Student Name : _____ Student Number: _____

**Practice Final II**

1. (10 points) Consider the function $f(x) = x^2e^x + e^x$.

   (a) (2 points) $f(x)$ has one horizontal asymptote. Compute this asymptote.

   (b) (2 points) For what values of $x$ is $f$ increasing? *Justify with either a table or a sentence.*

(c) (3 points) On what intervals is $f$ concave down? *Justify with a table or a sentence.*

(d) (2 points) State the points of inflection of $f$. *No justification necessary*

2. Let $C$ be the curve defined by the relation

$$\sin(x^2 + y^2) = e^{2y}$$

(a) (4 points) Find $\frac{dy}{dx}$ for $C$.

(b) (3 points) Find the equation of the tangent line to $C$ at the point $(x, y) = (\sqrt{\pi}, 0)$.

(c) (3 points) Use tangent line approximation at $(\sqrt{\pi}, 0)$ to estimate the number $a$ so that the point $(a, \frac{\pi}{10})$ is on the curve $C$. *To receive full credit use an exact answer.*

a=

3. Japan is moving through the ocean with parametric equations $(x, y) = (\pi t, \pi^t + \pi)$.

(a) (3 points) Is the tangent line to Japan ever horizontal? Justify your answer.

YES NO

(b) (2 points) Find the equation of the tangent line at the point $(\pi, 2\pi)$.

(c) (4 points) Is the graph of Japan's motion concave up or concave down at $(\pi, 2\pi)$? *You must show all work to get <u>any</u> credit.*

UP DOWN

4. (10 points) Matt is filling a spherical water balloon from a rigid cylindrical canister of radius 1 meter. Suppose that the moment the water balloon has radius $\sqrt{2}$ meters, the height of the water level in the canister is decreasing at a rate of $2\pi \frac{m}{min}$. At this instant, find the rate at which the radius of the water balloon is changing. ($V_{cylinder} = \pi r^2 h$, $V_{sphere} = \frac{4}{3}\pi r^3$)