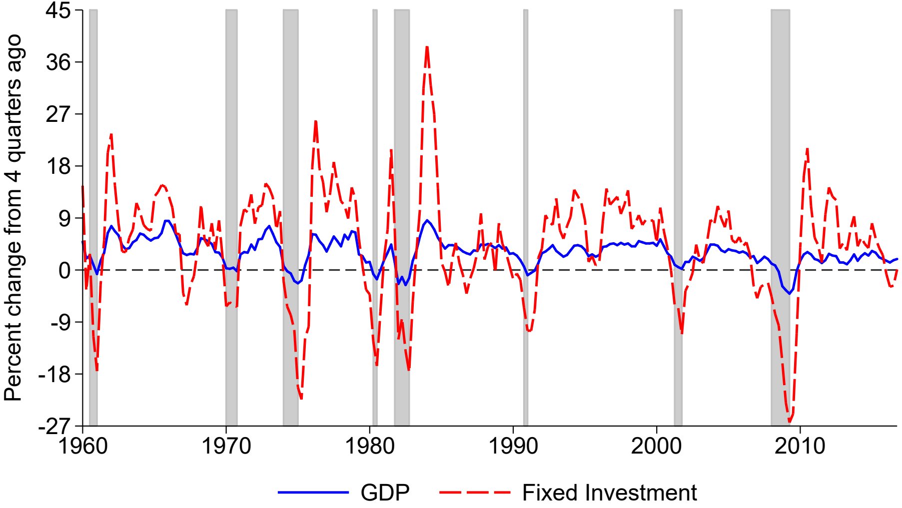
What is investment?

- Types of investment:
    1. Business fixed investment: equipment, structures, and intellectual property.
    2. Residential fixed investment.
    3. Inventory investment.
- Key property: invest today, payoff in the future.
- Investment is a *flow*. The accummulated *stock* of undepreciated fixed investment is the capital stock.
- Desired amount of capital and investment choices are closely related and we will study problem from both perspectives.

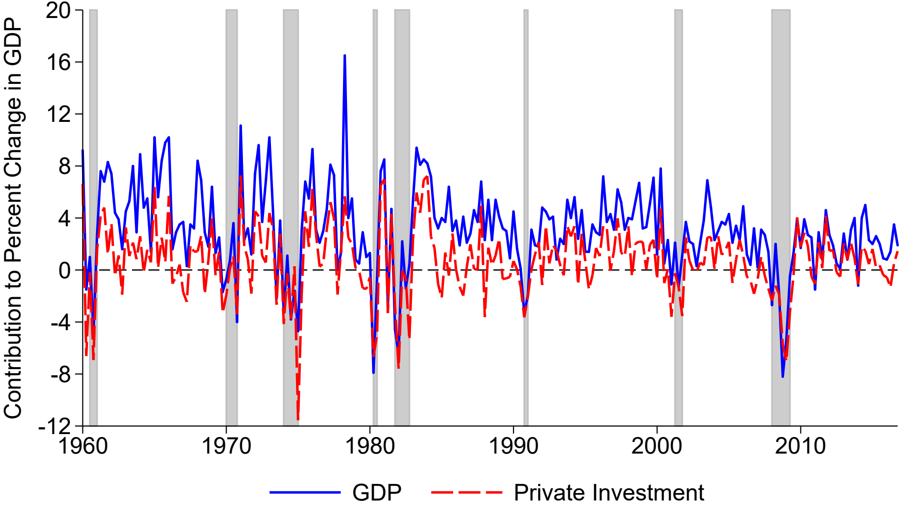
Types of investment



INVESTMENT IS VOLATILE



Investment arithmetically important



Investment versus capital

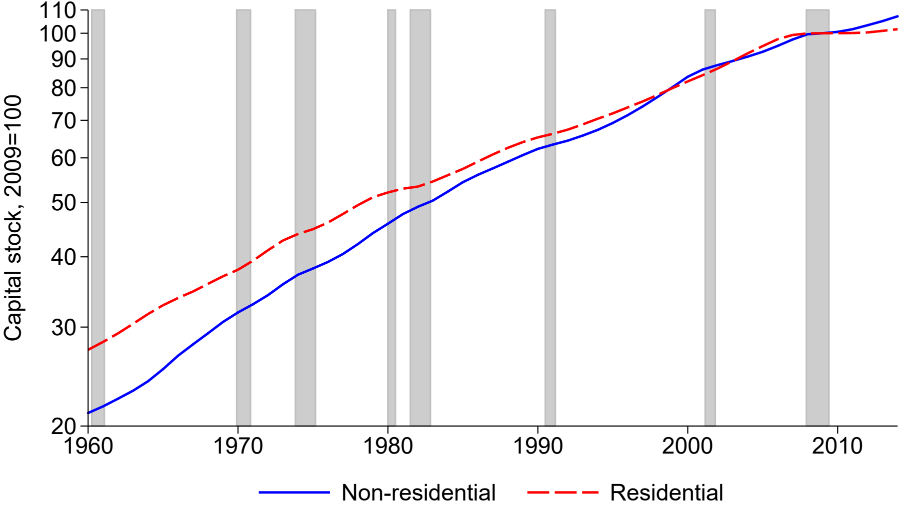
- Capital stock is accumulated investment net of depreciation.
- Suppose a firm buys a machine today. Over time the machine will lose value because of wear and tear, obselescence, accidental damage, and aging. This is depreciation.
- Rate of depreciation varies by investment type:

Type	Depreciation per year (%)
Office buildings	2.47
Aircraft	6.60
Metal working machinery	7.84
Prepackaged software	55.00

- Law of motion for capital stock:

$$\underbrace{K_t}_{\text{Capital stock}} = (1 - \underbrace{\delta}_{\text{Depreciation}})K_{t-1} + \underbrace{I_t}_{\text{Investment}}.$$

Capital stock smoother than investment



Today

- What determines investment decisions by firms?
- *Microfounded* model of firm behavior. Optimization, etc.
- Neoclassical model will make stark prediction: q is sufficient statistic for investment decision.
- Some empirical evidence on the importance of credit constraints.
- Investment and uncertainty.

Outline

STATIC PROBLEM

- Technology: $Y_t = F(K_t, L_t), F_K > 0, F_L > 0, F_{KK} < 0, F_{LL} < 0, F_{KL} > 0.$
- Firm rents capital at price r_t, hires labor at price w_t.
- Profix maximization: $\max_{K,L} F(K_t, L_t) - r_t K_t - w_t L_t$.

$$
\begin{aligned}
\text{FOC } (K_t)\text{:} \qquad & F_K = r_t, \\
\text{FOC } (L_t)\text{:} \qquad & F_L = w_t.
\end{aligned}
$$

- Let $K(r_t)$ be the capital that satisfies the capital FOC: $F_K(K(r_t)) = r_t$. Implicitly differentiate w.r.t. r_t:

Static problem

- Technology: $Y_t = F(K_t, L_t), F_K > 0, F_L > 0, F_{KK} < 0, F_{LL} < 0, F_{KL} > 0.$
- Firm rents capital at price r_t, hires labor at price w_t.
- Profix maximization: $\max_{K,L} F(K_t, L_t) - r_t K_t - w_t L_t$.

$$\begin{aligned} \text{FOC } (K_t): \quad & F_K = r_t, \\ \text{FOC } (L_t): \quad & F_L = w_t. \end{aligned}$$

- Let $K(r_t)$ be the capital that satisfies the capital FOC: $F_K(K(r_t)) = r_t$. Implicitly differentiate w.r.t. r_t:

$$F_{KK} \frac{\partial K(r_t)}{\partial r_t} = 1 \Longrightarrow \frac{\partial K(r_t)}{\partial r_t} = \frac{1}{F_{KK}} < 0.$$

  - Comparative static: demand for capital declines in the rental rate.

Dynamic problem, instantaneous adjustment

- Production technology: $Y_t = F(K_{t-1}), F_K > 0, F_{KK} < 0$.
  - Timing convention: capital used in production in t is chosen in $t-1$.
  - Ignore labor for simplicity.

- Firm owns its capital. Capital depreciates at rate δ, costs p_t^K per unit new investment.

- Dividends each period: $F(K_{t-1}) - p_t^K I_t$.

- Capital law of motion: $K_t = (1-\delta)K_{t-1} + I_t$.

- Firm discounts future dividends at rate $1+r$.

Dynamic maximization problem

$$\max_{\{I_s, K_s\}_{s=t}^{\infty}} \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \left[F(K_{s-1}) - p_s^K I_s\right]$$

s.t.

$$K_s = (1-\delta)K_{s-1} + I_s, \;\; K_{t-1} \text{ given.}$$

Lagrangian:

$$\mathscr{L} = \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \left\{F(K_{s-1}) - p_s^K I_s + \lambda_s \left[(1-\delta)K_{s-1} + I_s - K_s\right]\right\}$$

Dynamic maximization problem

$$\max_{\{I_s, K_s\}_{s=t}^{\infty}} \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} \left[F(K_{s-1}) - p_s^K I_s \right]$$

s.t.

$$K_s = (1-\delta) K_{s-1} + I_s, \;\; K_{t-1} \text{ given.}$$

Lagrangian:

$$\begin{aligned}
\mathscr{L} &= \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} \left\{ F(K_{s-1}) - p_s^K I_s + \lambda_s \left[(1-\delta) K_{s-1} + I_s - K_s \right] \right\} \\
&= F(K_{t-1}) - p_t^K I_t + \lambda_t \left[(1-\delta) K_{t-1} + I_t - K_t \right] \\
&+ \frac{1}{1+r} \left[F(K_t) - p_{t+1}^K I_{t+1} + \lambda_{t+1} \left[(1-\delta) K_t + I_{t+1} - K_{t+1} \right] \right] \\
&+ \left(\frac{1}{1+r} \right)^2 \left[F(K_{t+1}) - p_{t+2}^K I_{t+2} + \lambda_{t+2} \left[(1-\delta) K_{t+1} + I_{t+2} - K_{t+2} \right] \right] \\
&+ \dots
\end{aligned}$$

FOC

$$
\begin{aligned}
\text{FOC } (K_t): \quad & \frac{1}{1+r} F_K(K_t) + \frac{1}{1+r}\lambda_{t+1}(1-\delta) = \lambda_t \\
\text{FOC } (I_t): \quad & \lambda_t = p_t^K.
\end{aligned}
$$

- Combine:

$$\frac{F_K(K_t) + (1-\delta)p_{t+1}^K}{p_t^K} = 1+r.$$

- Euler equation interpretation:
  - RHS: keep \$1 in savings account and earn \$1+r between t and $t+1$.
  - LHS: buy $1/p_t^K$ units of capital, which generates F_K revenue in $t+1$, plus $1-\delta$ remains undepreciated and is worth p_{t+1}^K.

Comparative statics

$$\frac{F_K(K_t) + (1-\delta)p^K_{t+1}}{p^K_t} = 1 + r.$$

- Choose higher K_t when:
  - r is lower,
  - marginal product of capital is higher,
  - price of capital today is lower,
  - depreciation is lower,
  - price of capital tomorrow is higher.

Outline

SETUP

- Production technology: same as previous model.
- Firm owns its capital. Capital depreciates at rate δ, costs p_t^K per unit new investment.
- Firm pays adjustment cost $\Phi(I_t, K_{t-1})$ when it changes its capital stock, $\Phi(0,.) = 0$, $sign(\Phi_I) = sign(I)$. (e.g. $\Phi(I_t, K_{t-1}) = \phi I_t^2$.)
  - Negative investment allowed. Interpret as firm sells used capital for p_t^K.
- Dividends each period: $F(K_{t-1}) - p_t^K I_t - \Phi(I_t, K_{t-1})$.
- Capital law of motion: $K_t = (1-\delta)K_{t-1} + I_t$.
- Firm discounts future dividends at rate $1+r$.
- Lagrangian:

$$\mathscr{L} = \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} \Big\{ F(K_{s-1}) - p_s^K I_s - \Phi(I_s, K_{s-1}) \\ + \lambda_s \left[(1-\delta)K_{s-1} + I_s - K_s\right] \Big\}.$$

FOC

$$\text{FOC } (K_t): \qquad \frac{1}{1+r}\left[F_{K_t} - \Phi_{K_t} + \lambda_{t+1}(1-\delta)\right] = \lambda_t,$$
$$\text{FOC } (I_t): \qquad p_t^K + \Phi_{I_t} = \lambda_t.$$

- Interpret:
  - Cost to firm of installing additional unit of capital is market price plus marginal adjustment cost: $p_t^K + \Phi_{I_t} = \lambda_t$.
  - Benefit to firm: discounted additional profits F_{K_t}, less effect on $t+1$ adjustment costs Φ_{K_t}, plus remaining capital $(1-\delta)$ valued at λ_{t+1}.
  - Remaining capital valued at λ_{t+1} because alternative is to buy $1-\delta$ units in market, which costs firm $p_{t+1}^K + \Phi_{I_{t+1}} = \lambda_{t+1}$.
- Note: by same logic or derivation, for all $s = t, t+1, t+2, \ldots,$:

$$\text{FOC } (K_s): \qquad \frac{1}{1+r}\left[F_{K_s} - \Phi_{K_s} + \lambda_{s+1}(1-\delta)\right] = \lambda_s,$$
$$\text{FOC } (I_s): \qquad p_s^K + \Phi_{I_s} = \lambda_s.$$

Solve forward

$$\lambda_t = \frac{1}{1+r}\left[F_{K_t} - \Phi_{K_t} + \lambda_{t+1}(1-\delta)\right]$$

Solve forward

$$\lambda_t = \frac{1}{1+r}\left[F_{K_t} - \Phi_{K_t} + \lambda_{t+1}(1-\delta)\right]$$

$$= \frac{1}{1+r}\left(F_{K_t} - \Phi_{K_t}\right) + \left(\frac{1-\delta}{1+r}\right)\lambda_{t+1}$$

Solve forward

$$
\begin{aligned}
\lambda_t &= \frac{1}{1+r}\left[F_{K_t} - \Phi_{K_t} + \lambda_{t+1}(1-\delta)\right] \\
&= \frac{1}{1+r}\left(F_{K_t} - \Phi_{K_t}\right) + \left(\frac{1-\delta}{1+r}\right)\lambda_{t+1} \\
&= \frac{1}{1+r}\left(F_{K_t} - \Phi_{K_t}\right) + \left(\frac{1-\delta}{1+r}\right)\underbrace{\left[\frac{1}{1+r}\left(F_{K_{t+1}} - \Phi_{K_{t+1}}\right) + \left(\frac{1-\delta}{1+r}\right)\lambda_{t+2}\right]}_{\lambda_{t+1}}
\end{aligned}
$$

Solve forward

$$
\begin{aligned}
\lambda_t &= \frac{1}{1+r}\left[F_{K_t} - \Phi_{K_t} + \lambda_{t+1}(1-\delta)\right] \\
&= \frac{1}{1+r}\left(F_{K_t} - \Phi_{K_t}\right) + \left(\frac{1-\delta}{1+r}\right)\lambda_{t+1} \\
&= \frac{1}{1+r}\left(F_{K_t} - \Phi_{K_t}\right) + \left(\frac{1-\delta}{1+r}\right)\underbrace{\left[\frac{1}{1+r}\left(F_{K_{t+1}} - \Phi_{K_{t+1}}\right) + \left(\frac{1-\delta}{1+r}\right)\lambda_{t+2}\right]}_{\lambda_{t+1}} \\
&= \ldots = \frac{1}{1+r}\sum_{s=t}^{T}\left(\frac{1-\delta}{1+r}\right)^{s-t}\left(F_{K_s} - \Phi_{K_s}\right) + \left(\frac{1-\delta}{1+r}\right)^{T+1-t}\lambda_{T+1}.
\end{aligned}
$$

Solve forward

$$
\begin{aligned}
\lambda_t &= \frac{1}{1+r}\left[F_{K_t} - \Phi_{K_t} + \lambda_{t+1}(1-\delta)\right] \\
&= \frac{1}{1+r}\left(F_{K_t} - \Phi_{K_t}\right) + \left(\frac{1-\delta}{1+r}\right)\lambda_{t+1} \\
&= \frac{1}{1+r}\left(F_{K_t} - \Phi_{K_t}\right) + \left(\frac{1-\delta}{1+r}\right)\underbrace{\left[\frac{1}{1+r}\left(F_{K_{t+1}} - \Phi_{K_{t+1}}\right) + \left(\frac{1-\delta}{1+r}\right)\lambda_{t+2}\right]}_{\lambda_{t+1}} \\
&= \ldots = \frac{1}{1+r}\sum_{s=t}^{T}\left(\frac{1-\delta}{1+r}\right)^{s-t}\left(F_{K_s} - \Phi_{K_s}\right) + \left(\frac{1-\delta}{1+r}\right)^{T+1-t}\lambda_{T+1}.
\end{aligned}
$$

- Take limit as $T \to \infty$, assuming $\lim_{T\to\infty}\left(\frac{1-\delta}{1+r}\right)^{T+1-t}\lambda_{T+1} = 0$:

$$
\lambda_t = \frac{1}{1+r}\sum_{s=t}^{\infty}\left(\frac{1-\delta}{1+r}\right)^{s-t}\left(F_{K_s} - \Phi_{K_s}\right).
$$

Solve forward

$$\begin{aligned}
\lambda_t &= \frac{1}{1+r}\left[F_{K_t} - \Phi_{K_t} + \lambda_{t+1}(1-\delta)\right] \\
&= \frac{1}{1+r}\left(F_{K_t} - \Phi_{K_t}\right) + \left(\frac{1-\delta}{1+r}\right)\lambda_{t+1} \\
&= \frac{1}{1+r}\left(F_{K_t} - \Phi_{K_t}\right) + \left(\frac{1-\delta}{1+r}\right)\underbrace{\left[\frac{1}{1+r}\left(F_{K_{t+1}} - \Phi_{K_{t+1}}\right) + \left(\frac{1-\delta}{1+r}\right)\lambda_{t+2}\right]}_{\lambda_{t+1}} \\
&= \ldots = \frac{1}{1+r}\sum_{s=t}^{T}\left(\frac{1-\delta}{1+r}\right)^{s-t}\left(F_{K_s} - \Phi_{K_s}\right) + \left(\frac{1-\delta}{1+r}\right)^{T+1-t}\lambda_{T+1}.
\end{aligned}$$

- Take limit as $T \to \infty$, assuming $\lim_{T\to\infty}\left(\frac{1-\delta}{1+r}\right)^{T+1-t}\lambda_{T+1} = 0$:

$$\lambda_t = \frac{1}{1+r}\sum_{s=t}^{\infty}\left(\frac{1-\delta}{1+r}\right)^{s-t}\left(F_{K_s} - \Phi_{K_s}\right).$$

- Interpret: λ_t is present discounted value (PDV) of all future dividends from an additional unit of capital.

Investment decision

- λ_t : PDV of future dividends from additional unit of investment = market value of additional unit of investment.
- $p_t^K + \Phi_{I_t}$: cost of additional unit of investment.
- Firm invests up to point $p_t^K + \Phi_{I_t} = \lambda_t$.
- Rearrange:

$$\frac{\Phi_{I_t}}{p_t^K} = \frac{\lambda_t}{p_t^K} - 1 = q_t - 1.$$

- Terminology:
  - λ_t: market value of marginal capital.
  - p_t^K: replacement cost of capital.
  - $q_t \equiv \lambda_t / p_t^K$: Tobin's (marginal) q, after Nobel Laureate James Tobin.
- $\Phi_{I_t} > 0$ iff $q_t > 1$. Since $\Phi_{I_t} > 0 \Rightarrow I_t > 0$, $I_t > 0$ iff $q_t > 1$.
- Summary: q_t is sufficient statistic for whether firm invests.

Comparative statics

$$q_t = \frac{1}{p_t^K} \frac{1}{1+r} \sum_{s=t}^{\infty} \left(\frac{1-\delta}{1+r} \right)^{s-t} \left(F_{K_s} - \Phi_{K_s} \right).$$

- Investment increasing in profitability F_K.
- Investment declining in cost of capital p_t^K.
- Investment declining in interest rate r.
- Investment declining in depreciation rate δ.
- Investment declining in dividend taxes.
- Investment increasing in investment tax credits (see problem set).

Outline

What is q?

- q_t is sufficient statistic for whether firm invests.
- We do not observe Tobin's *marginal* q. We do observe *average* q.
- Average q is the ratio of the market value of all of a firm's capital to its replacement cost.
- Under certain conditions (constant returns to scale in production, firm price taker in input markets), average q equals marginal q.
  - Proof is beyond scope of this class and you are not responsible for memorizing the conditions.
  - But relationship is useful because it suggests we might be able to use average q as an empirical proxy for marginal q.
- What is market value of all future profits from firm's capital stock?
- Claimants on profits from firm's capital stock:
  1. Debt holders.
  2. Equity holders.
- Sum of market value of debt and equity divided by replacement cost of capital gives average q.

Does q predict investment?

- Market value of equity of publicly traded firm: total market capitalization, equal to share price multiplied by number of shares outstanding.

- Market value of debt: reported on form 10K regulatory filing to SEC.

- Replacement cost of capital: reported as value of tangible assets on form 10K.

- Best estimates: examine response of investment to regulatory or tax induced change in q.

- Evidence mixed at best.

Do other variables predict investment?

- Model predicts *only* q matters.
- What about internal cash flow?
- Model: if $q < 1$ and firm has internal funds, it should pay them out to shareholders.
- Model: if $q > 1$ and firm lacks internal funds, it should issue debt or equity to finance new investment.
- But maybe raising external funds is more costly. For example, banks substantially reduced their supply of lending during the 2008 crisis.

Investigating cash flow constraints

- Ideal experiment: compare two otherwise identical firms (same investment opportunities, same managerial skill, etc.), but one can finance desired investment out of internal funds and the other one cannot.

- We could run this experiment: choose a sample of firms, and randomly gift half the firms large amounts of money.

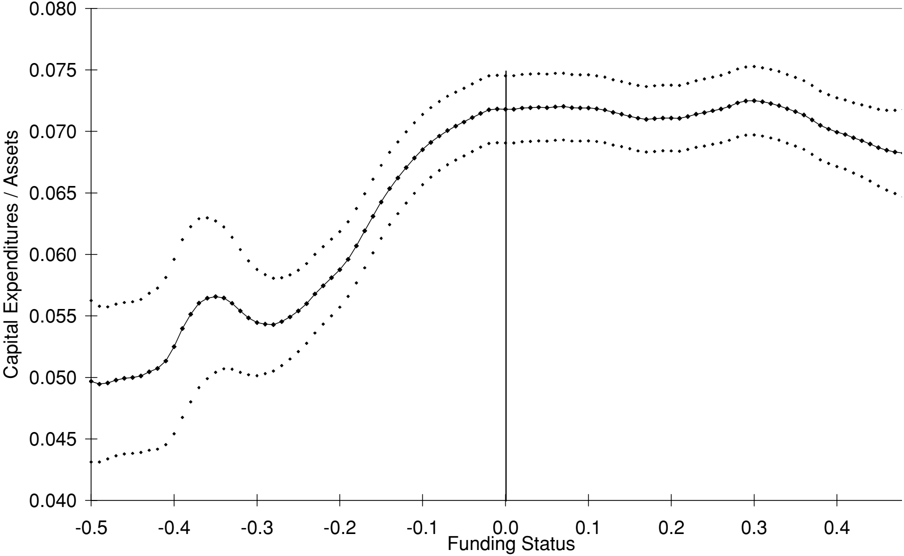
- But this would be expensive.

- Alternative: look for *natural experiment*: some quirk which subjects otherwise similar firms to different treatment.

Investigating cash flow constraints

- Joshua Rauh, Journal of Finance, 2006, "Investment and Financing Constraints": U.S. pension law creates such a quirk.
- Firms must make mandatory contributions to their defined benefit (DB) pension plan if the plan is deemed underfunded.
- Funding status based on a formula. Whether a firm is just over-funded or just under-funded is "as good as randomly assigned."
- Summary: firms' access to internal funds depends on the status of their pension plans, which is independent of their investment opportunities.

Rauh (JF 2006) Investment and Financing Constraints



Outline

What changes?

- Suppose future returns from investment are uncertain.
- If nothing else changes, previous analysis goes through replacing $1/(1+r)^{s-t}$ with the *stochastic discount factor* of the firm.
- Additional salient feature of investment is irreversability: once installed, costly to remove capital.
- This creates a *real options* effect.

Example

- Discount future with factor β.
- Firm has project that costs p^k.
- Investment in period t yields cash-flows x beginning in $t+1$.
- At date 0, x uncertain. Cash flow uncertainty resolved at date 1 with $x = x^u$ with probability π and x^ℓ with probability $1-\pi$, $x^u > x^\ell$.
- Assume profitable if $x = x^u$, not if $x = x^\ell$.
- Values of investing:

Invest at 0: $$V_0 = E_0 \sum_{t=1}^{\infty} \beta^t x - p^k = \frac{\beta}{1-\beta}\left(\pi x^u + (1-\pi)x^\ell\right) - p^k.$$

PV Invest at 1: $$\beta V_1(x^i) = \beta\left(\frac{\beta}{1-\beta}x^i - p^k\right), \ i \in \{\ell, u\}$$

PV Wait: $$\begin{aligned} V_{\text{wait}} &= \beta E_0 \max\{V_1(x^i), 0\} = \pi\beta V_1(x^u) \\ &= \pi\beta\left(\frac{\beta}{1-\beta}x^u - p^k\right). \end{aligned}$$

- $V_{\text{wait}} > V_0 > 0 \Rightarrow$ better to wait even though investment profitable at time 0.
- More uncertainty $\Rightarrow$ option value of waiting increases.