LAKSHYA JEE

LAKSHYA KO HAR HAAL ME PAANA HAI

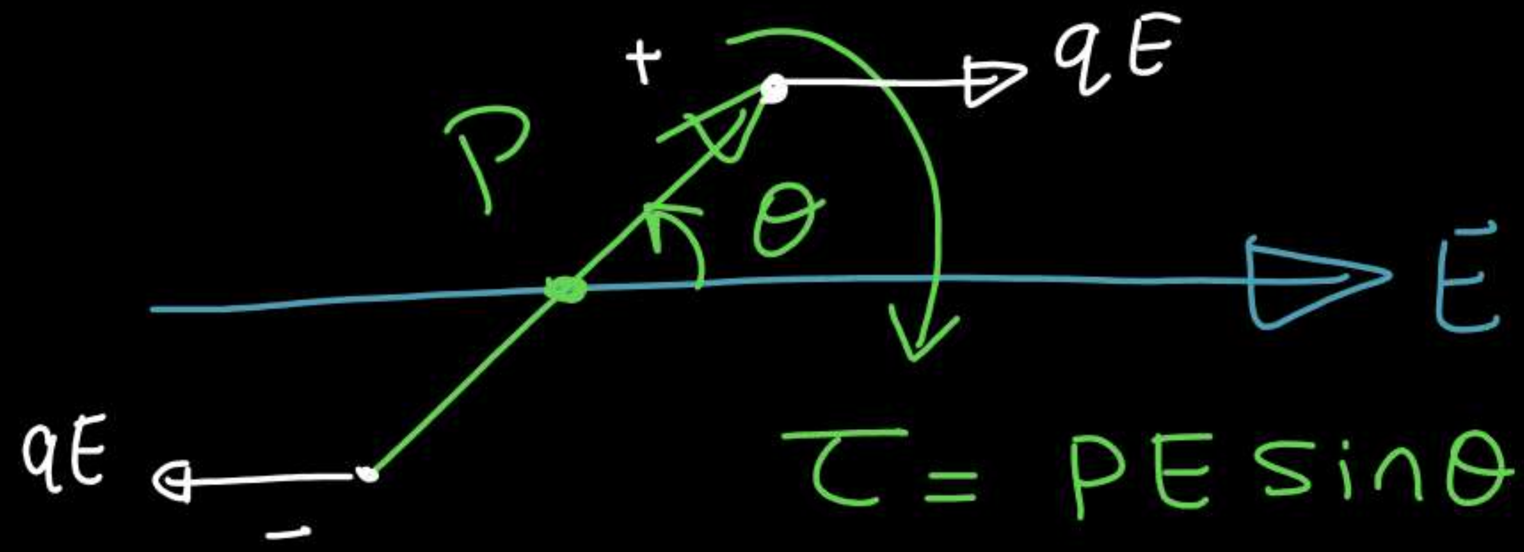
Electric Charges and Field

-Er. Rohit Gupta

Today's GOALS!

- ❑ Previous Year Problems

Dipole Oscillations



$\tau = PE\sin\theta = -I\alpha$ → moment of inertia

$\vec{P} \times \vec{E} = \vec{\tau}$

$\alpha = -\frac{PE}{I}\sin\theta$

If θ is very small then $\sin\theta \simeq \theta$

$\alpha \simeq -\frac{PE}{I}\theta$

$\alpha = -\omega^2\theta$ ← equation of angular S.H.M.

On comparision

$$\omega^2 = \frac{PE}{I}$$

$$\omega = \sqrt{\frac{PE}{I}}$$

$$T = \frac{2\pi}{\omega} = \boxed{2\pi\sqrt{\frac{I}{PE}}}\ *$$

I → moment of inertia

P → dipole moment.

How to do numericals?

Q

(i) Sabse pehla kam sundar diagram.

given, underline the give inf.

(ii) Mental strategy to solve the prob.

(iii) Try & execute the strategy you have made for solving the problem.

(iv) See the sol. & analyse the strategy.

(v) Compare the sol. with your sol. and find out your mistake!!

Let a total charge $2Q$ be distributed in a sphere of radius R, with the charge density given by $\rho(r) = kr$, where r is the distance from the centre. Two charges A and B, of $-Q$ each, are placed on diametrically opposite points, at equal distance a, from the centre. If A and B do not experience any force, then

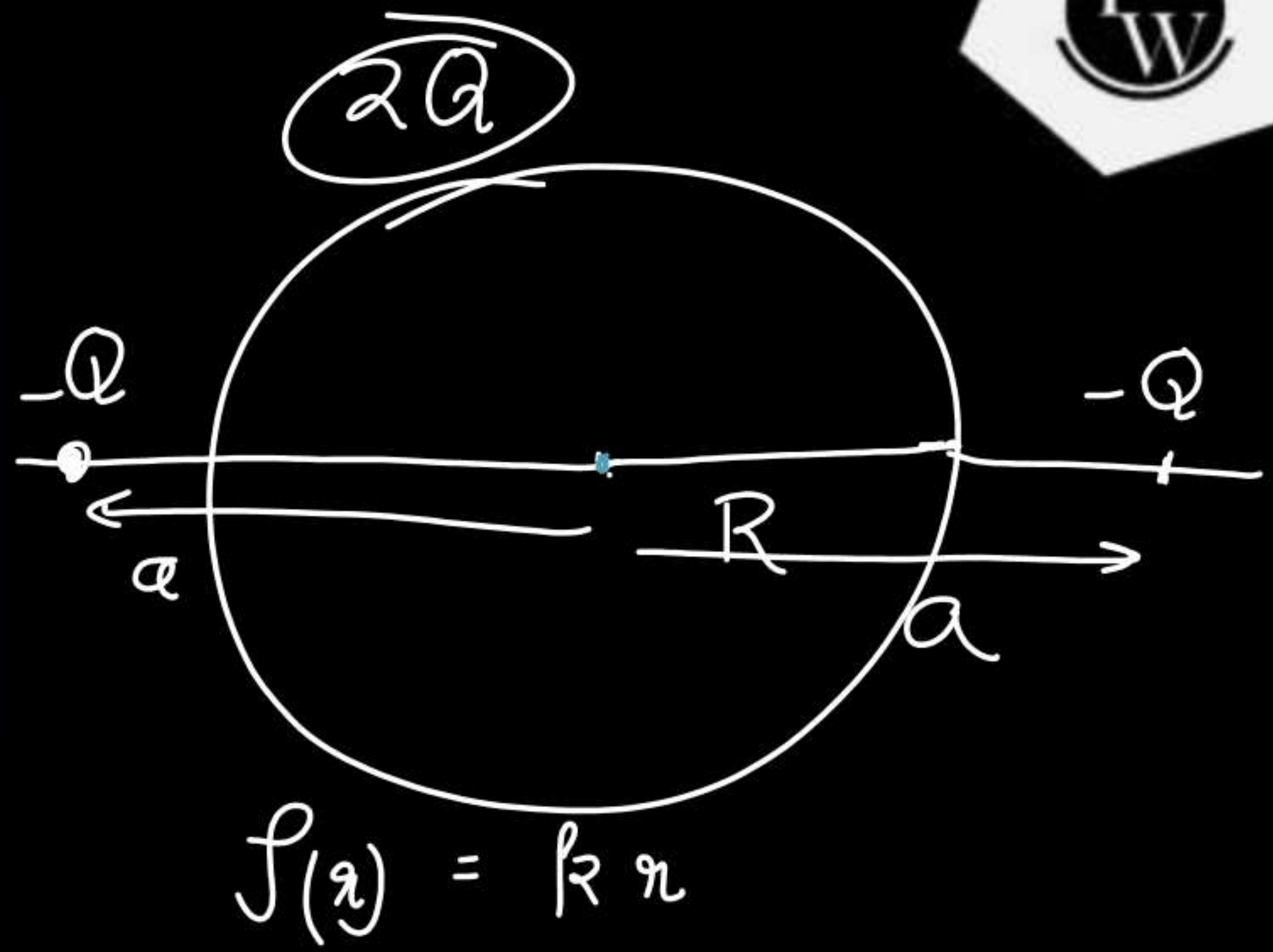
(Main 2019, 12 April II)

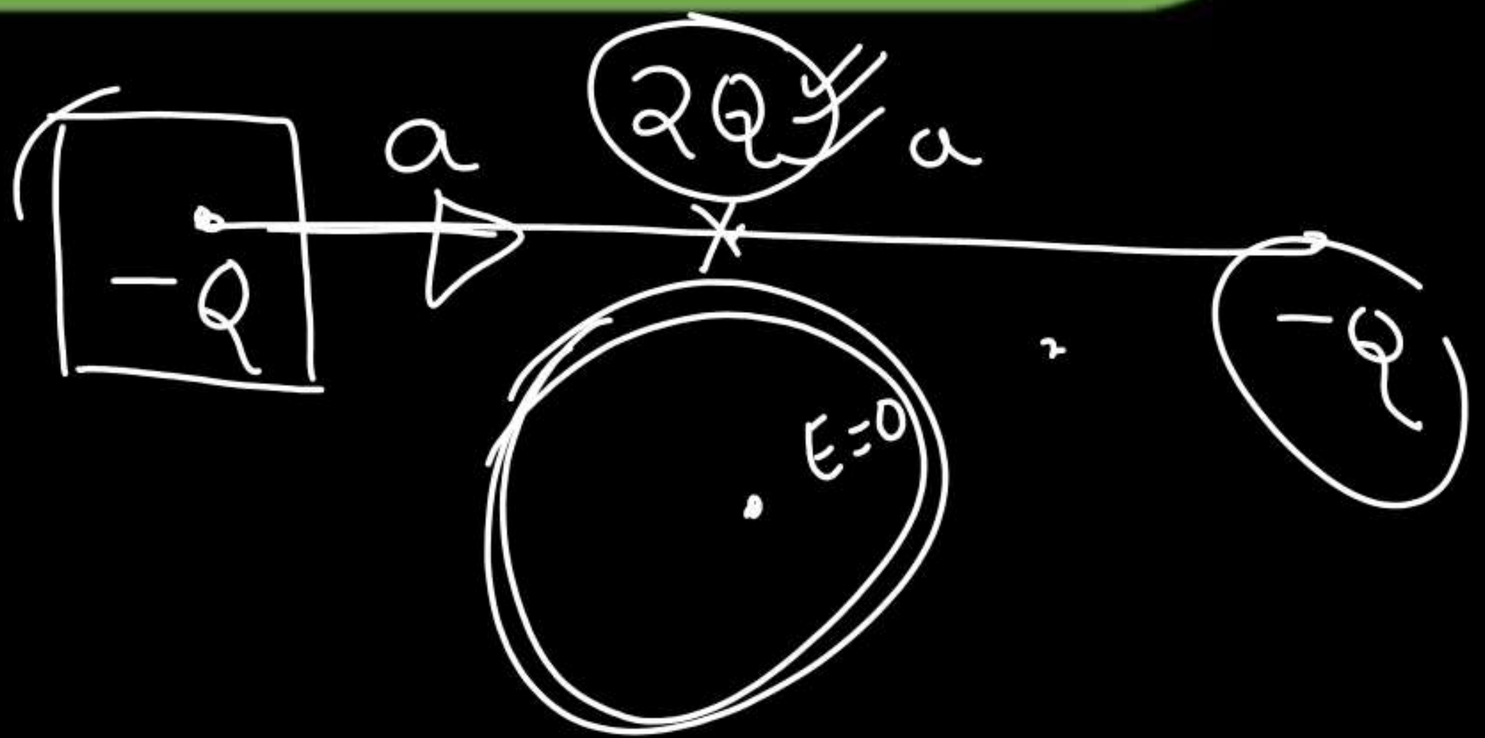
(a) $a = 8^{-1/4}R$

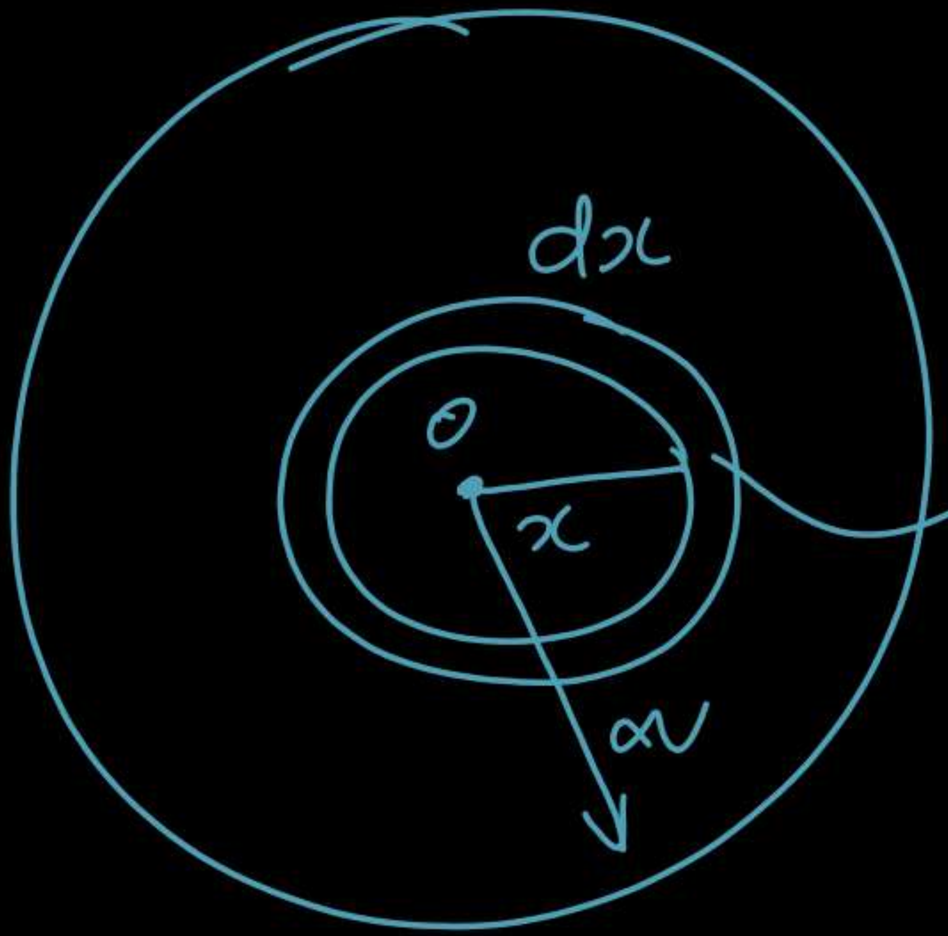
(b) $a = \dfrac{3R}{2^{1/4}}$

(c) $a = 2^{-1/4}R$

(d) $a = R/\sqrt{3}$







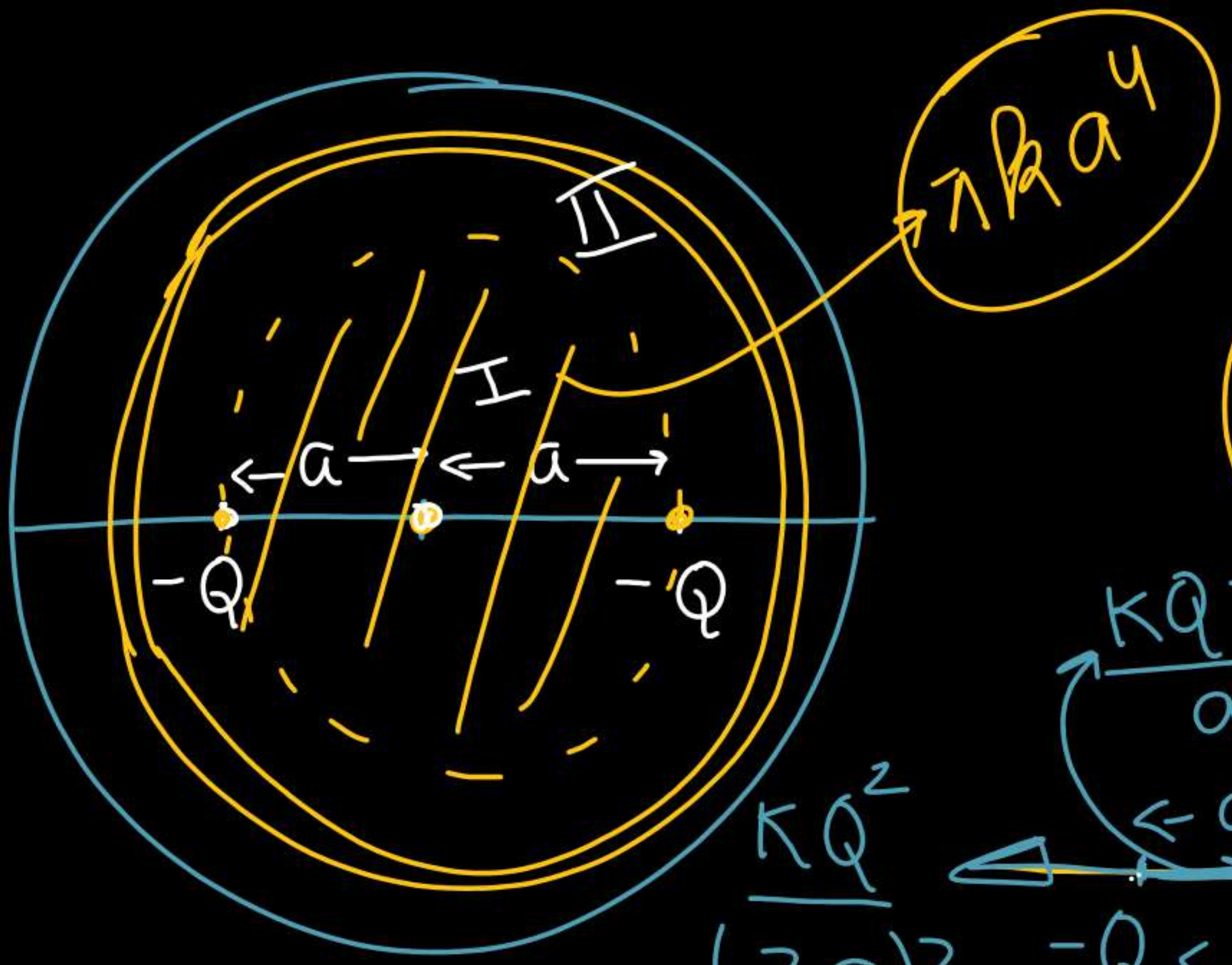
$$\int_0^r dq = \int (kx)(4\pi x^2 dx)$$

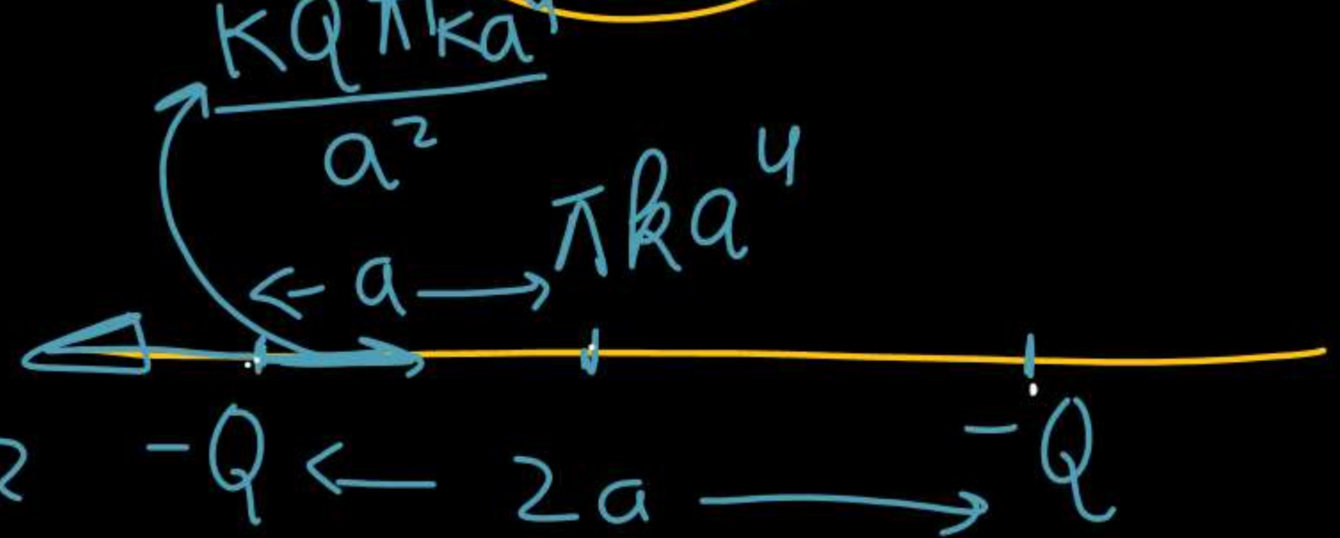
$$q = 4\pi k \frac{x^4}{4}\Big|_0^r$$

$$q = \pi k r^4 \Rightarrow q = \pi k a^4$$

$$* \; 2Q = \pi k R^4$$

$$k = \frac{2Q}{\pi R^4} \Rightarrow Q = \frac{\pi k R^4}{2}$$





$$\frac{KQ^2}{(2a)^2}$$

$$\frac{KQ^2}{4a^2} = \frac{KQ\pi k a^4}{a^2}$$

$$a^4 = \frac{Q}{4\pi k} = \frac{\pi k R^4}{2\,4\pi k}$$

$$a = R/8^{1/4}$$

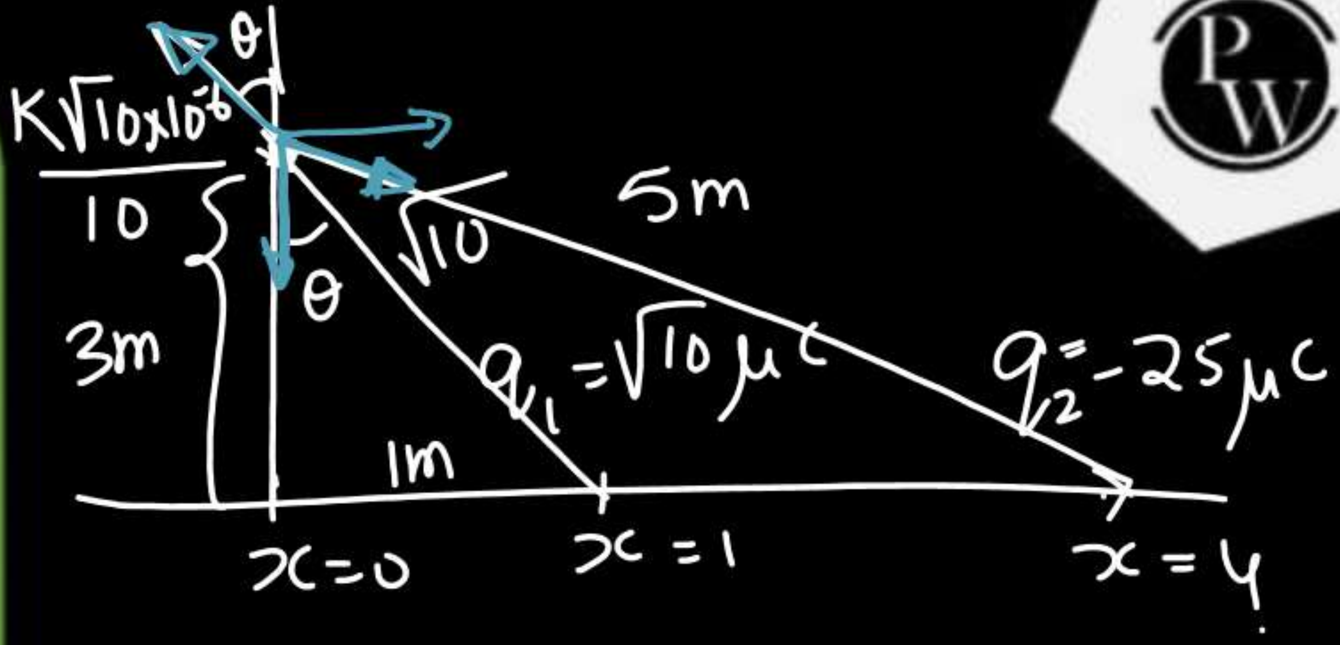
Two point charges q_1 ($\sqrt{10}\,\mu C$) and q_2 ($-25\,\mu C$) are placed on the x-axis at $x = 1\,m$ and $x = 4\,m$, respectively. The electric field (in V/m) at a point $y = 3\,m$ on Y-axis is

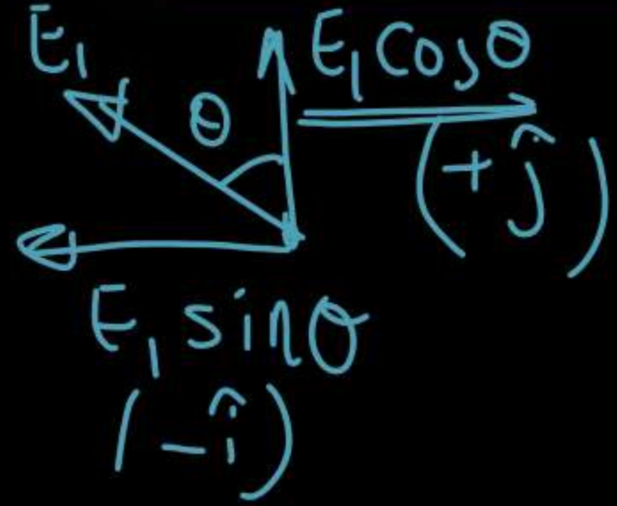
(Main 2019, 9 Jan II)

$\left(\text{Take, } \frac{1}{4\pi\varepsilon_0} = 9\times10^9\ \text{N-m}^2\text{C}^{-2}\right)$

(a) $(63\hat{i} - 27\hat{j})\times10^2$ (b) $(81\hat{i} - 81\hat{j})\times10^2$

(c) $(-81\hat{i} + 81\hat{j})\times10^2$ (d) $(-63\hat{i} + 27\hat{j})\times10^2$





$\vec{E}_1 = \left[\frac{(9\times10^9)\sqrt{10}\times10^{-6}}{10}\right]\left(\cos\theta\,\hat{j} - \sin\theta\,\hat{i}\right)$

$\cos\theta = \frac{3}{\sqrt{10}}$ $\sin\theta = \frac{1}{\sqrt{10}}$

$\vec{E}_2$ $\boxed{\vec{E}_{net} = \vec{E}_1 + \vec{E}_2}$

Charge is distributed within a sphere of radius R with a volume charge density $\rho(r)=\frac{A}{r^2}e^{\frac{-2r}{a}}$, where A and a are constants. If Q is the total charge of this charge distribution, the radius R is

(Main 2019, 9 Jan Shift II)

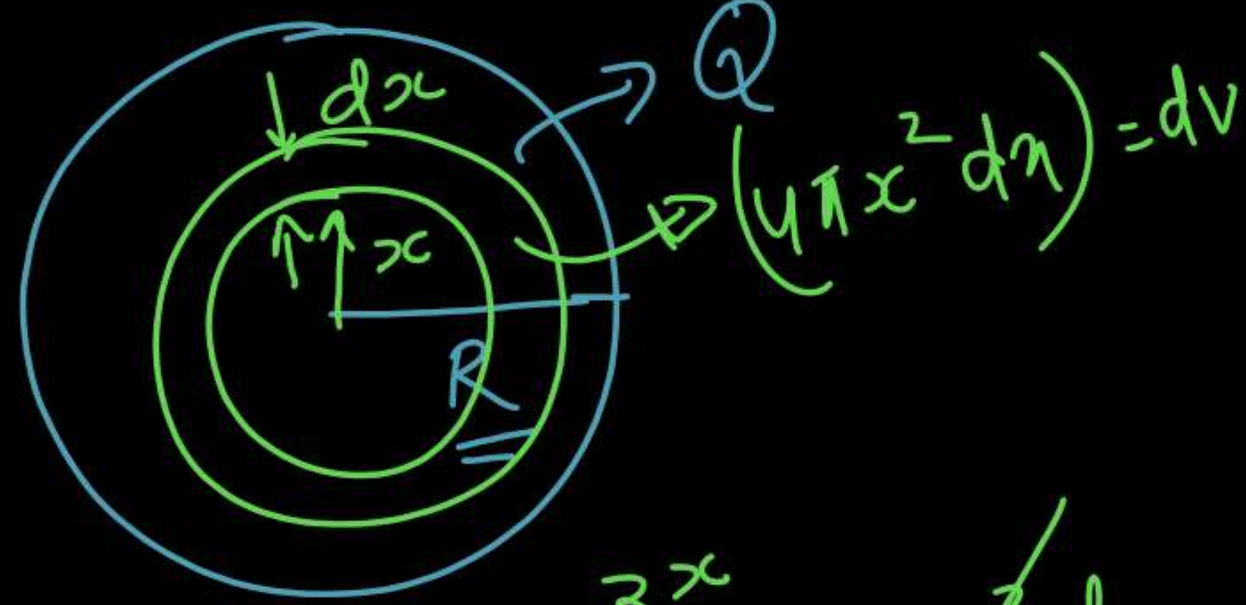
(a) $a\log\left(\frac{1}{1-\frac{Q}{2\pi aA}}\right)$

(b) $a\log\left(1-\frac{Q}{2\pi aA}\right)$

(c) $\frac{a}{2}\log\left(1-\frac{Q}{2\pi aA}\right)$

(d) $\frac{a}{2}\log\left(\frac{1}{1-\frac{Q}{2\pi aA}}\right)$

$\rho=\frac{A}{r^2}e^{\frac{-2r}{a}}$



$dq=\frac{A}{x^2}e^{-\frac{2x}{a}}\times 4\pi x^2 dx$

$Q=4\pi A\int_0^R e^{-\frac{2}{a}x}\,dx$

$Q=\frac{4\pi A\, e^{-\frac{2x}{a}}}{-2/a}\Big|_0^R$

$$Q = 2\pi a A\left[e^{0} - e^{-2R/a}\right)$$

$$\frac{Q}{2\pi a A} = 1 - e^{-2R/a}$$

$$e^{-2R/a} = 1 - \frac{Q}{2\pi a A}$$

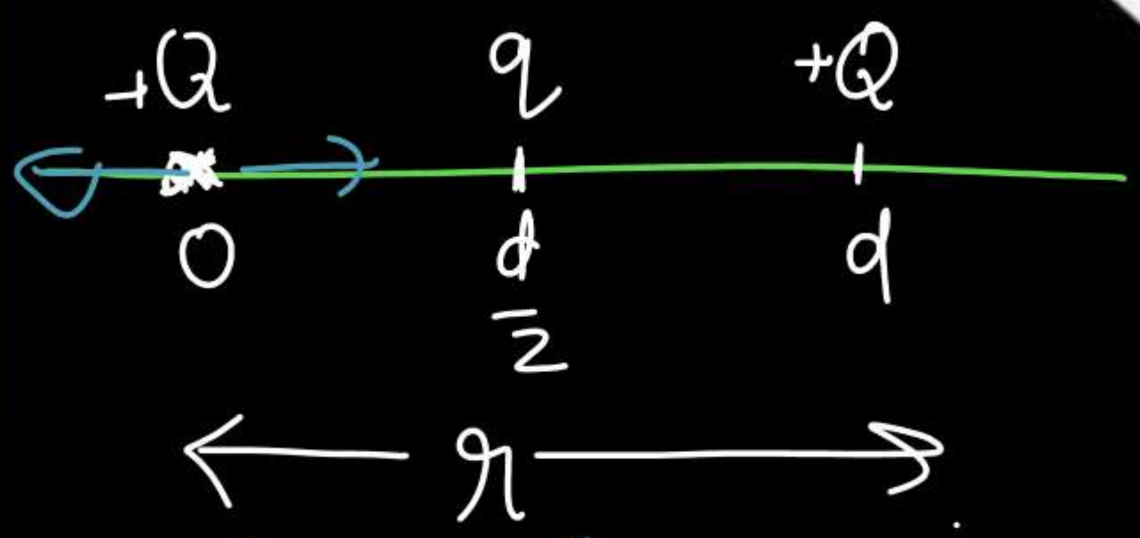
$$-2R/a = \ln\left(1 - Q/2\pi a A\right)$$

$$R = \frac{a}{2}\ln\left[\frac{1}{1 - \frac{Q}{2\pi a A}}\right]$$

Three charges $+Q, q, +Q$ are placed respectively at distance $0, \frac{d}{2}$ and d from the origin on the X-axis. If the net force experienced by $+Q$ placed at $x = 0$ is zero, then value of q is

(Main 2019, 9 Jan Shift I)

(a) $\frac{+Q}{2}$ (b) $\frac{+Q}{4}$ (c) $\frac{-Q}{2}$ (d) $\frac{-Q}{4}$

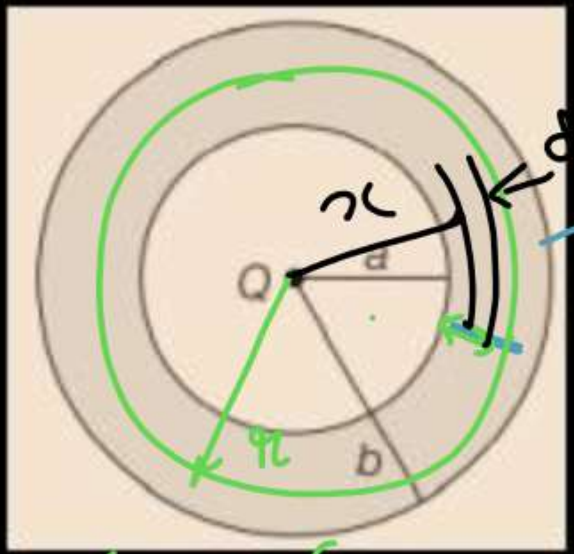


$$q = -\frac{Q^2}{(\sqrt{Q}+\sqrt{Q})^2} = -\frac{Q}{4}$$

$$\frac{KQ^2}{d^2} = \frac{Kq Q}{\left(\frac{d}{2}\right)^2}$$

$$q = -\frac{Q}{4}$$

The region between two concentric spheres of radii a and b, respectively (see the figure), has volume charge density $\rho = \frac{A}{r}$, where, A is a constant and r is the distance from the centre. At the centre of the spheres is a point charge Q. The value of A, such that the electric field in the region between the spheres will be constant, is **(2016 Main)**



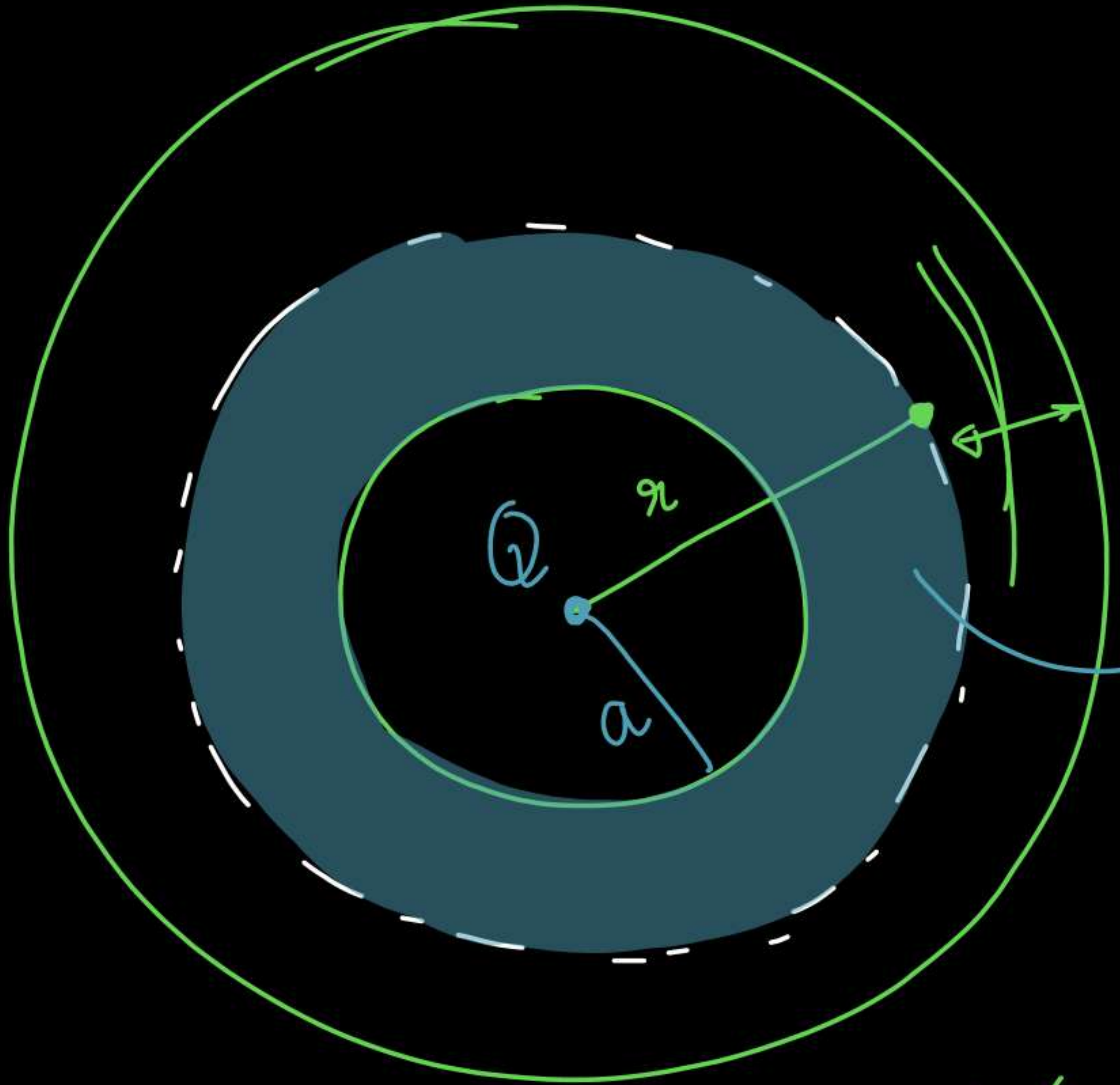
$\rho = \frac{A}{r}$

(a) $\frac{Q}{2\pi a^2}$ (b) $\frac{Q}{2\pi(b^2 - a^2)}$ (c) $\frac{2Q}{\pi(a^2 - b^2)}$ (d) $\frac{2Q}{\pi a^2}$

$$\int dq = \int_a^r (4\pi x^2 dx)\frac{A}{x}$$

$$= 4\pi A \left.\frac{x^2}{2}\right|_a^r$$

$$Q = 2\pi A(r^2 - a^2)$$



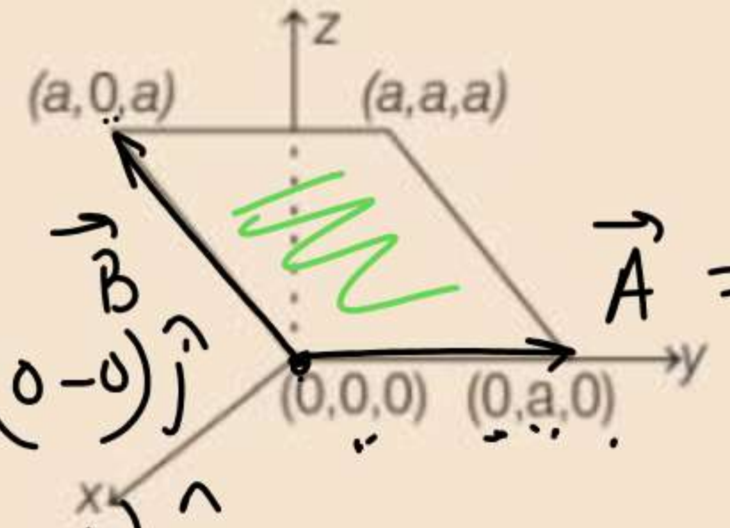
$\rightarrow 2\pi A(r^2-a^2)$

$$\frac{K\left(Q+2\pi A(r^2-a^2)\right)}{r^2}$$

$$K\left(\frac{Q}{r^2}+2\pi A-\frac{2\pi A a^2}{r^2}\right)$$

$$E \Rightarrow K\left(\frac{Q-2\pi A a^2}{r^2}+2\pi A\right) \Rightarrow Q = 2\pi A a^2$$

Consider an electric field $\mathbf{E} = E_0\,\hat{\mathbf{x}}$, where E_0 is a constant. The flux through the shaded area (as shown in the figure) due to this field is (2011)



(a) $2E_0a^2$ (b) $\sqrt{2}E_0a^2$

(c) E_0a^2 (d) $\frac{E_0a^2}{\sqrt{2}}$

$\vec{B} = (a-0)\hat{i} + (0-0)\hat{j} + (a-0)\hat{k}$

$\vec{A} = (0-0)\hat{i} + (a-0)\hat{j} + (0-0)\hat{k}$

$\vec{E} = E_0\hat{i}$

$|\vec{A}\times\vec{B}|$ = area of a parallelogram

$$(\vec{A}\times\vec{B}) = \begin{vmatrix} \hat{i} & -\hat{j} & \hat{k} \\ 0 & a & 0 \\ a & 0 & a \end{vmatrix}$$

$\hat{i}(a^2-0) - \hat{j}(0-0) + \hat{k}(0-a^2)$

$\phi = \vec{E}\cdot\vec{A}$

$= E_0\hat{i}\cdot(a^2\hat{i} - a^2\hat{k})$ $(a^2\hat{i} - a^2\hat{k})$

$\phi = E_0a^2$

LAKSHYA KO HAR HAAL ME PAANA HAI

A point dipole $\mathbf{p} = -p_0\,\hat{\mathbf{x}}$ is kept at the origin. The potential and electric field due to this dipole on the Y-axis at a distance d are, respectively [Take, $V = 0$ at infinity]

(Main 2019, 12 Apr I)

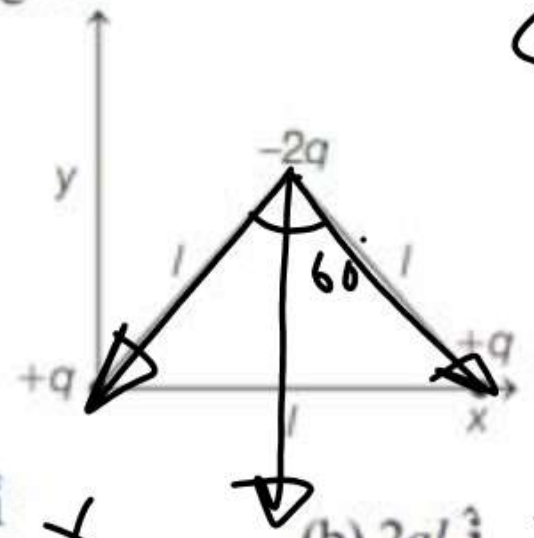
(a) $\dfrac{|\mathbf{p}|}{4\pi\varepsilon_0 d^2}, \dfrac{\mathbf{p}}{4\pi\varepsilon_0 d^3}$

(b) $0, \dfrac{-\mathbf{p}}{4\pi\varepsilon_0 d^3}$

(c) $0, \dfrac{\mathbf{p}}{4\pi\varepsilon_0 d^3}$

(d) $\dfrac{|\mathbf{p}|}{4\pi\varepsilon_0 d^2}, \dfrac{-\mathbf{p}}{4\pi\varepsilon_0 d^3}$

Determine the electric dipole moment of the system of three charges, placed on the vertices of an equilateral triangle as shown in the figure. (Main 2019, 12 Jan I)



(a) $\sqrt{3}\, ql \frac{\hat{j}-\hat{i}}{\sqrt{2}}$

(b) $2ql\,\hat{j}$

(c) $-\sqrt{3}\, ql\,\hat{j}$

(d) $(ql)\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

LAKSHYA KO HAR HAAL ME PAANA HAI

An electric field of 1000 V/m is applied to an electric dipole at angle of 45°. The value of electric dipole moment is 10^{-29} C-m. What is the potential energy of the electric dipole?

(Main 2019, 11 Jan II)

(a) -9×10^{-20} J
(b) -10×10^{-29} J
(c) -20×10^{-18} J
(d) -7×10^{-27} J

Thank You Lakshyians