

Exercise 5.3

Q.1 Use the remainder theorem to find the remainder when

(i) $3x^3 - 10x^2 + 13x - 6$ is divided by $(x - 2)$.

Solution:

$$P(x) = 3x^3 - 10x^2 + 13x - 6$$

Since $P(x)$ is divided by $(x - 2)$.

$$\therefore P(2) = R$$

$$R = 3(2)^3 - 10(2)^2 + 13(2) - 6$$

$$= 3(2)^3 - 10(2)^2 + 13(2) - 6$$

$$= 24 - 40 + 26 - 6$$

$$R = 4$$

Hence 4 is the remainder

(ii) $4x^3 - 4x + 3$ is divided by $(2x - 1)$

Solution:

$$P(x) = 4x^3 - 4x + 3$$

Since $P(x)$ is divided by $(2x - 1)$

$$\therefore R = P\left(\frac{1}{2}\right)$$

$$= 4\left[\frac{1}{2}\right]^3 - 4 \times \frac{1}{2} + 3$$

$$= 4 \times \frac{1}{8} - 2 + 3$$

$$= \frac{1}{2} - 2 + 3$$

$$= \frac{1 - 4 + 6}{2} = \frac{3}{2}$$

$$R = \frac{3}{2}$$

Hence $\frac{3}{2}$ is the remainder

(iii) $6x^4 + 2x^3 - x + 2$ is divided by $(x + 2)$ from $x + 2 = 0$

Solution: Given that

$$P(x) = 6x^4 + 2x^3 - x + 2$$

Since $P(x)$ is divided by $(x + 2)$

$$\therefore R = P(-2)$$

$$= 6(-2)^4 + 2(-2)^3 - (-2) + 2$$

$$= 96 - 16 + 2 + 2$$

$$R = 84$$

Hence 84 is the remainder

(iv) $(2x - 1)^3 + 6(3 + 4x)^2 - 10$ is divided by $2x + 1$ from $2x + 1 = 0$
 $x = -\frac{1}{2}$

Solution: Given that

$$P(x) = (2x - 1)^3 + 6(3 + 4x)^2 - 10$$

Since $P(x)$ is divided by $2x + 1$

$$\therefore R = P\left(-\frac{1}{2}\right)$$

$$= \left[2\left(-\frac{1}{2}\right) - 1\right]^3 + 6\left[3 + 4\left(-\frac{1}{2}\right)\right]^2 - 10$$

$$= [-1 - 1]^3 + 6[3 - 2]^2 - 10$$

$$= [-2]^3 + 6 - 10 = -8 + 6 - 10$$

$$R = -12$$

Hence -12 is the remainder

(v) $x^3 - 3x^2 + 4x - 14$ is divided by $(x + 2)$ from $x + 2 = 0, x = -2$

Solution: Given that

$$P(x) = x^3 - 3x^2 + 4x - 14$$

Since $P(x)$ is divided by $(x + 2)$

$$\therefore R = P(-2)$$

$$= (-2)^3 - 3(-2)^2 + 4(-2) - 14$$

$$= -8 - 12 - 8 - 14$$

$$R = -42$$

Hence -42 is the remainder

Q.2

- (i) If $(x+2)$ is a factor of $3x^2 - 4kx - 4k^2$ then find the values of k $x+2=0$ $x=-2$

Solution: Given that

$$P(x) = 3x^2 - 4kx - 4k^2$$

$$P(-2) = 3(-2)^2 - 4k(-2) - 4k^2$$

$$P(-2) = 12 + 8k - 4k^2$$

If $(x+2)$ is the factor then remainder is equal to zero

$$P(-2) = 0$$

$$12 + 8k - 4k^2 = 0$$

$$4(3 + 2k - k^2) = 0$$

$$-k^2 + 2k + 3 = \frac{0}{4}$$

$$-k^2 + 3k - k + 3 = 0$$

$$-k(k-3) - 1(k-3) = 0$$

$$(k-3)(-k-1) = 0$$

$$k-3 = 0 \quad -k-1 = 0$$

$$k = 3 \quad -1 = k$$

$$k = -1$$

- (ii) If $(x-1)$ is a factor of $x^3 - kx^2 + 11x - 6$ the find the value of k from $x-1=0$ $x=1$

Solution: Given that

$$P(x) = x^3 - kx^2 + 11x - 6$$

$$P(1) = (1)^3 - k(1)^2 + 11(1) - 6$$

$$P(1) = 1 - k + 11 - 6$$

$$P(1) = 6 - k$$

If $(x-1)$ is the factor then remainder is equal to zero

$$P(1) = 0$$

$$6 - k = 0$$

$$k = 6$$

Q.3 Without long division determine whether

- (i) $(x-2)$ and $(x-3)$ are factor of $P(x) = x^3 - 12x^2 + 44x - 48$ from $x-2=0$ $x=2$

Solution: Given that

$$P(x) = x^3 - 12x^2 + 44x - 48$$

If $(x-2)$ is the factor then remainder is equal to zero

$$P(2) = (2)^3 - 12(2)^2 + 44(2) - 48 = 8 - 48 + 88 - 48 = 0$$

Hence $x-2$ is a factor of $P(x)$

For $x-3$

$$R = P(3)$$

$$= (3)^3 - 12(3)^2 + 44(3) - 48$$

$$= (3)^3 - 12(3)^2 + 44(3) - 48$$

$$= 27 - 108 + 132 - 48$$

$$= 159 - 156$$

$$R = 3$$

3 is remainder hence $x-3$ is not factor of $P(x)$

$P(3)$ is not equal to zero then $x-3$ is not factor of $P(x) = x^3 - 12x^2 + 44x - 48$

- (ii) $(x-2)$, $(x+3)$ and $(x-4)$ are factor of $q(x) = x^3 + 2x^2 - 5x - 6$ from $x-2=0$, $x=2$

Solution: Given that

$$q(x) = x^3 + 2x^2 - 5x - 6$$

For $(x-2)$, putt $x-2=0$

$$x = 2$$

$$R = q(2)$$

$$= (2)^3 + 2(2)^2 - 5(2) - 6$$

$$R = 8 + 8 - 10 - 6$$

$$R = 16 - 16$$

$$R = 0$$

Hence $x-2$ is factor of

$$q(x) = x^3 + 2x^2 - 5x - 6$$

For $(x+3)$, putt $x+3=0$

$$x = -3$$

$$R = q(-3)$$

$$= (-3)^3 + 2(-3)^2 - 5(-3) - 6$$

$$= -27 + 18 + 15 - 6$$

$$R = 0$$

Hence $x-2$ is factor of

$$q(x) = x^3 + 2x^2 - 5x - 6$$

For $x-4$, $x-4=0$

$$x=4$$

$$R = q(4)$$

$$= (4)^3 + 2(4)^2 - 5(4) - 6$$

$$= 64 + 32 - 20 - 6$$

$$R=70$$

Hence $x-4$ is not a factor of

$$q(x) = x^3 + 2x^2 - 5x - 6$$

Q.4 For what value of m is the polynomial $P(x) = 4x^3 - 7x^2 + 6x - 3m$ exactly divisible by $x+2$?

Solution:

$$P(x) = 4x^3 - 7x^2 + 6x - 3m$$

From $x+2=0$, $x=-2$

$$P(-2) = 4(-2)^3 - 7(-2)^2 + 6(-2) - 3m$$

$$P(-2) = -32 - 28 - 12 - 3m = -72 - 3m$$

If $(x+2)$ is the factor then remainder is equal to zero

$$P(-2) = 0$$

$$-72 - 3m = 0$$

$$-72 = 3m$$

$$m = -\frac{72}{3}$$

$$m = -24$$

Q.5 Determine the value of k if $P(x) = kx^3 + 4x^2 + 3x - 4$ and $q(x) = x^3 - 4x + k$ leaves the same remainder when divided by $(x-3)$.

Solution:

$$q(x) = x^3 - 4x + k$$

from $x-3=0$ $x=3$

$$R_1 = q(3)$$

$$= (3)^3 - 4(3) + k$$

$$= 27 - 12 + k$$

$$= 15 + k$$

$$R_1 = 15 + k \quad \dots(i)$$

$$R_2 = P(3)$$

$$= k(3)^3 + 4(3)^2 + 3(3) - 4$$

$$= 27k + 36 + 9 - 4$$

$$R_2 = 27k + 41 \quad \dots(ii)$$

Since it leaves the same remainder.

$$\text{Hence } R_1 = R_2$$

$$15 + k = 27k + 41$$

$$15 - 41 = 27k - k$$

$$-26 = 26k$$

$$k = \frac{-26}{26}$$

$$k = -1$$

Q.6 The remainder after dividing the polynomial $P(x) = x^3 + ax^2 + 7$ by $(x+1)$ is $2b$ calculate the value of a and b if this expression leaves a remainder of $(b+5)$ on being dividing by $(x-2)$

Solution:

Let

$$P(x) = x^3 + ax^2 + 7$$

Since $P(x)$ is divided by $(x+1)$

$$\text{Put } x+1=0 \quad x=-1$$

$$R=P(-1)$$

$$= (-1)^3 + a(-1)^2 + 7$$

$$= -1 + a + 7$$

$$R = a + 6$$

According to first condition remainder is $2b$

$$2b = a + 6 \quad \dots(i)$$

Since $P(x)$ is divided by $(x-2)$

$$\text{Put } x-2=0$$

$$x=2$$

$$P(2) = (2)^3 + a(2)^2 + 7$$

$$= 8 + 4a + 7$$

$$R = 15 + 4a$$

According to second condition remainder is $(b+5)$

$$15 + 4a = b + 5$$

$$4a - b = 5 - 15$$

$$4a - b = -10 \quad \dots(ii)$$

Solving equations (i) and (ii)

From equation (ii) $b=10+4a$ putting the value of b in equation (i)

$$a+6=2(10+4a)$$

$$a=20+8a-6$$

$$-8a+a=14$$

$$-7a=14$$

$$a = \frac{14}{-7}$$

$$a=-2$$

Putting the value of a in equation (ii)

$$4a - b = -10$$

$$4(-2) - b = -10$$

$$-8 - b = -10$$

$$-8 + 10 = b$$

$$2 = b$$

$$b = 2$$

Q.7 The polynomial $x^3 + lx^2 + mx + 24$ has a factor $(x + 4)$ and it leaves a remainder of 36 when divided by $(x - 2)$

Find the values of l and m .

Solution:

Let

$$P(x) = x^3 + lx^2 + mx + 24$$

$$\text{From } x + 4 = 0 \quad x = -4$$

$$P(-4) = (-4)^3 + l(-4)^2 + m(-4) + 24$$

$$P(-4) = -64 + 16l - 4m + 24$$

$$P(-4) = 16l - 4m - 40$$

According to condition $(x+4)$ is the factor then

$$16l - 4m - 40 = 0$$

$$4[4l - m - 10] = 0$$

$$4l - m - 10 = 0 \quad (i)$$

$$\text{from } x - 2 = 0 \quad x = 2$$

$$\text{Now } P(2) = (2)^3 + l(2)^2 + m(2) + 24$$

$$P(2) = 8 + 4l + 2m + 24$$

$$P(2) = 4l + 2m + 32$$

According the condition

$$4l + 2m + 32 = 36$$

$$4l + 2m = 36 - 32$$

$$4l + 2m = 4$$

$$4l + 2m - 4 = 0 \quad (ii)$$

Subtracting (i) from (ii)

$$\cancel{4l} + 2m - 4 = 0$$

$$\cancel{4l} + 2m + 10 = 0$$

$$3m + 6 = 0$$

$$3m + 6 = 0$$

$$3m = -6$$

$$m = \frac{-6}{3}$$

$$m = -2$$

Putting the value of m in equation (i)

$$4l - (-2) - 10 = 0$$

$$4l + 2 - 10 = 0$$

$$4l - 8 = 0$$

$$4l = 8$$

$$l = \frac{8}{4}$$

$$l = 2$$

Q.8 The expression $lx^3 + mx^2 - 7$ leaves remainder of -3 and 12 when divided by $(x - 1)$ and $(x + 2)$ respectively. Calculate the value of l and m .

Solution:

$$P(x) = lx^3 + mx^2 - 7$$

$$\text{from } x - 1 = 0 \quad x = 1$$

$$P(1) = l(1)^3 + m(1)^2 - 7$$

$$P(1) = l + m - 7$$

According to conditions $l + m - 7 = -3$

$$l + m = 4 - 3$$

$$l = 1 - m \quad (i)$$

$$\text{From } x + 2 = 0 \quad x = -2$$

$$P(-2) = l(-2)^3 + m(-2)^2 - 7$$

$$P(-2) = -8l + 4m - 7$$

According to condition

$$-8l + 4m - 7 = 12$$

Putting the value of l in the equation

$$-8[1-m] + 4m = 16$$

$$-8 + 8m + 4m = 16$$

$$12m = 16 + 8$$

$$12m = 24$$

$$m = \frac{24}{12}$$

$$m = 2$$

Putting the value of m in equation (i)

$$l = 1 - 2$$

$$l = -1$$

$$m = 2$$

$$l = -1$$

Q.9 The expression $ax^3 - 9x^2 + bx + 3a$ is exactly divisible by $x^2 - 5x + 6$. Find the value of a and b .

Solution: Given that

$$P(x) = ax^3 - 9x^2 + bx + 3a$$

$$x^2 - 5x + 6 = x^2 - 2x - 3x + 6$$

$$= x[x - 2] - 3[x - 2]$$

$$= [x - 2][x - 3]$$

$(x - 2)(x - 3)$ divides the expression $ax^3 -$

$9x^2 + bx + 3a$ from $x - 2 = 0$, $x = 2$

$$P(2) = a(2)^3 - 9(2)^2 + b(2) + 3a$$

$$P(2) = 8a - 36 + 2b + 3a$$

$$P(2) = 11a + 2b - 36$$

According to condition $(x - 2)$ is the factor so

$$11a + 2b - 36 = 0 \quad (i)$$

From $x - 3 = 0$, $x = 3$

$$P(3) = a(3)^3 - 9(3)^2 + b(3) + 3a$$

$$P(3) = 27a - 81 + 3b + 3a$$

$$P(3) = 30a + 3b - 81$$

According to condition $(x - 3)$ is the factor so

$$30a + 3b - 81 = 0 \quad (ii)$$

$$3(10a + b - 27) = 0$$

$$10a + b - 27 = \frac{0}{3}$$

$$b = 27 - 10a \quad (iii)$$

Putting the value of b in equation (i)

$$11a + 2[27 - 10a] - 36 = 0$$

$$11a + 54 - 20a - 36 = 0$$

$$-9a + 18 = 0$$

$$+18 = 9a$$

$$a = \frac{+18}{9}$$

$$a = +2$$

Putting the value of a in equation (iii)

$$b = 27 - 10(+2)$$

$$b = 27 - 20$$

$$b = 7$$

$$a = 2$$