LAKSHYA JEE

LAKSHYA KO HAR HAAL ME PAANA HAI

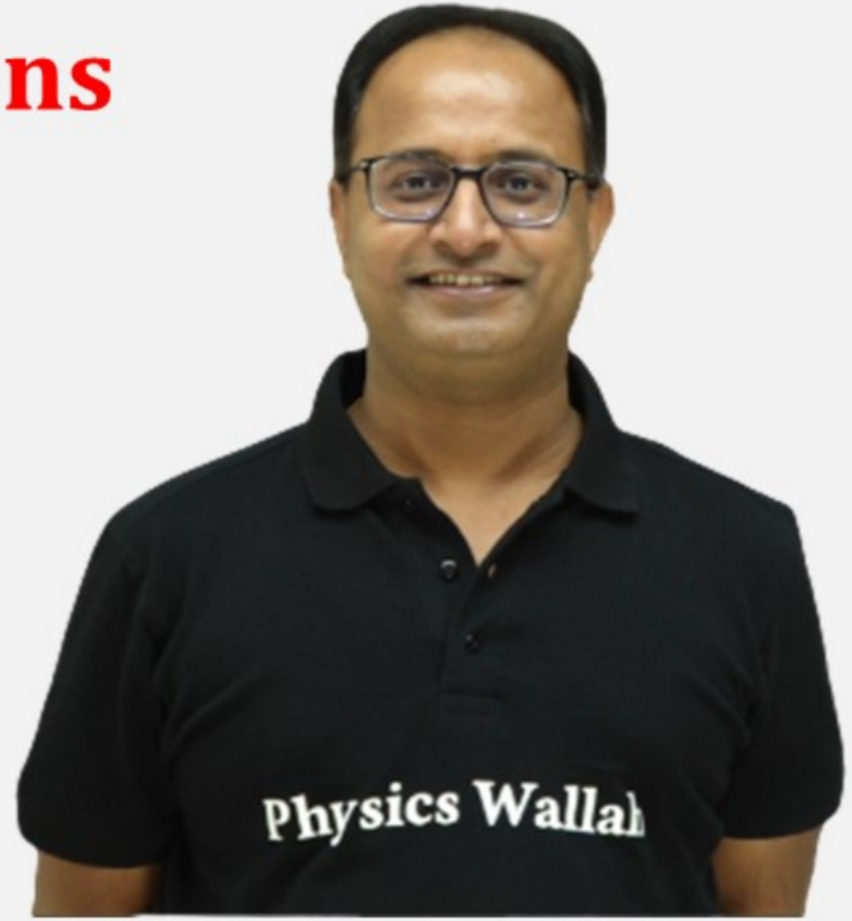
Relations & Functions

Lecture: 06

By: KUNDAN KUMAR

(B-Tech, IIT-BHU)

17+ years Teaching Experience



Today's Goal: :

Types of Basic Functions : (Continued)

Basic Problems of Domain of the Functions:

Basic Problems of Range of the Functions:

Types of functions

* Constant function $\Rightarrow f(x) = C$ eg $\begin{cases} f(x) \text{ is constt } f \\ \& \quad f(1) = 4 \\ \text{find} \quad f(2081) = ?4 \end{cases}$
* Identity function $\Rightarrow f(x) = x$

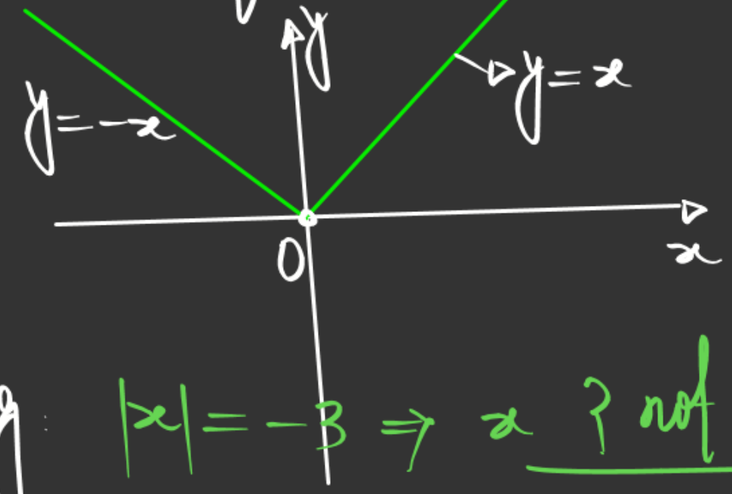
* Absolute value function / Modulus function $f(x) = |x|$ (always non-negative)

eg $|-3| = -(-3)$
$= 3$

$\Rightarrow f(x) = |x| = \begin{cases} x & ; x > 0 \\ -x & ; x < 0 \\ 0 & ; x = 0 \end{cases}$

* Modulus :

$f(x) = |x|$



eg: $|x| = -3 \Rightarrow x$? not possible

$|x| = 4 \Rightarrow \boxed{x = 4 \text{ OR } -4}$

Basic Properties: $(a > 0)$

* $|x| = a \Rightarrow x = \pm a$ } $x^2 = a^2 \Rightarrow x = \pm a$

* $|x| \geq a$ OR $x^2 \geq a^2$

$\Rightarrow \boxed{x \geq a \text{ OR } x \leq -a}$

* $|x| \leq a$ OR $x^2 \leq a^2$

$\Rightarrow \boxed{-a \leq x \leq a}$

* $|x| + |y| = |x+y|$ } possible iff $xy \geq 0$

Eg $|2x-3| = 5$ find x

$\Rightarrow 2x-3 = 5$ OR -5

$\Rightarrow 2x = 8$ OR -2

$\Rightarrow \boxed{x = 4 \text{ OR } -1}$

If $\left|4 - |4 - |4-x|\right| = 4$ find x

$4 - \left|4 - |4-x|\right| = 4$ OR -4

$\Rightarrow |4 - |4-x|| = 0$ OR 8

$\Rightarrow 4 - |4-x| = 0$ OR 8 OR -8

$\Rightarrow |4-x| = 4, -4, 12$

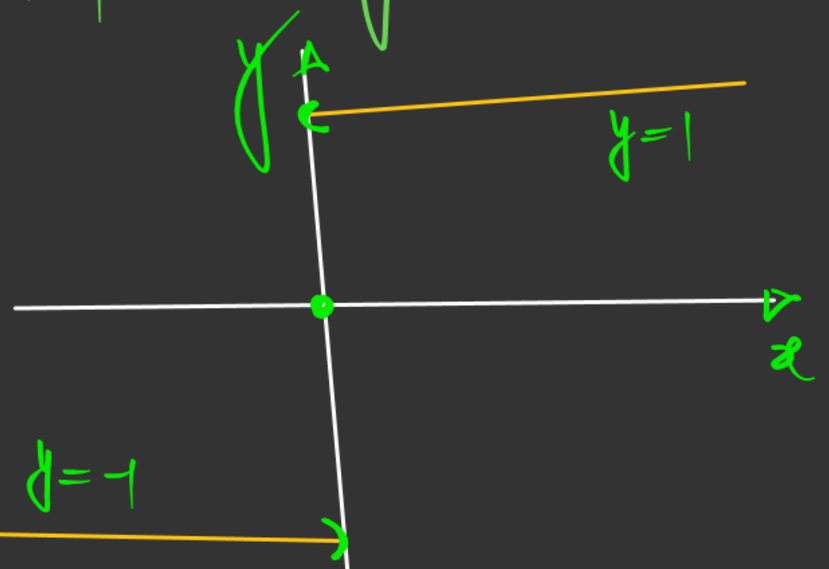
$\Rightarrow 4-x = 4, -4, 12, -12$

$\Rightarrow \boxed{x = 0, 8, -8, 16}$

Signum function : $f(x) = \frac{|x|}{x}$ OR $\frac{x}{|x|} = \text{Sgn}(x)$

$$\Rightarrow \text{Sgn}(x) = \begin{cases} 1 & ; x > 0 \\ -1 & ; x < 0 \\ 0 & ; x = 0 \end{cases}$$

(मीना गया)



Eg $\text{Sgn}(23) = 1$

$\text{Sgn}(x^2) = 0$ OR $1, \forall x \in R$

$\text{Sgn}(-2021) = -1$

$$\text{Sgn}(\sin x) = \begin{cases} 1 & ; \quad 0 < x < \pi/2 \\ -1 & ; \quad \frac{3\pi}{2} < x < 2\pi \end{cases}$$

Greatest Integer Functions:

* $f(x) = [x]$ → Maximum integral Value not greater than x

$\Rightarrow [x] \leq x$

Eg: $[2.4] = 2, [-3.6] = -4$

$[-3.4] = -4, [4] = 4$

$[x] = \text{integer} \leq x$

Eg $\left[\frac{1}{4}\right] + \left[\frac{1}{4}+\frac{1}{100}\right] + \left[\frac{1}{4}+\frac{2}{100}\right] + \cdots + \left[\frac{1}{4}+\frac{100}{100}\right] = ?$

$0 \quad 0 \quad 0$

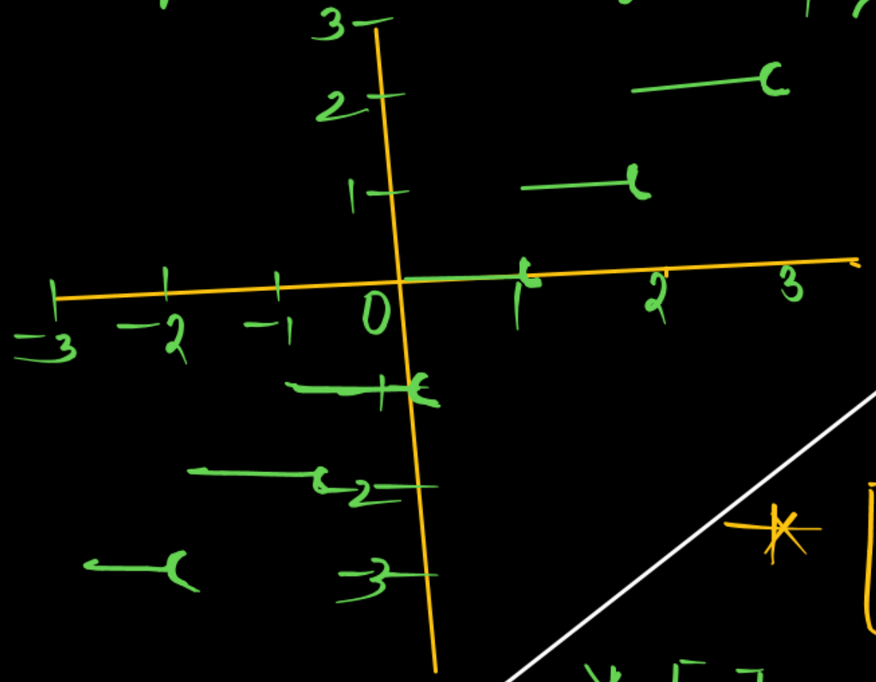
$\cdots + \left[\frac{1}{4}+\frac{74}{100}\right] + \left[\frac{1}{4}+\frac{75}{100}\right] + \cdots + \left[\frac{1}{4}+\frac{100}{100}\right]$

0

$= 26$

Greatest Integer Functions:

$f(x) = [x]$ is not Continuous at all integral Points



Properties ($n \in I$)

* $[x] = n \Rightarrow \boxed{n \leq x < n+1}$
* $x - 1 < [x] \leq x$
* $[x] + [-x] = \begin{cases} 0 ; & x \in I \\ -1 ; & x \notin I \end{cases}$
* $[x \pm I] = [x] \pm I$ although $[Ix] \neq I[x]$, $(I \neq 0, 1)$
* $[x+y] \geq [x] + [y]$
* $[x] \geq n \Rightarrow \boxed{x \geq n}$
* $[x] < n \Rightarrow \boxed{x < n}$
* $[x] > n \Rightarrow \boxed{x \geq n+1}$
* $[x] \leq n \Rightarrow \boxed{x < n+1}$

$$y = 2[x] + 3 \quad \& \quad y = 3[x-2] + 5$$

Find $[x+y] = ?$

$$[x+11] = [x] + 11 = 15$$

Solving, $2[x] + 3 = 3([x] - 2) + 5$

$\Rightarrow 2[x] + 3 = 3[x] - 1$

$\Rightarrow \boxed{[x] = 4}$

$y = 2 \times 4 + 3 = 11$

$n \in N$

$$* [x] + \left[x + \frac{1}{n}\right] + \left[x + \frac{2}{n}\right] + \cdots + \left[x + \frac{n-1}{n}\right] = [nx]$$

$$* \left[\frac{x+1}{2}\right] + \left[\frac{x+2}{4}\right] + \left[\frac{x+4}{8}\right] + \cdots = [x]$$

Greatest Integer Functions:

Find the value of the followings: (where [.] denotes the greatest integer function.)

$$\left(\left[\frac{2011+1}{2}\right]+\left[\frac{2011+2}{4}\right]+\left[\frac{2011+4}{8}\right]+....\infty\right)+\left([10]+\left[10+\frac{1}{10}\right]+\left[10+\frac{2}{10}\right]+....+\left[10+\frac{9}{10}\right]\right)$$

$[2011] = 2011$

$[10 \times 10] = 100$

$2011 + 100 = \boxed{2111}$

Least Integer Functions:

$f(x) = (x) \geq x$

not useful

Read (पढ़ा जाता है)

eg: $(9.4) = 3$

$(-4.2) = -4$

ceiling of $x = (x)$

floor of $x = [x]$

In general,

$x \in I \Rightarrow (x) = [x] = x$

$x \notin I \Rightarrow (x) = [x] + 1$

Thank You Lakshyians