

#1: Parametric Equations and Circles

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1 Welcome!

1.1 About Me

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(Ignore this stuff, its a side-effect of having a table of contents)

3 What is 124 All About?

3.1 Limits

Example:

$$\lim_{x \rightarrow 0} \frac{x}{\sin x}.$$

3.2 Tangent Lines

Example:

What is the equation of the tangent line to the curve $f(x) = \ln x$ when $x = 2$.

3.3 Curve Sketching

Example:

Without using a calculator draw the graph of $y = x^2 e^x$.

3.4 Rates of Change

Example:

A 10m ladder is being pulled away from a wall at a rate of 1 m/s. How fast is the top of the ladder sliding down the wall when the ladder makes an angle of $\pi/6$ with the ground?

3.5 Optimization

Example:

A plastic cup is made in the shape of a cylinder. The volume is to be 250 cm^3 . Find the radius and height of the cup which minimize the amount of plastic required.

4 Parametric Equations

Parametric equations are a way of precisely describing the position of an object in the plane.

Example:

Bug in the plane.

Diagram

! Parametric equations aren't a 'graph'. Instead they tell you an object's coordinates at a given time.

Example:

#1: Parametric Equations and Circles

Fred is walking in the woods with constant speed and direction. After 1 hour he is 4 miles east and 3 miles north of where he started. Find parametric equations describing his motion.

Diagram

$$x(t) = 4t, \quad y(t) = 3t.$$

Fact:

The formula for parametric equations of motion along a circle of radius R with angular velocity ω , center (A, B) and starting angle θ (relative to x -axis) is

$$x(t) = R \cos(\omega t + \theta) + A, \quad y(t) = R \sin(\omega t + \theta) + B.$$

Example:

You are riding a ferris wheel of radius 50 ft which is 3 ft above the ground. The wheel turns counterclockwise. You start at the bottom and it takes 10 seconds to get to the top. Write parametric equations describing your motion.

Diagram

$$x(t) = 50 \cos\left(\frac{\pi}{10}t - \frac{\pi}{2}\right), \quad y(t) = 50 \sin\left(\frac{\pi}{10}t - \frac{\pi}{2}\right) + 53.$$

5 The Essential Facts about Circles

There are a few things that you absolutely need to know about circles. First and foremost, you need to know the equation for a circle.

A circle with radius R and center (A, B) has equation:

$$(x - A)^2 + (y - B)^2 = R^2$$

The other must know fact for Math 124 is the special property satisfied by tangent lines.

Diagram

Fact: Any tangent line to a circle is perpendicular to the radial line from the circles center.

Diagram

Example:

Find the tangent line to the circle $x^2 + y^2 = 25$ at the point $(3,4)$.

Computation

Example:

Find the two tangent lines to the unit circle that go through the point $(2, 1)$.

Computation