125 MIDTERM ONE REVIEW OUTLINE ©

Outline

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   (d) Abstract Sums
2. Antiderivatives
   (a) General Antiderivatives
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3. Fundamental Theorem of Calculus (FTC)
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5. Applications
   (a) Integrating with respect to dx or with respect to dy
   (b) Area enclosed by curves
   (c) Volumes of Solids
      i. Washer
      ii. Shells

Examples

1. Integral As Area

   (a) Riemann Sums (Use $n = 4$ rectangles to compute the specified Riemann sum.)

      i. Left
      $$\int_{\pi}^{2\pi} \frac{\cos x}{x}\, dx$$
      ii. Right
      $$\int_{0}^{8} \sqrt{y - \sqrt{y}}\, dy$$
      iii. Midpoint
      $$\int_{e}^{3e} \ln(x)\, dx$$
      iv. Abstract Sums (Identify the value of the following sums)
      $$\lim_{n\to\infty} \sum_{k=1}^{n} \left(e^{\frac{\pi k}{n}}\right)\left(\frac{\pi}{n}\right)$$

   (b) Using Area Know-How to Compute an integral
   $$\int_{-1}^{1} \sqrt{1 - x^2}\, dx$$

2. Antiderivatives

   (a) General Antiderivatives

   Find a function $f(x)$ such that $\int f(x)\, dx = \pi^2 f(x) + C$.

   (b) Keeping track of units

   If x is in minutes, $g(x)$ describes the number of bugs per minute and $f(x)$ describes gallons per minute, what are the units of $\int f(x)^2 g(x)\, dx$?

   (c) Initial Conditions

   You and the moon are running a 120 km race. However, you have a 10 km headstart. The moon runs with constant acceleration $a(t) = 2$ (km/hr^2) and has starting velocity 1 km/hr. You run the race at a constant velocity of 10 km/hr. At what distance into the race does the moon pass you?

3. Fundamental Theorem of Calculus (FTC)

   (a) Differentiating under the integral sign
   $$f(x) = \int_{\sin x}^{\cos x} \tan t\, dt, \qquad \text{Find } f'(x).$$
   $$F(x) = \int_{\sin x}^{\cos x} \ln(t)\, dt, \quad G(x) = F(e^x), \qquad \text{Find } G'(x).$$
   $$g(t) = \int_{0}^{8} \sqrt{z} \sin z\, dz \qquad \text{Find } g'(t)$$

(b) Taking definite integrals

$$\int_1^4 \frac{4}{x+2}\,dx$$

(c) Total vs. Net vs. Absolute Change

The moon has gravitronity charge given by $g(t) = t^2 - 3t + 2$. What is the total gravitronity of the moon from $t = 0$ to $t = 3$ ($\int |g(t)|\,dt$)? What is the net gravitronity ($\int g(t)\,dt$)?

4. Techniques of Integration

(a) u-substitution

$$\int \frac{\sin 1/x}{x^2}\,dx$$

$$\int_0^{\pi/4} \frac{\sin t}{\cos^2 t}\,dt$$

Let $f(x)$ be a positive differentiable function. Compute $\int \frac{\ln(f(x))f'(x)}{f(x)}\,dx$

5. Applications

(a) Integrating with respect to dx or with respect to dy

Find $b > 0$ so that the area between the curve $y = x^2 - 1$ and the x-axis from $0 < x < 2$ is equal to the area between the curve $y = x^2 - 1$ and the y-axis from $0 < y < b$.

(b) Area enclosed by curves

(1). Compute the area in the first quadrant that lies between $y = \ln(x)$ and $x = e$.

(2). Find the area enclosed by the curves $y = 2x$ and $x = 4y - y^2$. Also, setup but do not evaluate the integral expressing the volume of the solid obtained by rotating this region about the y-axis.

(c) Volumes of Solids

(1). Let S be the region bounded by the curve $y = e^{x/3}$, the line $y = e$ and the y-axis.

i. Slices

Find the volume of the solid obtained by revolving S about the x-axis.

ii. Shells

Set up but do not evaluate the integral that represents the volume of the solid obtained by revolving S around the y-axis.

(2). Use both slices and shells to find the volume of the solid of revolution obtained by rotating a right triangle with vertices $(0,0)$, $(2,2)$ and $(4,0)$ about the y-axis