Missouri Longitudinal Study of Mathematical Development and Disability: The First Five Years

David C. Geary
Department of Psychological Sciences
University of Missouri

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Overview

General Theoretical Perspective

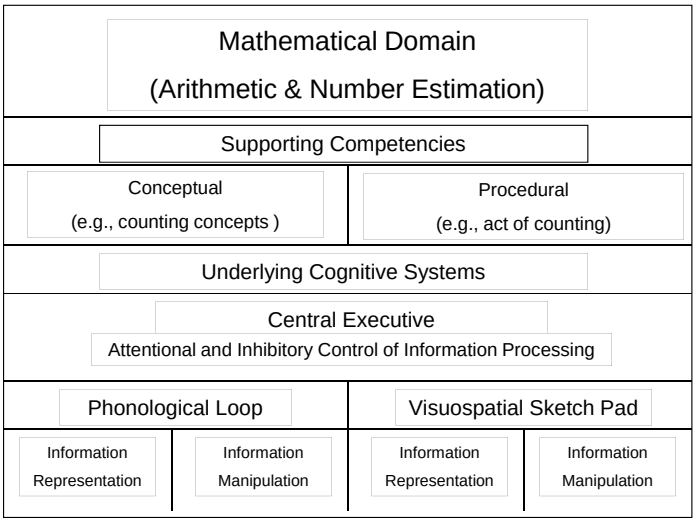
Sample and Study Design

Measures

Results:

- **First grade predictors of mathematics growth**
- **Development and delay in children with mathematical learning disability (MLD) and persistent low achievement (LA)**

Conceptual Approach



Sample and Design

- **289 children from 12 schools started the study at the end of kindergarten; more than 200 remain in the study at the end of 7th grade**
- **Goal: Follow their mathematical development through high school algebra and identify the sources of poor mathematics development and achievement**

Kindergarten	First Grade		Second Grade		Third Grade		Fourth Grade		Fifth Grade	
Spring	Fall	Spring	Fall	Spring	Fall	Spring	Fall	Spring	Fall	Spring
IQ Achievement Tests	Math RAN	IQ Achieve	Math RAN	Achieve	Math RAN	Achieve	Math RAN	Achieve	Math RAN	Achieve
	Working Memory		In-Class Attention (SWAN)						Working Memory	

Sample and Design

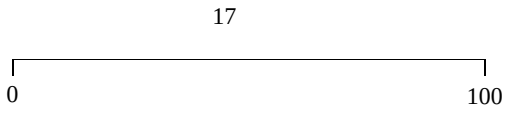
- Intelligence
  Raven's Coloured Progressive Matrixes
  Wechsler Abbreviated Intelligence Scale
- Achievement: Wechsler Individual Achievement Test – Word Reading
  Wechsler Individual Achievement Test – Numerical Operations
- Rapid automatized naming of number words and letters (RAN)
- The Strength and Weaknesses of ADHD-Symptoms and Normal-Behavior (SWAN) measure of in-class attention was used (Swanson et al., 2008).
- Working Memory Test Battery for Children (Pickering & Gathercole, 2001)
  4 Measures of Phonological Loop (e.g., digit span)
  2 Measures of Visuospatial Sketch Pad (e.g., Corsi)
  3 Measures of Central Executive (e.g., listening span)

Mathematical Cognition

- Counting Knowledge:
  - Detection of Counting Errors that violate basic counting principles (e.g., order irrelevance)
- Addition: Trial-by-trial assessment of problem solving for simple (5+4) and more complex (17+6) problems
  - Retrieval, Decomposition (e.g., 17+6 = 17+3 = 20 + 3 = 23), and Counting procedures (e.g., 17, 18, 19, …)
  - Frequency of use of memory based processes of retrieval and decomposition
  - Procedural competence:
    - [(2 x Min counts) + (1 x Sum counts)] – Counting errors

Mathematical Cognition

- Number Line estimation
  - Absolute error

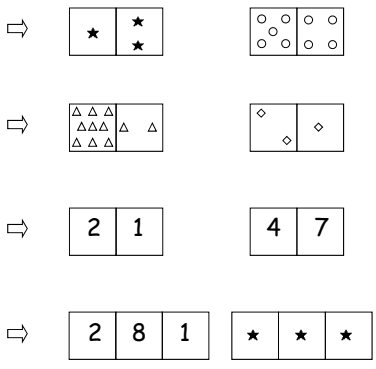


Mathematical Cognition

- Number Sets Fluency:
  - Z-scores for hits – false alarms

Circle all of the groups that add up to 3.
Work as quickly as you can.

3



Mathematical Cognition

- Added assessment of conceptual understanding of fractions to the spring assessments, beginning in 6th grade

- Paper and pencil measures of computational skills with whole numbers, fractions, and word problems administered in 7th grade; scheduled again for 9th
  - Predictive of employability, wages, and on the job productivity in young adults

First Grade Predictors of Mathematics Growth

- To identify basic number and arithmetic skills at the beginning of 1st grade that predict mathematical achievement and achievement growth through the end of 5th grade, controlling for domain general abilities

Mathematical Cognition

Variable	Task	Coding
Mathematical Cognition		
Counting error	Counting Knowledge	Number of counting errors detected as errors
Counting pseudo	Counting Knowledge	Number of pseudo counting errors detected as correct counts
Simple addition retrieval	Addition Strategy Choice	Frequency of problems correctly solved with direct retrieval
Simple addition procedural competence	Addition Strategy Choice	Sophistication and accuracy of using counting procedures
Complex addition decomposition	Addition Strategy Choice	Frequency of problems correctly solved with decomposition
Complex addition procedural competence	Addition Strategy Choice	Sophistication and accuracy of using counting procedures
Number line	Number Line	Mean of absolute difference between correct placement and child's actual placement
d-prime	Number Sets	*z* score for hits - *z* score for false alarms

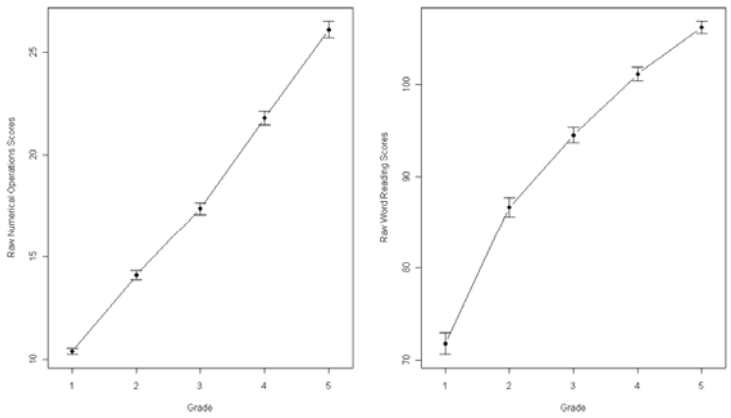
Sample Achievement: Standard Scores

Grade											
Intelligence		First		Second		Third		Fourth		Fifth	
Numerical Operations											
M	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
102	14	94	13	96	16	95	14	96	16	99	17
Word Reading											
		108	16	106	14	104	12	104	12	104	11

Analytic Approach

- Multilevel modeling
  - Linear and quadratic grade effects; random slopes and intercepts
  - First fit domain general (IQ, WM, Processing speed) predictors and interactions with grade
  - Non-significant effects were dropped to get a reduced set of predictors
  - The reduced set was the baseline for the math cognition predictors
  - Non-significant effects were dropped to get a reduced set of math cognition predictors above and beyond the domain general ones

Results



Results

- IQ and Processing Speed – numbers for math and letters for reading – are related to start point
  - Across grades IQ becomes more important for math but less important for word reading
- Phonological loop and visuospatial sketch pad related to reading and math start point, respectively
- Central executive is important for early word reading but becomes less important across grades – opposite pattern for math
- Speed of letter retrieval important for word reading growth, but speed of number retrieval not related to math growth

	Numerical Operations		Word Reading	
Effect	Final Domain General Model	*t* value	Final Domain General Model	*t* value
Intercept	7.81 (0.14)	57.83	52.10 (0.72)	72.08
Grade (linear slope)	2.61 (0.15)	17.80	20.54 (0.46)	45.08
Grade^2 (quadratic slope)	0.21 (0.03)	6.84	-1.99 (0.07)	-27.01
Intelligence on Intercept	0.38 (0.13)	2.87	4.01 (0.77)	5.23
Processing Speed on Intercept	0.35 (0.13)	2.78	4.36 (0.81)	5.40
Phonological Loop on Intercept	---	---	1.62 (0.54)	3.01
Visuospatial Sketch Pad on Intercept	0.49 (0.13)	3.71	---	---
Central Executive on Intercept	-0.05 (0.16)	< 1	3.43 (0.87)	3.93
Intelligence on Slope	0.16 (0.06)	2.43	-0.36 (0.16)	-2.30
Central Executive on Slope	0.40 (0.06)	6.18	-0.42 (0.17)	-2.45
Processing Speed on Slope	---	---	1.36 (0.46)	2.93
Processing Speed on Slope^2	---	---	-0.34 (0.07)	-4.67

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Results

- Fluency in processing simple number set (d-prime) information related to start point but not growth in math
- Number line accuracy not related to start point, but becomes increasingly important across grades for math
- Frequency of fact retrieval related to word reading but not math start point
  - Fact retrieval becomes important across grades in math, but not reading
- Use of decomposition and counting procedures important for math start point, but become less important across grades
- Higher sensitivity to counting errors, associated with lower math start point

	Numerical Operations		Word Reading	
Effect	Final Full Model	*t* value	Final Full Model	*t* value
Intercept	7.81 (0.13)	59.20	52.10 (0.72)	72.53
Grade (linear slope)	2.61 (0.15)	16.92	20.54 (0.46)	45.08
Grade^2 (quadratic slope)	0.21 (0.03)	6.57	-1.99 (0.07)	-27.01
Intelligence on Intercept	0.24 (0.11)	2.23	3.63 (0.78)	4.68
Processing Speed on Intercept	0.26 (0.10)	2.48	4.19 (0.80)	5.22
Phonological Loop on Intercept	---	---	1.82 (0.53)	3.45
Visuospatial Sketch Pad on Intercept	0.26 (0.11)	2.31	---	---
Central Executive on Intercept	-0.29 (0.14)	-2.03	3.19 (0.87)	3.68
Intelligence on Slope	---	---	-0.36 (0.16)	-2.30
Central Executive on Slope	0.34 (0.06)	5.53	-0.42 (0.17)	-2.45
Processing Speed on Slope	---	---	1.36 (0.46)	2.93
Processing Speed on Slope^2	---	---	-0.34 (0.07)	-4.67
Number Line on Intercept	0.14 (0.14)	< 1	---	---
d-prime on Intercept	0.47 (0.14)	3.41	---	---
Addition retrieval on Intercept	-0.16 (0.16)	-1.04	1.40 (0.44)	3.16
Addition decomposition on Intercept	0.67 (0.15)	4.50	---	---
Complex addition procedural on Intercept	0.36 (0.12)	3.15	---	---
Pseudo Counting on Intercept	-0.19 (0.09)	-2.00	---	---
Number Line on Slope	0.13 (0.07)	2.04	---	---
Addition retrieval on Slope	0.38 (0.08)	4.78	---	---
Addition decomposition on Slope	-0.22 (0.07)	-2.99	---	---

Summary and Implications

- Domain general abilities affect word reading acquisition and early mathematics learning, but the mix differs
  - IQ and speed of processing numbers for math and letters for reading are contributors to beginning-of-schooling differences
  - IQ becomes increasingly important in math and less important for word reading
  - Phonological memory is important for word reading, and visuospatial for math
  - Executive control become increasingly important for math and less so for word reading
- There are numerical competencies that contribute to math start point and growth above and beyond domain general abilities and that are largely unrelated to word reading
  - Children who start ahead are fluent in the composition and decomposition of number sets
  - Children who show more growth than others start with a better understanding of the number line and know more basic facts at the beginning of 1st grade

Development and Delay in Children with MLD and Their LA Peers

Document mathematical cognition deficits and delays in children with mathematical learning disability and persistent low achievement in mathematics from 1st to 5th grade, inclusive

Determine the extent to which these contribute to mathematics achievement test performance above and beyond the influence of domain-general abilities and in-class attention

Achievement, Domain General, and In-Class Attention Measures

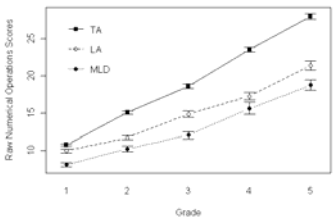
Variable	Task	Coding
Mathematics	Numerical Operations WIAT-II (Wechsler, 2001)	Raw Scores
Word Reading	Word Reading WIAT-II (Wechsler, 2001)	Raw Scores
Verbal Intelligence	Vocabulary subtest of WASI (Wechsler, 1999)	Standard scores from national norms (M = 100, SD = 15)
Non-Verbal Intelligence	Matrix Reason subtest of WASI (Wechsler, 1999); CPM (Raven et al., 1993)	Mean of scores from national WASI norms and standardized CPM scores ((M = 100, SD = 15)
Central Executive	WMTB-C (Pickering & Gathercole, 2001)	The mean of subtest span scores for first and fifth grade
Phonological Loop	WMTB-C (Pickering & Gathercole, 2001)	The mean of subtest span scores for first and fifth grade
Visuospatial Sketch Pad	WMTB-C (Pickering & Gathercole, 2001)	The mean of subtest span scores for first and fifth grade
Processing speed	Rapid Automatized Naming (Denckla & Rudel, 1976)	The mean of number naming and letter naming RTs for first to fifth grade, inclusive
In-class attention	SWAN (J. M. Swanson et al., 2008)	Mean teacher ratings across second to fourth grade, inclusive

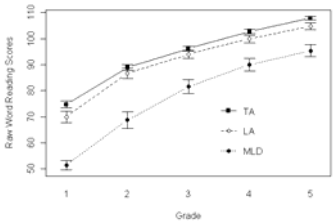
Mathematical Cognition

Variable	Task	Coding
Mathematical Cognition		
Simple addition retrieval	Addition Strategy Choice	Frequency of problems correctly solved with direct retrieval
Simple addition procedural competence	Addition Strategy Choice	Sophistication and accuracy of using counting procedures
Complex addition decomposition	Addition Strategy Choice	Frequency of problems correctly solved with decomposition
Complex addition procedural competence	Addition Strategy Choice	Sophistication and accuracy of using counting procedures
Number line	Number Line	Mean of absolute difference between correct placement and child's actual placement
Number Sets Fluency	Number Sets	*z* score for hits - *z* score for false alarms

Achievement Trends

- To identify groups; clustering, and free growth curve models with uncorrelated intercepts and slopes
  - Only one group emerged
  - Based on intercept and slope below 25th percentile
    - MLD = low start point and slow growth (n = 16); achievement %tile ~10th
    - LA = average start point and slow growth (n = 29); achievement % tile ~20th
    - TA = average in both (n = 132)

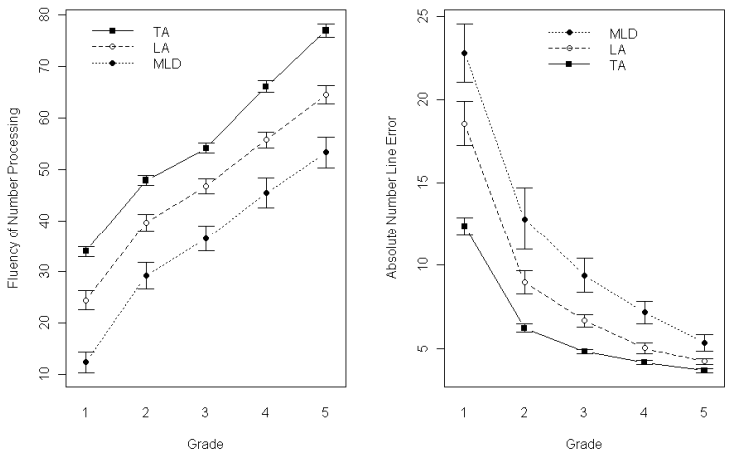




Results

			First Grade Working Memory Spans						Fifth Working Memory Spans					
	Verbal IQ	Non-Verbal IQ	Phonological Loop		Visuospatial Sketch Pad		Central Executive		Phonological Loop		Visuospatial Sketch Pad		Central Executive	
Group	*M*	*M*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
MLD	90	95	2.3	0.7	2.0	0.6	1.4	0.4	3.6	0.5	3.3	0.8	2.3	0.4
LA	99	98	3.4	0.6	2.6	0.6	1.9	0.5	3.9	0.6	3.8	0.8	2.9	0.4
TA	106	104	3.4	0.5	2.9	0.5	2.3	0.5	4.0	0.6	4.4	0.7	3.2	0.5

Results



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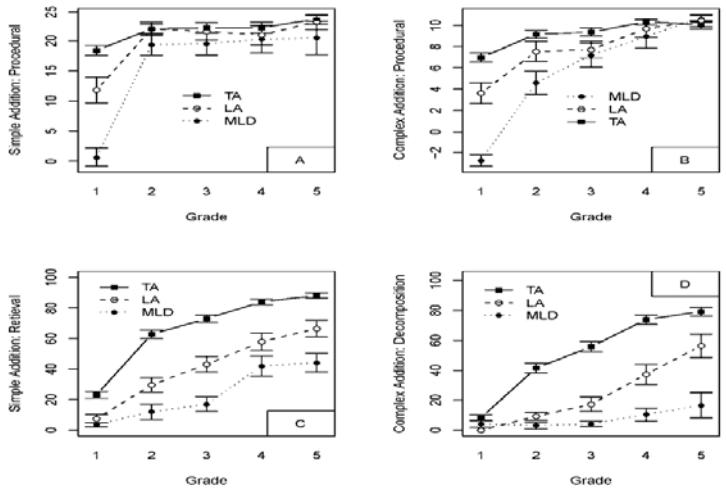

Results

- Mediational Analyses:
  - Contrast Numerical Operations Intercept of TA and MLD groups and slopes of TA and MLD/LA groups
  - First ran stepwise regressions to isolate best predictors of intercept and slope – included domain general, in-class attention, and math cognition
- Three variables predicted Numerical Operations intercept (R^2 = .40) - these were considered potential mediators of the group difference;
  - 1st Grade Number Sets Fluency, ß = .40, t = 4.92, p<.0001
  - 1st Grade Complex Addition Decomposition, ß = .20, t = 2.76, p<.01
  - 1st Grade Simple Addition Retrieval, ß = .15, t = 2.19, p<.05
- Five variables predicted Numerical Operations slopes (R^2 = .58);
  - 1st Grade Number Sets Fluency, ß = .14, t = 2.03, p<.05
  - In-Class Attention, ß = .23, t = 3.76, p<.0005
  - 1st Grade Simple Addition Retrieval, ß = .30, t = 5.11, p<.0001
  - 1st Grade Central Executive, ß = .24, t = 3.70, p<.0005
  - Complex Decomposition Slope, ß = .17, t = 2.85, p<.005

Results

- **Numerical Operations Intercept:**
  - **1st Grade Number Sets Fluency was full mediator** (z = 5.91, p<.0001)
- **Numerical Operations Slope:**
  - **All five variables were partial mediators (**zs > 3.87, ps<.0001)
  - **Group difference remained with simultaneous control of all 5**
  - **%Variance explained by group contrast dropped from 49% to 9%** (t = 14.41, p<.0001)

Summary and Implications

- **Mathematical achievement start point and growth appear normally distributed, with no distinct MLD and LA classes**
  - **Cut-offs are nonetheless critical for educational and remediation decisions**
  - **MLD – low start point, low growth; achievement scores < 10th percentile**
  - **LA – average start point, low growth; achievement scores ~ 20th percentile**
- **For MLD start point: The implication is some combination of a) poor access to representations of quantities associated with small sets and Arabic numerals, b) deficits in the ability to map numerals to quantities, c) or poor ability to add these representations are core deficits that should be addressed in kindergarten.**
- **Growth deficits are more complex and may require multiple types of interventions. Those that address**
  - **Numerical deficits and delays**
  - **Working memory deficits**
  - **On task behavior in the classroom**