6 Transformers

6.1. INTRODUCTION

Electrical energy is generated at places where it is easier to get water head, oil or coal for hydro-electric, diesel or thermal power stations respectively. Then energy is to be transmitted at considerable distances for use in villages, towns and cities located at distant places. Since transmission of electrical energy at high voltages is economical, therefore, some means are required for stepping up the voltage at generating stations and stepping down the same at the places where it is to be used. Electric machine used for this purpose is "*transformer*". In our country the electrical energy is usually generated at 6.6 or 11 or 33 kv, stepped up to 132, 220, 400, or 765 kv with the help of step-up transformers for transmission and then stepped down to 66 kv or 33 kv at grid substations for feeding various substations, which further step down the voltage to 11 kv for feeding distributing transformers stepping down the voltage further to 400|230 volts for the consumer uses.

Transformer is an ac machine that (*i*) transfers electrical energy from one electric circuit to another (*ii*) does so without a change of frequency (*iii*) does so by the principle of electro-magnetic induction and (*iv*) has electric circuits that are linked by a common magnetic circuit. The energy transfer usually takes place with a change of voltage, although this is not always necessary. When the transformer raises the voltage *i.e.* when the output voltage of a transformer is higher than its input voltage, it is called the *step-up transformer* and when it lowers the voltage it is called the *step-down transformer*.

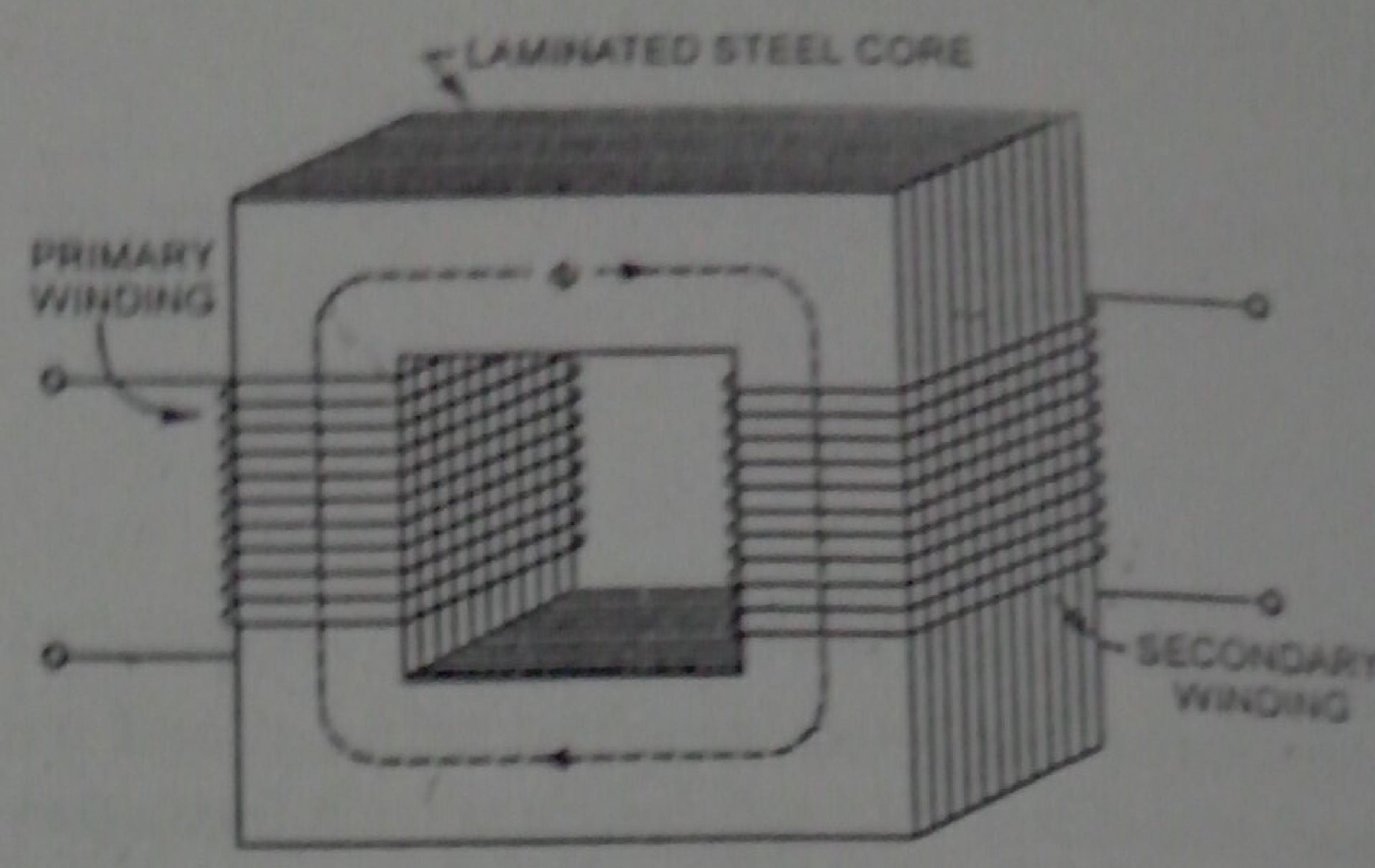
Since its basic construction requires no moving parts, so it is often called the '*static transformer*' and it is very rugged machine requiring the minimum amount of repair and maintenance. Owing to the lack of rotating parts there are no friction or windage losses. Further, the other losses are relatively low, so that the efficiency of a transformer is high. Typical transformer efficiencies at full load lie between 96 % and 97 % and with extremely large capacity transformers the efficiencies are as high as 99 %.

6.2. BASIC CONSTRUCTION AND WORKING PRINCIPLE OF TRANSFORMER

An elementary transformer consists of a soft iron or silicon steel core and two windings placed on it. The windings are insulated from both the core and each other. The core is built up of thin soft iron or silicon steel laminations to provide a path of low reluctance to the magnetic flux. The winding connected to the supply main is called the *primary* and the winding connected to the load circuit is called the *secondary*. The winding connected to higher voltage circuit is called the *high-voltage (hv) winding* while that connected to the lower voltage circuit is called the *low-voltage (lv) winding*. In case of a step-up transformer, low-voltage winding is the primary and high voltage winding is the secondary while in case of a step-down transformer the high-voltage winding is the primary and low-voltage winding is the secondary.

The action of a transformer is based on the principle that energy may be efficiently transferred by induction from one set of coils to another by means of a varying magnetic flux, provided that both the sets of coils are on a common magnetic circuit. In a transformer, the coils and magnetic circuit are all stationary with respect to one another. The emfs are induced by the variation in the magnitude of flux with time, as illustrated in fig 6.1.

Although in the actual construction the two windings are usually wound one over the other, for the sake of simplicity, the figures for analysing transformer theory show the windings on opposite sides of the core as in fig 6.1.



Simple Transformer
Fig. 6.1

When the primary winding is connected to an a c supply mains, a current flows through it. Since this winding links with an iron core, so current flowing through this winding produces an alternating flux ϕ in the core. Since this flux is alternating and links with the secondary winding also, so induces an emf in the secondary winding. The frequency of induced emf in secondary winding is the same as that of the flux or that of the supply voltage. The induced emf in the secondary winding enables it to deliver current to an external load connected across it. Thus the energy is transformed from primary winding to the secondary winding by means of electro-magnetic induction without any change in frequency. The flux ϕ of the iron core links not only with the secondary winding but also with the primary winding, so produces self induced emf in the primary winding. This induced emf in the primary winding opposes the applied voltage and, therefore, sometimes it is known as *back emf* of the primary. In fact the induced emf in the primary winding limits the primary current in much the same way that the back emf in a dc motor limits the armature current.

6.3. TRANSFORMER ON DC

A transformer cannot operate on dc supply and never be connected to a dc source. If a rated dc voltage is applied to the primary of a transformer, the flux produced in the transformer core will not vary but remain constant in magnitude and, therefore, no emf will be induced in the secondary winding except at the moment of switching on. Thus the transformer is not capable of raising or lowering the dc voltage. Also there will be no self induced emf in the primary winding, which is only possible with varying flux linkage, to oppose the applied voltage and since the resistance of primary winding is quite low, therefore, a heavy current will flow through the primary winding which may result in the burning out of the primary winding. This is the reason that *dc is never applied to a transformer*.

6.4. IDEAL TRANSFORMER

For a better understanding and an easier explanation of a practical transformer, certai idealizing assumptions are made which are close approximations for a practical transforme A transformer having these ideal properties is hypothetical (has no real existence) and referred to as the *ideal transformer*. It possesses certain essential features of a real trar former but some details of minor significance are ignored which will be introduced step-step while analysing a transformer. The idealizing assumptions made are as follows:

(*i*) *No winding resistance i.e.* the primary and secondary windings have zero resista It means that there is no ohmic power loss and no resistive voltage drop in an i transformer.

(*ii*) *No magnetic leakage i.e.* there is no leakage flux and all the flux set up is conf to the core and links both the windings.

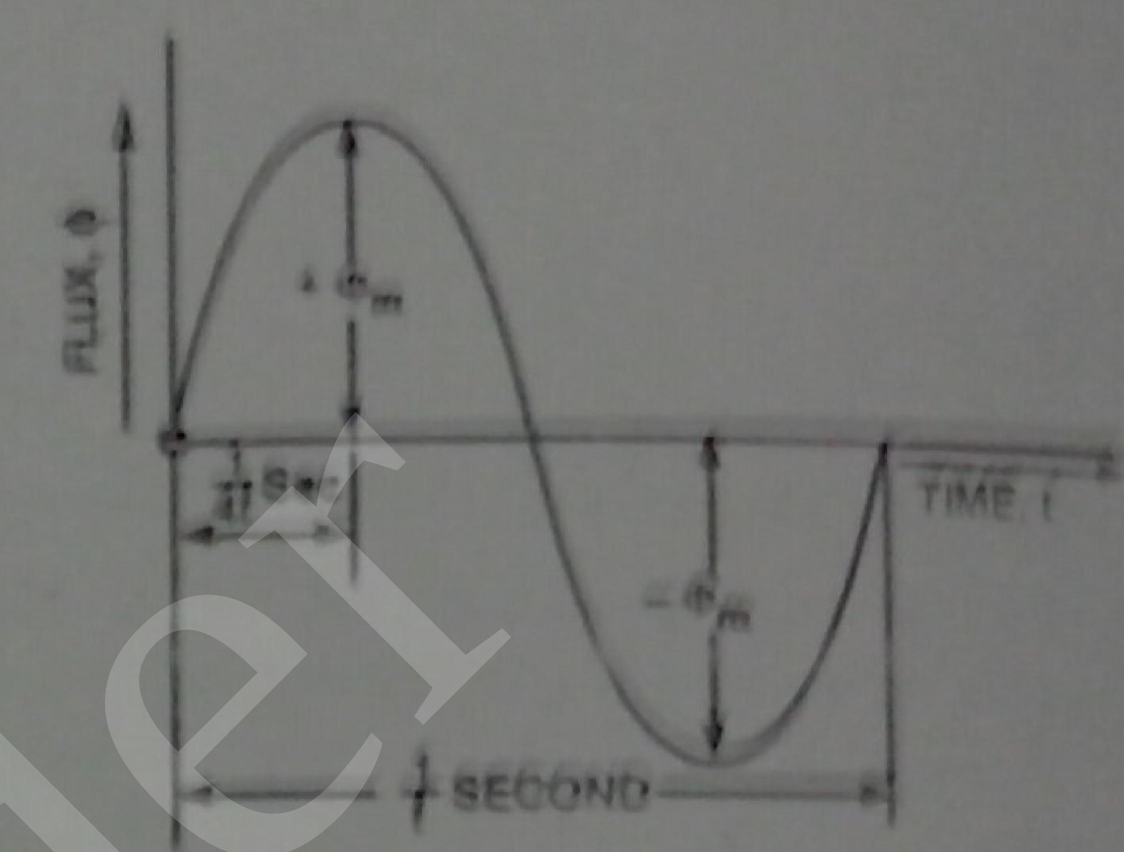
(iii) *No iron loss i.e.* hysteresis and eddy current losses in transformer core are zero.

(iv) *Zero-magnetizing current i.e.* the core has infinite permeability and zero reluctance so that zero magnetizing current is required for establishing the requisite amount of flux in the core.

From the above discussion an ideal transformer is supposed to consists of two purely inductive coils wound on a loss-free core.

6.5 EMF EQUATION

When an alternating (sinusoidal) voltage is applied to the primary winding of a transformer, an alternating (sinusoidal) flux, as shown is fig 6.2., is set up in the iron core which links both the windings (primary and secondary windings).



Sinusoidal Variation of Flux With Time
Fig 6.2

Let ϕ_{max} = Maximum value of flux in webers

and f = Supply frequency in hertz.

As illustrated in fig 6.2, the magnetic flux increases from zero to its maximum value ϕ_{max} in one-fourth of a cycle *i.e.* in $\frac{1}{4f}$ second.

So average rate of change of flux, $\frac{d\phi}{dt} = \frac{\phi_{max}}{1/4f} = 4f\phi_{max}$

Since average emf induced per turn in volts is equal to the average rate of change of flux,

so average emf induced per turn $= 4f\phi_{max}$ volts

Since flux ϕ varies sinusoidally, so emf induced will be sinusoidal and form factor for sinusoidal wave is 1.11 *i.e.* the rms or effective value is 1.11 times the average value.

$\therefore$ RMS value of emf induced per turn $= 1.11 \times 4f\phi_{max}$ volts ...(6.1)

If the number of turns on primary and secondary windings are N_1 and N_2 respectively, then

RMS value of emf induced in primary, E_1 = EMF induced per turn × number of primary turns

$$= 4.44f\phi_{max} \times N_1 = 4.44f N_1 \phi_{max} \text{ volts} \quad ...(6.2)$$

Similarly rms value of emf induced in secondary,

$$E_2 = 4.44f\phi_{max} \times N_2 \text{ volts} \quad ...(6.3)$$

In an ideal transformer the voltage drops in primary and secondary windings are negligible, so

EMF induced in primary winding, E_1 = Applied voltage to primary, V_1

and terminal voltage, V_2 = EMF induced in secondary, E_2

Note: If B_{max} is the maximum allowable flux density in Wb/m^2 (or T) and a is the area of x-section of iron core in square metres, then in equations (6.1), (6.2) and (6.3), ϕ_{max} is given as

$$\phi_{max} = B_{max}\, a \text{ webers}$$

6.6 VOLTAGE AND CURRENT TRANSFORMATION RATIOS

Referring to equation (6.1), it is clear that the *volts per turn* is exactly the same for both the primary and secondary windings *i.e.* in any transformer, the secondary and primary induced emfs are related to each other by the ratio of the number of secondary and primary turns. Thus

$$\frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{N_2}{N_1} = K \qquad ...(6.4)$$

The same relationship can be derived by dividing equation (6.3) by equation (6.2).

The constant K in equation (6.4) is called the *voltage transformation ratio.*

For step-up transformer, $V_2 > V_1$ or voltage transformation ratio, $K > 1$.

For step-down transformer, $V_2 < V_1$ or voltage transformation ratio, $K < 1$.

In an ideal transformer, the losses are negligible, so the volt-ampere input to the primary and volt-ampere output from secondary can be approximately equated *i.e.*

$$\text{Output VA} = \text{Input VA}$$

$$\text{or } V_2 I_2 = V_1 I_1$$

$$\text{or } \frac{I_2}{I_1} = \frac{V_1}{V_2} = \frac{E_1}{E_2} = \frac{N_1}{N_2} = \frac{1}{K} \qquad ...(6.5)$$

i.e. Primary and secondary currents are inversely proportional to their respective turns.

Example 6.1. The emf per turn of a single phase 10 KVA, 2,200/220 V, 50 Hz transformer is 10 V. Calculate (i) the number of primary and secondary turns, (ii) the net cross-sectional area of core for a maximum flux density of 1.5 T.

Solution:

EMF per turn = 10 V

Primary induced emf, $E_1 = V_1 = 2{,}200$ V

Secondary induced emf, $E_2 = V_2 = 220$ V

Supply frequency, $f = 50$ Hz

Maximum flux density, $B_{max} = 1.5$ T

(*i*) Number of primary turns, $N_1 = \frac{E_1}{\text{EMF per turn}} = \frac{2{,}200}{10} = 220$ **Ans.**

Number of secondary turns, $N_2 = \frac{E_2}{\text{EMF per turn}} = \frac{220}{10} = 22$ **Ans.**

Maximum value of flux, $\phi_{max} = \frac{\text{EMF per turn}}{4.44 f} = \frac{10}{4.44 \times 50} = 0.045$ Wb

...refer equation (6.1)

(*ii*) Net cross-sectional area of core, $a = \frac{\phi_{max}}{B_{max}} = \frac{0.045}{1.5} = 0.03\ m^2$ **Ans.**

Example 6.2. A single phase transformer has 400 primary and 1,000 secondary turns. The net cross-sectional area of the core is 60 cm². If the primary winding be connected to a 50 Hz supply at 500 V, calculate (i) the peak value of the flux density in the core, and (ii) the voltage induced in the secondary winding.

Solution:

Primary induced emf, $E_1 = V_1 = 500$ V

Supply frequency, $f = 50$ Hz

Primary turns, $N_1 = 400$

Net cross-sectional area of core, $a = 60\ cm^2 = 0.006\ m^2$

Maximum value of flux, $\phi_{max} = \dfrac{E_1}{4.44 f N_1}$ ...refer equation (6.2)

$$= \frac{500}{4.44 \times 50 \times 400} = 0.00563 \text{ Wb}$$

(*i*) Peak value of flux density in the core, $B_{max} = \dfrac{\phi_{max}}{a} = \dfrac{0.00563}{0.006} = 0.9384$ T **Ans.**

(*ii*) Voltage induced in the secondary winding, $E_2 = E_1 \times \dfrac{N_2}{N_1}$ ...refer equation (6.4)

$$= 500 \times \frac{1,000}{400} = 1,250 \text{ V}$$ **Ans.**

Example 6.3. A 3,000/200 V, 50 Hz single phase transformer is built on a core having an effective cross-section area of 150 cm² and has 80 turns in low voltage winding. Calculate (a) the value of maximum flux density in the core (b) number of turns in high-voltage winding.

Solution: Maximum value of flux, $\phi_{max} = \dfrac{\text{Induced emf in low-voltage winding}}{4.44 \times \text{supply frequency} \times \text{low-voltage winding turns}}$

$$= \frac{200}{4.44 \times 50 \times 80} = 0.01126 \text{ Wb}$$

(*i*) Value of maximum flux density in the core,

$$B_{max} = \frac{\phi_{max}}{a} = \frac{0.01126}{150 \times 10^{-4}} = 0.75 \text{ T}$$ **Ans.**

(*ii*) Number of turns on high voltage winding

$$= \frac{\text{Induced emf in hv winding} \times \text{lv winding turns}}{\text{induced emf in lv winding}}$$

$$= \frac{3,000}{200} \times 80 = 1,200$$ **Ans.**

Example 6.4. A 200 kva, 6600/400 V, 50 Hz single phase transformer has 80 turns on the secondary. Calculate (i) the approximate values of the primary and secondary currents (ii) the approximate number of primary turns and (iii) the maximum value of flux.

Solution: Output = 200 kva

(*i*) Approximate value of primary current on full load,

$$I_1 = \frac{\text{Rated kva} \times 1,000}{V_1} = \frac{200 \times 1,000}{6,600} = 30.3 \text{ A}$$ **Ans.**

Approximate value of secondary current on full load,

$$I_2 = \frac{\text{Rated kva} \times 1,000}{V_2} = \frac{200 \times 1,000}{400} = 500 \text{ A}$$ **Ans.**

(*ii*) Approximate number of primary turns,

$$N_1 = N_2 \times \frac{E_1}{E_2} = N_2 \times \frac{V_1}{V_2} = 80 \times \frac{6,600}{400} = 1,320$$ **Ans.**

(*iii*) Maximum value of flux in the core,

$$\phi_{max} = \frac{E_1}{4.44 f N_1} = \frac{6,600}{4.44 \times 50 \times 1,320} = 0.0225 \text{ Wb or } 22.5 \text{ m Wb}$$ **Ans.**

Example 6.5. **A single phase, 50 Hz core type transformer has square cores of 20 cm side. The permissible flux density in the core is 1.0 Wb/m^2. Calculate the number of turns per limb on the high and low voltage sides for a 3,000/220 V ratio. To allow for insulation of stampings, assume the net iron length to be 0.9 × gross iron length.**

Solution: Iron-length on one side of the core

$= 0.9 \times 20 = 18$ cm $= 0.18$ m

Iron-length on the other side of the core

$= 20$ cm or 0.20 m

because only one side of the core is affected by insulation and other side remains unaffected (fig 6.3).

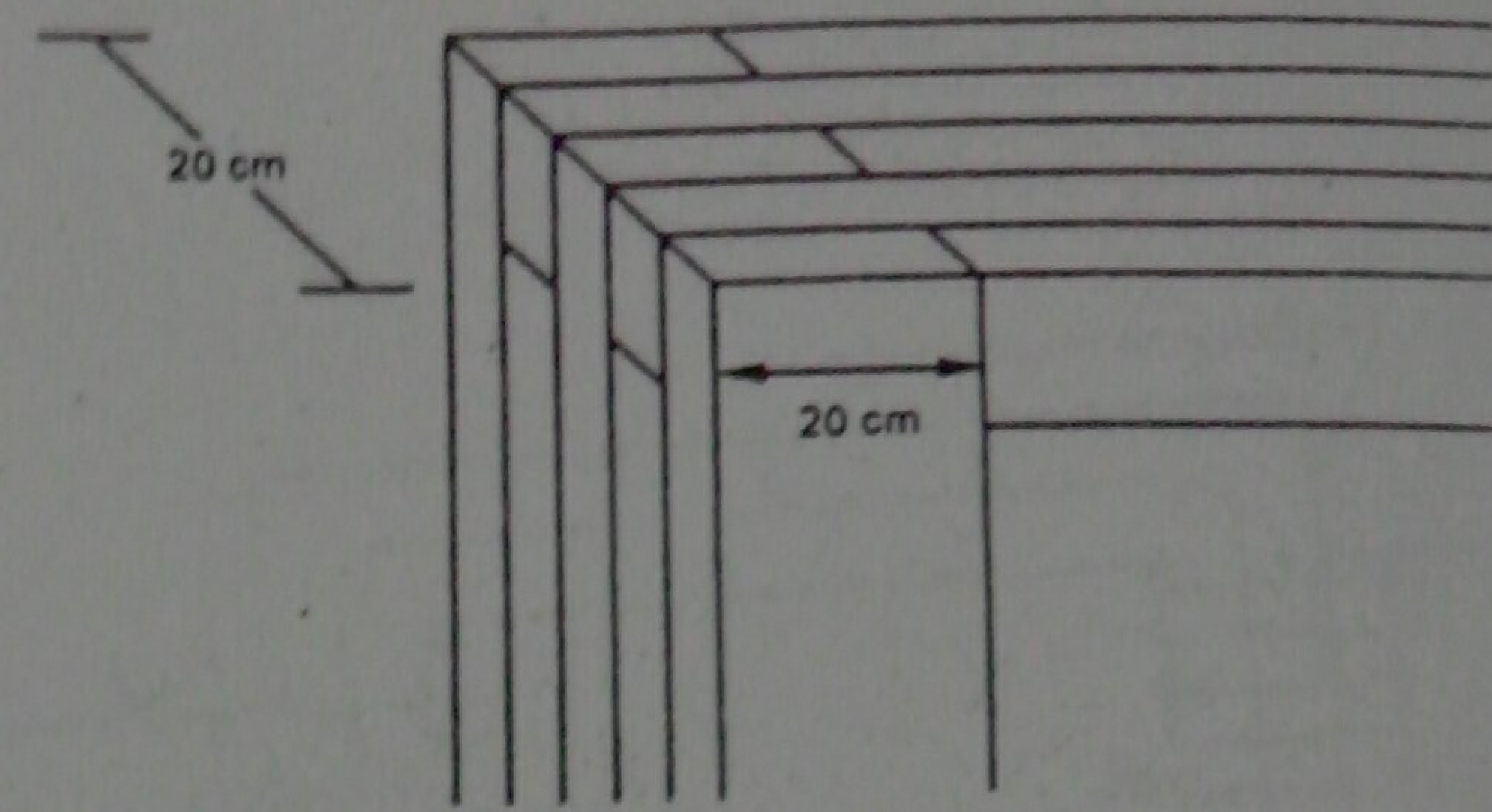


Fig. 6.3

Core area, $a = 0.18 \times 0.20 = 0.036$ m^2

Permissible flux density, $B_{max} = 1.0$ Wb/m^2

Maximum value of flux,

$$\phi_{max} = B_{max} \times a = 1.0 \times 0.036 = 0.036 \text{ Wb}$$

$$\text{Number of turns on hv (primary) side, } N_1 = \frac{E_1}{4.44 f \phi_{max}} = \frac{3,000}{4.44 \times 50 \times 0.036}$$

$$= 375.4 \simeq 376 \text{ Ans.}$$

$$\text{Number of turns on lv (secondary) side, } N_2 = \frac{E_2}{E_1} \times N_1 = \frac{220}{3,000} \times 376 = 27.53 \simeq 28 \text{ Ans.}$$

Example 6.6. **A single phase transformer with 10 : 1 turn-ratio and rated at 50 KVA, 2400/240 V, 50 Hz is used to step down the voltage of a distribution system. The low tension voltage is to be kept constant at 240 V. Find the value of load impedance of the *lt* side so that the transformer will be loaded fully. Find also the value of maximum flux inside the core if the *lt* side has 23 turns.**

Solution : Low tension (secondar y) voltage, $V_2 = 240$ volts

$$\text{Full-load secondary current, } I_2 = \frac{\text{Rated KVA} \times 1,000}{V_2} = \frac{50 \times 1,000}{240} = 208.33 \text{ A}$$

$$\text{Load impedance connected on secondary side, } Z = \frac{V_2}{I_2} = \frac{240}{208.33} = 1.152\ \Omega \text{ Ans.}$$

$$\text{Maximum value of flux in the core, } \phi_{max} = \frac{E_2}{4.44 f N_2} = \frac{240}{4.44 \times 50 \times 23} = 0.047 \text{ Wb Ans.}$$

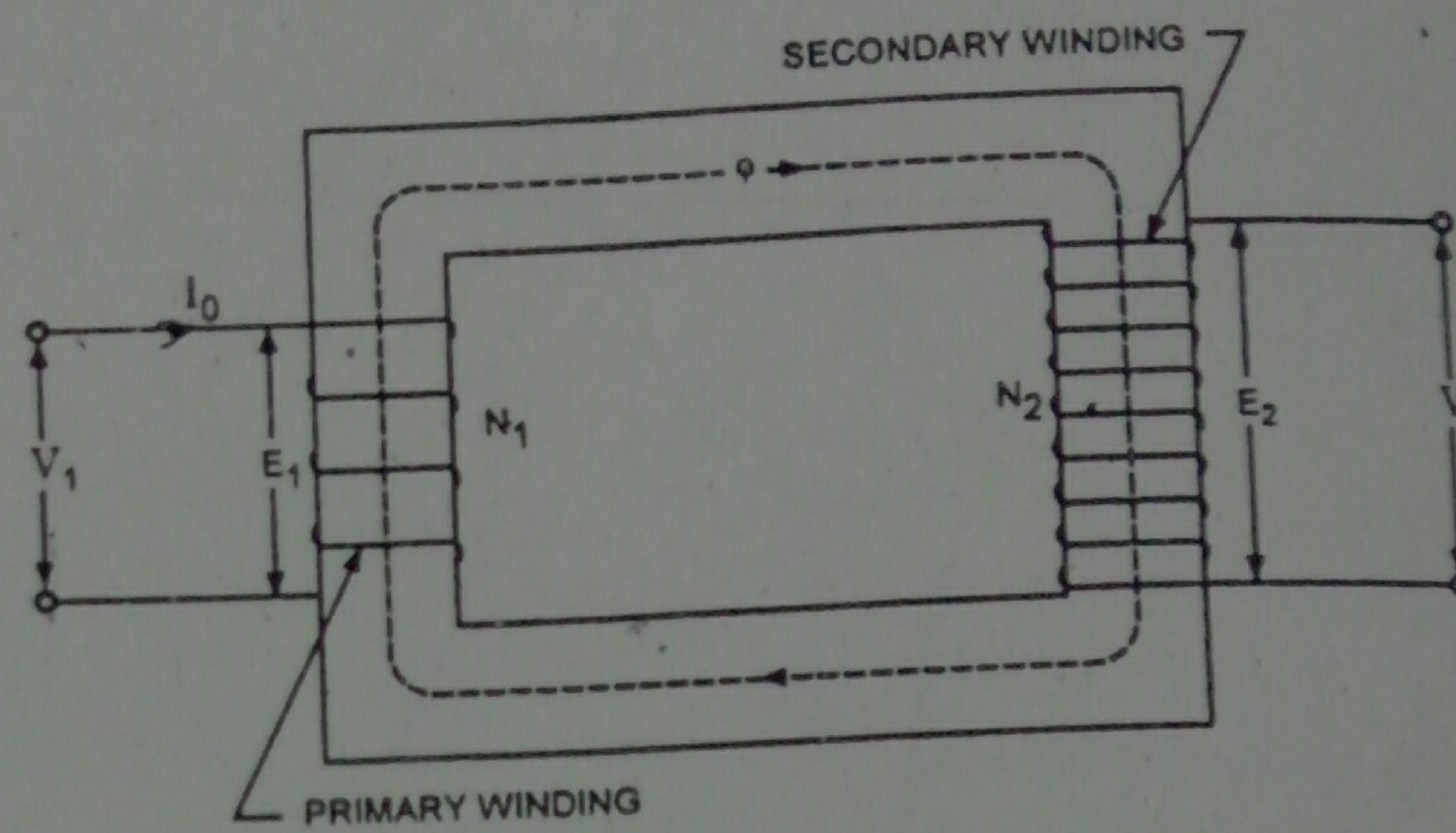
6.7. TRANSFORMER ON NO LOAD

When the primary of a transformer is connected to the source of ac supply and the secondary is open, the transformer is said to be at no-load (there is no load on secondary).

Consider an ideal transformer whose secondary side is open and the primary winding is connected to a sinusoidal alternating voltage V_1. The alternating voltage applied to the primary winding will cause flow of alternating current in the primary winding. Since the primary coil is purely inductive and there is no output (secondary being open) the primary draws the magnetising current I_m only. The function of this current is merely to magnetize the core. If the transformer is truely ideal, the magnitude of I_m should be zero by virtue of assumption (*iv*) made in Art 6.4. Since the reluctance of the magnetic circuit is never zero, I_m has definite magnitude. The magnetising current, I_m is small in magnitude and lags behind supply voltage V_1 by 90°. This magnetising current I_m produces an alternating flux ϕ which is, at all times, proportional to the current (assuming permeability of the magnetic circuit to be constant) and, hence, in phase with it.

Let the instantaneous linking flux be given as

$$\phi = \phi_{max} \sin \omega t \qquad ..(6.6)$$



Transformer on No Load
Fig. 6.4

The varying flux is linked with both of the windings (primary and secondary) and so induces emfs in the primary and secondary windings. The instantaneous values of induced emfs in the primary and secondary windings will be

$$e_1 = -N_1 \frac{d\phi}{dt} = -N_1 \frac{d}{dt} (\phi_{max} \sin \omega t)$$

$$= -N_1 \omega \phi_{max} \cos \omega t = N_1 \omega \phi_{max} \sin \left(\omega t - \frac{\pi}{2}\right) \quad ...(6.7)$$

$$\text{Similarly, } e_2 = N_2 \omega \phi_{max} \sin \left(\omega t - \frac{\pi}{2}\right) \quad ...(6.8)$$

Since primary winding has no ohmic resistance, (as assumed), therefore, applied voltage to primary winding is to only oppose the induced emf in the primary winding, hence instantaneous applied voltage to primary will be given by

$$v_1 = -e_1 = -N_1 \omega \phi_{max} \sin \left(\omega t - \frac{\pi}{2}\right) \quad ...(6.9)$$

Comparing expressions (6.6), (6.7), (6.8), and (6.9) we conclude that

(*i*) Induced emfs in primary and secondary windings, E_1 and E_2 lag behind the main flux ϕ by $\frac{\pi}{2}$, so these emfs (E_1 and E_2) are in phase with each other, as shown in fig. 6.5 vectorially.

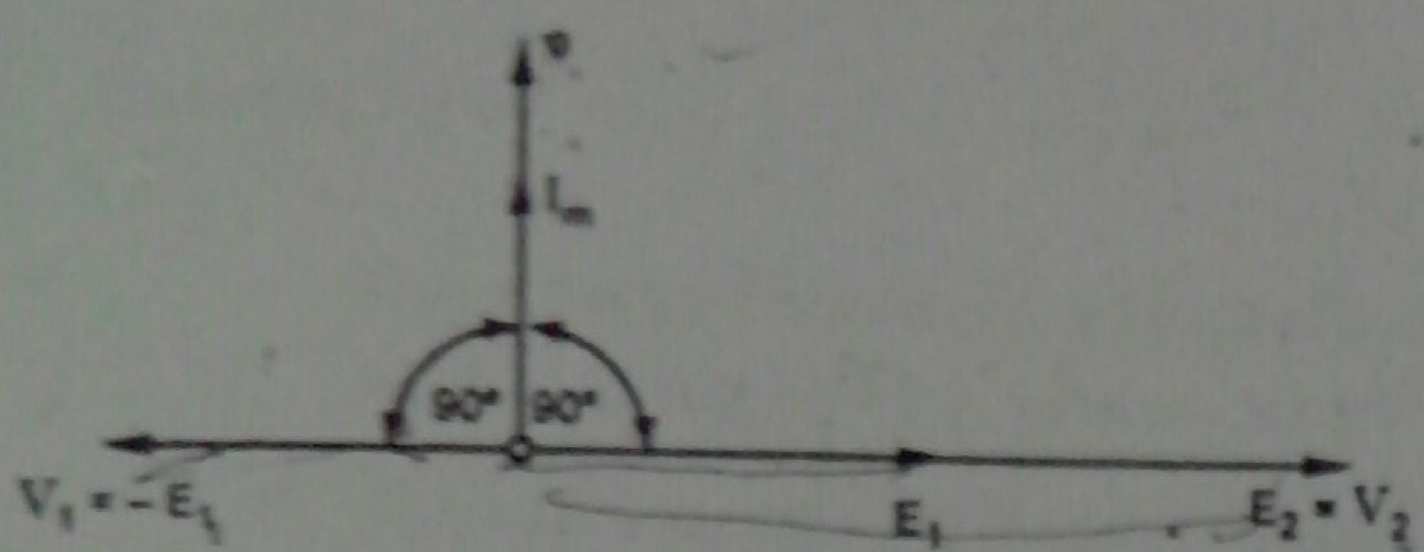
(*ii*) Applied voltage to the primary winding leads the main flux by $\frac{\pi}{2}$ and is in phase opposition to the induced emf in the primary winding, as shown in fig 6.5 vectorially.

(*iii*) Secondary voltage $V_2 = E_2$ as there is no voltage drop in secondary.

The instantaneous values of applied voltage, induced emfs, flux and magnetising current, in case of an ideal transformer, are illustrated by sinusoidal waves in fig 6.6.

However, when a varying flux is set up in magnetic material, there will be power loss, called the *iron or core loss*. So the input current to the primary under no-load condition has also to supply the hysteresis and eddy current losses (iron losses) occuring in the core in addition to small amount of copper loss occuring in primary winding (no copper loss occurs in secondary winding on open circuit or on no-load). Hence, the no-load primary current I_0 does not lag behind applied voltage V_1 by 90° but lags behind V_1 by angle $\phi_0 < 90°$..

Input power on no-load, $P_o = V_1 I_0 \cos \phi_0$ where $\cos \phi_0$ is the primary power factor under no-load conditions.



No-Load Phasor Diagram For An Ideal Transformer
Fig. 6.5

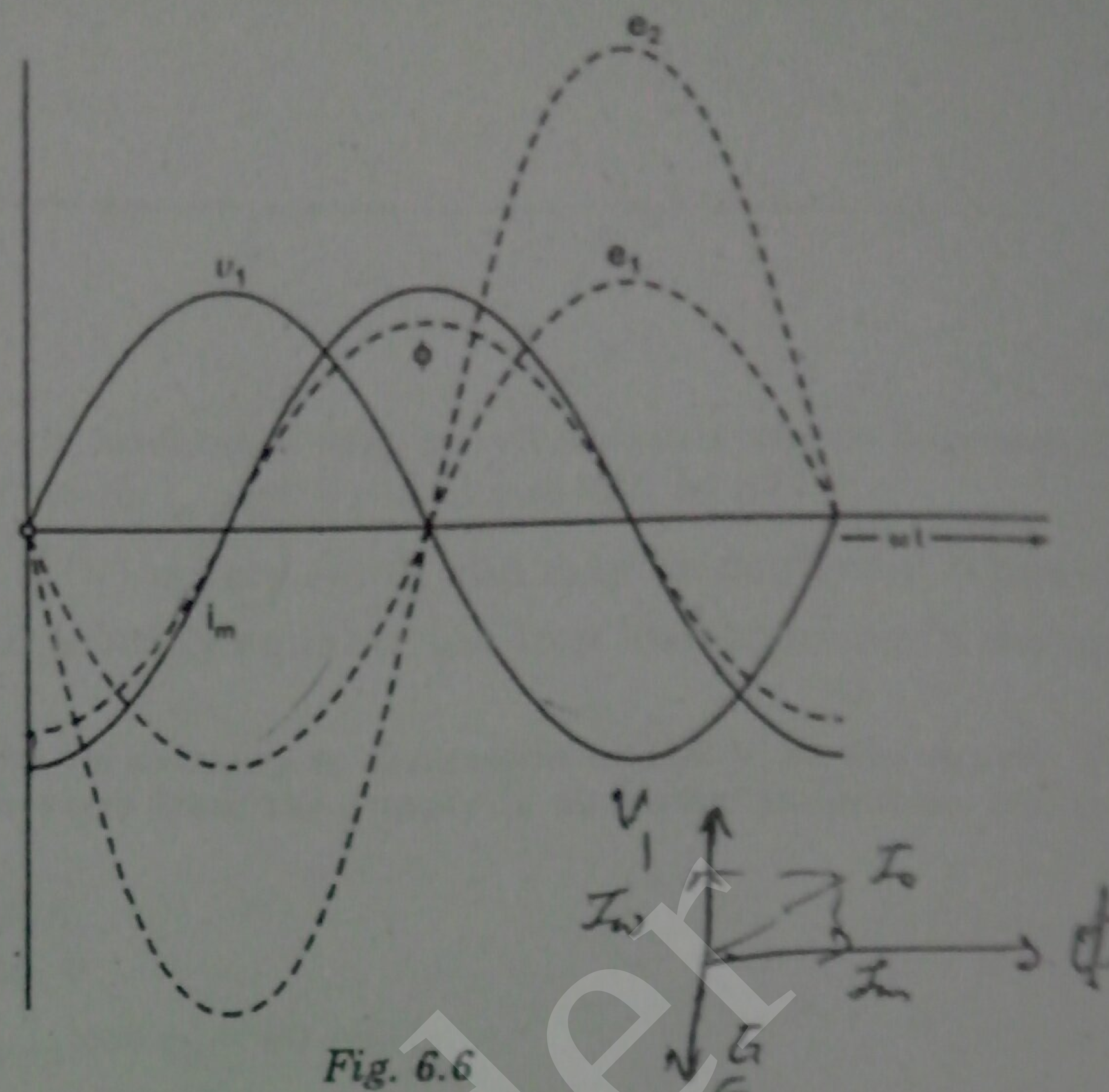


Fig. 6.6

As seen from phasor diagram shown in fig 6.7, input current to the primary I_0, called the *exciting current*, has two components (*i*) in-phase, active or energy component, I_e used to meet the iron loss in addition to small amount of copper loss occurring in the primary winding and (*ii*) quadrature component or wattless component, called the magnetizing component, I_m used to create the alternating flux in the core.

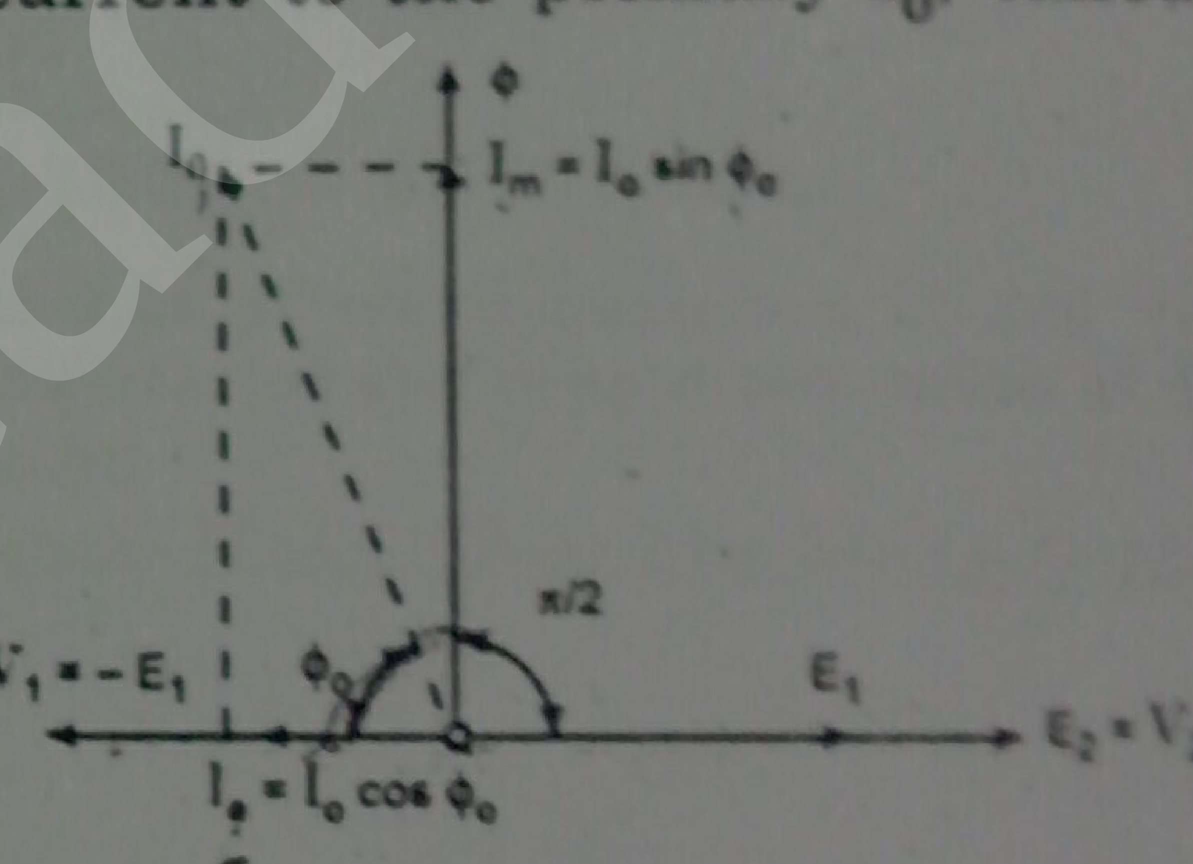


Fig. 6.7

Thus, $I_e = I_0 \cos \phi_0$

and $I_m = I_0 \sin \phi_0$

and $\sqrt{I_e^2 + I_m^2} = I_0$

Angle of lag, $\phi_0 = \text{Tan}^{-1} \dfrac{I_m}{I_e}$

The equivalent circuit of a transformer on no-load is illustrated in fig 6.8, where in two components of no-load current; I_e and I_m are represented by currents drawn by a non-inductive resistance R_0 and a pure inductive reactance X_0 respectively. Both these currents are drawn at induced emf $E_1 = V_1$ for resistance-less, no-leakage primary coil; even otherwise $E_1 \simeq V_1$

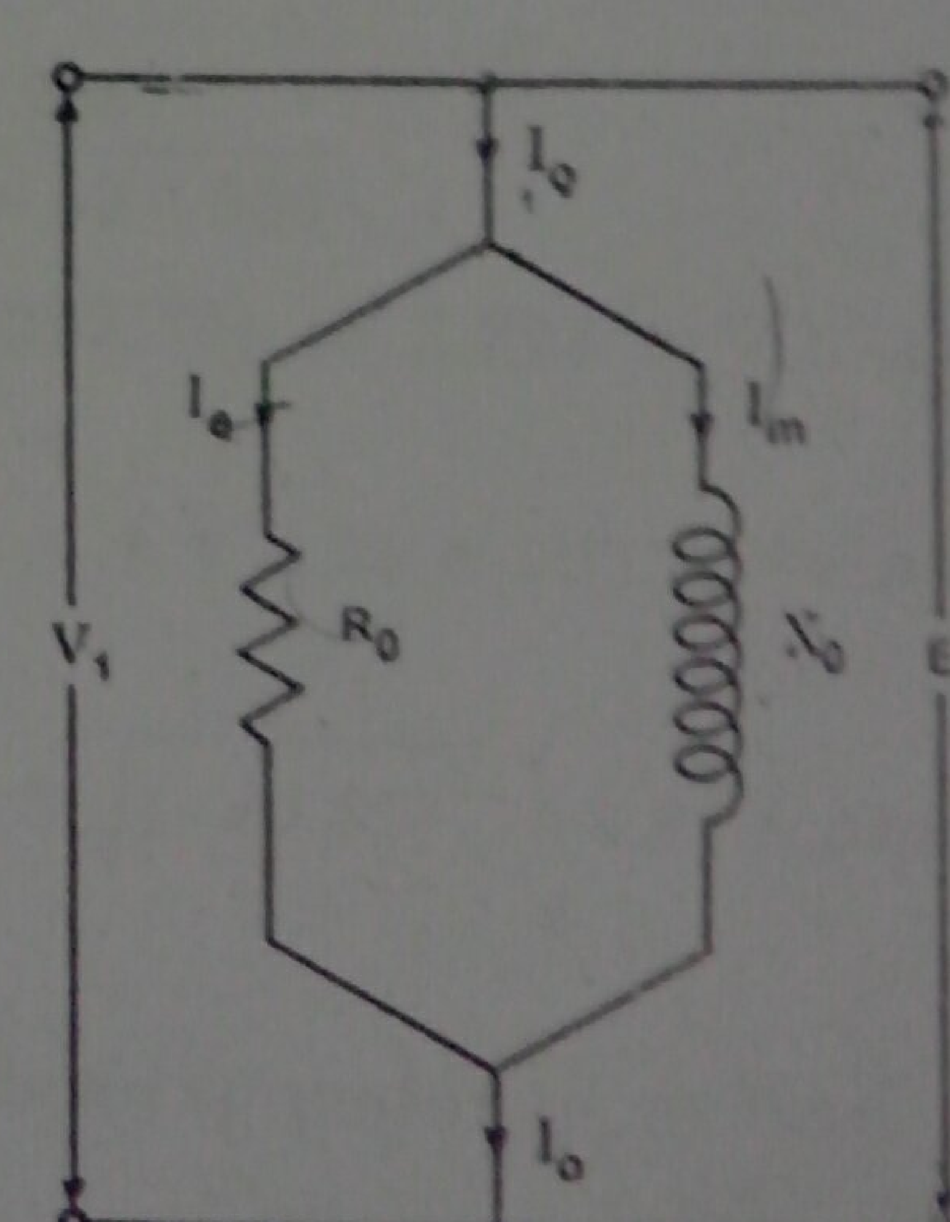


Fig. 6.8

The worthnoting points are given below:

1. The no-load primary current I_0, called the *exciting current*, is very small in comparison to the full-load primary current. It ranges from 2 to 5 per cent of full-load primary current.
2. The exciting or no-load current I_0 is made up of a relatively large quadrature or magnetizing component I_m and a comparatively small in-phase or energy component I_e so

the power factor of a transformer on no-load is very small (usually varies between 0.1 and 0.2 lag). The phase angle between I_0 and V_1 is about 78° to 87°.

3. No-load primary copper loss $\left(i.e\ I_0^2 R_1\right)$ is very small and may be neglected. Thus the no-load primary input power is practically equal to the iron loss occuring in the core of the transformer.

Example 6.7. A transformer takes 0.8 A when its primary is connected to 200 V, 50 Hz supply. The secondary is open-circuited. The power absorbed from the supply is 60 watts. Determine the iron loss current and magnetising current.

Solution:

$$\text{No-load current, } I_0 = 0.8 \text{ A}$$
$$\text{Primary voltage, } V_1 = 200 \text{ V}$$
$$\text{Iron loss} = V_1 I_0 \cos \phi_0 = 60 \text{ watts}$$
$$\text{Iron loss current, } I_e = I_0 \cos \phi_0 = \frac{I_m}{I_e} = \frac{60}{200} = 0.3 \text{ A Ans.}$$
$$\text{Magnetising current, } I_m = \sqrt{I_0^2 - I_e^2} = \sqrt{(0.8)^2 - (0.3)^2} = 0.742 \text{ A Ans.}$$

Example 6.8. The no-load current of a transformer is 5 A at 0.25 pf when supplied at 235 V, 50 Hz. The number of turns on the primary winding is 200. Calculate (a) the maximum value of flux in the core, (ii) the core loss (iii) the magnetizing component.

[*Nagpur Univ. Elec. Machines-I, 1993*]

Solution :

$$\text{No-load current, } I_0 = 5 \text{ A}$$
$$\text{No-load power factor, } \cos \phi_0 = 0.25$$
$$\text{Primary induced emf, } E_1 = V_1 = 235 \text{ V}$$
$$\text{Supply frequency, } f = 50 \text{ Hz}$$
$$\text{Number of primary turns, } N_1 = 200$$

(*i*) Maximum value of flux in the core, $\phi_{max} = \dfrac{E_1}{4.44 f N_1} = \dfrac{235}{4.44 \times 50 \times 200}$

$= 0.005293$ Wb or 5.293 m Wb **Ans.**

(*ii*) Core loss = Input on no-load

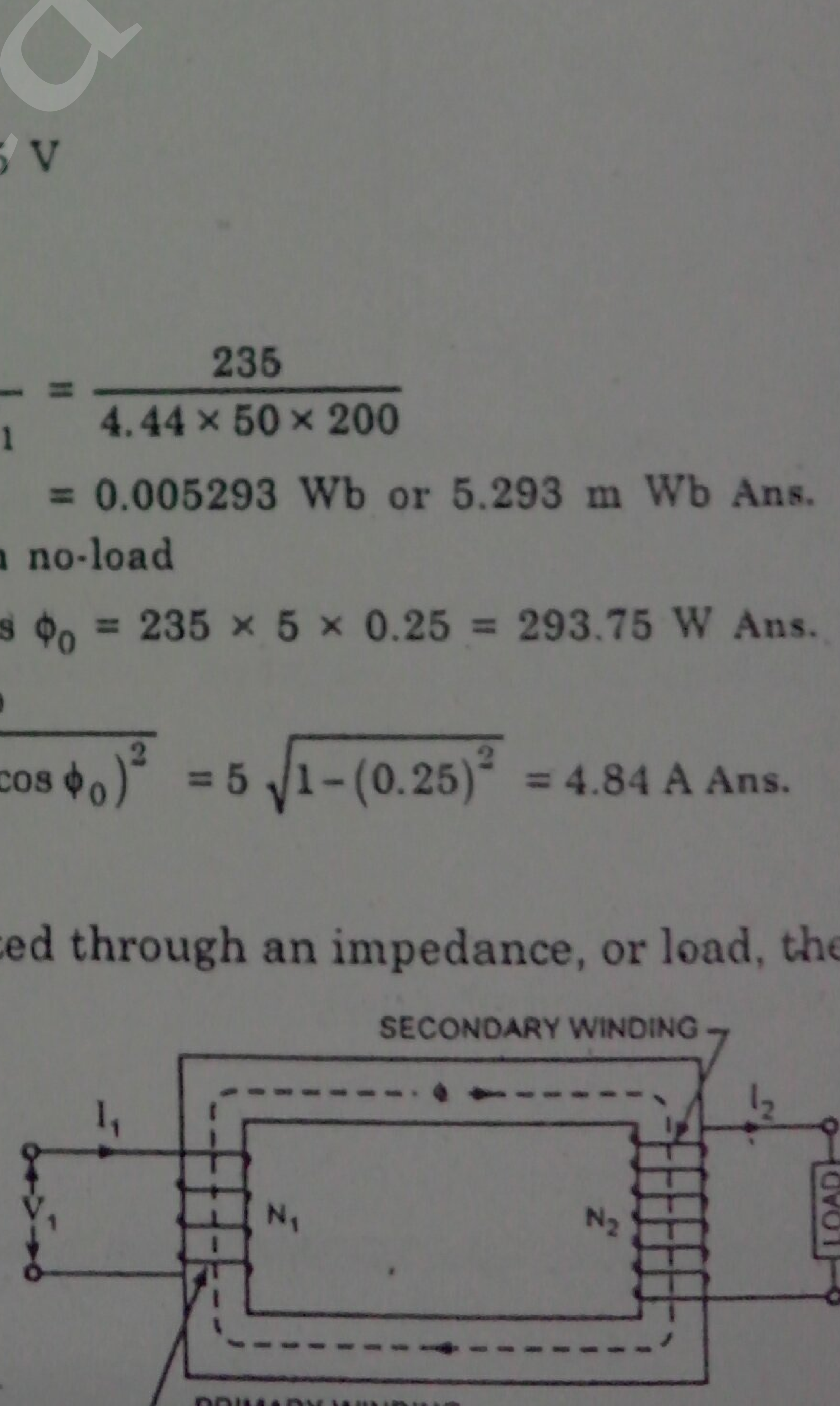
$= V_1 I_0 \cos \phi_0 = 235 \times 5 \times 0.25 = 293.75$ W **Ans.**

(*iii*) Magnetizing component, $I_m = I_0 \sin \phi_0$

$= I_0 \sqrt{1 - (\cos \phi_0)^2} = 5\sqrt{1 - (0.25)^2} = 4.84$ A **Ans.**

6.8. TRANSFORMER ON LOAD

When the secondary circuit of a transformer is completed through an impedance, or load, the transformer is said to be loaded, and current flows through the secondary winding and the load. The magnitude and phase of secondary current I_2 with respect to secondary terminal voltage V_2 will depend upon the characteristic of load *i.e.* current I_2 will be in phase, lag behind and lead the terminal voltage V_2 respectively when the load is non-inductive, inductive and capacitive.

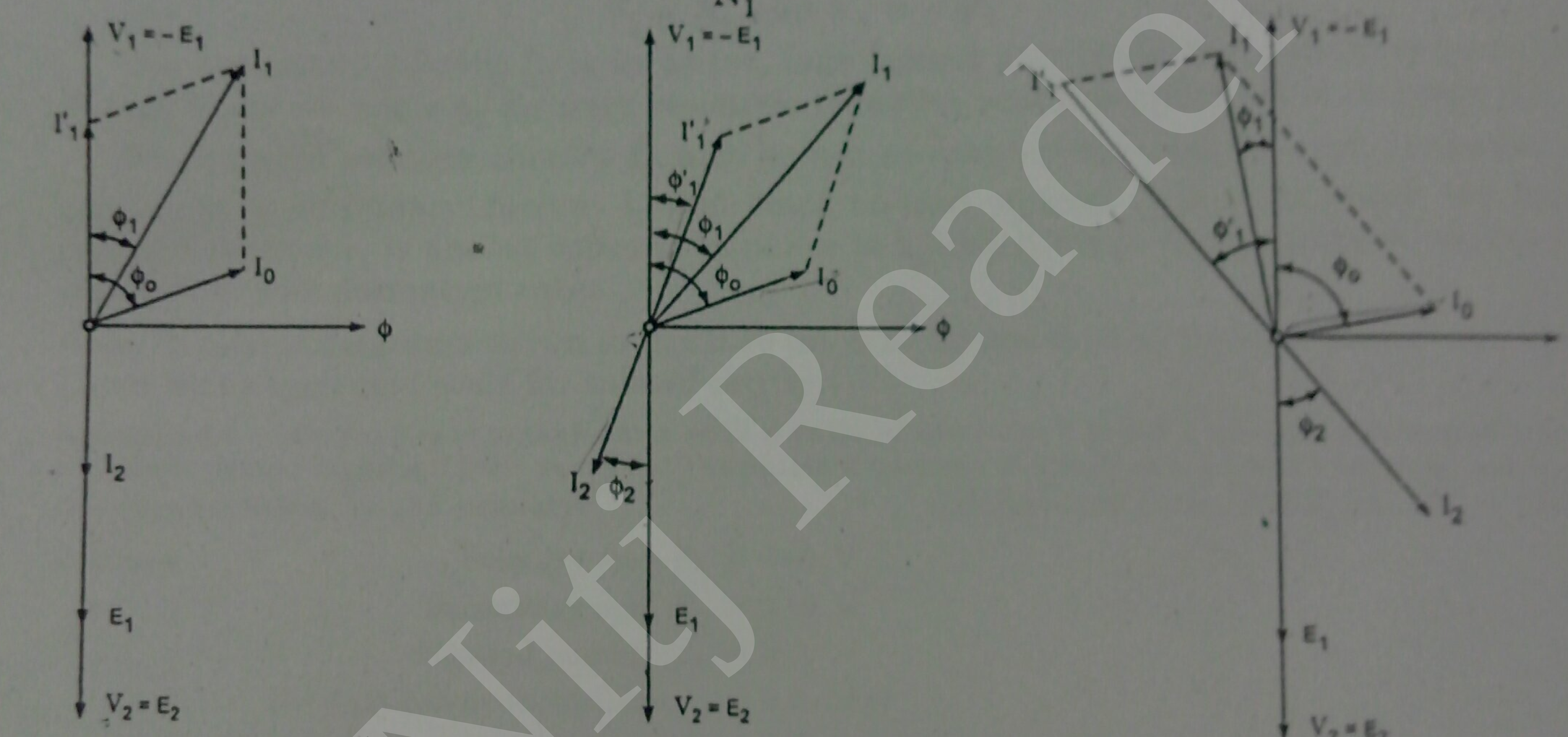


An Ideal Transformer On Load
Fig. 6.9

When the transformer is on no load, as shown in fig 6.4, it draws no-load current I_0 from the supply mains. The no-load current I_0 sets up an mmf $N_1 I_0$ which produces flux ϕ in the core. When an impedance is connected across the secondary terminals, as shown in fig 6.9, current I_2 flows through the secondary winding. The secondary current I_2 sets up its own mmf and hence creates a secondary flux ϕ_2. The secondary flux ϕ_2 opposes the main flux ϕ set up by the exciting current I_0 according to Lenz's law. The opposing secondary flux ϕ_2 weakens the main flux ϕ momentarily, so primary back emf E_1 tends to be reduced. So difference of applied voltage V_1 and back emf E_1 increases, therefore, more current is drawn from the source of supply flowing through the primary winding until the original value of flux ϕ is obtained. It again causes increase in back emf E_1 and it adjusts itself as such that there is a balance between applied voltage V_1 and back emf E_1. Let the additional primary current be I_1'. The current I_1' is in phase opposition with secondary current I_2 and is called the *counter-balancing current*. The additional current I_1' sets up an mmf $N_1 I_1'$ producing flux ϕ_1' in the same direction as that of main flux ϕ and cancels the flux ϕ_2 produced by secondary mmf $N_2 I_2$ being equal in magnitude.

$$\text{So } N_1 I_1' = N_2 I_2$$

$$\text{or } I_1' = \frac{N_2}{N_1} I_2$$



(a) For Pure Resistive Load *(b) For Inductive Load* *(c) For Capacitive Load*

Phasor Diagram For An Ideal Transformer On Load

Fig. 6.10

The total primary current I_1 is, therefore, phasor sum of primary counter-balancing current I_1' and no-load current I_0, which will be approximately equal to I_1' as I_0 is usually very small in comparison to I_1'.

$$\therefore I_1 = I_1' = \frac{N_2}{N_1} I_2$$

$$\text{or } \frac{I_1}{I_2} = \frac{N_2}{N_1} = K \quad \text{(transformation ratio)}$$

Hence primary and secondary currents are inversely proportional to their respective turns.

Since the secondary flux ϕ_2 produced by secondary mmf $N_2 I_2$ is neutralized by the flux ϕ'_1 produced by mmf $N_1 I_1'$ set up by counter-balancing primary current I'_1, so the flux in the transformer core remains almost constant from no-load to full load.

The phasor diagrams for a transformer on non-inductive, inductive and capacitive loads are shown in figures 6.10 (*a*), (*b*) and (*c*) respectively.

Since the voltage drops in both of the windings of the transformer are assumed to be negligible, therefore

$$V_2 = E_2 \text{ and } V_1 = -E_1$$

The secondary current I_2 is in phase, lags behind and leads the secondary terminal voltage V_2 by an angle ϕ_2 for pure resistive, inductive and capacitive loads respectively.

The induced primary current I_1', also known as *counterbalancing current*, is always in opposition to secondary current I_2 and since no-load current I_0 is very small, the total primary current I_1 is almost opposite in phase to I_2 and K times the secondary current I_2, where K is transformation ratio.

Note. In phasor diagrams shown in figs 6.10 (a), 6.10 (b) and 6.10 (c) no-load current has been drawn on exaggerated scale for sake of clarity.

Example 6.9. A single phase transformer with a ratio of 440 / 110 V takes a no-load current of 5 A at 0.2 power factor lagging. If the secondary supplies a current of 120 A at a pf of 0.8 lagging, estimate the current taken by the primary. *[Bangalore Univ. 1992; Pb. Univ. 1991]*

Solution :

Primary emf, $E_1 = 440$ V

Secondary emf, $E_2 = 110$ V

No-load current, $I_0 = 5$ A

No-load power factor, $\cos \phi_0 = 0.2$ (lag)

Secondary current, $I_2 = 120$ A

Load power factor, $\cos \phi_2 = 0.8$ (lag)

Transformation ratio, $K = \frac{V_2}{V_1} = \frac{110}{440} = \frac{1}{4}$

Primary counter–balancing current, $I_1' = K I_2 = \frac{1}{4} \times 120 = 30$ A

lagging behind the applied voltage V_1 by an angle ϕ_1' where

$$\phi_1' = \phi_2 = \text{Cos}^{-1}\, 0.8 = 36.87°$$

No-load current, $I_0 = 5$ A lagging behind the applied voltage V_1 by an angle

$$\phi_0 = \text{Cos}^{-1}\, 0.2 = 78.46°$$

Primary current, $I_1 = I_1' + I_0$

$$= (I_1' \cos \phi_1 + I_0 \cos \phi_0) + j\,(I_1' \sin \phi_1 + I_0 \sin \phi_0)$$

$$= (30 \times 0.8 + 5 \times 0.2) + j\,(30 \times 0.6 + 5 \times 0.98)$$

$$= (25 + j\,22.9) \text{ A}$$

Magnitude of primary current, $I_1 = \sqrt{25^2 + 22.9^2} = 33.9$ A **Ans.**

Phase angle, $\phi_1 = \text{Tan}^{-1} \frac{22.9}{25} = 42.49°$ (lag) **Ans.**

6.9. RESISTANCE AND LEAKAGE REACTANCE

In preceding discussions we considered an ideal transformer, which according to our assumptions, has got no resistance in the windings and no leakage flux but in actual practice it is impossible to obtain such an ideal transformer.

In actual transformer both the windings, primary and secondary windings have finite resistances R_1 and R_2 which cause copper losses and voltage drops in them. The result is that:

(*i*) The secondary terminal voltage V_2 is less than the secondary induced emf E_2 and is equal to phasor difference of secondary induced emf E_2 and voltage drop in the secondary winding $I_2 R_2$, if magnetic leakage is negligible, *i.e.*

$$V_2 = E_2 - I_2 R_2$$

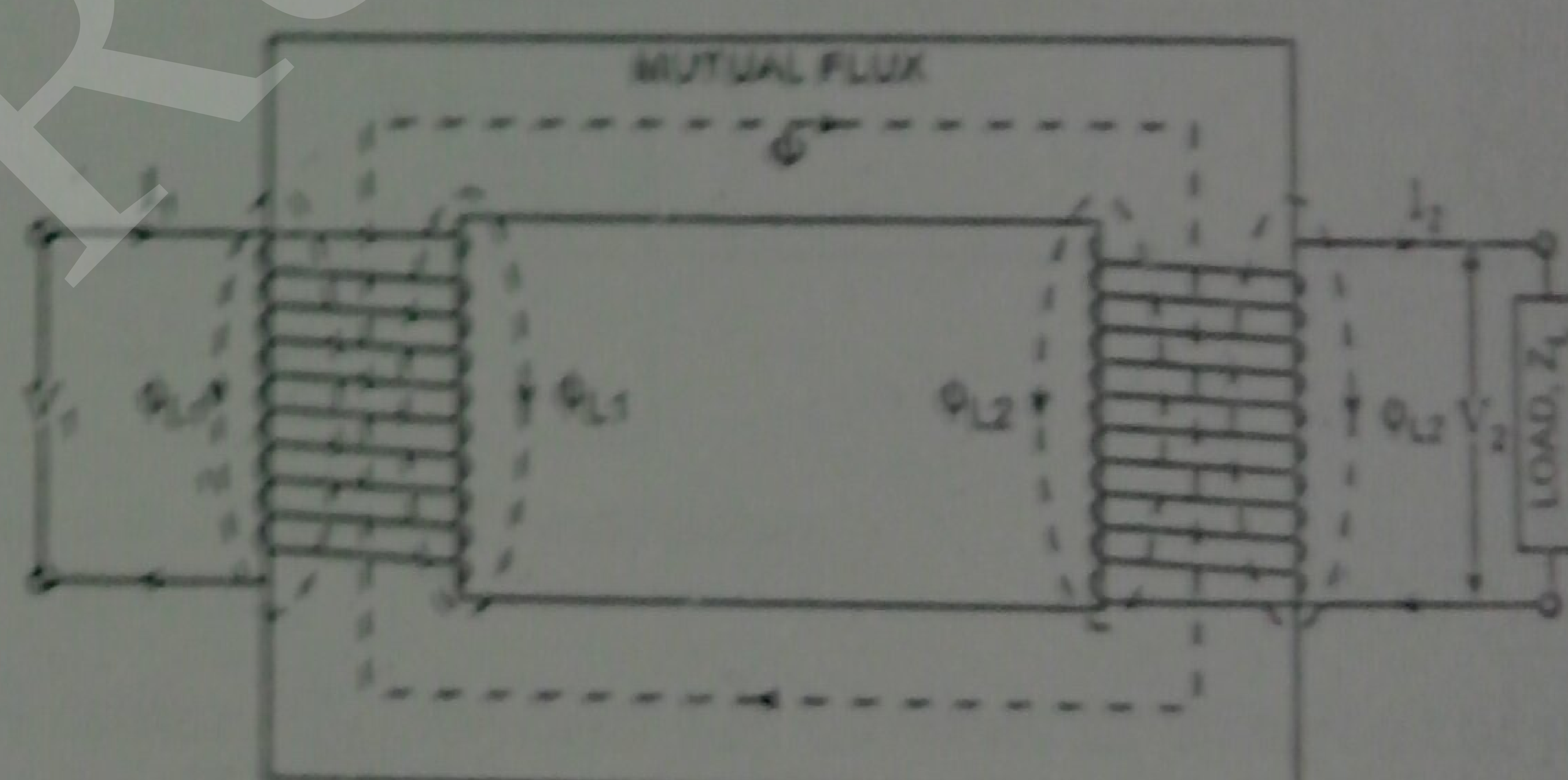
where I_2 is the secondary current and R_2 is the secondary winding resistance.

(*ii*) Similarly the counter-emf of primary, $-E_1$ is equal to phasor difference of voltage applied to the primary winding V_1 and voltage drop in the primary winding $I_1 R_1$, provided magnetic leakage is negligible, i.e.,

$$-E_1 = V_1 - I_1 R_1$$

where I_1 is the primary current and R_1 is the primary resistance.

It was previously assumed that the entire flux ϕ, developed by the primary winding, links with and cut every turn of both the primary and secondary windings. In practice, it is impossible to realize this condition. However, part of the flux set up by the primary winding links only the primary turns, as illustrated in fig 6.11 by flux ϕ_{L1}. Also, some of the flux set up by the secondary winding links only the secondary turns, as illustrated in fig 6.11 by ϕ_{L2}. These two fluxes ϕ_{L1} and ϕ_{L2} are known as leakage flux *i.e. that flux which leaks out of the core and does not link both windings*. The flux which does pass completely through the core and links both the windings is known as the *mutual flux* and is illustrated as ϕ.



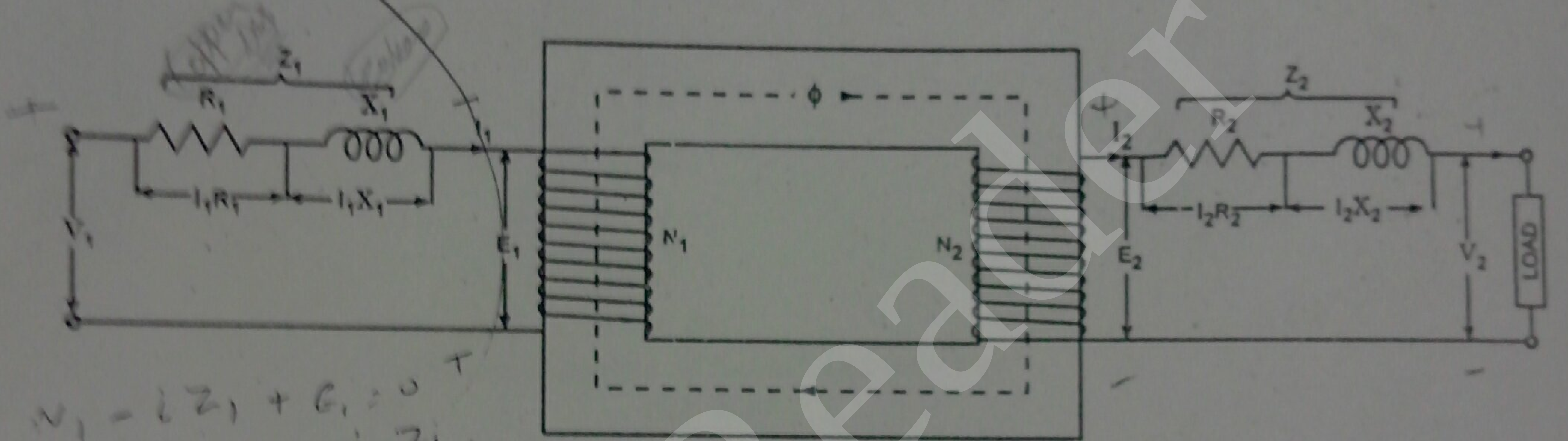
Magnetic Fluxes in a Transformer
Fig. 6.11

The primary leakage flux ϕ_{L1} linking with the primary winding is produced by primary ampere-turns only, therefore, it is proportional to primary current, number of primary turns being fixed. On no load primary current is so small that leakage flux ϕ_{L1} produced by it can be neglected but on load primary current increases resulting in increase in ampere-turns and hence leakage flux ϕ_{L1} increases. The primary leakage flux ϕ_{L1} is in phase with I_1 and produces self induced emf E_{L1} given by $E_{L1} = 2\pi f L_1 I_1$ in primary winding, where L_1 is the self inductance of the primary winding produced by primary leakage flux ϕ_{L1}.

The self induced emf E_{L1}, due to primary leakage flux, in the primary winding must lag leakage flux ϕ_{L1} and primary current I by 90°. The emf necessary to balance this counter-emf is opposite and equal to it and, therefore, leads the primary current I_1 by 90°. As this emf, induced by the primary leakage flux, is proportional to the current and lags it by 90°, it is nothing more than a reactance voltage and is denoted by $-I_1 X_1$. The component of line voltage that balances this emf is $+I_1 X_1$. The effect of the primary leakage flux, therefore, is to induce an emf that opposes the flow of current to the transformer. The reactance of the primary winding, X_1 can be obtained by dividing self induced emf E_{L1} by the primary current I_1 *i.e.*

$$X_1 = \frac{E_{L1}}{I_1} = \frac{2\pi f L_1 I_1}{I_1} = 2\pi f L_1$$

Similarly secondary leakage flux ϕ_{L2} is set up by secondary ampere-turns and is proportional to secondary current I_2. On no load there is no current in secondary winding and, therefore, no leakage flux exists across the secondary winding on no load. On load leakage flux ϕ_{L2} in phase with secondary current I_2 and produces self-induced emf $E_{L2} = 2\pi f L_2 I_2$ in the secondary winding where L_2 is self-inductance of secondary winding due to leakage flux ϕ_{L2}. This is also a reactance voltage, and the component that balances it leads the



An Equivalent Diagram of Actual Transformer
Fig. 6.12

secondary current by 90°. The secondary reactance X_2 opposes the current flowing out of the transformer and can be obtained by dividing self induced emf in secondary winding, E_{L2} by the secondary current I_2 *i.e.*

$$X_2 = \frac{E_{L2}}{I_2} = \frac{2\pi f L_2 I_2}{I_2} = 2\pi f L_2$$

The effect of magnetic leakage is, thus to produce in their respective windings emfs of self-inductance which are proportional to the current, and are, therefore, equivalent in effect to the addition of an inductive coil in series with each winding, the reactance of which is called the leakage reactance.

A transformer with magnetic leakage and winding resistance is equivalent to an ideal transformer (having no resistance and leakage reactance) having inductive and resistive coils connected in series with each windinng as shown in fig 6.12.

6.10. PHASOR DIAGRAM OF ACTUAL TRANSFORMER ON LOAD

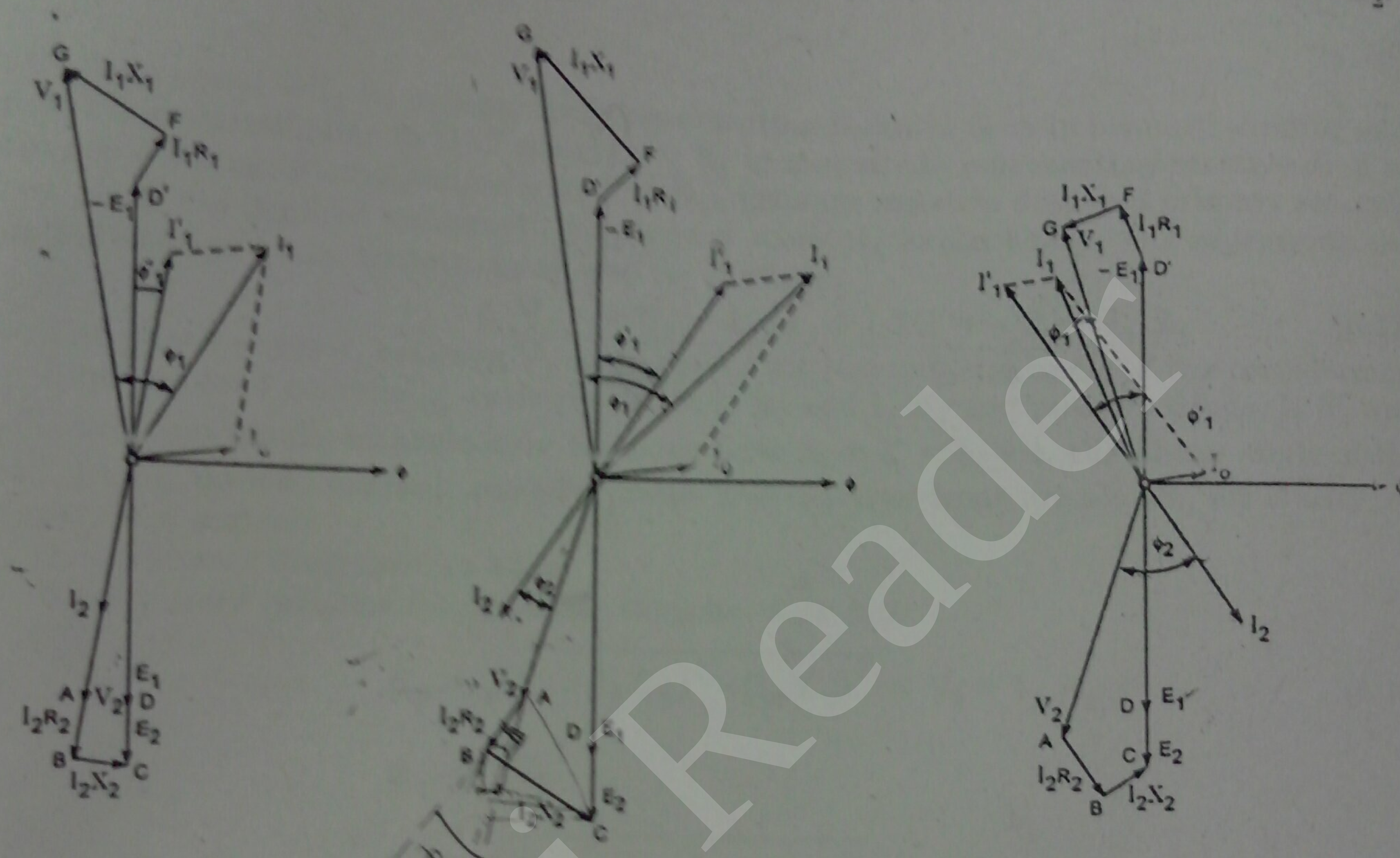
Consider a transformer shown in fig 6.12 having primary and secondary windings of resistances R_1 and R_2 and reactances X_1 and X_2 respectively. The impedance of primary winding is given by $Z_1 = R_1 + j X_1$ and impedance of secondary winding is given by $Z_2 = R_2 + j X_2$.

The phasor diagrams of above transformer on (*i*) pure resistive, (*ii*) resistive-inductive, and (*iii*) resistive-capacitive loads are shown in fig 6.13 (*a*), (*b*) and (*c*) respectively.

Draw *OA* representing secondary terminal voltage V_2 and OI_2 representing secondary current I_2 in phase as well as magnitude. Since voltage drops due to secondary winding resistance and reactance are $I_2 R_2$ in phase with current I_2 and $I_2 X_2$ leading current I_2 by

∴ I_0 is very nearly in phase with ϕ being $I_m >> I_e$ [Magnetizing current > loss current].

$\frac{\pi}{2}$ respectively, so draw AB parallel to OI_2 and equal to $I_2 R_2$ in magnitude representing resistive drop in secondary winding and draw BC perpendicular to AB and equal to $I_2 X_2$ in magnitude representing reactive drop of secondary winding. Since phasor sum of terminal voltage V_2, secondary resistive drop $I_2 R_2$ and secondary reactive drop $I_2 X_2$ is equal to induced emf E_2 in secondary windng so phasor OC represents secondary induced emf E_2. Hence we have



Phasor Diagram of Actual Transformer

Fig. 6.13

$$E_2 = V_2 + I_2 (R_2 + j X_2) = V_2 + I_2 Z_2 \qquad ...(6.10)$$

The induced emf E_1 in primary winding is in phase with E_2 and equal to $\frac{N_1}{N_2} E_2$ in magnitude, so take $OD = \frac{N_1}{N_2} OC$ representing E_1. Produce DO to D′ taking OD′ = OD hence representing $(-E_1)$.

The induced primary current I_1' is equal to $- I_2 \frac{N_2}{N_1}$ so draw OI_1' equal to $OI_2 \times \frac{N_2}{N_1}$ by producing line I_2 O. Draw line OI_0 representing no-load current in magnitude as well as in phase. The phasor sum of induced primary current I_1' and no-load current I_0 gives primary current represented by phasor OI_1 in fig 6.13.

Since voltage drop due to primary winding resistance and reactance are $I_1 R_1$ in phase with primary current I_1 and $I_1 X_1$ leading current I_1 by $\frac{\pi}{2}$ respectively, so draw D′F parallel

V2 will always lag E2 as the circuit is inductive

to OI_1 and equal to $I_1 R_1$ in magnitude representing resistive drop in primary winding and draw FG perpendicular to D' F equal to $I_1 X_1$ in magnitude representing reactive drop in primary winding. As the phasor sum of $(-E_1)$, primary resistive drop and primary reactive drop gives the applied voltage V_1 to primary winding, hence phasor OG represents the applied voltage V_1 in magnitude as well as in phase.

$$i.e. \quad V_1 = -E_1 + I_1 (R_1 + jX_1) = -E_1 + I_1 Z_1 \quad ...(6.11)$$

The phase angle ϕ_1 between V_1 and I_1 gives the power factor angle of the transformer.

Since no-load current I_0, resistive drops $I_1 R_1$ and $I_2 R_2$ and reactive drops, $I_1 X_1$ and $I_2 X_2$ are very small, so neglecting these we have $\phi_2 = \phi_1' = \phi_1 = \phi$, the phase angle of the load. In fig. no-load current, resistive drops and reactive drops are shown, for clarity, on exaggerated scales.

From phasor diagrams we have

(a) *For pure resistive load [phasor diagram 6.13 (a)]*

$$E_2 = \sqrt{(V_2 + I_2 R_2)^2 + (I_2 X_2)^2} \approx V_2 + I_2 R_2 \quad ...(6.12)$$

$$E_1 = \frac{E_2}{K}$$

$$\text{and } V_1 = \sqrt{(E_1 + I_1 R_1)^2 + (I_1 X_1)^2} \approx E_1 + I_1 R_1 \quad ...(6.13)$$

(b) *For resistive-inductive load [phasor diagram 6.13 (b)]*

$$E_2 = \sqrt{(V_2 + I_2 R_2 \cos\phi + I_2 X_2 \sin\phi)^2 + (I_2 X_2 \cos\phi - I_2 R_2 \sin\phi)^2}$$

$$\approx V_2 + I_2 R_2 \cos\phi + I_2 X_2 \sin\phi \quad ...(6.14)$$

$$E_1 = \frac{E_2}{K}$$

$$\text{and } V_1 = \sqrt{(E_1 + I_1 R_1 \cos\phi + I_1 X_1 \sin\phi)^2 + (I_1 X_1 \cos\phi - I_1 R_1 \sin\phi)^2}$$

$$\approx E_1 + I_1 R_1 \cos\phi + I_1 X_1 \sin\phi \quad ...(6.15)$$

(c) *For resistive-capacitive load [phasor diagam 6.13 (c)]*

$$E_2 = \sqrt{(V_2 + I_2 R_2 \cos\phi - I_2 X_2 \sin\phi)^2 + (I_2 X_2 \cos\phi + I_2 R_2 \sin\phi)^2}$$

$$\approx V_2 + I_2 R_2 \cos\phi - I_2 X_2 \sin\phi \quad ...(6.16)$$

$$E_1 = \frac{E_2}{K}$$

$$\text{and } V_1 = \sqrt{(E_1 + I_1 R_1 \cos\phi - I_1 X_1 \sin\phi)^2 + (I_1 X_1 \cos\phi + I_1 R_1 \sin\phi)^2}$$

$$= E_1 + I_1 R_1 \cos\phi - I_1 X_1 \sin\phi \quad ...(6.17)$$

Example 6.10. A 230/460 V transformer has a primary resistance of 0.2 Ω and a reactance of 0.5 Ω and the corresponding values for the secondary are 0.75 Ω and 1.8 Ω respectively. Find the secondary terminal voltage when supplying (*a*) 10 A at 0.8 pf lagging (*b*) 10 A at pf 0.8 leading.

[Allahabad Univ. Elec. Machines-I, 1993]

Solution: Transformation ratio, $K = \frac{V_2}{V_1} = \frac{460}{230} = 2$

(i) **When Load is 10 A at 0.8 pf lagging** i.e. when

$I_2 = 10$ A and $\cos\phi = 0.8$ lagging.

From equation (6.15)

Primary current $I_1 = K I_2 = 2 \times 10 = 20$ A

Primary induced emf, $E_1 = V_1 - (I_1 R_1 \cos\phi + I_1 X_1 \sin\phi)$

$= 230 - (20 \times 0.2 \times 0.8 + 20 \times 0.5 \times 0.6) = 220.8$ V

From equation (6.14)

Secondary induced emf, $E_2 = K E_1 = 2 \times 220.8 = 441.6$ V

Secondary terminal voltage, $V_2 = E_2 - (I_2 R_2 \cos\phi + I_2 X_2 \sin\phi)$

$= 441.6 - (10 \times 0.75 \times 0.8 + 10 \times 1.8 \times 0.6) = 424.8$ V Ans.

(ii) **When load is 10 A at 0.8 pf leading**

From equation (6.17)

Primary induced emf, $E_1 = V_1 - (I_1 R_1 \cos\phi - I_1 X_1 \sin\phi)$

$= 230 - (20 \times 0.2 \times 0.8 - 20 \times 0.5 \times 0.6) = 232.8$ V

and from equation (6.16)

Secondary terminal voltage, $V_2 = K E_1 - (I_2 R_2 \cos\phi - I_2 X_2 \sin\phi)$

$= 2 \times 232.8 - (10 \times 0.75 \times 0.8 - 10 \times 1.8 \times 0.6) = 470.4$ V Ans.

6.11. EQUIVALENT RESISTANCE AND REACTANCE

The two independent circuits of a transformer can be resolved into an equivalent circuit to make the calculations simple.

Let resistances and reactances of primary and secondary windings be R_1 and R_2 and X_1 and X_2 ohms respectively and let transformation ratio be K.

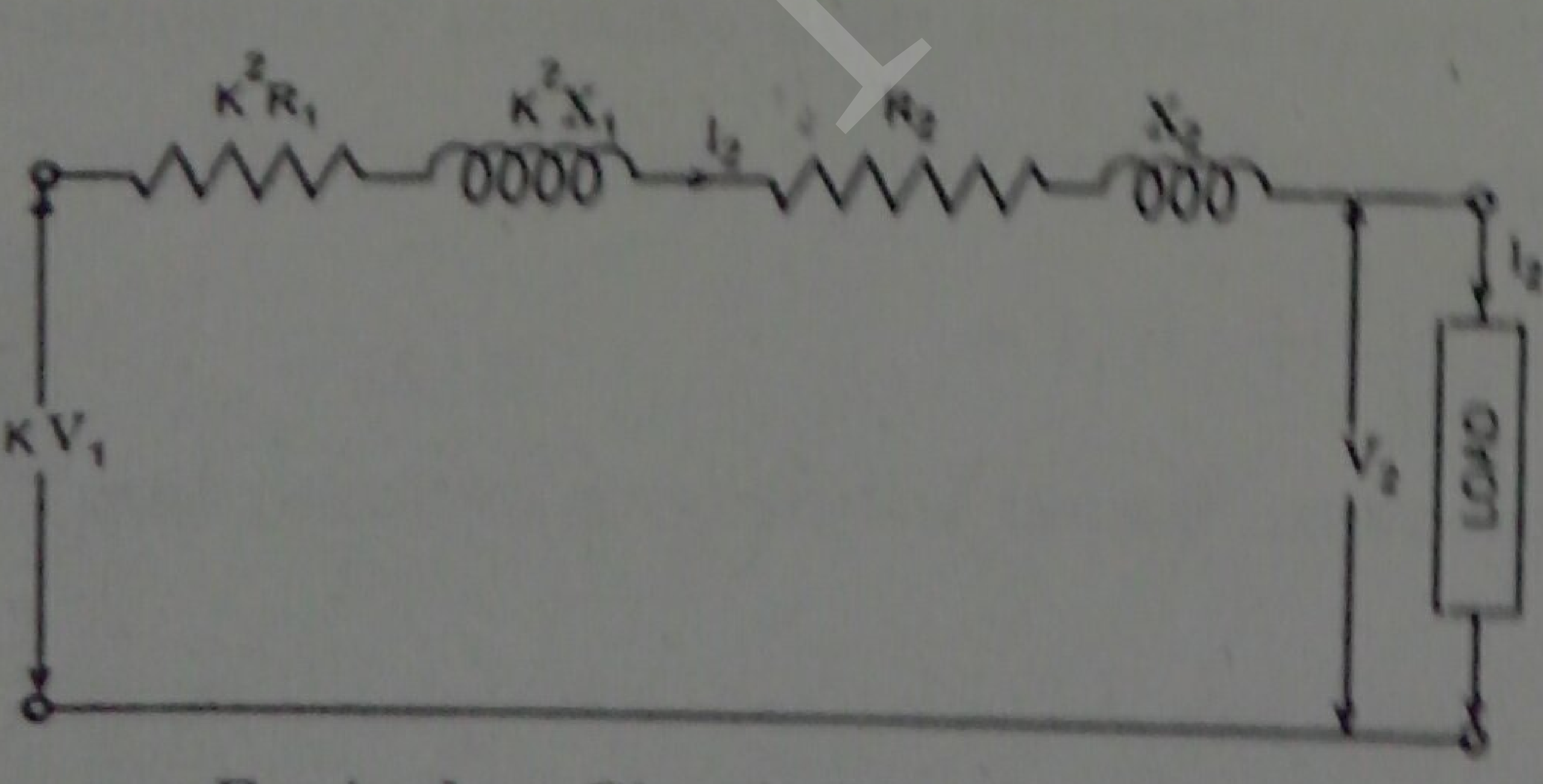
Resistive drop in secondary winding $= I_2 R_2$

Reactive drop in secondary winding $= I_2 X_2$

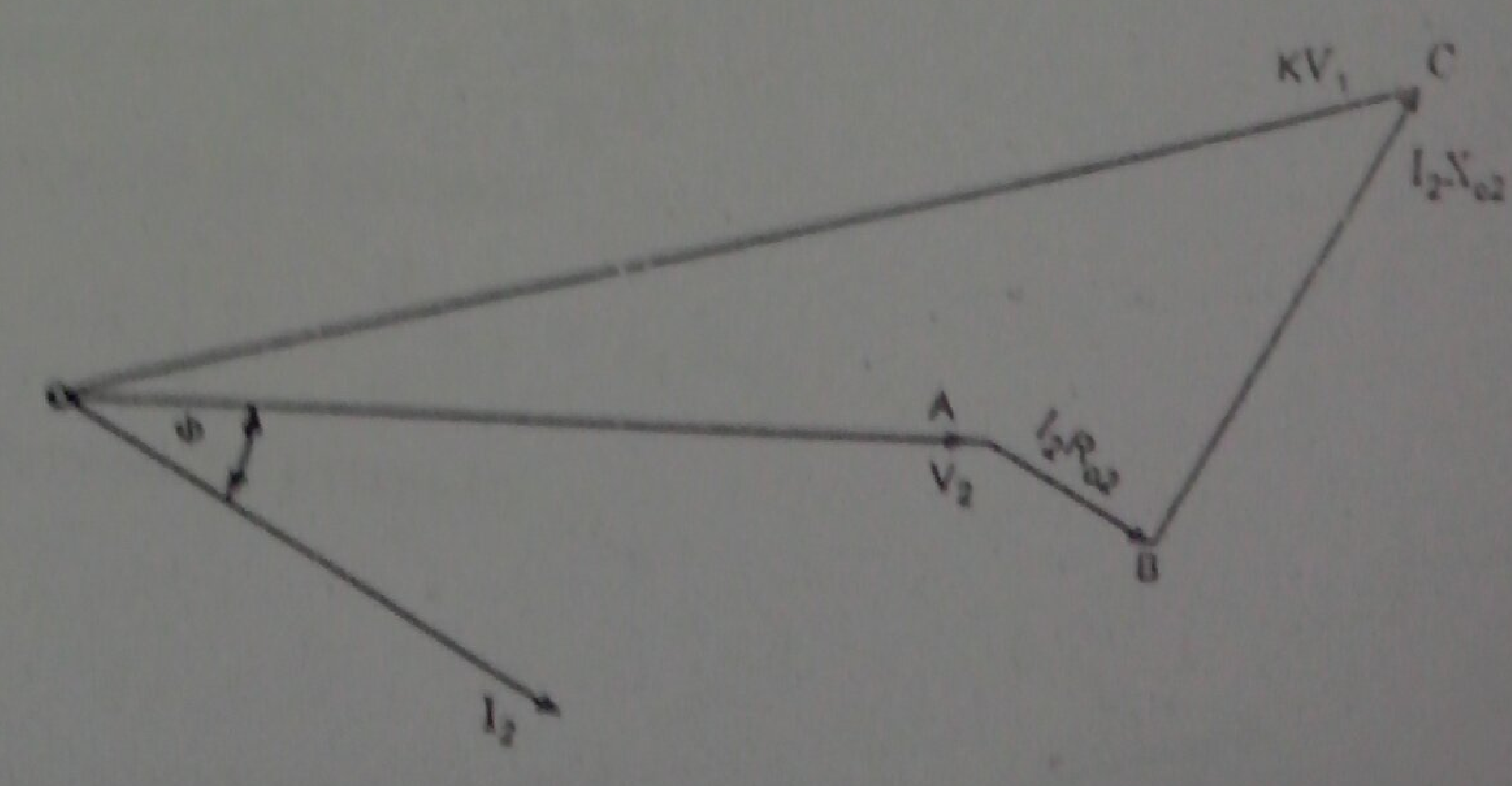
Resistive drop in primary winding $= I_1 R_1$

Reactive drop in primary winding $= I_1 X_1$

Referred To Secondary Side. Since transformation ratio is K, so primary resistive and reactive drops as referred to secondary will be K times, *i.e.* $K I_1 R_1$ and $K I_1 X_1$ respectively. If I_1 is substituted equal to KI_2 then we have primary resistive and reactive drops as referred



Equivalent Circuit Of a Transformer Referred To Secondary
Fig. 6.14



Phasor Diagram
Fig. 6.15

to secondary equal to $K^2 I_2 R_1$ and $K^2 I_2 X_1$ respectively.

Total resistive drop in a transformer $= K^2 I_2 R_1 + I_2 R_2 = I_2 (K^2 R_1 + R_2) = I_2 R_{02}$

Total reactive drop in a transformer $= K^2 I_2 X_1 + I_2 X_2 = I_2 (K^2 X_1 + X_2) = I_2 X_{02}$

The term $(K^2 R_1 + R_2)$ and $(K^2 X_1 + X_2)$ represent the equivalent resistance and reactance respectively of the transformer referred to secondary and let these be represented by R_{02} and X_{02} respectively. Equivalent circuit referred to secondary has been shown in fig 6.14.

From phasor diagram (fig 6.15)

$$K V_1 = \sqrt{(V_2 + I_2 R_{02} \cos\phi + I_2 X_{02} \sin\phi)^2 + (I_2 X_{02} \cos\phi - I_2 R_{02} \sin\phi)^2}$$

where V_2 is secondary terminal voltage, I_2 is secondary current lagging behind the terminal voltage V_2 by ϕ.

Since term $(I_2 X_{02} \cos\phi - I_2 R_{02} \sin\phi)$ is very small as compared to the term $(V_2 + I_2 R_{02} \cos\phi + I_2 X_{02} \sin\phi)$, so neglecting the former we have

$$K V_1 = V_2 + I_2 R_{02} \cos\phi + I_2 X_{02} \sin\phi$$

$$\text{or } V_2 = KV_1 - I_2 R_{02} \cos\phi - I_2 X_{02} \sin\phi \qquad \ldots (6.18)$$

where V_1 is applied voltage to primary winding.

If load is pure resistive, $\phi = 0$ and $V_2 = KV_1 - I_2 R_{02}$ ...(6.19)

If load is capacitive then ϕ should be taken as –ve hence we have

$$V_2 = KV_1 - I_2 R_{02} \cos\phi + I_2 X_{02} \sin\phi \qquad \ldots(6.20)$$

Referred To Primary Side. Secondary resistive drop referred to primary

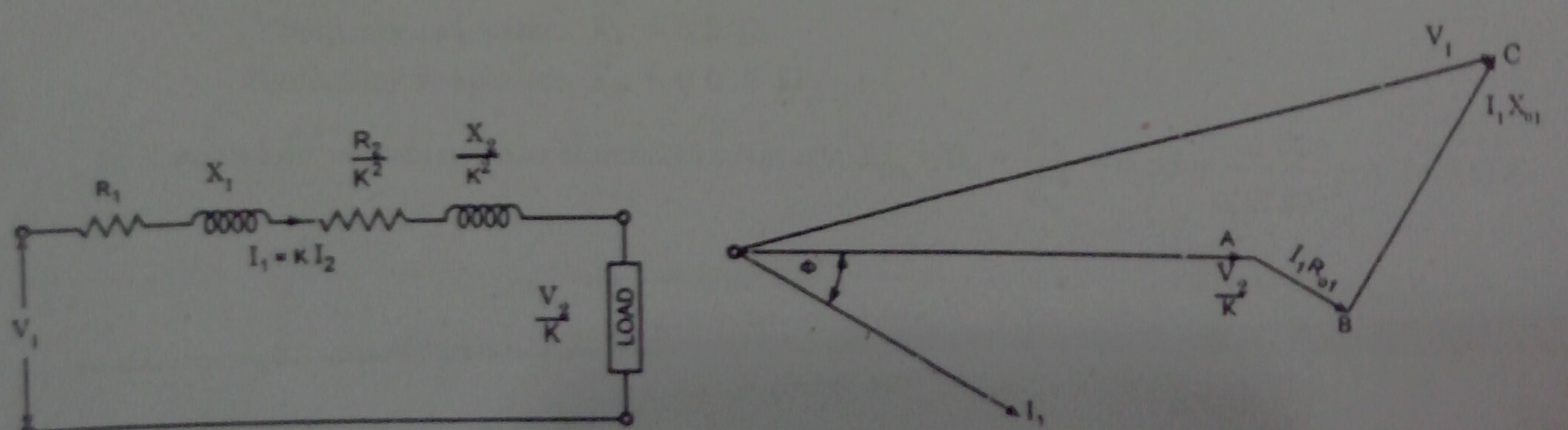
$$= \frac{I_2 R_2}{K} = \frac{I_1 R_2}{K^2} \qquad \text{since } I_2 = \frac{I_1}{K}$$

Secondary reactive drop referred to primary $= \dfrac{I_2 X_2}{K} = \dfrac{I_1 X_2}{K^2}$

Total resistive drop in the transformer referred to primary

$$= I_1 R_1 + \frac{I_1 R_2}{K^2} = I_1 \left(R_1 + \frac{R_2}{K^2}\right) = I_1 R_{01}$$

Total reactive drop in the transformer referred to primary



(a) Equivalent Circuit of a Transformer Referred To Primary

(b) Phasor Diagram

Fig. 6.16

$$= I_1 X_1 + \frac{I_1 X_2}{K^2} = I_1 \left(X_1 + \frac{X_2}{K^2} \right) = I_1 X_{01}$$

The terms $\left(R_1 + \frac{R_2}{K^2} \right)$ and $\left(X_1 + \frac{X_2}{K^2} \right)$ represent the total resistance and reactance of the transformer referred to primary respectively. Let these be represented by R_{01} and X_{01} respectively.

Equivalent circuit referred to primary is shown in fig. 6.16 (*a*)

(*a*) *Equivalent Circuit of Transformer Referred To Primary*

From phasor diagram shown in fig 6.16 (*b*) we have

$$V_1 = \sqrt{\left(\frac{V_2}{K} + I_1 R_{01} \cos\phi + I_1 X_{01} \sin\phi \right)^2 + (I_1 X_{01} \cos\phi - I_1 R_{01} \sin\phi)^2}$$

Since term $(I_1 X_{01} \cos\phi - I_1 R_{01} \sin\phi)$ is very small as compared to the term $\left(\frac{V_2}{K} + I_1 R_{01} \cos\phi + I_1 X_{01} \sin\phi \right)$ so neglecting the former we have

$$V_1 = \frac{V_2}{K} + I_1 R_{01} \cos\phi + I_1 X_{01} \sin\phi \quad \text{... (6.21)}$$

If load is pure resistive *i.e.* $\phi = 0$ then $V_1 = \frac{V_2}{K} + I_1 R_{01}$ ... (6.22)

If load is capacitive then ϕ should be taken as –ve so we have

$$V_1 = \frac{V_2}{K} + I_1 R_{01} \cos\phi - I_1 X_{01} \sin\phi \quad \text{... (6.23)}$$

Example 6.11. A 50 kva, 4,400/220 V transformer has R_1 = 3.45 Ω, R_2 = 0.009 Ω. The values of reactances are X_1 = 5.2 Ω and X_2 = 0.015 Ω. Calculate for the transformer (*i*) equivalent resistance as referred to primary (*ii*) equivalent resistance referred to secondary (*iii*) equivalent reactance as referred to both primary and secondary (*iv*) equivalent impedance as referred to both primary and secondary (*v*) total copper loss, first using individual resistances of the two windings and secondly using equivalent resistances as referred to each side.

[*Nagpur Univ. Elec. Engineering-I, 1993*]

Solution: Transformation ratio, $K = \frac{V_2}{V_1} = \frac{220}{4,400} = \frac{1}{20}$

Full–load secondary current, $I_2 = \frac{50 \times 1,000}{220} = 227.3$ A

Full–load primary current, $I_1 \doteq KI_2 = \frac{1}{20} \times 227.3 = 11.36$ A

Primary resistance, $R_1 = 3.45\ \Omega$

Secondary resistance, $R_2 = 0.009\ \Omega$

Primary reactance, $X_1 = 5.2\ \Omega$

Secondary reactance, $X_2 = 0.015\ \Omega$

(*i*) Equivalent resistance as referred to primary, $R_{01} = R_1 + \frac{R_2}{K^2} = 3.45 + \frac{0.009}{(1/20)^2} = 7.05\ \Omega$ Ans.

(*ii*) Equivalent resistance as referred to secondary,

$$R_{02} = R_2 + K^2 R_1 = 0.009 + \left(\frac{1}{20}\right)^2 \times 3.45 = 0.017625\ \Omega \text{ Ans.}$$

$$\text{or } R_{02} = K^2 R_{01} = \left(\frac{1}{20}\right)^2 \times 7.05 = 0.017625\ \Omega, \text{ as above}$$

(*iii*) Equivalent reactance as referred to primary, $X_{01} = X_1 + \frac{X_2}{K^2} = 5.2 + \frac{0.015}{(1/20)^2} = 11.2\Omega$ Ans.

Equivalent reactance as referred to secondary, $X_{02} = K^2 X_{01} = \left(\frac{1}{20}\right)^2 \times 11.2 = 0.028\ \Omega$ Ans.

(*iv*) Equivalent impedance as referred to primary,

$$Z_{01} = R_{01} + j\, X_{01} = (7.05 + j\ 11.2)\ \Omega \text{ or } 13.23\ \angle 57.81^\circ\ \Omega \text{ Ans.}$$

Equivalent impedance as referred to secondary,

$$Z_{02} = R_{02} + j\, X_{02} = (0.017625 + j\ 0.028)\ \Omega \quad \text{or } 0.0331 \angle 57.81^\circ\ \Omega \text{ Ans.}$$

(*v*) Copper loss $= I_1^2 R_1 + I_2^2 R_2 = (11.36)^2 \times 3.45 + (227.3)^2 \times 0.009$

$= 910.4$ watts Ans.

Also copper loss $= I_1^2 R_{01} = (11.36)^2 \times 7.05 = 910.4$ watts Ans.

Also copper loss $= I_2^2 R_{02} = (227.3)^2 \times 0.017625 = 910.4$ watts Ans.

} The same as expected

Example 6.12. A 10 KVA, 500/100 V transformer has the following circuit parameters referred to primary; Resistance R_{01} = 0.3 Ω; Reactance X_{01} = 5.2 Ω. When supplying power to a lagging load, the current, power and voltage measured on primary side were 20 A, 8 kw and 500 V respectively. Calculate voltage on secondary terminals under these conditions.

Solution: Primary voltage, $V_1 = 500$ V

Secondary voltage, $V_2 = 100$ V

$$\text{Transformation ratio, } K = \frac{V_2}{V_1} = \frac{100}{500} = 0.2$$

Equivalent resistance of transformer referred to secondary,

$$R_{02} = K^2 R_{01} = (0.2)^2 \times 0.3 = 0.012\ \Omega$$

Equivalent reactance of transformer referred to secondary,

$$X_{02} = K^2 X_{01} = (0.2)^2 \times 5.2 = 0.208\ \Omega$$

$$\text{Secondary current, } I_2 = \frac{I_1}{K} = \frac{20}{0.2} = 100 \text{ A}$$

$$\text{Power factor, } \cos\phi = \frac{8 \times 1{,}000}{500 \times 20} = 0.8$$

$$\text{Sin } \phi = \text{Sin } (\cos^{-1} 0.8) = 0.6$$

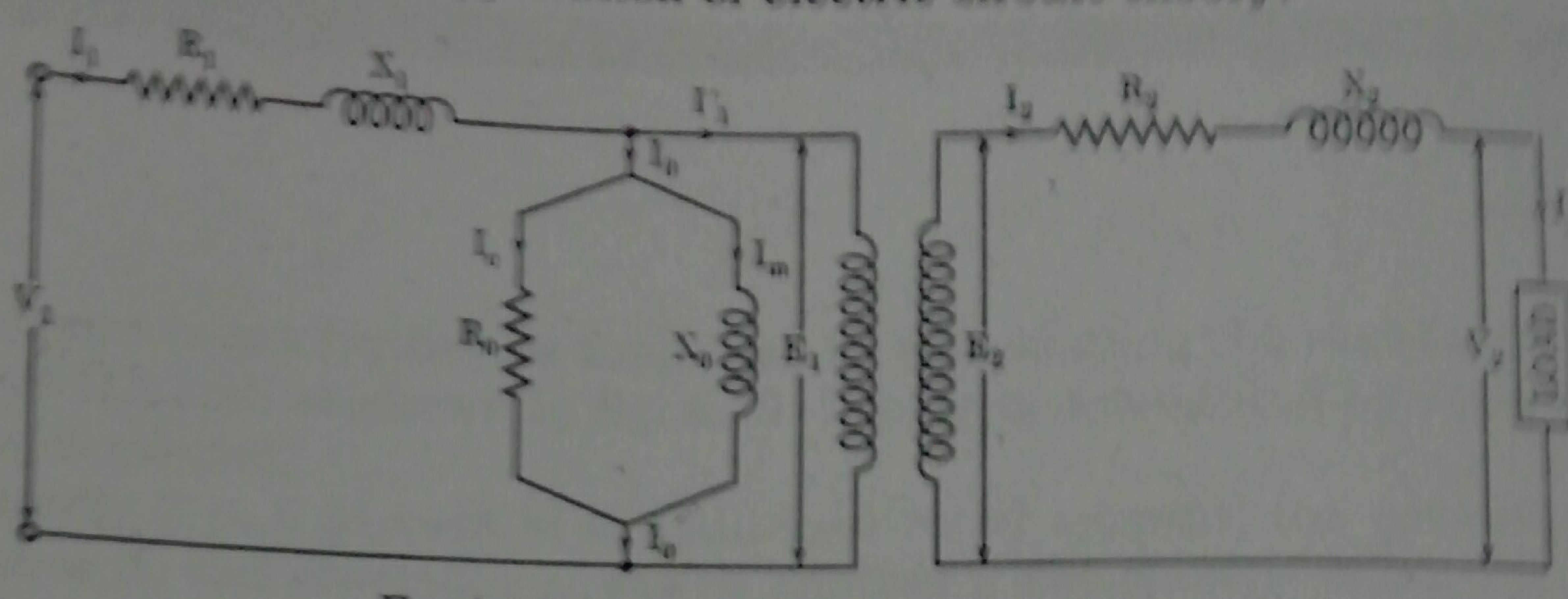
Secondary terminal voltage,

$$V_2 = KV_1 - I_2 R_{02} \cos\phi - I_2 X_{02} \sin\phi$$
$$= 0.2 \times 500 - 100 \times 0.012 \times 0.8 - 100 \times 0.208 \times 0.6$$
$$= 86.56 \text{ V Ans.}$$

6.12. EQUIVALENT CIRCUIT OF A TRANSFORMER

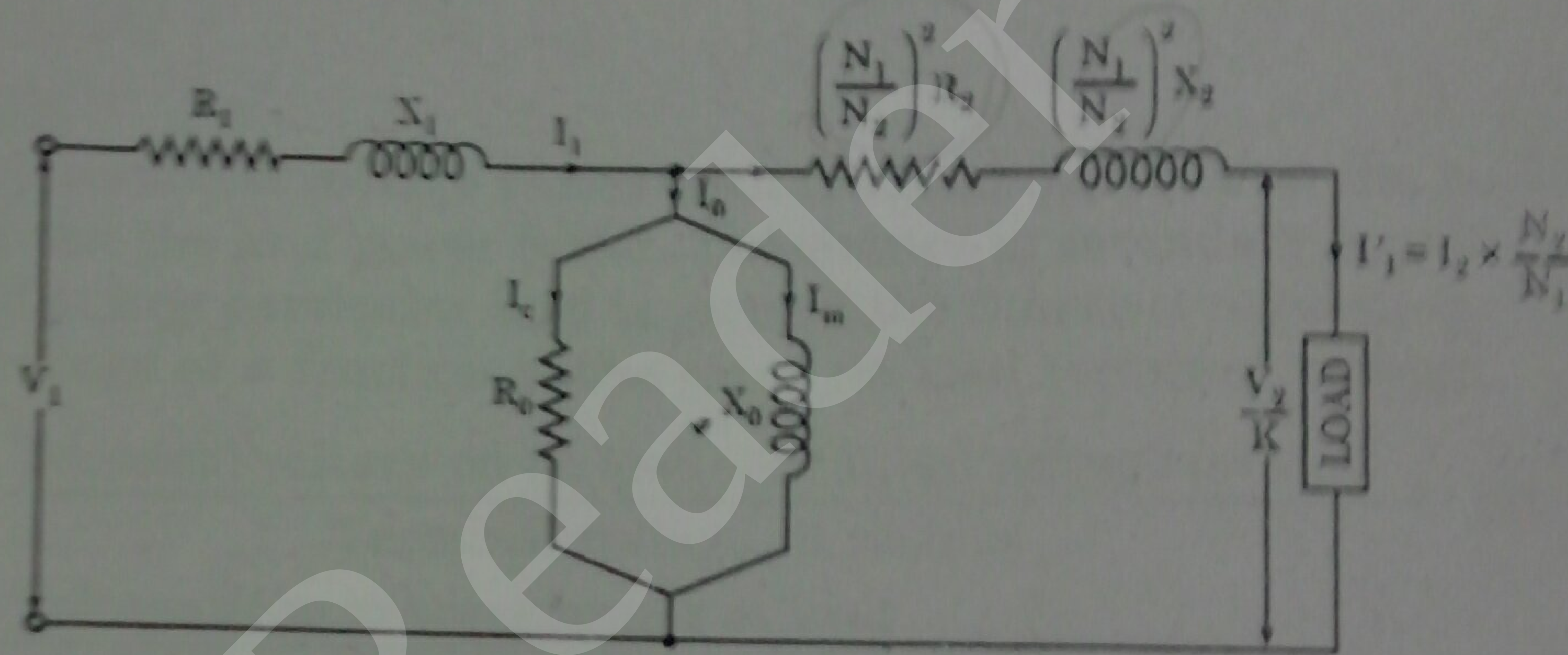
The equivalent circuit of any device can be quite helpful in predetermination of the behaviour of the device under various conditions of operation and it can be drawn if the equations describing its behaviour are known. If any electrical device is to be analysed and investigated further for suitable modifications, its appropriate equivalent circuit is necessary. The equivalent circuit for electromagnetic devices consists of a combination of resistances, inductances, capacitances, voltage etc. Such an equivalent circuit (or circuit model) can, therefore, be analysed and studied easily by the direct application of electric circuit theory.

As stated above, equivalent circuit is simply a circuit representation of the equations describing the performance of the device. Equations (6.10) and (6.11) describe the behaviour of the transformer under load and are helpful in arriving at the transformer equivalent circuit.



Equivalent Circuit of a Transformer
Fig. 6.17

Equivalent circuit of a transformer having transformation ratio $K = \frac{E_2}{E_1}$ is shown in fig 6.17.



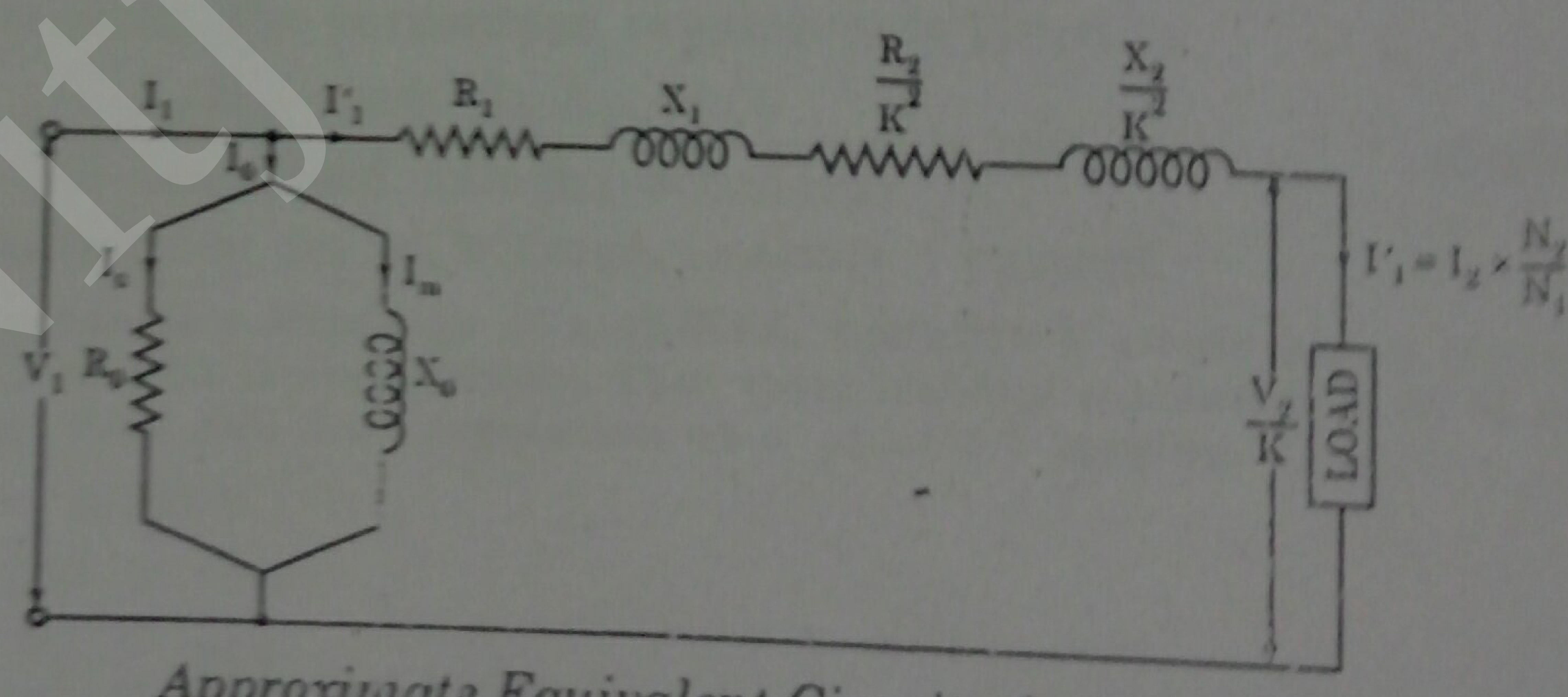
Equivalent Circuit of a Transformer With All Secondary Impedances Transferred To Primary Side
Fig. 6.18

The induced emf in primary winding E_1 is primary applied voltage V_1 less primary voltage drop. This voltage causes iron loss current I_0 and magnetising current I_m and we can, therefore, represent these two components of no-load current by the current drawn by a non-inductive resistance R_0 and pure reactance X_0 having the voltage E_1 (or V_1—primary voltage drop) applied across them, as shown in fig. 6.17.

Secondary current,

$$I_2 = \frac{I'_1}{K} = \frac{I_1 - I_0}{K}$$

Terminal voltage V_2 across load is induced emf E_2 in secondary winding less voltage drop in secondary winding.



Approximate Equivalent Circuit of a Transformer
Fig. 6.19

The equivalent circuit can be simplified by transferring the voltage, current and impedance to the primary side. After transferring the secondary voltage, current and impedance to primary side equivalent circuit is reduced to that shown in fig. 6.18.

The equivalent circuit diagram can further be simplified by transferring the resistance R_0 and reactance X_0 towards left end, as shown in fig. 6.19. The error introduced by doing so is very small and can be neglected.

No-load current I_0 is hardly 3 to 5 percent of the full-load rated current, the parallel branch consisting of resistance R_0 and reactance X_0 can be omitted without introducing any appreciable error in the behaviour of the transformer under loaded condition. Such a circuit is shown in fig 6.16 (*a*). The equivalent circuit referred to secondary side (neglecting no-load current I_0) is illustrated in fig 6.14.

6.13. VOLTAGE REGULATION

The way in which the secondary terminal voltage varies with the load depends on the load current, the internal impedance and the load power factor. The change in secondary terminal voltage from no-load to full load at any particular load is termed the *inherent* regulation. It is usually expressed as a percentage or a fraction of the rated no-load terminal voltage.

$$\text{i.e. Percentage regulation} = \frac{\text{Terminal voltage on no load} - \text{terminal voltage on load}}{\text{Terminal voltage on no load}} \times 100$$

$$= \frac{\text{Voltage drop in transformer at load}}{\text{No-load rated voltage (secondary)}} \times 100$$

$$= \frac{I_2 R_{02} \cos\phi + I_2 X_{02} \sin\phi}{\text{No-load rated voltage (secondary)}} \times 100 \qquad \text{...(6.24)}$$

When the power factor is leading, the percentage regulation is given by

$$\%\ \text{regulation} = \frac{I_2 R_{02} \cos\phi - I_2 X_{02} \sin\phi}{\text{No-load rated voltage (secondary)}} \times 100 \qquad \text{...(6.25)}$$

Voltage regulation of a transformer, on an average, is about 4 percent.

Example 6.13. The primary and secondary windings of a 40 KVA, 6,600/250 V single phase transformer have resistances of 10 Ω and 0.02 Ω respectively. The total leakage reactance is 35 Ω as referred to the primary winding. Find full-load regulation at a pf of 0.8 lagging.

Solution : Primary voltage, $V_1 = 6{,}600$ V

Secondary voltage $V_2 = 250$ V

$$\text{Transformation ratio, } K = \frac{V_2}{V_1} = \frac{250}{6{,}600} = 0.03788$$

Equivalent resistance of transformer referred to secondary,

$$R_{02} = K^2 R_1 + R_2 = (0.03788)^2 \times 10 + 0.02 = 0.03435\ \Omega$$

Equivalent leakage reactance of transformer referred to secondary,

$$X_{02} = K^2 X_{01} = (0.03788)^2 \times 35 = 0.05022\ \Omega$$

$$\text{Secondary rated current, } I_2 = \frac{\text{Rated KVA} \times 1{,}000}{V_2} = \frac{40 \times 1{,}000}{250} = 160\ \text{A}$$

$$\text{Power factor, } \cos\phi = 0.8 \text{ and } \sin\phi = \sqrt{1-0.8^2} = 0.6$$

$$\text{Full load regulation} = \frac{I_2 R_{02} \cos\phi + I_2 X_{02} \sin\phi}{E_2} \times 100$$

$$= \frac{160 \times 0.03435 \times 0.8 + 160 \times 0.05022 \times 0.6}{250} \times 100 = 3.687\%\ \text{Ans.}$$

Example 6.14. A single phase transformer on full load has an impedance drop of 20 V and resistance drop of 10 V. Calculate the value of power factor when its regulation will be zero.

Solution : Regulation of a transformer is given as $\frac{I_2 R_{02} \cos \phi + I_2 X_{02} \sin \phi}{E_2}$.

Regulation will be zero when the numerator of the expression for regulation is zero

i.e. $I_2 R_{02} \cos \phi + I_2 X_{02} \sin \phi = 0$ ...(*i*)

Resistance drop, $I_2 R_{02} = 10$ V

Impedance drop, $I_2 Z_{02} = 20$ V

Reactance drop, $I_2 X_{02} = \sqrt{(I_2 Z_{02})^2 - (I_2 R_{02})^2} = \sqrt{20^2 - 10^2} = 17.32$ V

Substituting $I_2 R_{02} = 10$ V and $I_2 X_{02} = 17.32$ V in equation (*i*) we have

$10 \cos \phi + 17.32 \sin \phi = 0$

or phase angle, $\phi = \text{Tan}^{-1} \frac{-10}{17.32} = -30°$ or $30°$ (leading)

Power factor $= \text{Cos } \phi = \text{Cos } 30° = 0.866$ (leading) **Ans.**

Example 6.15. Calculate the regulation of a transformer in which ohmic drop is 1% and reactance drop 5 % of the voltage at the following power factors (*i*) 0.8 lagging (*ii*) 0.8 leading.

Solution:

$$\text{Percentage resistive drop} = \frac{I_2 R_{02}}{E_2} \times 100 = 1$$

$$\text{Percentage reactive drop} = \frac{I_2 X_{02}}{E_2} \times 100 = 5$$

(*i*) When power factor $\cos \phi = 0.8$ lagging; $\sin \phi = 0.6$

$$\text{Voltage regulation} = \frac{I_2 R_{02} \cos \phi + I_2 X_{02} \sin \phi}{E_2} \times 100$$

$$= \frac{I_2 R_{02}}{E_2} \times 100 \cos \phi + \frac{I_2 X_{02}}{E_2} \times 100 \times \sin \phi$$

$$= 1 \times 0.8 + 5 \times 0.6 = 3.8 \textbf{ Ans.}$$

(*ii*) When power factor $\cos \phi = 0.8$ (leading); $\sin \phi = -0.6$

$$\text{Voltage regulation} = \frac{I_{02} R_{02}}{E_2} \times 100 \cos \phi + \frac{I_2 X_{02}}{E_2} \times 100 \sin \phi$$

$$= 1 \times 0.8 + 5 \times (-0.6) = -2.2\ \% \textbf{ Ans.}$$

6.14. TRANSFORMER LOSSES

1. Iron or Core Losses. Iron loss is caused by the alternating flux in the core and consists of hysteresis and eddy current losses.

(a) Hysteresis Loss. The core of a transformer is subjected to an alternating magnetizing force and for each cycle of emf a hysteresis loop is traced out. The hysteresis loss per second is given by the equation

$$\text{Hysteresis loss, } P_h = \eta' (B_{max})^x f V \text{ joules per second or watts} \quad ...(6.26)$$

where f is the supply frequency in Hz, V is the volume of core in cubic metres, η' is the hysteresis coefficient, B_{max} is peak value of flux density in the core and x lies between 1.5 and 2.5 depending upon the material and is often taken as 1.6.

(b) Eddy Current Loss. We have seen that whenever the flux linkage with a closed electric circuit changes, an emf is induced in the circuit and a current flows, the value of which depends on the emf around the circuit and the resistance of the circuit. It is not necessary that the circuit be a wire and that the flux passes entirely through it. If a solid block of metal is traversed by a varying flux, metallic circuits in the block itself, which are linked by the flux, will carry current. If the magnetic circuit is made up of iron and if the flux in the circuit is variable, currents will be induced by induction in the iron circuit itself. All such currents are known as *eddy currents*.

The eddy current loss is given by equation

$$P_e = K_e (B_{max})^2 f^2 t^2 V \text{ watts}$$

2. Copper or Ohmic Losses. These losses occur due to ohmic resistance of the transformer windings. If I_1 and I_2 are the primary and secondary currents respectively and R_1 and R_2 are the respective resistances of primary and secondary windings then copper losses occuring in primary and secondary windings will be $I_1^2 R_1$ and $I_2^2 R_2$ respectively. So total copper losses will be $(I_1^2 R_1 + I_2^2 R_2)$. These losses vary as the square of the load current or kva. For example if the copper losses at full load are P_c then copper losses at one-half or one-third of full load will be $\left(\frac{1}{2}\right)^2 P_c$ or $\left(\frac{1}{3}\right)^2 P_c$ i.e. $\frac{P_c}{4}$ or $\frac{P_c}{9}$ respectively.

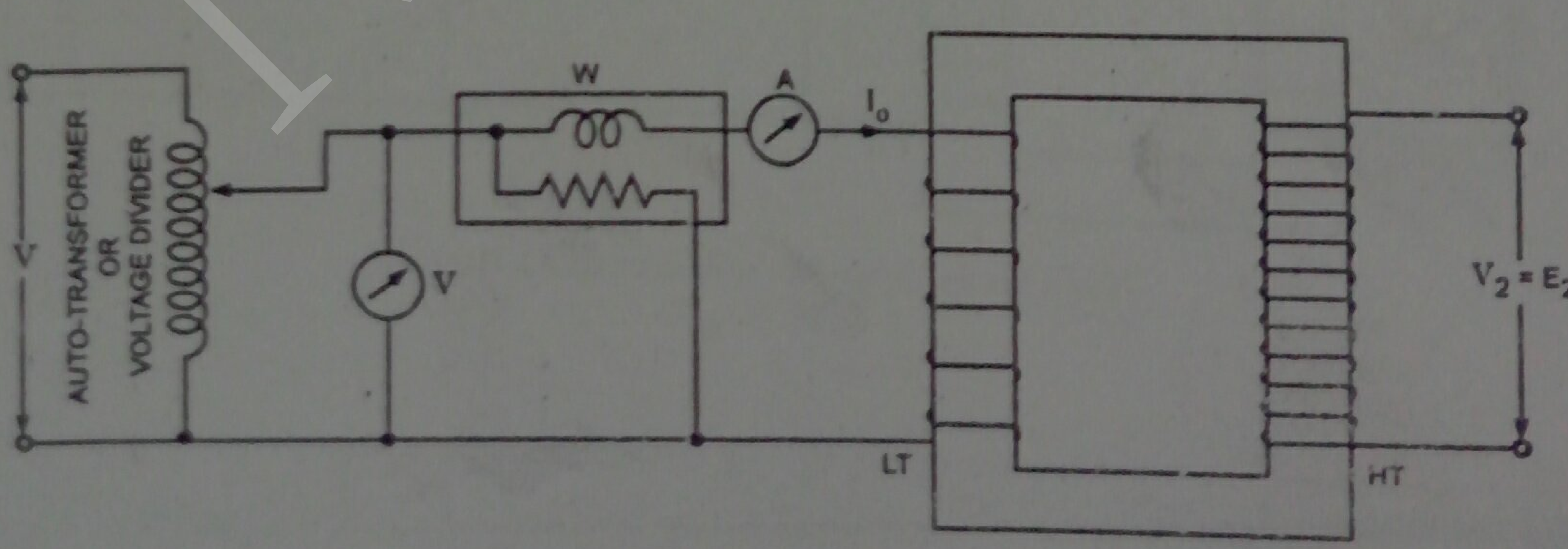
Copper losses are determined on the basis of constant equivalent resistance R_{eq} determined from the short-circuit test (refer Art 6.16) and then corrected to 75 °C (since the standard operating temperature of electrical machines is taken 75 °C).

6.15. OPEN-CIRCUIT TEST (OR NO-LOAD TEST)

The purpose of this test is to determine the core (or iron or excitation) loss, P_i and no-load current I_0 and thereby the shunt branch parameters R_0 and X_0 of the equivalent circuit.

In this test, one of the windings (usually high voltage winding) is kept open-circuited and the rated voltage at rated frequency is applied to the other winding, as shown fig 6.20. No doubt, the core loss will be the same whether the measurements are made on lv winding or hv winding so long as the rated voltage of that winding is applied to it but in case the measurements are made on hv winding, the voltage required to be applied would often be inconveniently large while the current I_0 would be inconveniently small.

Either an auto-transformer or a voltage divider (VD) is used for varying the voltage applied to the low-voltage winding. Ammeter A and wattmeter W are connected to measure no-load current I_0 and input power P_0. Voltmeter V is connected to measure the applied voltage.



Circuit Diagram For Open-Circuit Test

Fig. 6.20

Since no current flows in the open-circuited secondary the current in the primary will be merely that necessary to magnetize the core at normal voltage. Moreover, this magnetising current is a very small fraction of the full-load current (usually 3 to 10 % of full-load current) and may be neglected as far as the copper loss is concerned consequently, the test gives core loss alone practically.

With normal voltage applied to the primary, normal flux will be set up in the core and, therefore, normal iron (or core) loss will occur which are recorded by a wattmeter W.

The open-circuit test gives enough data to compute the equivalent circuit constants R_0, X_0, no-load power factor cos ϕ_0, no-load current I_0 and no-load power loss (iron loss) of a transformer.

$$\text{Iron loss, } P_i = \text{Input power on no load} = P_0 \text{ watts (say)}$$

$$\text{No-load current} = I_0 \text{ amperes}$$

$$\text{Applied voltage to primary} = V_1 \text{ volts}$$

$$\text{Angle of lag, } \phi_0 = \text{Cos}^{-1} \frac{P_0}{V_1 I_0} \quad ...(6.27)$$

$$\text{No-load current energy component, } I_e = I_0 \cos \phi_0 = \frac{P_0}{V_1} \quad ...(6.28)$$

$$\text{No-load current magnetizing component, } I_m = \sqrt{I_0^2 - I_e^2} \quad ...(6.29)$$

$$\text{Equivalent circuit parameter, } R_0 = \frac{V_1}{I_e} = \frac{V_1^2}{P_0} \quad ....(6.30)$$

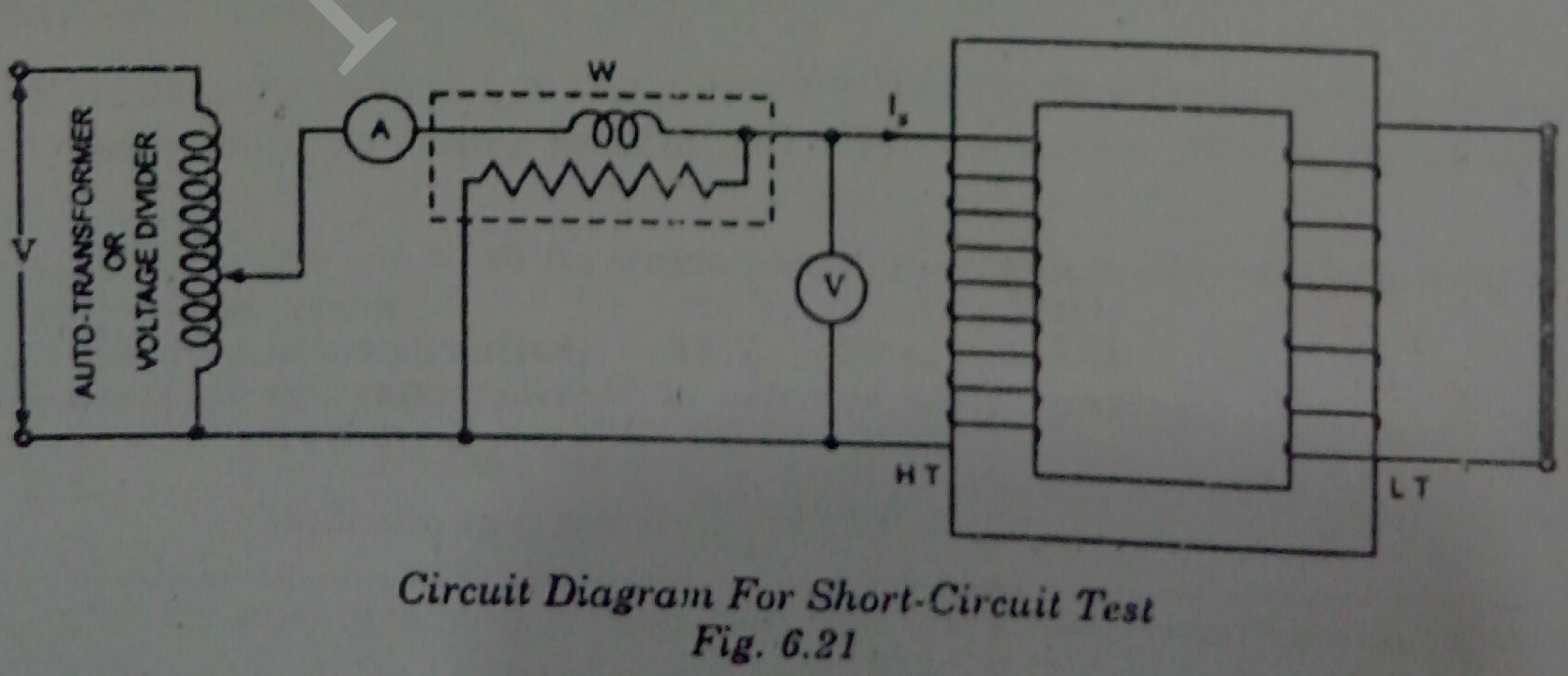
$$\text{Equivalent circuit parameter, } X_0 = \frac{V_1}{I_m} = \frac{V_1}{\sqrt{I_0^2 - I_e^2}} \quad ...(6.31)$$

Note. 1. Since no-load current I_0 is very small, therefore, pressure coils of wattmeter and the voltmeter should be connected such that the currents drawn by them do not flow through the current coils of the wattmeter and ammeter.

2. Since power factor at no-load is quite low (in the range of 0.1 – 0.2 lag) a low power factor wattmeter should be used to ensure accurate measurements.

3. The error due to power loss in ammeter can be eliminated by short-circuiting the ammeter while reading wattmeter.

4. Sometimes a high resistance voltmeter is connected across the secondary to indicate the emf induced in the secondary (hv winding). This helps in determination of transformation ratio K.



Circuit Diagram For Short-Circuit Test
Fig. 6.21

5. It must, however, be remembered that in making this test, hv side is hot and, therefore, its terminals must be properly insulated.

6.16. SHORT-CIRCUIT TEST (OR IMPEDANCE TEST)

The purpose of this test is to determine full-load copper loss and equivalent resistance and equivalent reactance referred to metering side.

In this test, the terminals of secondary winding (usually of low-voltage winding*) are short-circuited by a thick wire or strip or through an ammeter (which may serve the additional purpose of indicating secondary rated load current) and variable low voltage is applied to the primary through an auto-transformer or potential divider, as shown in fig 6.21. The transformer now becomes equivalent to a coil having an impedance equal to impedance of both the windings.

The applied voltage, V_s to the primary is gradually increased till the ammeter A indicates the full-load (rated) current of the metering side. Since applied voltage is very low (5 – 8 % of the rated voltage) so flux linking with the core is very small and, therefore, iron losses are so small that these can be neglected. Thus the power input (reading of wattmeter W) gives total copper loss at rated load, output being nil. Let the readings of voltmeter, ammeter and wattmeter be V_s, I_s and W_s respectively.

$$\text{Full-load copper loss, } P_c = I_s^2 R_{eq} = W_s \quad \text{...(6.32)}$$

$$\text{Equivalent resistance, } R_{eq} = \frac{W_s}{I_s^2} \quad \text{...(6.33)}$$

$$\text{Equivalent impedance, } Z_{eq} = \frac{V_s}{I_s} \quad \text{...(6.34)}$$

$$\text{Equivalent reactance, } X_{eq} = \sqrt{Z_{eq}^2 - R_{eq}^2} \quad \text{...(6.35)}$$

The above values are referred to the metering side (high voltage side in above case). If desired, the values could be easily determined referred to the other side, as discussed in Art 6.11.

Example 6.16. A 10 KVA, 200/ 400 V, 50 Hz single phase transformer gave the following test results.

OC test (hv winding open)	**200 V,**	**1.3 A,**	**120 W**
SC test (lv winding short-circuited)	**22 V,**	**30 A,**	**200 W**

Find parameters of equivalent circuit as referred to lv winding.

Solution :

No-load current, $I_0 = 1.3$ A

No-load input power, $P_0 = 120$ W

---

* Voltage required for the short-circuit test is about 5 per cent of the rated value. For a 200 KVA, 2,200/220 V transformer, test on high voltage side would need the voltage (to be applied) of $2{,}200 \times \frac{5}{100}$ i.e. 110 V (which is standard voltage for instrument coils) and a current of $\frac{200 \times 1{,}000}{2{,}200} = 91$ A.

If the test is conducted on low voltage side of the above transformer, the voltage needed would be $220 \times \frac{5}{100} = 11$ V and the current would be $\frac{200 \times 1{,}000}{220} = 910$ A (very high). At this low voltage, high precision would not be readily obtainable with ordinary instruments.

Thus we see that the if the measurements were made on low voltage side, the voltage needed would be inconveniently low, while the current would often be inconveniently large.

Primary (lv side) voltage, $V_1 = 200$ V

No-load power factor, $\cos\phi_0 = \frac{P_0}{V_1 I_0} = \frac{120}{1.3 \times 200} = 0.4615$ (lagging)

Energy component of no-load current, $I_e = I_0 \cos\phi_0 = 1.3 \times 0.4615 = 0.6$ A

Magnetising component of no-load current, $I_m = \sqrt{I_0^2 - I_e^2} = \sqrt{(1.3)^2 - (0.6)^2} = 1.1533$ A

Equivalent circuit parameter, $R_0 = \frac{V_1}{I_e} = \frac{200}{0.6} = 333.3\ \Omega$ Ans.

Equivalent circuit parameter, $X_0 = \frac{V_1}{I_m} = \frac{200}{1.1533} = 173.42\ \Omega$ Ans.

Equivalent impedance referred to hv side, $Z_{02} = \frac{V_s}{I_s} = \frac{22}{30} = 0.733\ \Omega$

Equivalent resistance referred to hv side, $R_{02} = \frac{W_s}{I_s^2} = \frac{200}{(30)^2} = 0.222\ \Omega$

Equivalent reactance referred to hv side, $X_{02} = \sqrt{(Z_{02})^2 - (R_{02})^2} = \sqrt{(0.733)^2 - (0.222)^2}$
$= 0.699\ \Omega$

Transformation ratio, $K = \frac{V_2}{V_1} = \frac{400}{200} = 2$

Equivalent resistance referred to lv winding, $R_{01} = \frac{R_{02}}{K^2} = \frac{0.222}{2^2} = 0.0555\ \Omega$ Ans.

Equivalent reactance referred to lv winding, $X_{01} = \frac{X_{02}}{K^2} = \frac{0.699}{2^2} = 0.175\ \Omega$ Ans.

6.17. DETERMINATION OF REGULATION OF TRANSFORMER FROM OPEN-CIRCUIT AND SHORT-CIRCUIT TESTS

Percentage regulation is given as

$$\%\text{ age regulation} = \frac{I_2 R_{02} \cos\phi \pm I_2 X_{02} \sin\phi}{\text{No-load rated secondary voltage}} \times 100$$

...refer equations (6.24) and (6.25)

Equivalent resistance R_{02} and reactance X_{02} referred to secondary can be determined from short-circuit test, as explained in Art 6.16. I_2 and $\cos\phi$ are the load current and power factor (lagging or leading) of the load, so known. No-load secondary terminal voltage is equal to emf induced in the secondary E_2.

$$\text{So percentage regulation} = \frac{I_2 R_{02} \cos\phi \pm I_2 X_{02} \sin\phi}{E_2} \times 100$$

Note: *Open-circuit test datas are not needed for determination of voltage regulation of transformer.*

Example 6.17. A short-circuit test when performed on the hv side of a 10 kva 2,000/400 V single phase transformer, gave the following data — 60 V, 4 A, 100 W

If the lv side is delivering full-load current at 0.8 pf lag and at 400 V, find the applied voltage to hv side. *[Nagpur Univ. Elec. Machines-I, 1993]*

Solution:

Equivalent impedance referred to primary, $Z_{01} = \frac{V_s}{I_s} = \frac{60}{4} = 15\ \Omega$

$\because$ Short-circuit test has been conducted on the hv (primary) side

Equivalent resistance referred to primary, $R_{01} = \frac{P_s}{I_s^2} = \frac{100}{4^2} = 6.25\ \Omega$

Equivalent reactance referred to primary, $X_{01} = \sqrt{(Z_{01})^2 - (R_{01})^2} = \sqrt{(15)^2 - (6.25)^2} = 13.64\ \Omega$

Full-load primary current, $I_1 = \frac{\text{KVA (rated)} \times 1,000}{V_1} = \frac{10 \times 1,000}{2,000} = 5\text{ A}$

Total voltage drop in transformer winding as referred to primary

$= I_1 (R_{01} \cos\phi + X_{01} \sin\phi)$

$= 5\ (6.25 \times 0.8 + 13.64 \times 0.6) = 65.9\text{ V}$

Applied voltage to the hv side = Load voltage + voltage drop

$= 2,000 + 65.9 = 2,065.9\text{ V}$ **Ans.**

6.18. TRANSFORMER EFFICIENCY

The rated capacity of a transformer is defined as the product of rated voltage and full-load (rated) current on the output side. The power output depends upon the power factor of the load.

The efficiency (η) of a transformer, like that of any other apparatus, is defined as the ratio of useful power output to the input power, the two being measured in same units (either in watts or killowatts).

i.e. Transformer efficiency, $\eta = \frac{\text{Output}}{\text{Input}} = \frac{\text{Output}}{\text{Output + losses}}$

$$= \frac{\text{Output}}{\text{Output + iron loss + copper loss}}$$

$$= 1 - \frac{\text{iron loss + copper loss}}{\text{Output + iron loss + copper loss}}$$

Now power output $= V_2 I_2 \cos\phi$

where V_2 is the secondary terminal voltage on load, I_2 is the secondary current at load and $\cos\phi$ is the power factor of the load.

Iron loss, P_i = Hysteresis loss + eddy current loss

Copper loss $= I_1^2 R_1 + I_2^2 R_2 = I_1^2 R_{01} = I_2^2 R_{02}$

6.18.1. Determination of Transformer Efficiency. The ordinary transformer has a very high efficiency (in the range of 96—99%). Hence the transformer efficiency cannot be determined with high precision by direct measurement of output and input, since the losses are of the order of only 1-4 %. The difference between the readings of output and input instruments is then so small that an instrument error as low as 0.5 % would cause an error of the order of 15 % in the losses. Further, it is inconvenient and costly to have the necessary loading devices of the correct current and voltage ratings and power factor to load the transformer. There is also a wastage of large amount of power (equal to that of power output + losses) and no information is available from such a test about the proportion of copper and iron losses.