Review Exercise 4

Q.1 Multiple type questions?

(i) **is an algebraic …**

(a) Expression **(b)** Sentence
(c) Equation **(d)** In-equation

(ii) **The degree of polynomial** $4x^4 + 3x^2y$ **is**

(a) 1 **(b)** 2
(c) 3 **(d)** 4

(iii) $a^3 + b^3$ **is equal to**

(a) $(a-b)(a^2+ab+b^2)$ **(b)** $(a+b)(a^2-ab+b^2)$
(c) $(a-b)(a^2-ab+b^2)$ **(d)** $(a-b)(a^2+ab+b^2)$

(iv) $(3+\sqrt{2})(3-\sqrt{2})$ **is equal to**

(a) 7 **(b)** -7
(c) -1 **(d)** 1

(v) **Conjugate of surd** $a+\sqrt{b}$ **is;**

(a) $-a+\sqrt{b}$ **(b)** $a-\sqrt{b}$
(c) $\sqrt{a}+\sqrt{b}$ **(d)** $\sqrt{a}-\sqrt{b}$

(vi) $\frac{1}{a-b} - \frac{1}{a+b}$ **is equal to**

(a) $\frac{2a}{a^2-b^2}$ **(b)** $\frac{2b}{a^2-b^2}$
(c) $\frac{-2a}{a^2-b^2}$ **(d)** $\frac{-2b}{a^2-b^2}$

(vii) $\frac{a^2-b^2}{a+b}$ **is equal to**

(a) $(a-b)^2$ **(b)** $(a+b)^2$
(c) $a+b$ **(d)** $a-b$

(viii) $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})$ **is equal to**

(a) a^2+b^2 **(b)** a^2-b^2
(c) $a-b$ **(d)** $a+b$

ANSWER KEY

i	ii	iii	iv	v	vi	vii	viii
a	d	b	a	b	b	d	c

Q.2 Fill in the blanks

(i) **The degree of polynomial** $x^2y^2+3xy+y^3$ **is** __________

(ii) x^2-4 __________

(iii) $x^3+\frac{1}{x^3}=\left[x+\frac{1}{x}\right](\_\_\_\_\_\_\_\_\_\_)$

(iv) $2\left(a^2+b^2\right)=(a+b)^2+(\_\_\_\_\_\_\_\_\_\_)^2$

(v) $\left[x-\frac{1}{x}\right]^2=$ __________

(vi) **Order of surd** $\sqrt[3]{x}$ **is** __________

(vii) $\frac{1}{2-\sqrt{3}}=$ __________

ANSWER KEY

(i) 4

(ii) $(x-2)(x+2)$

(iii) $x^2-1+\frac{1}{x^2}$

(iv) $a-b$

(v) $x^2+\frac{1}{x^2}-2$

(vi) 3

(vii) $2+\sqrt{3}$

Q.3 If $x+\frac{1}{x}=3,$ **find**

(i) $x^2+\frac{1}{x^2}$

Solution: Given that $x+\frac{1}{x}=3$

$\therefore (a+b)^2=a^2+b^2+2ab$

Putting the values

$$\left[x+\frac{1}{x}\right]^2=(x)^2+\left(\frac{1}{x}\right)^2+2(x)\left(\frac{1}{x}\right)$$

$$(3)^2=x^2+\frac{1}{x^2}+2$$

$$9=x^2+\frac{1}{x^2}+2$$

$$9-2=x^2+\frac{1}{x^2}$$

$x^2+\frac{1}{x^2}=7$ **Ans**

(ii) $x^4+\dfrac{1}{x^4}$

Solution: Given that $x^2+\dfrac{1}{x^2}=7$

$$\left(x^2+\frac{1}{x^2}\right)^2=\left(x^2\right)^2+\left(\frac{1}{x^2}\right)^2+2\left(x^2\right)\left(\frac{1}{x^2}\right)$$

$$(7)^2=x^4+\frac{1}{x^4}+2$$

$$49=x^4+\frac{1}{x^4}+2$$

$$x^4+\frac{1}{x^4}=49-2$$

$$x^4+\frac{1}{x^4}=47 \textbf{ Ans}$$

Q.4 **If** $x-\dfrac{1}{x}=2$ **find**

(i) $x^2+\dfrac{1}{x^2}$ $(ii)\, x^4+\dfrac{1}{x^4}$

Solution (i)

Given that $x-\dfrac{1}{x}=2$

$\therefore (a+b)^2=a^2+b^2+2ab$

Putting the values

$$\left(x-\frac{1}{x}\right)^2=(x)^2+\left(\frac{1}{x}\right)^2-2(\cancel{x})\left(\frac{1}{\cancel{x}}\right)$$

$$(2)^2=x^2+\frac{1}{x^2}-2$$

$$4+2=x^2+\frac{1}{x^2}$$

$$x^2+\frac{1}{x^2}=6 \textbf{ Ans}$$

Solution (ii)

Given that $x^2+\dfrac{1}{x^2}=6$

$$\left(x^2+\frac{1}{x}\right)=x^4+\frac{1}{x^4}+2\left(\cancel{x^2}\right)\left(\frac{1}{\cancel{x^2}}\right)$$

$$(6)^2=x^4+\frac{1}{x^4}+2$$

$$x^4+\frac{1}{x^4}=36-2$$

$$x^4+\frac{1}{x^4}=34 \textbf{ Ans}$$

Q.5 **Find the value of** x^3+y^3 **and** xy **if** $x+y=5$ **and** $x-y=3$ **.**

Solution: Given that $x+y=5$

$x-y=3$

As we know that

$\therefore (x+y)^2-(x-y)^2=4xy$

Putting the values

$4xy=(5)^2-(3)^3$

$4xy=25-9$

$4xy=16$

$xy=\dfrac{\cancel{16}^{4}}{\cancel{4}}$

$xy=4$ **Ans**

As we know that

$(x+y)^3=x^3+y^3+3xy(x+y)$

Putting the values

$(5)^3=x^3+y^3.3\times4\times5$

$125=x^3+y^3+60$

$x^3+y^3=125-60$

$x^3+y^3=65$

$x^3+y^3=65$ **Ans**

Q.6 If $P = 2+\sqrt{3}$, find

(i) $P+\frac{1}{P}$

Solution: Given that $P = 2+\sqrt{3}$

$$\frac{1}{P} = \frac{1}{2+\sqrt{3}}$$

$$\frac{1}{P} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$\frac{1}{P} = \frac{2-\sqrt{3}}{(2)^2-\left(\sqrt{3}\right)^2}$$

$$\frac{1}{P} = \frac{2-\sqrt{3}}{4-3}$$

$$\frac{1}{P} = \frac{2-\sqrt{3}}{1}$$

$$\frac{1}{P} = 2-\sqrt{3}$$

$$P+\frac{1}{P} = 2+\cancel{\sqrt{3}}+2-\cancel{\sqrt{3}}$$

$$P+\frac{1}{P} = 4 \text{ **Ans**}$$

(ii) $P-\frac{1}{P}$

As we know that

$\frac{1}{P} = 2-\sqrt{3}$ and

$P = 2+\sqrt{3}$

$$P-\frac{1}{P} = 2+\sqrt{3}-\left(2-\sqrt{3}\right)$$

$$= \cancel{2}+\sqrt{3}-\cancel{2}+\sqrt{3}$$

$$= 2\sqrt{3} \text{ **Ans**}$$

(iii) $P^2+\frac{1}{P^2}$

Solution: Given that $P+\frac{1}{P} = 4$

$$\therefore (a+b)^2 = a^2+b^2+2ab$$

$$\left(P+\frac{1}{P}\right)^2 = (P)^2+\left(\frac{1}{P}\right)^2+2\left(\cancel{P}\right)\left(\frac{1}{\cancel{P}}\right)$$

$$(4)^2 = P^2+\frac{1}{P^2}+2$$

$$16-2 = P^2+\frac{1}{P^2}$$

$$P^2+\frac{1}{P^2} = 14 \text{ **Ans**}$$

(iv) $P^2-\frac{1}{P^2}$

Solution:

$$P^2-\frac{1}{P^2} = \left(P+\frac{1}{P}\right)\left(P-\frac{1}{P}\right)$$

$$P^2-\frac{1}{P^2} = (4)\left(2\sqrt{3}\right)$$

$$= 8\sqrt{3} \text{ **Ans**}$$

Q.7 If $q = \sqrt{5}+2$ find.

(i) $q+\frac{1}{q}$

Solution: Given that $q = \sqrt{5}+2$

$$\frac{1}{q} = \frac{1}{\sqrt{5}+2} \times \frac{\sqrt{5}-2}{\sqrt{5}-2}$$

$$= \frac{\sqrt{5}-2}{\left(\sqrt{5}\right)^2-(2)^2}$$

$$= \frac{\sqrt{5}-2}{5-4}$$

$$= \sqrt{5}-2$$

$$q+\frac{1}{q} = \sqrt{5}+\cancel{2}+\sqrt{5}-\cancel{2}$$

$$q+\frac{1}{q} = 2\sqrt{5} \text{ **Ans**}$$

(ii) $q-\frac{1}{q}$

Solution: Given that $q=\sqrt{5}+2$

$$\frac{1}{q}=\sqrt{5}-2$$

$$q-\frac{1}{q}=\sqrt{5}+2-\left(\sqrt{5}-2\right)$$

$$=\cancel{\sqrt{5}}+2-\cancel{\sqrt{5}}+2$$

$$q-\frac{1}{q}=4 \ \textbf{Ans}$$

(iii) $q^2+\frac{1}{q^2}$

Solution: Given that $q-\frac{1}{q}=4$

Squaring both sides

$$\left(q-\frac{1}{q}\right)^2=(4)^2$$

$$q^2+\frac{1}{q^2}-2=16$$

$$q^2+\frac{1}{q^2}=16+2$$

$$q^2+\frac{1}{q^2}=18 \ \textbf{Ans}$$

(iv) $q^2-\frac{1}{q^2}$

Solution: Given that $q+\frac{1}{q}=2\sqrt{5}$

$$q-\frac{1}{q}=4$$

By using formula

$$q^2-\frac{1}{q^2}=\left(q+\frac{1}{q}\right)\left(q-\frac{1}{q}\right)$$

$$=\left(2\sqrt{5}\right)(4)$$

$$=8\sqrt{5} \ \textbf{Ans}$$

Q.8 Simplify

(i) $\frac{\sqrt{a^2+2}+\sqrt{a^2-2}}{\sqrt{a^2+2}-\sqrt{a^2-2}}$

Solution:

$$=\frac{\sqrt{a^2+2}+\sqrt{a^2-2}}{\sqrt{a^2+2}-\sqrt{a^2-2}}\times\frac{\sqrt{a^2+2}+\sqrt{a^2-2}}{\sqrt{a^2+2}+\sqrt{a^2-2}}$$

$$=\frac{\left(\sqrt{a^2+2}+\sqrt{a^2-2}\right)^2}{\left(\sqrt{a^2+2}\right)^2-\left(\sqrt{a^2-2}\right)^2}$$

$$=\frac{\left(\sqrt{a^2+2}\right)^2+\left(\sqrt{a^2-2}\right)^2+2\left(\sqrt{a^2+2}\right)\left(\sqrt{a^2-2}\right)}{a^2+2-a^2+2}$$

$$=\frac{a^2+\cancel{2}+a^2-\cancel{2}+2\left(\sqrt{a^4\cancel{-2a^2}+\cancel{2a^2}-4}\right)}{4}$$

$$=\frac{2a^2+2\sqrt{a^4-4}}{4}$$

$$=\frac{\cancel{2}\left(a^2+\sqrt{a^4-4}\right)}{\cancel{4}\,2}$$

$$=\frac{a^2+\sqrt{a^4-4}}{2} \ \textbf{Ans}$$

(ii) $\frac{1}{a-\sqrt{a^2-x^2}}-\frac{1}{a+\sqrt{a^2-x^2}}$

$$=\left(\frac{1}{a-\sqrt{a^2-x^2}}\times\frac{a+\sqrt{a^2-x^2}}{a+\sqrt{a^2-x^2}}\right)$$

$$-\left(\frac{(1)}{\left(a+\sqrt{a^2-x^2}\right)}\frac{\left(a-\sqrt{a^2-x^2}\right)}{\left(a-\sqrt{a^2-x^2}\right)}\right)$$

$$=\left(\frac{a+\sqrt{a^2-x^2}}{(a)^2-\left(\sqrt{a^2-x^2}\right)^2}\right)-\left(\frac{a-\sqrt{a^2-x^2}}{(a)^2-\left(\sqrt{a^2-x^2}\right)^2}\right)$$

$$=\left(\frac{a+\sqrt{a^2-x^2}}{a-\left(a^2-x^2\right)}\right)-\left(\frac{a-\sqrt{a^2-x^2}}{a-\left(a^2-x^2\right)}\right)$$

$$=\left(\frac{a+\sqrt{a^2-x^2}}{a^2-a^2+x^2}\right)-\left(\frac{a-\sqrt{a^2-x^2}}{\cancel{a^2}-\cancel{a^2}+x^2}\right)$$

$$= \left(\frac{a+\sqrt{a^2-x^2}}{x^2}\right)-\left(\frac{a-\sqrt{a^2-x^2}}{x^2}\right)$$

$$= \frac{\cancel{a}+\sqrt{a^2-x^2}-\cancel{a}+\sqrt{a^2-x^2}}{x^2}$$

$$= \frac{2\sqrt{a^2-x^2}}{x^2}$$ **Ans**