

CALCULATING WITHOUT READING? COMMENTS ON COHEN AND DEHAENE (2000)

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Cohen and Dehaene (*Cognitive Neuropsychology*, 2000, Vol. 17, pp. 563–583) reported the case of a pure alexic patient who preserved some calculation abilities despite severely impaired Arabic numeral reading. They argued that these residual abilities and the general pattern of performance of the patient can be fully explained within their anatomo-functional triple-code model. Here, we show how the lack of specification of the assumed architecture, the failure to provide sufficiently detailed data, and the absence of adequate refutation of alternative accounts make this study unsuitable for constraining theories of numerical cognition.

INTRODUCTION

In *Calculating without reading*, Cohen and Dehaene (2000; hereafter, C&D) report the case of a pure alexic patient (VOL) whose performance pattern suggests complex relations between the representations and processes involved in mapping Arabic onto verbal numerals and those involved in retrieving various aspects of numerical knowledge from Arabic numerals. The case is used to address specific aspects of numerical knowledge involved in various tasks, and how they are related within a general architecture of number processing and calculation.

In this comment we argue that although C&D offer *plausible* theoretical propositions related to this issue, their study does not meet two basic requirements for a single-case study to *constrain* theories of normal cognition. The first unfulfilled requirement is that it must be possible to derive a

patient's pattern of performance from the characteristics of the assumed architecture (i.e., the functional components and their relations) and the nature of representations and computational principles of its components, especially those assumed to be impaired (Caramazza, 1986, 1992). We think that, in C&D's study, the nature of representations and computations in the hypothetical numerical architecture is not sufficiently detailed to allow VOL's pattern of performance to be interpreted reliably. The second unfulfilled requirement bears upon a more general principle of scientific argumentation. Data support a theory not only if they *fit* this theory, but if they *compel* us to prefer it to other plausible ones. In a single-case study, this entails not only accounting for a patient's performance within one's own set of assumptions, but also rejecting, with adequate evidence, accounts based on other plausible postulates. C&D never consider alternative accounts for their data and, accordingly,

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neither seek nor report evidence to evaluate other possible accounts. We will show how the failure to meet these two basic requirements makes C&D's study unable to constrain models of numerical cognition. We first summarise C&D's observations and the derived theoretical claims. Then we examine each of these claims and ask whether they are commensurate to the level of detail of the theoretical framework and to the amount and nature of evidence provided.

C&D'S ACCOUNT OF VOL'S PATTERN OF PERFORMANCE

Although her comprehension of Arabic numerals and her production of verbal numerals were correct per se, VOL was impaired at reading aloud single-digit Arabic numerals. When solving arithmetical problems, she correctly read aloud the operands in about 20% of the cases. Yet, she gave the result of the target problem with a relatively good level of accuracy in subtraction, addition, and division (72%, 61%, and 66% correct, respectively), but not in multiplication (only 13% correct). Interestingly, she gave the correct result for the problem erroneously read in 77% of the cases (e.g., 5×9 was read as *four times six* and answered *twenty-four*). This contrast between impaired naming and multiplication and relatively spared comparison and other arithmetic operations led C&D (2000, p. 580) to propose two distinct routes for Arabic numeral processing, "one that is the mandatory input pathway to naming and multiplication processes, and the other that is able to supply comparison, addition, subtraction, and simple division routines." In patient VOL, only the direct Arabic-to-verbal mapping route would be impaired, whereas the indirect semantic route (i.e., through access to and manipulation of the quantities represented by numerals) would be spared. Multiplication problems could only be solved by retrieval of the corresponding facts stored as verbally coded associations linking operands and products. Arabic-to-verbal mapping would thus be the *prerequisite* for multiplications, which explains answers stemming from erroneous reading of operands through this route.

C&D also tentatively link these functional proposals with an anatomical organisation of their "triple-code" model (Dehaene & Cohen, 1995), according to which, digit structural representation and quantity information are processed by both hemispheres, whereas only the left hemisphere processes verbal numerals. VOL's lesion damaged the left-hemispheric digit recognition processes, precluding the transcoding of Arabic numerals into words (hence, the retrieval of stored multiplication facts) and the access to the left-hemispheric quantity representation. Her residual numerical abilities would reflect intact right-hemispheric processes.

C&D'S THEORETICAL CLAIMS AND THE SIGNIFICANCE OF THE DATA

Do Arabic numeral reading errors reflect functional Arabic-to-verbal mapping impairments?

C&D hypothesise that VOL's naming difficulties arise from the verbal system being deprived of its normal input, but without specifying whether the damage was to the Arabic recognition system or to the Arabic-to-verbal mapping system. Here, we focus on the functional aspects of this claim, and discuss its anatomo-functional aspects in the last part of this section.

At a functional level, structural processing of digits seemed relatively spared. VOL could discriminate digits from letters, and her errors did not reflect visual confusions. Therefore, the deficit causing VOL's naming difficulties would lie in the asemantic Arabic-to-verbal mapping system. However, this conclusion stems from C&D's prior assumption that Arabic numerals are named without semantic access, which is quite controversial (for a discussion, see McCloskey, 1992; Seron & Noël, 1995). It might turn out to be right but, to be stated as a *motivated* account of VOL's naming difficulties, alternative assumptions must be empirically ruled out, the more straightforward being reading digits through semantic access. Then, VOL's naming errors would stem from an inability to access semantics from the structural representation of Arabic digits. Unfortunately, C&D do not

specify the characteristics of their asemantic transcoding system, and they do not derive nor test specific predictions about the pattern of naming errors generated when the system is not properly addressed. For instance, in case of damage to the asemantic route, should one expect substitutions between *phonologically* similar verbal numerals, or between *semantically* similar (i.e., close in magnitude) numerals as expected in case of damage to the semantic route? In the reading tasks, VOL was presented with one- to four-digit Arabic numerals, and she made almost exclusively digit substitution errors. If the visual representation computed on the basis of Arabic numerals cannot properly address the Arabic-to-verbal mapping system, one should expect VOL, besides substitutions, to make syntactic errors due to loss or exchange of digit positions during transfer. Such errors were not observed, which shows that she at least retained the ability to analyse the positions of multi-digit numerals, to maintain and transfer these positions to the transcoding system, and to map them into the appropriate multi-word numeral structure. This seems rather inconsistent with a “destruction or deafferentation” (C&D, 2000, p. 573) of the processing component whose output is used by the Arabic-to-verbal mapping system. C&D should at least explain why only the lexical part (identity of the digit) of the structural representation was not correctly transferred and processed, while its syntactic part (positional structure) was.

Thus, the data reported do not allow the reader to decide whether Arabic naming errors stem from impairment to the asemantic or to the semantic route. C&D argue that their hypothesis naturally follows from observations made with other tasks. Since VOL could access quantity information from Arabic numerals but could not read them aloud, then naming *necessarily* relies on an asemantic processing route. In the next two sections, we discuss the relevance of the evidence supporting the

premise of this argument and explain why it is inadequate.

Distinct processing routes for naming and comparing Arabic numerals?

VOL could accurately compare two Arabic numerals though she could not read them aloud. This would support the idea of two processing routes if it is shown that a single route cannot yield, when damaged, different levels of performance reflecting *inherently* different levels of difficulty of the tasks.

If naming and comparing rely on the same processing route, the structural representation of Arabic numerals would give access to their meaning and, thereafter, to the verbal numerals retrieved from the phonological lexicon. Assuming that VOL can compute structural and magnitude representations, the functional damage would then be located in the structural-to-magnitude representation addressing procedures, causing weak activation of the numeral meaning, and resulting in approximate/degraded quantity information. This could nevertheless allow VOL to give accurate responses in a magnitude comparison task since it is logically possible to compare numerals without accessing their exact magnitude (as demonstrated by Dehaene & Cohen, 1991). This is especially true for the comparison task used in C&D’s study, where the numerical distance between the numbers in the pairs ranged from 2 to 26 (mean distance: 8.8). Accessing only approximate quantity information might thus suffice in most cases. Moreover, accurate responses might be given even when the quantity information accessed is totally wrong, provided that the relative size of the two numbers is not reversed¹. This account would not hold if it had been shown that VOL could access exact quantities (e.g., verifying whether there is a correct match between Arabic numerals and arrangements of poker chips of various numerical values, compari-

¹ C&D rejected in advance this objection by arguing that VOL made very few reversing errors when reading aloud pairs of two-digit numerals. In our view, this only suggests that if VOL had based her responses in comparison on the quantity information she accessed in reading, the probability of errors in comparison would be very low. Note that data in comparison and in naming were collected with different stimuli on different occasions. It would have been more convincing if VOL had had to read aloud and immediately after to compare the pairs.

sons with splits one or two, etc.). In this study, nothing compels us to accept that VOL “showed a better access to quantity information than could be expected on the basis of her number reading performance” (C&D, 2000, p. 567). The data are compatible with the hypothesis of two distinct processing routes, but they do not *need* to make this hypothesis.

We showed that, in VOL’s performance, nothing allows one to characterise the naming route as a non-quantity-based one. In our opinion, there is no strong basis for characterising the comparison process as *mandatorily* involving quantity information either. That numerical comparison *normally* relies on quantity information is a widely shared assumption. However, aspects of VOL’s performance question whether quantity information is *absolutely* required to compare numerals. Indeed, VOL correctly accessed the knowledge of the rank of Arabic numerals and the corresponding rank of verbal numerals (she could retrieve the correct verbal numeral by reciting the conventional sequence of number words up to the rank retrieved from the presented digit). This raises the question of the status of rank information in number semantics. Rank and quantity information might be two logically dissociable aspects of number semantics whose integration forms the core meaning of a numerical concept (for a dissociation between these two aspects, see Delazer & Butterworth, 1997; see also Fuson, 1988; Gallistel & Gelman, 1992; Wynn, 1992, 1995). Thus, that VOL performed the comparison task on the basis of rank information because of weak activation of quantity information cannot be dismissed on the basis of the results reported for the comparison task. There is evidence that VOL did not compare Arabic numerals by (covert) counting, but she might directly access information about their rank value (e.g., some representation of their “sequence meaning”; cf. Fuson, 1988, 1992). In such conditions, short latencies and distance effects might also be expected, as is found when comparing the order of letters (Hamilton & Sanford, 1978; Taylor, Kim, & Sudevan, 1984). A possible account for the dissociation observed between naming and comparing is that digit naming (without verbal sequence recitation)

is dependent on (impaired) quantity information whereas comparing (and naming after verbal recitation) might be based on (spared) rank information only. Such an account does not entail that rank and quantity are represented and processed by two distinct systems. Rank information associated with numerals is acquired earlier than the quantity they represent (Wynn, 1990, 1992) and might just be more easily accessible within the number semantic system or/and less vulnerable to brain damage.

A single processing route for comparing, adding, subtracting, and dividing Arabic numerals?

Because VOL performed almost perfectly the comparison task *and*, with reasonable accuracy, simple subtractions, divisions, and additions, C&D propose that the same processing route (access to and manipulation of quantities) was used in all cases. However, observing a good performance in two tasks does not suffice to infer that both rely on the same processing route or functional system. One needs independent evidence that the same sort of knowledge—here, quantities—is used, with specific additional knowledge or processing for each task.

As we understand it, C&D propose that, in VOL as well as in healthy subjects, simple subtractions could rely only on quantity manipulations. However, they do not specify the kind of computation involved in these manipulations (i.e., how quantities encoded as distributions of activation on an internal number line are manipulated). Therefore, it is not clear how errors arising from faulty manipulations could be distinguished from errors arising from faulty access to stored facts. For example, the fact that “most erroneous responses to subtractions were false by only 1 unit” (C&D, 2000, p. 578) might reflect a faulty access to the quantity of one of the operands rather than errors in quantity manipulation during calculation.

C&D (2000, p. 576) propose “two parallel circuits” for additions: “rote verbal retrieval and strategical manipulations” or “back-up strategies”

such as counting or “referring to $10 (6 + 5 = 6 + 4 + 1 = 10 + 1 = 11)$ ”². However, it is unclear whether these circuits are two equally efficient means to reach correct results or if they must be recruited in a complementary manner. In the above-mentioned example, is $6 + 4$ retrieved from memory or computed? The same ambiguity holds for divisions. C&D (2000, p. 577) propose that, for large problems, the corresponding multiplication fact is retrieved, but “the simplest division problems, such as those presented to VOL, can also be solved using semantic procedures. For instance, halving a number can be performed by referring to the corresponding tie addition fact ($6 \div 2 = 3$ because $3 + 3 = 6$).” In this example, how is the result of $3 + 3$ retrieved? These ambiguities prevent precise predictions in case of damage to the verbal retrieval circuit resulting in compensatory reliance on quantity manipulations. If verbal retrieval and quantity manipulation are equally efficient, then subtractions, divisions, and additions should yield the same level of performance in VOL—as was actually observed. If rote verbal retrieval and quantity manipulation are complementary, then subtractions should yield better performance than additions and divisions in VOL—which was not observed. If VOL solved additions by strategical counting and/or decomposition, her response times should be longer for additions than for subtractions and divisions—which was not reported.

Predictions are even more obscure concerning the problems that VOL should solve accurately/erroneously. If quantity manipulation is the rule, poorer performance should be observed for large than for small problems, whatever the operation. If quantity manipulation and verbal retrieval are both required for additions and divisions, the size of the problems should affect error rates for subtractions only. If additions require both procedures, VOL should resort either to quantity manipulations for *some* problems and rote retrieval for *some* others, or to quantity manipulations *and* rote retrieval for most if not all problems (e.g., $3 + 4 = \underline{3 + 3 + 1}$). Finally, regarding divisions, VOL should be able to

cope either with problems with 2 as the divisor, whatever the size of the first operand and result (e.g., better performance both for $6 \div 2$ and $10 \div 2$ than for $6 \div 3$ and $15 \div 3$), or with problems with 2 as the result, whatever the size of the operands ($6 \div 3$ or $18 \div 9$), because these problems can be solved by counting.

Given the lack of specifications on VOL’s assumed back-up strategies, and the lack of demonstration that her pattern of performance can be derived from these specifications, it is not possible to evaluate the claim that she solved subtractions, additions, and divisions by manipulating quantities. For the same reasons, other accounts cannot be dismissed. For instance, one can assume that arithmetical facts are stored in memory as abstract declarative (instead of verbal) knowledge directly accessed from the structural representation of Arabic numerals. Then, VOL would solve subtractions, additions, and divisions by directly retrieving the answers, and her errors would result from relatively *impaired* access to this stored knowledge. Otherwise, VOL could solve these problems by relying on both direct memory retrieval and quantity manipulation, and her errors would result from impaired access to quantities.

Mandatory verbal retrieval for multiplication facts?

Multiplication errors were generally the correct answers for the operands (erroneously) read aloud, which clearly shows that multiplication was more dependent on the verbal coding of the operands than other operations. However, it does not imply that the products were retrieved from a verbal store—only that they were retrieved *after* or *on the basis* of the verbal coding of the operands. Furthermore, even if VOL actually retrieved the results from verbal memory, it might not be the only available routine. Indeed, healthy subjects use back-up strategies based on calculation when memory retrieval fails to provide an answer (LeFevre, Sadeski, & Bisanz, 1996). Why does VOL not

² It is unclear whether “quantity manipulation”, “semantic routines”, “strategical quantity manipulations”, and “back-up strategies” have the same status in C&D’s proposals.

resort to such strategies, as she does for other operations? C&D argue that these back-up strategies cannot be distinguished from retrieval because they involve other stored facts (e.g., computing 7×8 as $8 \times 8 - 8$). However, the set of multiplications presented to VOL also included problems that do not resort to fact retrieval ($N \times 1$ problems, $N \times 12$ problems potentially solvable by addition), and virtually all problems could be solved by counting by 2, by 3, etc.³. Yet, different rates of errors were not reported for these problems. On the contrary, problems matched in result magnitude (mainly $N \times 1$ problems) gave rise to as many errors (C&D, Table 2). No explanation is given for this result: Why was VOL able to access quantities and strategically manipulate them to retrieve the results for additions and divisions, and not for at least some multiplications? A possible interpretation is that, in fact, quantity-based and compensatory strategies were no longer available to VOL, because access to quantity information from Arabic numerals was damaged. This conclusion is reasonable since no strong evidence was provided that VOL resorted to quantity-based or compensatory strategies for subtractions, additions, and divisions. Thus, let us again assume that arithmetical facts (for all operations) are stored as declarative knowledge, access to which can proceed directly from the structural representation of the operands, and that solving arithmetic problems requires both retrieval and quantity manipulations, the latter being specially recruited for less familiar problems. If access to quantities from Arabic numerals was impaired while access to stored facts was spared, VOL would be more impaired at multiplication, because the set of multiplication problems probably comprised less familiar problems. To compensate for these difficulties, VOL could choose systematically to first transcode the operands verbally so as to access the quantity system and the stored multiplication facts from a verbal input (i.e., by the same route as when solving orally presented multiplications). Then, errors for other operations should reflect the impaired access to quantity information as well—which is not inconsistent with the data.

Because they are learned as verbal associations, multiplication facts might be represented in a verbal as well as in an abstract knowledge store. Then, VOL would retrieve not only the operands, but also the results of multiplications through the impaired verbal route (the direct verbal route or the quantity-to-verbal route). However, this additional point is not required by the observations. C&D (2000, p. 570) note that, in the occasional errors (as a function of the read operands) made by VOL in multiplications, “the results fell close to within the correct row or column of the spoken problem within the multiplication table.” Are such table errors predicted by the hypothesis of rote verbal retrieval for multiplications? How are multiplication facts organised within the verbal store? Should errors showing phonologically based confusions not be expected? Here again, without more detailed specifications about the representation and processing principles of the rote verbal memory for multiplication facts, it is not possible to distinguish equally reasonable and compatible accounts.

Intact abilities of the right hemisphere or residual abilities of the partially impaired left hemisphere?

C&D proposed that VOL’s residual abilities in processing quantities reflect the normal functioning of intact numerical processes in the right hemisphere (RH) since her left occipito-temporal damage was assumed to alter processing of Arabic numerals, depriving the left-hemispheric (LH) Arabic-to-verbal mapping and quantity systems from their normal input.

As previously shown, the study provides no evidence likely to constrain this argument. C&D *assume* that the functional implication of VOL’s LH damage is to prevent processing of Arabic numerals but without specifying whether the damage was to the left Arabic recognition system or to the Arabic-to-verbal connection pathway. Within their theoretical framework, this has distinct implications. Damage to the LH recognition system

³ Unfortunately, it is not known whether the patient is able to use such counting sequences.

itself would result in incorrect input to both the LH verbal and semantic systems. Damage altering the connection between the recognition and verbal systems would spare the access to the left semantic component. Hence, VOL's residual abilities could be accounted for without assuming RH routes, and her errors might simply reflect the residual abilities of only partially damaged left components. Damage to the left Arabic recognition system predicts that (1) the digit substitution errors made by VOL in naming should reflect confusions between similarly shaped digits (i.e., errors reflecting visual proximity between target and response or/and other visuospatial features), and (2) if presented with Arabic numerals to her right hemifield, VOL should be unable to access semantics, or should also provide responses reflecting visually based confusions. Such predictions were not tested, although the main hypothesis underlying the anatomo-functional proposal was damage to the visual recognition system.

Even if we accept C&D's assumptions about the functional locus of impairment in VOL, and even if there was independent evidence for RH numerical capacities in other patients, this case would not demonstrate RH numerical capacities. This issue includes three main aspects. First, up to which point is the RH able to process Arabic numerals from perception? We agree that the RH seems able to build structural representations of Arabic numerals. RH processing capacities might end there, with the output of this process being transferred to the LH, as C&D state in their conclusion. However, they also claim that "both the left- and the right-hemispheric visual systems are able to build a structural representation of Arabic numerals ... and to access their meaning, i.e., the quantity to which they refer" (C&D, 2000, p. 573) or that "quantity manipulation abilities are distributed over the two hemispheres" (C&D, 2000, p. 576). This cannot be inferred either from VOL's performance

or from other pure alexic patients (Cohen & Dehaene, 1995; McNeil & Warrington, 1994; Miozzo & Caramazza, 1998) who "showed spared comparison abilities, whereas sparing of arithmetic problem solving seems at first sight to be more variable across patients and operation types" (C&D, 2000, p. 573). Even leaving aside this inter-patient variability, it remains to show that the residual abilities reflected RH rather than LH capacities after transfer of structural representations computed in the RH⁴. Second, are RH processing components different or a mere duplication of the left ones? C&D only speculate that the right and left processing routes involve the same kind of knowledge and computations, but the right route does it less accurately: There might be a LH advantage for visual identification of digits, the right quantity system might be inherently less precise than the left, or both hemispheres might be equally accurate, but there could be some loss of information due to callosal transfer. Finally, if the left and right components are different, what are their respective functions during normal number processing? Unfortunately, the nature of computation and respective contributions of RH and LH components during normal processing, and the means to distinguish RH *intact* processing of *approximate* quantity from LH *impaired* processing of (more precise) quantity information are not clearly formulated.

ANOTHER POSSIBLE ACCOUNT OF VOL'S PERFORMANCE

It might be objected that C&D's explanation of VOL's entire pattern is more elegant and coherent than our multiple alternative accounts. However, putting together these alternatives would not necessarily yield a less coherent explanation. Let us assume a number processing system composed of

⁴ Admittedly, data from split-brain patients (e.g., Cohen & Dehaene, 1996) speak against the view that the RH is only able to compute digit structural representations. With Arabic numerals presented to their left hemifield, these patients are able to perform comparison tasks. However, to the best of our knowledge, performance is generally not normal.

one functional component⁵, building a structural representation of Arabic numerals, one processing number semantics, and one representing stored arithmetic knowledge (Figure 1). The “Number semantics” system represents and processes the core meaning of numerals integrating their rank and quantity properties. The “Stored arithmetic knowledge” represents, in an abstract format, simple, highly familiar, and frequently used arithmetical facts (for all operations), as well as usual arithmetic properties (like parity, square roots, prime num-

bers, etc.; it is not relevant here whether these various aspects of stored arithmetic knowledge are represented in partly independent networks). In this context, VOL’s pattern of performance could be accounted for as follows. The Arabic-to-semantic processing route was impaired, so that only rank and approximate quantities information were accessed. From this partial meaning, VOL could retrieve the correct verbal numeral by reciting the verbal sequence up to its rank, and perform comparisons but not tasks requiring exact number

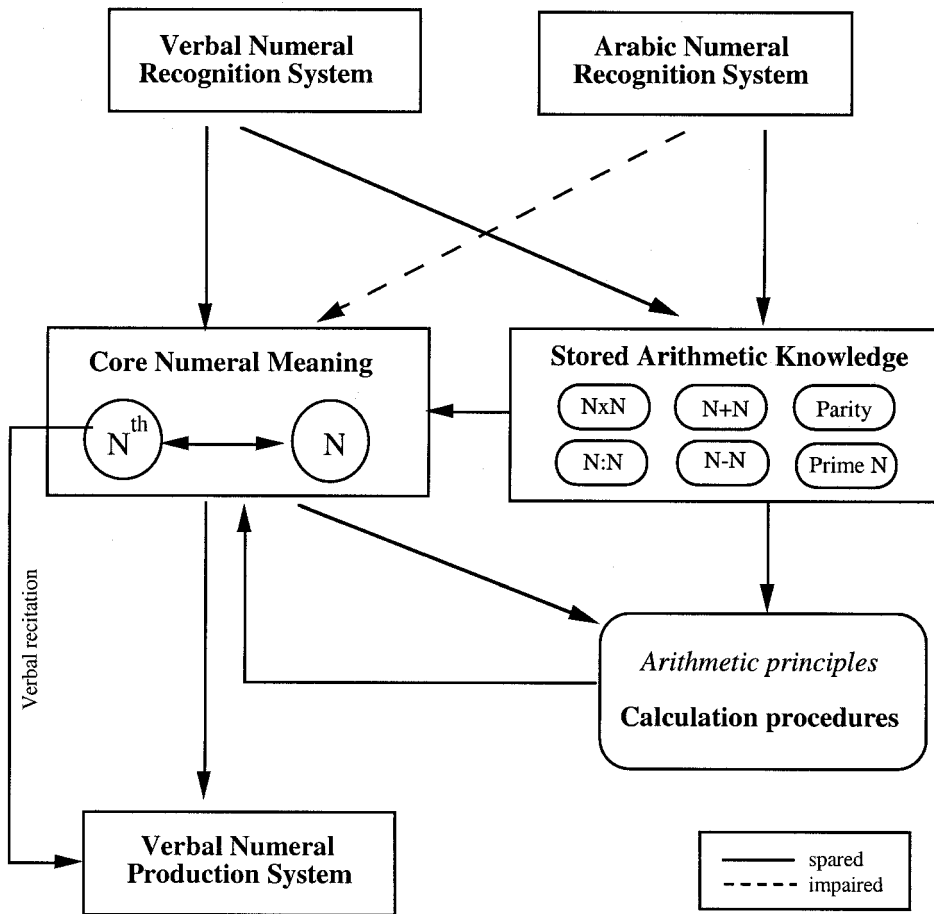


Figure 1. A possible alternative account of VOL’s performance (plain lines: spared routes; dashed lines: impaired route).

⁵ By “functional” system, we mean a component within a functional architecture that is assumed to be involved in specific operations. It does not matter, within this definition, whether this processing component is located within one hemisphere or distributed over both hemispheres. In case of a functional component distributed over the two hemispheres, if the *normal* functioning of the assumed operations required *intact* processing of both, then only one functional component is assumed.

meaning such as reading aloud Arabic numerals. Access to stored arithmetic knowledge would be spared. If only familiar arithmetic knowledge is stored and/or reliably retrieved, then less familiar arithmetic problems (and arithmetic properties) would require access to both stored arithmetical knowledge and quantity manipulations. Therefore, solving simple problems and accessing parity information, though not perfect, would yield a better performance than naming. The need to rely on quantity manipulations for less familiar facts would account for the different levels of accuracy across the four operations. In order to compensate for her difficulties in multiplication, VOL would strategically attempt to retrieve the results on the basis of the verbal coding of the operands so that she could access the number knowledge system (number semantics and stored arithmetical knowledge) through the spared verbal-to-semantic route.

Let us stress, in order to avoid any misunderstanding, that we are neither advocating here this particular architecture of number processing nor claiming that it could account for a larger range of observations than just this study. Providing fine specifications and discussing the whole range of evidence to date would fall beyond the scope of this commentary. We are not even defending this specific account of VOL's performance. Instead, we want to show that, with the evidence reported in this study and other reasonable assumptions about a possible architecture, it is possible to draw a conclusion exactly opposite to that of C&D.

CONCLUSION

Deriving claims about normal processing systems from impaired performance requires clear and motivated hypotheses about the nature and functional locus of the deficits. Although C&D do put forward a hypothesis about the locus of VOL's damage, this hypothesis is ambiguous and not motivated by adequate evidence. The data reported generally support the proposed theory, but they do not compel us to prefer it to other accounts. We have shown that, due to a lack of specification of some aspects of the model and the absence of criti-

cal tests of alternative assumptions, it is possible to account for VOL's performance in exactly the opposite way as C&D did. If so, their study clearly does not contribute to *constraining* theories about number processing. We also hope to have shown that single-case studies in neuropsychology, if they aim at constraining theories of normal cognition, should not neglect the old precept: "To give support to your theory, assume just the opposite of what you want to demonstrate, and then show that this opposite view cannot work."

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