

BLIMP 3



User's Guide

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1 Introduction

1.1 Blimp Overview

Blimp is an all-purpose data analysis and latent variable modeling program that harnesses the flexible power of Bayesian estimation in a user-friendly application that requires minimal scripting and no deep-level knowledge about Bayes. The application, which is available for macOS, Windows, and Linux, was developed with funding from Institute of Educational Sciences awards R305D150056 and R305D190002. The application began as a platform for implementing multilevel multiple imputation via fully conditional specification (Enders, Keller, & Levy, 2018), and its second release transitioned the software to a full-featured multilevel analysis package (Enders, Du, & Keller, 2020). Blimp 3 introduces wide ranging and powerful capabilities for multivariate analyses with latent variables (e.g., path models, measurement models, structural equation models), including many models not available in other software packages.

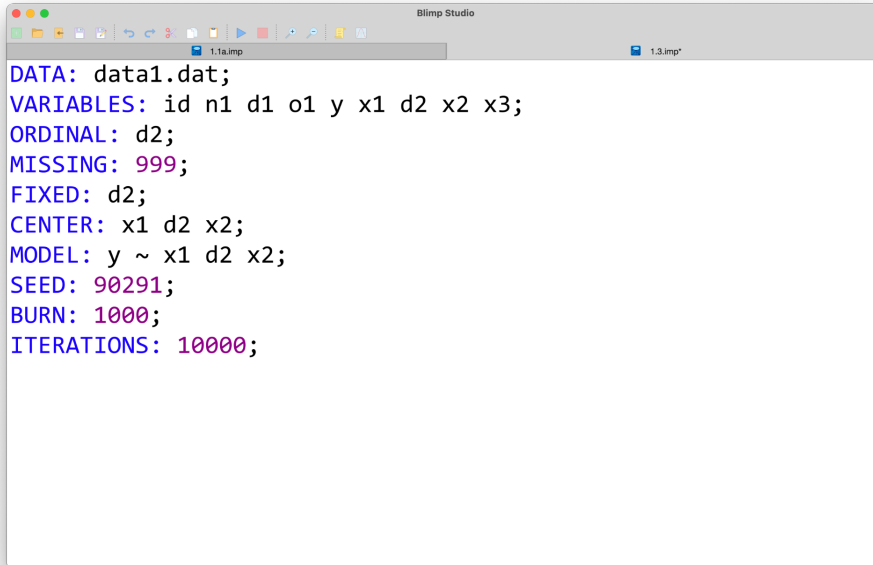
The development team's philosophy for Blimp is to bring easy Bayes estimation to the masses; the program offers some opportunities for "getting under the hood", but algorithmic tweaks and nuanced model specifications are not as customizable as they are in specialized (but less user-friendly) programs such as Stan or JAGS. To this end, Blimps implements a reasonable set of diffuse or noninformative prior distributions with a handful of "off-the-shelf" alternatives described in Bayesian texts. In line with our overarching philosophy, complex models can be specified with minimal coding by simply listing variable names in a format that resembles a regression equation (e.g., $y \sim x_1 \ x_2 \ x_3$). In most cases, Blimp automatically introduces means, variances, and

covariances (or correlations) with no additional specifications required, and the software also adds any supporting models needed for missing data handling.

Blimp's primary purpose is to provide researchers with a powerful tool for analyzing data, with or without missing values. Blimp 3 offers a commercial-grade user experience with the flexibility to estimate complex latent variable models, many of which are not available in other software packages. Models can include up to three levels with mixtures of binary, ordinal, multicategorical nominal, normal, skewed continuous variables, and count variables. Chapters 4 through 7 of this guide provides numerous examples. Separate from its data analytic core, Blimp continues to offer the fully conditional specification routines introduced in Version 1. Blimp's implementation of fully conditional specification parallels van Buuren's popular MICE program (van Buuren & Groothuis-Oudshoorn, 2011), but it uses latent response variable framework to treat categorical variables and uses latent group means to preserve multilevel data structures. Enders (2022) describes fully conditional specification with latent variables, and Chapters 4 through 6 of this guide provides illustrations.

1.2 Working in Blimp

One of the major features in Blimp 3 is a redesigned graphical user interface called Blimp Studio. The Studio application features a tabbed interface that makes it easy to work with multiple scripts and projects at the same time. The graphic below shows the Blimp Studio interface.

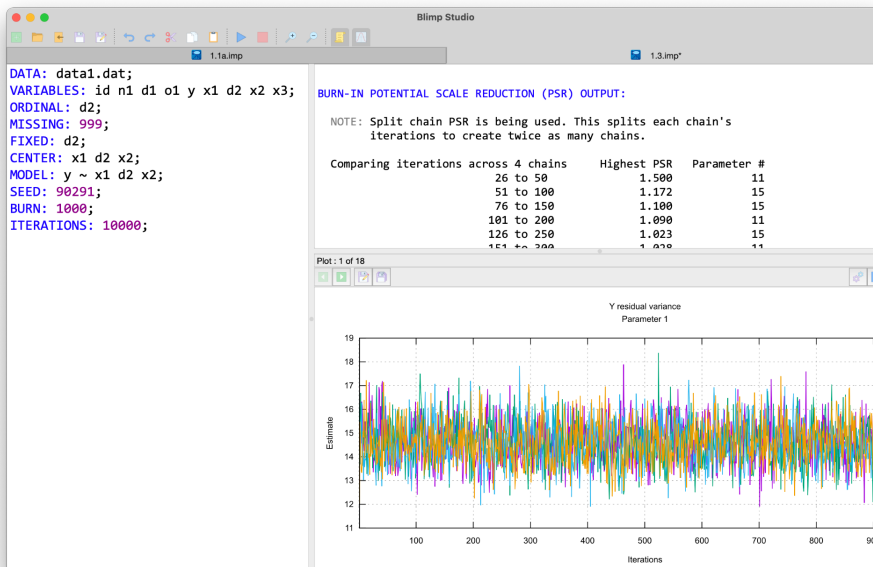


```

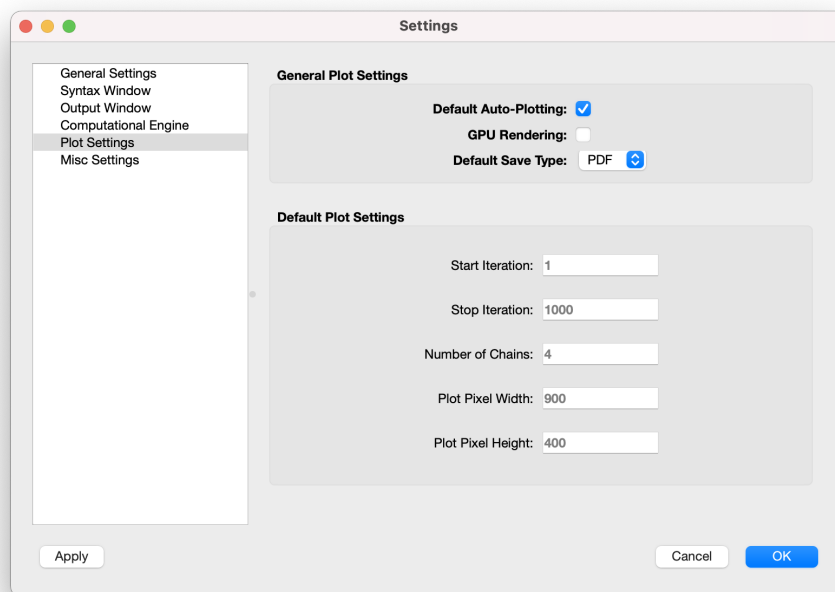
DATA: data1.dat;
VARIABLES: id n1 d1 o1 y x1 d2 x2 x3;
ORDINAL: d2;
MISSING: 999;
FIXED: d2;
CENTER: x1 d2 x2;
MODEL: y ~ x1 d2 x2;
SEED: 90291;
BURN: 1000;
ITERATIONS: 10000;

```

Clicking the blue arrow button on the toolbar executes a script and spawns a paned interface that adds an output window containing the analysis results and a plotting window displaying trace (time series) plots for every model parameter.

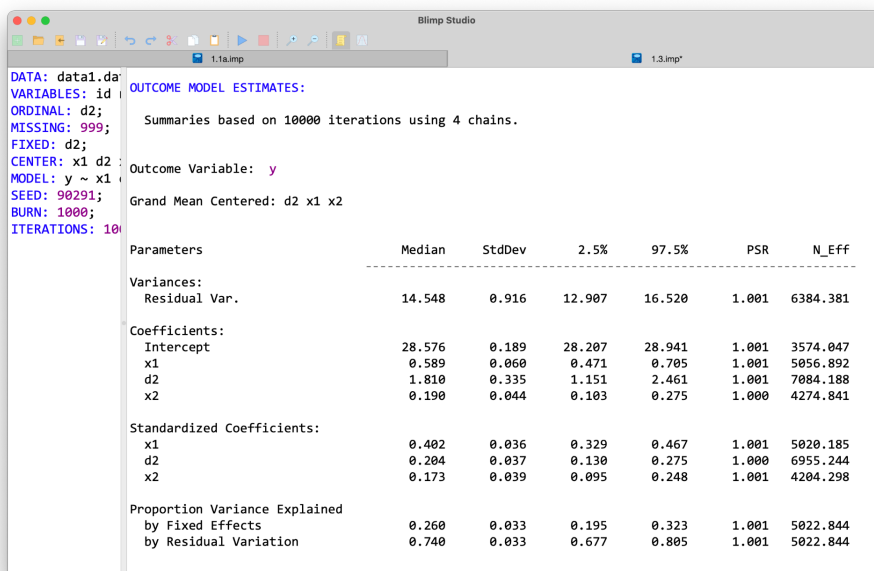


Clicking on the normal distribution icon in the toolbar hides the plotting window, which can also be disabled completely in the application's Preferences, located under the **Blimp Studio > Preferences** pull-down menu. Other visual settings such as fonts and the orientation of the paned windows can also be set in the Preferences pane, shown below.



The Blimp output includes regression model parameters for each variable, including standardized estimates and variance explained effect sizes (Rights & Sterba, 2019). Certain types of analyses spawn additional output (e.g., odds ratios in a logistic regression; transformation parameters for skewed variables). By default, Blimp uses the posterior median and standard deviation as a point estimate and measure of uncertainty, respectively, and it provides 95% credible interval limits. Other summary statistics such as posterior mean and median absolute deviation are also available (see

OUTPUT command). The graphic below shows a typical tabular display of the analysis results with a vertical split between the scripting and output window.



OUTCOME MODEL ESTIMATES:

Summaries based on 10000 iterations using 4 chains.

Outcome Variable: *y*

Grand Mean Centered: *d2* *x1* *x2*

Parameters	Median	StdDev	2.5%	97.5%	PSR	N_Eff
Variances:						
Residual Var.	14.548	0.916	12.907	16.520	1.001	6384.381
Coefficients:						
Intercept	28.576	0.189	28.207	28.941	1.001	3574.047
<i>x1</i>	0.589	0.060	0.471	0.705	1.001	5056.892
<i>d2</i>	1.810	0.335	1.151	2.461	1.001	7084.188
<i>x2</i>	0.190	0.044	0.103	0.275	1.000	4274.841
Standardized Coefficients:						
<i>x1</i>	0.402	0.036	0.329	0.467	1.001	5020.185
<i>d2</i>	0.204	0.037	0.130	0.275	1.000	6955.244
<i>x2</i>	0.173	0.039	0.095	0.248	1.001	4204.298
Proportion Variance Explained						
by Fixed Effects	0.260	0.033	0.195	0.323	1.001	5022.844
by Residual Variation	0.740	0.033	0.677	0.805	1.001	5022.844

Blimp automatically saves the output file to the same directory as the analysis script. Blimp output is saved in a text file with a `.blimp-out` extension. The outputs are linked to their analysis scripts, such that double-clicking on one of the files opens both in the Blimp Studio interface.

1.3 Blimp's Modeling Framework

The major feature that distinguishes Blimp's estimation architecture from other latent variable modeling software packages is that it does not work with the joint distribution of the analysis variables. Rather, the multivariate distribution is factored into the product of multiple univariate distributions. To illustrate, consider an analysis involving Y , X , and M . The trivariate distribution factors into the product of three univariate distributions, each of which corresponds to a univariate regression model.

$$f(Y, X, M) = f(Y|X, M) \times f(X|M) \times f(M)$$

Blimp estimates the models on the right of the equals sign without assuming anything about the form or shape of the multivariate distribution on the left. The advantage of this specification is that the individual regression equations can feature mixtures of categorical and normal variables, continuous variables with skewed distributions, interactive or nonlinear terms, and other complex constructions. In such cases, the multivariate distribution on the left doesn't have a known or simple form, and model misspecifications (e.g., treating such data as multivariate normal) can introduce bias.

The theory for Blimp's model specification is given by Ibrahim and colleagues (Ibrahim, Chen, & Lipsitz, 2002; Ibrahim, Lipsitz, & Chen, 1999; Lipsitz & Ibrahim, 1996), and the software extends these ideas to latent variable models with up to three levels. More recent literature refers to this model specification as fully Bayesian estimation, the sequential specification, and factored regression (Enders et al., 2020; Erler, Rizopoulos, Jaddoe, Franco, & Lesaffre, 2019; Erler et al., 2016; Lüdtke, Robitzsch, & West, 2020a, 2020b; Zhang & Wang, 2017).

To illustrate Blimp's modeling framework more concretely, consider a research scenario where the focal analysis model is a linear regression of Y on X and M . The factorization above translates into the following linear regression models, where all residuals are normal and have constant variance.

$$Y = \beta_0 + \beta_1 X_i + \beta_2 M_i + \varepsilon_i$$

$$X_i = \gamma_{01} + \gamma_{11} M_i + r_{1i}$$

$$M_i = \gamma_{02} + r_{2i}$$

The X and M equations are essentially nuisance models in this example, and their role is to link incomplete predictors to one another as well as to any complete regressors.

Any univariate equation can feature mixtures of categorical and normal variables, continuous variables with skewed distributions, and interactive or nonlinear terms, among other things. For example, the following equations include an interaction between X and M in the focal model and a quadratic association between X and M in the supporting predictor model.

$$Y = \beta_0 + \beta_1 X + \beta_2 M + \beta_3 (X \times M) + \varepsilon$$

$$X = \gamma_{01} + \gamma_{11} M + \gamma_{21} M^2 + r_1$$

$$M = \gamma_{02} + r_2$$

If either X or M has missing values, a joint modeling framework is inappropriate and would produce biased estimates because the incomplete predictor distributions are complicated nonlinear functions of the outcome; such associations are fundamentally incompatible with off-the-shelf distributions such as the multivariate normal (Bartlett, Seaman, White, & Carpenter, 2015; Liu, Gelman, Hill, Su, & Kropko, 2014). Specifying a model as a sequence of factored regressions bypasses the problematic joint distribution altogether. These ideas readily extend to latent variables, which Blimp views as missing data to be estimated (imputed).

1.4 Specifying Models for Incomplete Predictors

Throughout the guide, we use the term “predictors” to refer to exogenous variables—in a path diagram, variables that do not have incoming arrows. When predictors are complete, there is usually no reason to specify a distribution for these variables; instead, the covariate data essentially function as known constants, as in ordinary least

squares. In contrast, incomplete predictors require an explicit distribution for imputation. Blimp allows these distributions to be many different things (e.g., normal, skewed, discrete). In most cases, assigning a distribution to a predictor means making that covariate a dependent variable in its own regression model. These supporting models can be explicitly specified, or Blimp can create them automatically. These two strategies are somewhat different and have different strengths and weaknesses. We use the following multiple regression to illustrate the two model specification strategies, and we assume that all variables are incomplete.

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \varepsilon$$

To begin, consider the situation where Blimp automatically constructs supporting regression models for the predictors. The MODEL statement is as follows.

MODEL :

```
y ~ x1 x2 x3;
```

Throughout the guide, we refer to this specification as reflecting **unspecified associations** among the predictors. The underlying regression models follow a round robin pattern where each predictor is regressed on all other predictors.

$$X_1 = \mu_1 + \gamma_{11} (X_2 - \mu_2) + \gamma_{21} (X_3 - \mu_3) + r_1$$

$$X_2 = \mu_2 + \gamma_{12} (X_1 - \mu_1) + \gamma_{22} (X_3 - \mu_3) + r_2$$

$$X_3 = \mu_3 + \gamma_{13} (X_1 - \mu_1) + \gamma_{23} (X_2 - \mu_2) + r_3$$

The regressions above are linear and assume normally distributed residuals, but this specification also allows for binary, ordinal, and multicategorical nominal predictors, in which case Blimp adopts a latent response variable formulation (Albert & Chib, 1993;

Carpenter & Kenward, 2013; Enders et al., 2018; Johnson & Albert, 1999). Variable metrics are specified using the `ORDINAL` and `NOMINAL` commands described in Chapter 2. Enders et al. (2020) describe the multilevel version of this specification.

More formally, adopting unspecified associations for the predictors invokes a model that factors the joint distribution into the product of a univariate distribution for the analysis model and a multivariate distribution for the predictors.

$$f(Y, X_1, X_2, X_3) = f(Y|X_1, X_2, X_3) \times f(X_1, X_2, X_3)$$

We refer to this setup as a **partially factored specification** because it leaves the rightmost term—a multivariate normal distribution for continuous predictors and latent response variables—unfactored. With mixed response types, the multivariate distribution's covariance matrix is difficult to model because it could include a mixture of fixed constants, variances and covariances, and correlations. The round robin regression equations above simplify estimation by leveraging the property that a multivariate normal distribution spawns an equivalent set of linear regression models (Arnold, Castillo, & Sarabia, 2001; Liu, Gelman, Hill, Su, & Kropko, 2014). Importantly, the normal distribution assumption precludes the possibility of nonlinear associations among the predictors, as such relations are incompatible with normal data.

Next, consider the situation where the user explicitly specifies the regression equations for the predictors. This specification leverages the probability chain rule to factorize the joint distribution of the analysis variables into the product of several univariate conditional distributions, each of which corresponds to a regression model.

$$f(Y, X_1, X_2, X_3) = f(Y|X_1, X_2, X_3) \times f(X_1|X_2, X_3) \times \\ f(X_2|X_3) \times f(X_3)$$

The corresponding regression equations follow the same cascading pattern where the first predictor's model is empty, the second predictor is regressed on the first, the third on the first and second, and so on.

$$X_1 = \gamma_{01} + \gamma_{11}X_2 + \gamma_{21}X_3 + r_1$$

$$X_2 = \gamma_{02} + \gamma_{12}X_3 + r_2$$

$$X_3 = \gamma_{03} + r_3$$

The MODEL statement for this specification is

```
MODEL :
# predictor models
x3 ~ 1;
x2 ~ x3;
x1 ~ x2 x3;
# focal model
y ~ x1 x2 x3;
```

and the code block below illustrates a syntax shortcut for this specification that lists all predictors to the left of a tilde.

```
MODEL :
# predictor models
x1 x2 x3 ~ 1;
# focal model
y ~ x1 x2 x3;
```

We refer to this setup as a **factored regression specification** or **sequential specification** (Erler et al., 2016; Lüdtke et al., 2020b).

Blimp's output produces a tabular summary for each estimated model (there are four in the previous example). By default, the summary tables are alphabetized according to the outcome's name. Some users may prefer to order the tables so the primary analysis results appear first. This is accomplished by using labels ending in a colon to define blocks of models that appear in the order listed. The following example creates two model blocks, such that the focal model summary table appears first on the printed output.

MODEL :

focal.model:

y ~ x1 x2 x3;

predictor.models:

x1 x2 x3 ~ 1;

The sequential specification for predictors differs in important ways. First, the predictor's equation can have any metric allowed by Blimp—normal, skewed continuous, binary (probit or logit link), ordinal (probit link), multicategorical nominal (logistic link), or count (negative binomial link). Second, associations among the predictors need not be linear. For example, the following equations include the quadratic effect of X_3 on X_2 .

$$X_1 = \gamma_{01} + \gamma_{11}X_2 + \gamma_{21}X_3 + r_1$$

$$X_2 = \gamma_{02} + \gamma_{12}X_3 + \gamma_{12}X_3^2 + r_2$$

$$X_3 = \gamma_{03} + r_3$$

The corresponding MODEL statement is as follows.

MODEL :

```
# predictor models
x3 ~ 1;
x2 ~ x3 (x3^2);
x1 ~ x2 x3;
# focal model
y ~ x1 x2 x3;
```

When using a sequential specification, ordering the predictors in a particular way can facilitate estimation and reduce the impact of model misspecifications. Lüdtke et al. (2020b, pp. 171-172) recommend ordering variables from left to right by their missingness rates, with categorical variables before continuous variables. To illustrate, suppose that X_1 is an incomplete binary variable, X_2 is complete, and X_3 is an incomplete continuous variable. Their recommended specification would be as follows

$$f(Y|X_1, X_2, X_3) \times f(X_3|X_1, X_2) \times f(X_1|X_2) \times f(X_2)$$

and the corresponding model specification is

```
ORDINAL: x1;
MODEL:
# predictor models
x2 ~ 1;
x1 ~ x2;
x3 ~ x1 x2;
# focal model
y ~ x1 x2 x3;
```

or simply as follows.

```
ORDINAL: x1;
MODEL:
# predictor models
```

```
x3 x1 x2 ~ 1;
# focal model
y ~ x1 x2 x3;
```

Finally, when predictors are complete, there is usually no reason to specify a distribution for these variables; instead, the covariate data essentially function as known constants, as in ordinary least squares. With either specification for the predictors, listing complete predictors on the FIXED command line indicates that the variable does not require a model (or distribution). Continuing with the previous example where X_2 is complete, the sequential specification moves the complete variable from the left to the right of the tilde, as follows.

```
FIXED: x2;
MODEL:
# predictor models
x3 x1 ~ x2;
# focal model
y ~ x1 x2 x3;
```

The partially factored specification with unspecified predictor associations is as follows.

```
FIXED: x2;
MODEL:
y ~ x1 x2 x3;
```

The examples in Chapters 4 through 7 generally treat predictor distributions as unspecified. This setup is easy to specify, and it is also convenient for centering because the means are explicit model parameters that MCMC iteratively estimates. This approach does not limit the composition of the focal analysis model, which can

include interactive or nonlinear terms. However, predictor regressions are necessarily additive, as interactions and similar nonlinearities are incompatible with a multivariate normal distribution. As mentioned previously, unspecified predictor associations accommodate normal, binary, ordinal, and multicategorical variables via a latent response variable framework. Blimp can apply a Yeo-Johnson (Yeo & Johnson, 2000) distribution to skewed variables and negative binomial regression to count variables, but these features require a sequential specification.

The next section provides a complete description of the Blimp command language, and Chapters 4 through 7 provide numerous analysis examples. The examples span a wide variety of single-level and multilevel analyses with manifest and latent variables, including analyses for missing not at random processes.

1.5 Missing Data Handling

As detailed in Section 1.3, Blimp's estimation architecture factorizes a multivariate distribution into the product of univariate distributions. This factorization carries through to missing data handling, where the distributions of missing values rely on a collection of univariate models. The advantage of this specification is that Blimp can generate appropriate imputations from models that are fundamentally incompatible with known multivariate distributions. Examples include models with incomplete interactive or polynomial effects, multilevel models with random effects, and models with skewed variables or mixtures of discrete and numeric variables.

To illustrate missing data handling, consider an analysis involving Y , X , and M . To refresh, the trivariate distribution factors into the product of three univariate distributions, each of which corresponds to a regression model.

$$f(Y, X, M) = f(Y|X, M) \times f(X|M) \times f(M)$$

Blimp estimates the models on the right of the equals sign without assuming anything about the form or shape of the multivariate distribution on the left. In a simple scenario where all three variables are continuous, the factorization corresponds to the following linear regression models.

$$Y = \beta_0 + \beta_1 X_i + \beta_2 M_i + \varepsilon_i$$

$$X_i = \gamma_{01} + \gamma_{11} M_i + r_{1i}$$

$$M_i = \gamma_{02} + r_{2i}$$

In a factored regression framework, the distributions of missing values depend on every model in which a variable appears. For example, the distribution of missing Y values depends only on the analysis model, and MCMC samples imputations from a normal curve with center and spread equal to a predicted value and residual variance, respectively.

$$f(Y|X, M) = N(\beta_0 + \beta_1 X_i + \beta_2 M_i, \sigma_\varepsilon^2)$$

Because it appears in two models—once as a predictor and once as an outcome—the conditional distribution of missing X values is proportional to the product of two normal distributions.

$$f(X|Y, M) \propto f(Y|X, M) \times f(X|M) =$$

$$N(\beta_0 + \beta_1 X_i + \beta_2 M_i, \sigma_\varepsilon^2) \times N(\gamma_{01} + \gamma_{11} M_i, \sigma_{r1}^2)$$

In a similar vein, the conditional distribution of the missing M values is proportional to the product of three normal distributions.

$$f(M|Y, X) \propto N(\beta_0 + \beta_1 X_i + \beta_2 M_i, \sigma_\varepsilon^2) \times N(\gamma_{01} + \gamma_{11} M_i, \sigma_{r1}^2) \times N(\gamma_{02}, \sigma_{r2}^2)$$

These conditional distributions have analytic solutions in many cases (Levy & Enders, 2021), but Blimp's MCMC algorithm uses Metropolis sampling to draw imputations from composite functions such as these.

With a collection of additive models like those above, the distributions of missing values are equivalent to the distributions implied by a joint modeling framework or fully conditional specification. The same is not true for models with nonlinearities, skewed variables, or mixtures of discrete and numeric variables. To illustrate, suppose the analysis model includes an interaction between X and M . The factorization and the corresponding regression models are as follows.

$$f(Y, X, M) \propto f(Y|X, M, X \times M) \times f(X|M) \times f(M)$$

$$Y = \beta_0 + \beta_1 X_i + \beta_2 M_i + \beta_3 (X_i \times M_i) + \varepsilon$$

$$X_i = \gamma_{01} + \gamma_{11} M_i + r_{1i}$$

$$M_i = \gamma_{02} + r_{2i}$$

As before, the distributions of missing values depend on every model in which a variable appears. For example, the distribution of missing X values is again the product of two normal distributions, as follows.

$$f(X|Y, M) \propto f(Y|X, M, X \times M) \times f(X|M) =$$

$$N(\beta_0 + \beta_1 X_i + \beta_2 M_i + \beta_3 (X_i \times M_i), \sigma_\varepsilon^2) \times N(\gamma_{01} + \gamma_{11} M_i, \sigma_{r1}^2)$$

Importantly, the conditional distribution of missing values is incompatible with multivariate normality because its variance is heteroscedastic function (Enders et al.,

2020, Eq. 8). The same issue applies more broadly to models with polynomial or nonlinear terms and multilevel models with random effects, among others.

Basing imputations on factored regression specification is guaranteed to produce a set of compatible univariate regressions, whereas conventional modeling frameworks that create imputations based on a multivariate distribution are prone to bias (Bartlett, Seaman, White, & Carpenter, 2015; Liu, Gelman, Hill, Su, & Kropko, 2014). More generally, the univariate models described above could feature discrete variables (binary, ordinal, multicategorical nominal, count), skewed continuous variables, and even latent variables, which Blimp views as missing data to be estimated (imputed).

1.6 New Features

The following is a list of new features and functionality available in Version 3.2.

- ❖ Multiple-equation models (e.g., path models) with up to three levels
- ❖ Latent variables and latent variable regressions
- ❖ Latent variables with random effects, interactions, and nonlinear effects
- ❖ Single-level and multilevel selection and pattern mixtures models for missing not at random processes
- ❖ Multivariate regression models
- ❖ Parameter constraints
- ❖ Auxiliary parameters that are functions of estimated parameters
- ❖ Latent variable imputation
- ❖ Yeo-Johnson modeling for skewed continuous variables
- ❖ Binary and multinomial logistic regression
- ❖ Negative binomial regression for count outcomes
- ❖ Estimation with sampling weights

- ❖ Facilities for computing new variables with numerous built-in functions
- ❖ Built-in functions embedded within regression equations
- ❖ Facilities for introducing custom univariate prior distributions
- ❖ New Blimp Studio graphical user interface
- ❖ Redesigned output with numerous enhancements and additional printing options
- ❖ Better optimization and many algorithmic improvements
- ❖ Enhanced user guide with dozens of new examples and analysis scripts
- ❖ Enhanced TEST command for Bayesian Wald tests in all models Blimp estimates
- ❖ DIC and WAIC information criteria for model selection
- ❖ Correlations among the residuals of all outcome variables for evaluating local sources of model misfit

The following is a list of features and functionality that were introduced in Version 2.

- ❖ TEST command for Bayesian Wald tests (Asparouhov & Muthén, 2021)
- ❖ Simplified scripting language and redesigned output
- ❖ Graphical interface with automatic updates when new features become available
- ❖ Graphical engine that creates trace plots for all model parameters
- ❖ Bayesian estimation of single-level, multilevel (up to three levels), and multiple group regression models with complete or incomplete data
- ❖ Posterior summaries of all model parameters from Bayesian estimation (posterior mean, median, standard deviation, and credible interval)
- ❖ Single-level and multilevel R-squared measures (Rights & Sterba, 2019)
- ❖ Bayesian estimation for interactive and nonlinear effects with missing data
- ❖ Bayesian estimation with grand mean centering (all models) and group mean centering (two- and three-level models)
- ❖ Post-hoc probing of interaction effects with continuous or categorical moderators
- ❖ Bayesian estimation of conditional effects (simple slopes) in regression models with interaction effects

- ❖ Incomplete binary, ordinal, or nominal predictor variables
- ❖ Discrete and latent imputations for binary, ordinal, and nominal variables
- ❖ FCS or Bayesian estimation with level-2 and level-3 cluster means modeled as latent variables
- ❖ Contextual effects models with latent group means or manifest group means
- ❖ Interaction effects with latent group means
- ❖ Various algorithmic and interface enhancements (eg, random starting values, options for saving various estimates and output)

1.7 Running From Terminal

Blimp scripts can also be executed from the terminal without the graphical interface. This is useful when conducting computer simulations, for example. The most basic specification includes a file path to the Blimp executable file followed by a file path to the script to be executed. To illustrate, the following line of code executes a script located on the desktop.

```
/Applications/Blimp/blimp ~/desktop/myscript.imp
```

Similarly, the following line uses the Blimp beta engine to execute the same file.

```
/Applications/Blimp/blimp-beta ~/desktop/myscript.imp
```

Several parts of the Blimp script can be specified via command line arguments. The general form of an argument includes a double dash followed by a keyword and an input parameter. For example, the following code block uses a command line argument to specify the input data set.

```
BLIMPPATH=/Applications/Blimp/blimp
```

```
${BLIMPPATH} ~/desktop/myscript.imp --data ~/desktop/mydata.dat
```

Note that any parameters specified as command line arguments replace the current contents of the script (e.g., the file specified on the DATA command is replaced by the file `~/desktop/mydata.dat`).

In addition to the input data, command line arguments include the random number seed and most quantities exported using the SAVE command. The code block below shows the full array of command line arguments. The backslash is the Linux command continuation character; the arguments would otherwise need to appear on a single line separated by a space.

```
/Applications/Blimp/blimp ~/desktop/myscript.imp \  
  --seed {seed value} \  
  --data {filepath to input data} \  
  --output {filepath to blimp-out output file} \  
  --stacked {filepath to stacked imputation data} \  
  --stacked0 {filepath to stacked original + imputed data} \  
  --separate {filepath to separate imputation data sets} \  
  --estimates {filepath to save estimate summary tables} \  
  --burn {filepath to save all burn-in estimates} \  
  --iterations {filepath to save all post burn-in estimates} \  
  --psr {filepath to save burn-in psr values} \  
  --waldtest {filepath to save Wald statistics} \  
  --
```

2 Blimp Command Language

2.1 Overview

This chapter gives a detailed account of the Blimp's scripting language. Blimp commands can be entered in the Blimp Studio syntax editor or in a plain text file with `.imp` as the file extension. The code block below shows a typical script with many of Blimp's major commands.

```
DATA: data.dat;  
VARIABLES: id a1:a4 y m x1:x3 z1 z2;  
ORDINAL: x1;  
NOMINAL: x3;  
MISSING: 999;  
FIXED: x3;  
CENTER: grandmean = x1 x2;  
MODEL:  
# x1-x3 and x2-x3 interaction predicting m;  
m ~ x1 x2 x3 x2*x3;  
# m and x1-x3 predicting y;  
y ~ m x1:x3;  
SEED: 90291;  
BURN: 2000;  
ITERATIONS: 10000;
```

The Blimp command language uses the following general conventions, most of which are shown in the previous code block.

- ❖ Upper and lower case are equivalent, no case sensitivity
- ❖ Command names (e.g., DATA, VARIABLES) end in a colon
- ❖ Subcommands or specifications following a command in a semicolon
- ❖ Commands and subcommands can span multiple lines

- ❖ A colon can be used to specify a range of variables with the same prefix and suffix
- ❖ A # symbol indicates a comment that Blimp ignores until the end of the line
- ❖ Three symbols are needed to specify models: (a) ~ or <- denotes a regression equation, (b) <-> or ~~ denote variances and covariances, and (c) -> or =~ assigns indicators to a latent variable
- ❖ Mathematical operator symbols are * for multiplication, / for division, + for addition, - for subtraction, ^ or ** to raise a variable or quantity to a power, and parentheses for specifying order of operations

Blimp also provides a number of built-in functions that work in conjunction with certain commands. The TRANSFORM command can use these functions to create new variables, the PARAMETERS command can use these routines to compute auxiliary parameters that are functions of the estimated model parameters, and functions can be embedded within regression equations listed in the MODEL statement. The list of functions is below.

- ❖ `abs(x)` = absolute value of x
- ❖ `sqrt(x)` = square root of x
- ❖ `exp(x)` = exponential function applied to x
- ❖ `logit(x)` = logit function applied to x
- ❖ `sigm(x)` = sigmoid function applied to x
- ❖ `log(x)` or `ln(x)` = natural log of x
- ❖ `log1p(x)` = $\log(1 + x)$
- ❖ `expm1(x)` = $\exp(x) - 1$
- ❖ `phi(x)` = normal cumulative distribution function of x
- ❖ `iphi(x)` or `probit(x)` = inverse normal cumulative distribution function of x
- ❖ `yjt(x, lambda)` = Yeo-Johnson transformation of x with optional shape parameter

- ❖ `iyjt(x, lambda)` = inverse Yeo-Johnson transformation of `x` with optional shape parameter
- ❖ `mean(x)` = returns the mean of `x`
- ❖ `mean(x, idvar)` = returns the cluster means of `x` computed within the grouping variable `idvar`
- ❖ `sd(x)` = returns the standard deviation of `x`
- ❖ `sd(x, idvar)` = returns the cluster standard deviation of `x` computed within the grouping variable `idvar`
- ❖ `stand(x)` or `scale(x)` = returns `x` standardized as a z-score
- ❖ `stand(x, idvar)` or `scale(x, idvar)` = returns `x` standardized as a z-score within the grouping variable `idvar`
- ❖ `center(x)` = returns `x` but centered. Equivalent to $(x - \text{mean}(x))$
- ❖ `center(x, idvar)` = returns `x` but centered within the grouping variable `idvar`. Equivalent to $(x - \text{mean}(x, idvar))$
- ❖ `max(x)` = returns the maximum of `x`
- ❖ `max(x, y)` = returns the row-wise maximum between `x` and `y`
- ❖ `min(x)` = returns the minimum of `x`
- ❖ `min(x, y)` = returns the row-wise minimum between `x` and `y`
- ❖ `vec(x)` = creates a variable filled with the scalar `x`

The following built-in functions are only available in the TRANSFORM command:

- ❖ `ismissing(x)` = returns missing data indicator for `x`
- ❖ `lag1(x, time)` = returns the lag 1 of `x` based on the `time` variable
- ❖ `lag1(x, time, idvar)` = returns the lag 1 of `x` within the grouping variable `idvar`, based on the `time` variable
- ❖ `ifelse(condition, value if true, value if false)` recodes a variable `X` into a new variable with two values

2.2 Blimp Commands

DATA Command

The DATA command specifies the input data set, which must be saved as a .csv (comma separated values) format or a whitespace (including tab) delimited file (e.g., .dat or .txt). Blimp accepts only numeric characters for data values (e.g., a nominal variable cannot have alphanumeric labels as score values), although alphanumeric characters (e.g., NA) can be used for missing value codes. Variable names can appear in the column headers, but the VARIABLE command (described next) must be omitted. No file path is needed if the Blimp script (the .imp file) is located in the same directory as the data. The following code block illustrates this specification.

```
DATA: mydata.dat;
```

The DATA command requires a full file path to the input data set that is located in a directory other than the one that contains the Blimp script. The file path should not be enclosed in quotations. The following code block reads a data file located in a directory named “research project” located on the desktop. In line with macOS and other Unix-based systems conventions, a tilde can be used to reference the user’s home directory. The following input line reads a data file from a directory within the desktop folder.

```
DATA: ~/desktop/research project/mydata.dat;
```

VARIABLE Command

The VARIABLES command specifies the variable names for the data set listed on the DATA command. in the input file. This command should not be used if the data file has

variable names as column headers. The variable list may include variables that are not used in an analysis model or imputation model. The code block below illustrates a basic specification with five variables.

```
VARIABLES: y x1 x2 x3 x4;
```

A colon can be used to specify a range of variables with the same prefix but different numeric suffixes, as follows.

```
VARIABLES: y x1:x4;
```

The colon specification also works if a group of variables has a common alphanumeric string following the numeric values (e.g., a set of variables and their recoded counterparts).

```
VARIABLES: y x1:x4 x1r:x4r;
```

ORDINAL Command

The ORDINAL command identifies ordinal variables that appear in a MODEL statement. For computational efficiency, we recommend listing binary variables on the ORDINAL line, but these variables could also be treated as nominal. A colon can be used to specify a range of ordinal variables, as follows.

```
ORDINAL: x1:x5;
```

By default, Blimp uses a latent response variable (i.e., probit regression) framework for ordinal variables (Albert & Chib, 1993; Carpenter & Kenward, 2013; Enders et al.,

2018; Johnson & Albert, 1999), and a logistic link is an option for binary variables (Asparouhov, T., & Muthén, B. (2021; Polson, Scott, & Windle, 2013).

NOMINAL Command

The NOMINAL command specifies nominal variables that appear in a MODEL statement. Nominal variables must be represented as a single variable with numeric codes. Blimp automatically recodes the discrete responses into a set of dummy codes (or latent response difference scores, in some cases) during estimation. By default, Blimp assigns the first (lowest) code as the reference category. To change the reference category, list the numeric code of the desired reference group in parentheses following the variable's name. To illustrate, consider two nominal variables X and Z , each with codes 1, 2, and 3. The following example assigns $X = 3$ and $Z = 1$ as the reference groups.

```
NOMINAL: x(3) z;
```

For predictors with unspecified associations, Blimp uses a latent difference score (i.e., multinomial probit regression) framework for nominal variables (Albert & Chib, 1993; Carpenter & Kenward, 2013; Enders et al., 2018; Johnson & Albert, 1999), and it uses a logistic link for multicategorical nominal variables on the left side of a tilde (Asparouhov, T., & Muthén, B. (2021; Polson, Scott, & Windle, 2013).

COUNT Command

The COUNT command identifies count variables that appear in a MODEL statement. Count dependent variables have a negative binomial regression (Asparouhov, T., & Muthén, B. (2021; Polson, Scott, & Windle, 2013). Currently, incomplete count

variables can only be outcomes, not predictors. Additionally, the COUNT command requires the `#!/beta` shebang at the top of the Blimp script (see Example 4.15).

```
COUNT: y x3;
```

CLUSTERID Command

The CLUSTERID command specifies cluster-level identifier variable(s) needed for a multilevel analysis or multilevel imputation. Two-level analyses require a single identifier for the level-2 sampling unit (cluster), and three-level analyses require level-2 and level-3 identifier variables. The order of the identifier variables does not matter, as Blimp automatically determines variable levels. To illustrate, the following code block specifies a single cluster-level identifier for a two-level analysis.

```
VARIABLES: level2id y x1 x2;  
CLUSTERID: level2id;  
MODEL: y ~ x1 x2;
```

The code block below illustrates a pair of cluster-level identifiers for a three-level analysis.

```
VARIABLES: level2id level3id y x1 x2;  
CLUSTERID: level2id level3id;  
MODEL: y ~ x1 x2;
```

Blimp currently does not allow cross-classified clustering schemes.

MISSING Command

The MISSING command is used to specify the missing value code. Missing values can be coded with a single numeric (e.g., 999) or alphanumeric value (e.g., NA). The following code block specifies a numeric value of 999 as the missing data code.

```
MISSING: 999;
```

LATENT Command

The LATENT command is used to define latent variables (e.g., factors in a measurement model) that will be referenced in the MODEL section. For example, the code block below illustrates the specification for a single latent factor with three manifest indicators.

```
LATENT: yfactor;  
MODEL:  
yfactor -> y1:y3;
```

The default scaling for latent factors is described in the MODEL command section. Blimp treats all latent variables as missing data to be imputed, and adding the `saveLatent` keyword to the OPTIONS line saves the estimated latent variable scores to the imputed data.

Latent variables can be specified at any level of a multilevel model. This specification references cluster-level identifier variables from the CLUSTERID line. For example, the code below illustrates the specification of a level-1 latent factor with three manifest indicators measured at level-1 and a level-2 latent factor with three indicators measured at level-2. The variable

```
CLUSTERID: level2id;  
LATENT: yfactor, level2id = xfactor;  
MODEL:  
yfactor -> y1:y3;  
xfactor -> x1:x3;
```

Latent variables can also be listed on separate lines as follows.

```
LATENT:  
yfactor;  
level2id = xfactor;
```

RANDOMEFFECT Command

The **RANDOMEFFECT** command is used to define new latent variables that equal the random intercepts and slopes from a multilevel regression model. These latent variables are referenced in the **MODEL** section, where they can be used to predict other variables in a multilevel path or structural equation model. The **RANDOMEFFECT** command is similar to the **LATENT** command except that the random intercept and slope residuals can only function as predictors and not outcomes like a latent factor.

The specification for a random effect latent variable has four components: (a) the new latent variable's name appears on the left side of the equation, (b) the target equation's outcome variable is listed after the equals sign, (c) the random slope predictor's name from the target model appears after the vertical pipe, and (d) the cluster-level identifier variable from the **CLUSTERID** command appears at the end of the line in square brackets. The generic specification is as follows.

```
RANDOMEFFECT:  
newlatent = outcome variable | random predictor [CLUSTERID var];
```


To illustrate more concretely, the code block below defines a pair of new latent variables equal to the random intercept and random slope residuals from a two-level model and uses the random effects to predict an outcome.

```
CLUSTERID: cluster;  
RANDOMEFFECT:  
ranicepts = y | 1 [cluster];  
ranslopes = y | x [cluster];  
MODEL:  
y ~ x | x;  
z ~ ranicepts ranslopes;
```

TRANSFORM Command

The TRANSFORM command creates new variables that are functions of existing variables. If imputations are requested, the new variable is saved to the output data sets. The general form of the TRANSFORM command is as follows.

```
TRANSFORM:  
newvar1 = expression or function;  
newvar2 = expression or function;
```

Mathematical operator symbols are * and / for multiplication and division, + and – for addition and subtraction, and ^ to raise a variable or quantity to a power. The following examples apply these operators to create new variables from an existing variable X.

```
TRANSFORM:  
newvar1 = x * 2;  
newvar2 = x / 2;  
newvar3 = x + 2;  
newvar4 = x - 2;
```

```
newvar5 = x^2;
```

Blimp also offers three ways to recode an existing variable into a new variable. The switch function is the most convenient in many situations.

TRANSFORM:

```
newvar = switch(condition1, value if true,  
  condition2, value if true,  
  condition3, value if true,  
  ...  
  value if all conditions are false);
```

To illustrate, consider the creation of a new binary variable that equals 1 if X is less than 50 and 2 if X is greater than or equal to 50. The TRANSFORM command for recoding X into a binary variable is as follows.

TRANSFORM:

```
newvar = switch(x < 50, 1, 2);
```

As a second example, suppose X is a multicategorical variable with four groups coded 1 through 4. The TRANSFORM command below creates a new three-category variable that combines codes 3 and 4 into a single group.

TRANSFORM:

```
newvar = switch(x == 1, 1, x == 2, 2, 3);
```

The second recording strategy uses Boolean statements that evaluate to a 1 or 0. To illustrate, consider the creation of a new binary variable that equals 1 if X is less than 50 and 2 if X is greater than or equal to 50. The TRANSFORM command for recoding X into a binary variable is as follows.

TRANSFORM:

```
newvar = (x < 50)*1 + (x >= 50)*2;
```

The Boolean operator $(x < 50)$ evaluates to 1 if X is less than 50 and 0 otherwise.

Multiplying by 1 establishes the score value that is assigned if the first condition is true. The Boolean operator $(x \geq 50)$ similarly evaluates to a 1 or 0, and multiplying by 2 establishes the score value that is assigned if the second condition is true. Summing the two components produces the desired binary variable because only one of the two terms is non-zero.

As a second example, suppose X is a multicategorical variable with four groups coded 1 through 4. The TRANSFORM command below creates a new three-category variable that combines codes 3 and 4 into a single group.

TRANSFORM:

```
newvar = (x == 1)*1 + (x == 2)*2 + (x == 3)*3 + (x == 4)*3;
```

Following the previous example, the command consists of four Boolean operators, each of which evaluates to 1 or 0 if the condition is true or false, respectively. Each operator is multiplied by the score value that is assigned if the condition is true. Finally, summing the result of each product recodes the variable because only one condition is non-zero.

Multiple Boolean operators can be combined into a single condition by separating two operators with and/or . For example, the TRANSFORM command below creates a new three-category variable that uses an or logical argument to recode codes 3 and 4 into a single category.

TRANSFORM:

```
newvar = (x == 1)*1 + (x == 2)*2 + ((x == 3) or (x == 4))*3;
```

The `ifelse` function is a third method for recoding variables. The general form of the command has three arguments: a condition to be evaluated, the value of the new variable if the condition is true, and the value of the new variable if the condition is false.

TRANSFORM:

```
newvar = ifelse(condition, value if true, value if false);
```

To illustrate, reconsider the creation of a new binary variable that equals 1 if X is less than 50 and 2 if X is greater than or equal to 50. The TRANSFORM command for recoding X into a binary variable is as follows.

TRANSFORM:

```
newvar = ifelse(x >= 50, 2, 1);
```

The condition being evaluated is whether X is greater than or equal to 50, and the new variable is assigned a value of 2 if the condition is true and 1 otherwise. Multiple `ifelse` statements can be nested within each other, but the `switch` function is generally more convenient for recoding variables into three or more score values.

Global functions are listed in Section 2.1. The following functions are specific to the TRANSFORM command and cannot be used elsewhere:

- ❖ `ismissing(x)` = returns missing data indicator for a variable X
- ❖ `lag1(x, timescores, cluster)` = in a multilevel longitudinal structure, shifts all rows of variable X down by one row, as indexed by a level-1 temporal predictor `timescores` nested within a cluster-level identifier variable `cluster`

- ❖ `switch(condition1, value if true, condition2, value if true, ..., value if all conditions false)` recodes existing variables
- ❖ `ifelse(condition, value if true, value if false)` recodes a variable *X* into a new variable with two values

BYGROUP Command

The BYGROUP command is used to perform fully conditional specification imputation (when used in conjunction with the FCS command) or model estimation (when used in conjunction with the MODEL command) for observed subgroups in the data. For example, consider a manifest (and complete) grouping variable *G* with three categories. The following code block specifies fully condition specification imputation separately for each level of *G*.

```
BYGROUP: g;  
FCS: y x1 x2;
```

Similarly, the following code block estimates a separate multiple regression model for each subgroup of *G*.

```
BYGROUP: g;  
MODEL: y ~ x1 x2;
```

Only a single categorical variable is allowed on the BYGROUP command, although this limitation can be bypassed by recoding multiple categorical variables into a single variable, sample size permitting. Additionally, the BYGROUP variable should not appear on the ORDINAL, NOMINAL, or MODEL lines. Trace plots are currently unavailable with BYGROUP processing. Finally, you can use this command to fit multiple-group models, but Blimp does not allow between-group constraints.

FIXED Command

The **FIXED** command identifies complete predictor variables that do not require a distribution. Incomplete variables and outcome variables (variables that appear to the left of a tilde) must be random variables with a distribution. With relatively few exceptions, we recommend listing complete variables on the fixed line, as doing so speeds computations and convergence. Fixed variables listed on the **CENTERING** line will be centered at the means of the observed data (i.e., the means will not be treated as random variables to be estimated). The following code block illustrates a multiple regression analysis with two complete fixed variables, X_1 and X_2 .

```
VARIABLES: y x1 x2 x3 x4;  
FIXED: x1 x2;  
MODEL: y ~ x1 x2 x3;
```

CENTER Command

The **CENTERING** command is used to center predictor variables in regression equations. This command affects Blimp's printed estimates but has no bearing on imputations generated by the **SAVE** command. For complete variables listed on the **FIXED** line, Blimp centers variables at arithmetic averages (grand mean or group means). For all variables assigned a distribution, the **CENTERING** command treats grand means and group means as random variables to be estimated at each MCMC iteration (Enders & Keller, 2019). Any product terms specified on the **MODEL** line automatically reflect the specified centering method.

In a single-level model, there is no need to specify the type of centering because centering at the grand means is the only option. The code block below shows a basic grand mean centering specification for a single-level multiple regression model.

```
CENTER: x1 x2;  
MODEL: y ~ x1 x2 x3;
```

The equivalent specification below explicitly requests grand mean centering.

```
CENTER: grandmean = x1 x2;  
MODEL: y ~ x1 x2 x3;
```

Predictor variables in a multilevel regression model can be centered at the grand means or group-level cluster means (lower-level regressors only). In this case, the type of centering must be explicitly specified. The following code block illustrates a two-level regression model with a cross-level interaction where a level-1 predictor X_1 is centered at the level-2 latent group means, and a level-2 predictor X_2 is centered at its grand mean (group mean centering is not an option for variables at the highest level).

```
CLUSTERID: level2id;  
CENTER: groupmean = x1.i, grandmean = x2.j;  
MODEL: y ~ x1.i x2.j x1.i*x2.j | x1.i;
```

Centering specifications can also be spread over multiple lines, as follows.

```
CLUSTERID: level2id;  
CENTER:  
groupmean = x1.i;  
grandmean = x2.j;
```

```
MODEL: y ~ x1.i x2.j x1.i*x2.j | x1.i;
```

Importantly, group mean centering reflects deviations between the raw scores and latent group means (unless the variable is complete and listed on the FIXED line, in which case the group means are arithmetic averages). Further, group mean centering is always performed by subtracting the latent group means at the next level of the data hierarchy. For example, if the previous analysis was a three-level model, the centering procedure would subtract X_1 scores from the level-2 latent group means. The group means themselves can be included in the analysis model, and these latent variables can be centered just like any other predictor.

Categorical variables can also be centered (Enders & Tofighi, 2007; Yaremych & Preacher, 2021). As mentioned elsewhere, categorical predictors (binary, ordinal, or nominal) are modeled as underlying normal latent response variables. The grand and group means are also modeled on the latent metric, and listing categorical variables on the CENTERING command invokes a transformation that converts the latent mean to the metric required by the analysis model (Enders & Keller, 2019). For example, centering a binary predictor converts the latent grand mean to a “manifest” mean equal to the model-implied proportion of ones in the data. Applying centering to nominal variables with three or more categories can be computationally intensive because the latent mean conversion requires Monte Carlo integration at each MCMC step.

MODEL Command

The MODEL command typically consists of one or more univariate regression models. Blimp’s modeling framework can accommodate a wide range of analyses ranging from basic multiple regression models to complicated multilevel structural equation models

with interactions involving latent variables. This section describes the command, and Chapters 4 through 7 provide numerous examples.

Regression Models

Univariate regression models are the building blocks for specifying more complex multivariate models involving networks of variables—Blimp's modeling framework simply defines any multivariate model as a collection of individual univariate regressions (see Section 1.3). A univariate regression is specified by listing an outcome variable to the left of the tilde symbol and predictors (or perhaps just an intercept) to the right of the tilde. The code block below illustrates a multiple regression analysis with three predictors.

MODEL :

```
y ~ 1 x1 x2 x3;
```

Outcome variables that appear to the left of a tilde can be latent factors or manifest variables with a variety of different metrics (normal, skewed continuous, binary, ordinal, multicategorical nominal, count). With the exception of latent outcomes where means are set equal to 0, Blimp estimates the intercept by default, and the above specification can be shortened as follows.

MODEL :

```
# unspecified predictor models  
y ~ x1 x2 x3;
```

As explained in Section 1.4, supporting regression models for incomplete predictors can be explicitly specified (i.e., they can appear as outcomes to the left of a tilde), or Blimp can create them automatically. The previous code block does not list models for

the regressors, so Blimp constructs them as needed for missing data handling. The examples in Chapter 3 generally adopt this specification because it is easy to implement and accommodates normal, binary, ordinal, and multicategorical nominal variables. Leaving predictor associations unspecified also facilitates centering because grand means and latent group means (multilevel models) are iteratively estimated parameters. To reiterate, regressions among the predictors are simply a device for assigning distribution to and preserving associations among incomplete covariates. These models usually are not the substantive focus, and they need not have a logical causal construction.

Alternatively, predictor models can be invoked with a sequential specification that features a cascading pattern of univariate regressions, where the first predictor's model is empty, the second predictor is regressed on the first, the third on the first and second, and so on.

MODEL :

```
# sequentially specified predictor models
x3 ~ 1;
x2 ~ x3;
x1 ~ x2 x3;
# focal analysis model
y ~ x1 x2 x3;
```

Sequential models can be specified more succinctly by listing all predictors on the left side of the same tilde.

MODEL :

```
# sequentially specified predictor models
x1 x2 x3 ~ 1;
```

```
# focal analysis model
y ~ x1 x2 x3;
```

When using the **FIXED** command to identify complete predictor variables that do not require a distribution, those predictors should only appear on the right side of a tilde in a sequential specification.

```
FIXED: x2;
MODEL:
# sequentially specified predictor models
x1 x3 ~ x2;
# focal analysis model
y ~ x1 x2 x3;
```

A sequential specification is primarily useful for two scenarios: modeling nonlinear associations among predictors and modeling skewed or count distributions. In most other situations, unspecified and sequentially specified predictor models are equivalent. To illustrate, the code below depicts a scenario where X_2 is a quadratic function of X_3 (see the later section on interactive and polynomial effects).

```
MODEL:
x3 ~ 1;
x2 ~ x3 (x3^2);
x1 ~ x2 x3;
y ~ x1 x2 x3;
```

As a second example, the following code block assigns a Yeo-Johnson normal distribution (Yeo & Johnson, 2000) that allows X_2 's distribution to be positively or negatively skewed (see the later section on functions embedded within equations).

MODEL :

```
x3 ~ 1;
yjt(x2) ~ x3;
x1 ~ x2 x3;
y ~ x1 x2 x3;
```

Lüdtke et al. (2020b) provide recommendations for ordering variables when adopting a sequential specification (see Section 1.4).

Blimp prints a table of estimates for each outcome variable in a model (i.e., every variable to the left of a tilde. By default, the tables are printed in alphabetical order. Users can specify a custom order for tables by defining equation blocks within the MODEL statement. Equation blocks are defined by specifying an arbitrary name for the block (which will appear on the output) followed by a colon. For example, the code below defines two equation blocks, such that the focal regression output would be the first table of results. Within a given block, order is alphabetic.

MODEL :

```
focal.regression:
y ~ x1 x2 x3;
predictor:models:
x3 ~ 1;
yjt(x2) ~ x3;
x1 ~ x2 x3;
```

With the exception of latent dependent variables, Blimp automatically estimates each equation's intercept and residual variance. In some situations, it may be necessary to explicitly mention these parameters (e.g., when imposing a constraint or labeling a parameter). The code block below explicitly references the intercept by including a 1

on the right of the tilde (the keyword `intercept` can be used in lieu of the `1`), and it uses a double-headed arrow to reference the residual variance

MODEL :

```
y ~ 1 x1 x2 x3;
y <-> y;
```

Variances can also be specified with double tildes, as follows.

MODEL :

```
y ~ 1 x1 x2 x3;
y ~~ y;
```

Discrete Outcomes

Discrete outcomes are defined on the `ORDINAL`, `NOMINAL`, and `COUNT` lines. In general, little or no further specification is needed. For example, the following code block illustrates a probit regression for a binary outcome.

```
ORDINAL: y;
```

MODEL :

```
y ~ x1 x2 x3;
```

A logistic regression additionally applies the logit function to the dependent variable, as shown below.

```
ORDINAL: y;
```

MODEL :

```
logit(y) ~ x1 x2 x3;
```

Discrete Predictors

Discrete predictors are defined on the ORDINAL, NOMINAL, and COUNT lines (the latter is only available with a sequential specification). In general, little or no further specification is needed to invoke a discrete predictor. For example, the following code block illustrates a linear regression where X_2 is a binary dummy code.

```
ORDINAL: x2;  
MODEL:  
y ~ x1 x2 x3;
```

The discrete scores appear in the focal analysis model, but Blimp uses a latent response variable formulation for the predictor's supporting regression model (which is left unspecified above).

The specification for nominal variables is similar. To illustrate, the code block below specifies a linear regression where X_2 is a multicategorical predictor ($X_2 = 1, 2, 3$).

```
NOMINAL: x2;  
MODEL:  
y ~ x1 x2 x3;
```

Blimp uses a latent difference variable formulation (multinomial probit model) for the predictor's supporting regression (which is left unspecified above), but a set of dummy codes appear in the focal analysis model. By default, Blimp assigns the first (lowest) numeric code as the reference category. To override this default behavior, list the desired reference group's numeric code in parentheses on the NOMINAL line. To illustrate, the following code block assigns $X_2 = 3$ as the reference category.

```
NOMINAL: x2(3);
```

```
MODEL:
```

```
y ~ x1 x2 x3;
```

In some situations, it may be necessary to refer to a specific dummy code (e.g., when constraining or labeling a parameter). This specification uses a period and a numeric label following the variable's name. For example, the following code block assigns $X_2 = 3$ as the reference group, and it explicitly references the dummy codes for the $X_2 = 1$ and 2 categories in a MODEL statement.

```
NOMINAL: x2(3);
```

```
MODEL:
```

```
y ~ x1 x2.1 x2.3 x3
```

Interaction and Polynomial Terms

Traditional modeling frameworks that assume a multivariate distribution for the analysis variables (e.g., all structural equation models based on multivariate normal distribution) are fundamentally incompatible with incomplete nonlinear effects. This includes models with incomplete interaction effects, curvilinear effects, and random slopes (two- or three-level models). Practically speaking, incompatibility means that imputations generated by a multivariate distribution are mathematically impossible given the configuration of effects in the focal analysis model.

Blimp's estimation architecture avoids this problem by working with a set of univariate regression models that are guaranteed to be mutually compatible. Rather than imputing the product directly, Blimp uses a Metropolis sampling step to select imputations that are consistent with any nonlinear effects in the univariate regression

models. The methodological literature uniformly favors this strategy over so-called just-another-variable imputation schemes that apply normal distribution assumptions to incomplete nonlinear effects (Bartlett et al., 2015; Enders et al., 2020; Erler et al., 2016; Kim, Belin, & Sugar, 2018; Kim, Sugar, & Belin, 2015; Lüdtke, Robitzsch, & West, 2019; Zhang & Wang, 2017). The specifications described below are the same for single-level and multilevel regression models.

Interaction terms are specified by connecting two predictors in the same equation with an asterisk. The following code block illustrates a two-way interaction with lower-order terms. The supporting regressions for incomplete predictors are constructed automatically (see Section 1.4).

MODEL :

```
y ~ x1 x2 x1*x2;
```

Similarly, the code below shows a three-way interaction with all possible two-way interactions and lower-order terms.

MODEL :

```
y ~ x1 x2 x3 x1*x2 x1*x3 x2*x3;
```

Generally speaking, any variable to the left of a tilde (dependent variables, predictors in a factored regression specification) can have interaction effects in its regression model.

Blimp allows for interactions with categorical predictors defined on the NOMINAL and ORDINAL lines. Binary and ordinal predictors function as numeric variables when multiplied by another variable; the supporting regressor model again uses a latent response variable formulation. Interactions involving multicategorical nominal

variables require product terms for each dummy code in a set. By default, Blimp automatically creates a model that includes the necessary product terms. To illustrate, the code block below illustrates an interaction effect where X_2 is a multicategorical nominal predictor ($X_2 = 1, 2, 3$) and X_3 is continuous.

```
NOMINAL: x2;  
MODEL:  
y ~ x1 x2 x3 x2*x3;
```

In this case, Blimp automatically generates a model with two product terms, one for each of the two dummy codes (recall that $X_2 = 1$ is the reference group). In some situations, it may be necessary to refer to a specific component of the product (e.g., when constraining or labeling a parameter). The following specification is equivalent to the one above.

```
NOMINAL: x2;  
MODEL:  
y ~ x1 x2 x3 x2.2*x3 x2.3*x3;
```

A polynomial term in a curvilinear regression is just interaction between a variable and itself. As such, these terms can be specified by connecting a regressor with itself using an asterisk. The following code block illustrates a quadratic function with a lower-order term and a covariate.

```
MODEL:  
y ~ x1 x1*x1 x2;
```

Alternatively, the quadratic term can be specified by using a function embedded in a regression equation, as follows..

MODEL :

```
y ~ x1 (x1^2) x2;
```

Correlations and Residual Correlations

In Blimp, univariate regression models are always the building blocks for specifying more complex multivariate models involving networks of variables—the modeling framework simply defines a multivariate model as a collection of individual univariate regressions (see Section 1.3). Because Blimp does not work with the joint distribution of the variables (e.g., impose a multivariate normal distribution on the data), these univariate equations are uncorrelated by construction. For example, the code block below illustrates a bivariate analysis with two empty regression equations, but the correlation (or covariance) between the two dependent variables is not a byproduct of estimation.

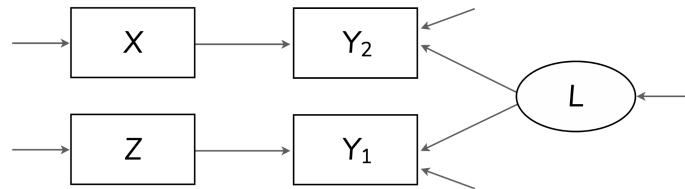
MODEL :

```
y1 ~ 1;
```

```
y2 ~ 1;
```

Blimp has two ways of estimating associations among a set of variables like those above. Whenever possible, the software assigns a multivariate distribution with a mean vector and covariance matrix as the parameters. The covariance matrix approach invokes a Wishart prior distribution. The second approach uses phantom latent factors to correlate dependent variables from different regression equations. Variances and correlations are the parameters of this specification. In a single-level model, the procedure is the “srs” specification described in Merkle and Rosseel (2018), and Blimp extends their approach to two- and three-level models. The phantom variable approach uses a so-called separation strategy (Barnard, McCulloch, & Meng, 2000; Liu,

Zhang, & Grimm, 2016) that assigns distinct priors to the diagonal and off-diagonal elements of the covariance matrix. A path diagram of the underlying model is shown below.



The multivariate structure of this specification consists of variances (or residual variances) and correlations (or residual correlations). If desired, covariances can be obtained by using the PARAMETERS command to define these quantities as auxiliary functions of the estimated parameters. The phantom variable approach can be manually specified by listing the use_phantom keyword on the OPTIONS line.

Like variances, correlations and residual correlations are specified with double-headed arrows or double tildes. The following code block illustrates a simple bivariate analysis with two empty regression models.

MODEL :

```

y1 ~ 1;
y2 ~ 1;
y1 <-> y2;

```

MODEL :

```

y1 ~ 1;
y2 ~ 1;
y1 ~~ y2;

```

The analysis can be specified more succinctly as follows.

MODEL :

```
y1 <-> y2;
```

The same specification applies to correlated residual terms from a multivariate regression.

MODEL :

```
y1 ~ x1 x2 x3;
y2 ~ x1 x2;
y1 <-> y2;
```

Finally, multiple correlations can be specified by listing a set of variables on each side of a double-headed arrow or double tilde. The following code block requests all possible correlations among a set of five variables.

MODEL :

```
y1:y3 x1 x2 <-> y1:y3 x1 x2;
```

Parameter Constraints

Blimp allows for many types of parameter constraints. These restrictions are imposed by listing the @ symbol and a numeric value or label following a variable's name. For example, the following code block uses a label "beta" to specify an equality constraint on X_1 and X_2 's regression slopes.

MODEL :

```
y ~ x1@beta x2@beta x3;
```

As a second example, the code below uses a numeric label to fix the regression intercept to zero during estimation.

MODEL :

```
y ~ 1@0 x1 x2;
```

Similarly, the code below fixes the variance of a variable to 1 during estimation.

MODEL :

```
y ~ x1 x2;
y <-> y@1;
```

Many, but not all model parameters can be constrained. For example, between-group constraints are not permissible when using BYGROUP processing.

Auxiliary Variables

Blimp uses a sequential specification to incorporate auxiliary variables into a model.

Associations among the auxiliary variables and analysis variables follow the same cascading pattern of univariate models used to connect regressors; the first auxiliary is regressed on the analysis variables, the second auxiliary variable is regressed on the first plus the analysis variables, the third is regressed on the first two, and so on. The code block below illustrates a multiple regression analysis with three auxiliary variables, A_1 to A_3 .

MODEL :

```
y ~ x1 x2;
a1 ~ y x1 x2;
a2 ~ a1 y x1 x2;
a3 ~ a1 a2 y x1 x2;
```

The auxiliary models can be specified more succinctly by listing all auxiliary variables on the left side of the same tilde.

MODEL:

```
y ~ x1 x2;  
a3 a2 a1 ~ y x1 x2;
```

Latent Variables

The LATENT command described earlier defines latent variables (e.g., factors in a measurement model) referenced in the MODEL section. To illustrate, the following code block shows a basic measurement model with a single latent factor and three normally distributed indicators (indicators can also be binary or ordinal).

```
LATENT: yfactor;
```

MODEL:

```
yfactor -> y1:y3;
```

By default, Blimp establishes identification by fixing the first factor loading to one and the latent mean (or intercept) to zero. The following code block uses univariate regression equations to achieve an identical specification.

```
LATENT: yfactor;
```

MODEL:

```
yfactor ~ 1@0;  
y1 ~ yfactor@1;  
y2 ~ yfactor;  
y3 ~ yfactor;
```

It may be beneficial to override the default identification settings in some cases. For example, convergence speed may be improved by scaling the latent factor to an indicator with complete data (or the indicator with the least amount of missing data) or fixing one of the regression intercepts instead of the latent mean to 0. To illustrate, the

code block below illustrates a specification with the following features: (a) Y_1 's loading is freely estimated, (b) the latent mean is estimated, (c) Y_3 's measurement intercept is constrained to 0, and (d) Y_3 's loading is constrained to 1.

```
LATENT: yfactor;
MODEL:
# estimate the latent mean
yfactor ~ 1;
# estimate loadings
y1 ~ yfactor;
y2 ~ yfactor;
# fix intercept to 0 and loading to 1
y3 ~ 1@0 yfactor@1;
```

Blimp's univariate modeling framework treats latent factors as incomplete variables to be imputed (adding the `saveLatent` keyword to the `OPTIONS` line saves the estimated latent scores to the imputed data sets). Imputing the latent scores opens up interesting opportunities not available in other software packages. For example, Blimp allows a latent variable to interact with a manifest variable or with another latent variable (Keller, 2021). The following code block illustrates a latent-by-manifest variable interaction.

```
LATENT: xfactor;
MODEL:
xfactor -> x1:x3;
y ~ xfactor z xfactor*z;
```

The manifest variable Z is normal in this example, but it could have any metric that Blimp supports. Similarly, two latent variables can interact with one another. The

following code block illustrates a latent-by-latent interaction involving two factors with three indicators each.

```
LATENT: xfactor mfactor;  
MODEL:  
xfactor -> x1:x3;  
zfactor -> z1:z3;  
y ~ xfactor zfactor xfactor*zfactor;
```

Finally, an outcome variable (manifest or latent) can be a polynomial function of a latent variable. The code below shows a latent variable with a quadratic effect on the outcome.

```
LATENT: xfactor;  
MODEL:  
xfactor -> x1:x3;  
y ~ xfactor (xfactor^2) z;
```

Multilevel Regression Models

Multilevel regression models require relatively few additional specifications beyond those for single-level regression models. Blimp automatically determines the level at which a variable is measured in a multilevel data set, so the user need only provide a basic model specification. The one exception is latent variables, the levels of which must be specified in the LATENT command. Enders et al. (2020) provide specific details about Blimp's multilevel modeling framework, which uses the same factored regression specification outlined for single-level models. Predictor variables can be centered at their grand means or group means (Enders & Tofighi, 2007) using the CENTERING command (discussed later).

Blimp automatically adds random intercepts residuals to all lower-level models (outcomes or predictors) whenever the CLUSTERID command is used to specify a cluster-level identifier variable. To illustrate, consider a two-level regression model where X_1 and X_2 are level-1 and level-2 predictors, respectively. The following code block illustrates a regression model with random intercepts.

```
CLUSTERID: level2id;  
MODEL:  
y ~ x1.i x2.j;
```

The estimated model includes a random intercept in the analysis model as well as in X_1 's supporting model. In some cases, it may be necessary to manually reference the random intercept (e.g., when labeling or constraining the parameter). In the code block below, the 1 to the right of the vertical pipe represents a random intercept.

```
CLUSTERID: level2id;  
MODEL:  
y ~ x1.i x2.j | 1;
```

Random slope coefficients are specified by listing lower-level predictors to the right of a vertical pipe. For example, the code block below illustrates a regression model with random intercepts (implicit) and a random slope for the level-1 predictor X_1 .

```
CLUSTERID: level2id;  
MODEL:  
y ~ x1.i x2.j | x1.i;
```

Blimp estimates an unstructured variance–covariance matrix for the random intercepts and random slopes. Adding the `saveLatent` keyword to the `OPTIONS` line saves the random effect estimates to the imputed data sets.

Random intercepts and slopes can also appear as regressors in other equations. To illustrate, the code block below uses the `RANOMEFFECT` command to define the intercepts and slopes as cluster-level latent variables that predict another variable Z .

```
CLUSTERID: level2id;
RANOMEFFECT:
ranicepts = y | 1 [level2id];
ranslopes = y | x1.i [level2id];
MODEL:
y ~ x1.i x2.j | x1.i;
z ~ ranicepts ranslopes;
```

Blimp can also estimate three-level models. To illustrate, consider a three-level model where X_1 and X_2 are level-1 and level-2 regressors, respectively, and X_3 is a level-3 predictor. As before, Blimp automatically detects the level at which a variable is measured. The following code block illustrates a three-level regression model with random intercepts induced by a pair of cluster-level identifier variables.

```
CLUSTERID: level2id level3id;
MODEL:
y ~ x1.i x2.j x3.k;
```

As a second example, the code block below illustrates a three-level random slope model where the influence of the level-1 regressor X_1 varies across level-2 and level-3 units and the influence of the level-2 predictor X_2 varies across level-3 units.

```
CLUSTERID: level2id level3id;
MODEL:
y ~ x1.i x2.j x3.k | x1.i x2.j;
```

By default, Blimp estimates an unstructured variance-covariance matrix of the random effects at all higher levels of the data hierarchy.

In some situations, it is desirable or necessary to override Blimp's default behavior and fix certain variance components to zero (or alternatively, select which variances get estimated). This is achieved by listing the desired random effects on the right side of the vertical pipe and appending to the effect's name a cluster-level identifier in square brackets. To illustrate, the following code block illustrates a three-level model with random intercepts at both levels and a random coefficient for X_1 at the second level.

```
CLUSTERID: level2id level3id;
MODEL:
y ~ x1.i x2.j x3.k | 1[level2id] 1[level3id] x1[level2id];
```

The resulting variance–covariance matrix at level-2 is an unstructured 2×2 matrix, and the level-3 covariance matrix reduces to a scalar with only a random intercept variance.

Multilevel regression models can also include cluster means as group-level predictors (i.e., contextual effects; Longford, 1989; Raudenbush & Bryk, 2002). Appending the `.mean` keyword to the end of a lower-level covariate's name references that variable's latent group means. To illustrate, the following code block specifies a two-level regression model that includes X_1 as a level-1 predictor and its group means as a level-2 predictor.

```
CLUSTERID: level2id;
MODEL:
y ~ x1 x1.mean x2;
```

Importantly, the group means are cluster-level latent variables rather than deterministic arithmetic averages of the level-1 scores. Methodology research favors a latent variable specification because it can reduce bias associated with arithmetic or “manifest” group means in some scenarios (Hamaker & Muthén, 2019; Lüdtke et al., 2008).

In a three-level model, appending the `.mean` suffix to a level-1 predictor automatically introduces the level-2 and level-3 latent group means as predictors. To specify the group means at one level but not the other, additionally append the cluster-level identifier variable in square brackets. For example, the following code block illustrates a three-level random intercept regression with X_1 's level-3 latent group means as a predictor but not its level-2 averages.

```
CLUSTERID: level2id level3id;
MODEL:
y ~ x1 x1.mean[level3id] x2 x3;
```

Functions Embedded in Equations

Blimp allows users to embed functions inside parentheses on the right side of regression equations and, in limited cases, on the left side as well. As an example, the following code block features a predictor centered at a constant value of 10.

```
MODEL:
y ~ (x1 - 10);
```

The next example uses an embedded function to specify a curvilinear regression where the outcome is a quadratic function of the predictor.

MODEL:

```
y ~ x (x^2);
```

Embedded functions can also reference multiple variables. For example, the following code block defines the predictor variable as the sum of four ordinal variables. A common application occurs with scale scores computed as the sum of several questionnaire items.

```
ORDINAL: x1:x4;
```

MODEL:

```
y ~ (x1 + x2 + x3 + x4);
```

Importantly, the embedded sum function does *not* imply a deterministic computation or passive imputation. Rather, the sum is essentially a random variable that varies as a function of its four components (e.g., items). Any missing data handling uses a Metropolis sampling step to impute the individual components while simultaneously accounting for the fact that the incomplete component is part of a larger summation. The procedure for treating missing data is described in Alacam, Enders, Du, and Keller (2023).

There are two equivalent ways to specify a sum function in a regression equation. The first is to separate the first and last variables in the sum with `:+:` as shown below.

```
ORDINAL: x1:x4;
```

MODEL:

```
y ~ x1+:x4;
```

The second method defines *the function or computation* as a new variable and then uses that new variable in a regression equation. The code block below illustrates this approach, which is equivalent to the previous two.

```
ORDINAL: x1:x4;
MODEL:
xsum = x1+:x4;
y ~ xsum;
```

Again, it is important to emphasize that `xsum` is itself a random variable rather than a deterministic computation or passive imputation.

Although computationally different, the previous sum functions are conceptually equivalent to placing equality constraints on item-level coefficients from the same scale, as follows.

```
ORDINAL: x1:x4;
MODEL:
y ~ x1@beta x2@beta x3@beta x4@beta;
```

Embedded functions can also be part of interactive effects. To illustrate, the following code block shows an interaction between a scale (sum) score involving five items and a continuous moderating variable *M* (manifest or latent).

```
ORDINAL: x1:x4;
MODEL:
y ~ x1+:x4 m ( ( x1+:x4 ) * m );
```

Alternatively, the same analysis can be expressed by first defining the sum function as a new random variable as follows.

```
ORDINAL: x1:x4;
MODEL:
xsum = x1+:x4;
y ~ xsum m xsum*m;
```

Although computationally different, the embedded function above is conceptually equivalent to placing equality constraints on products involving items and the moderator, as follows.

```
ORDINAL: x1:x5;
MODEL:
y ~ x1@beta1 x2@beta1 x3@beta1 x4@beta1 m x1*m@beta3 x2*m@beta3
  x3*m@beta3 x4*m@beta3;
```

Extending the previous idea, the code below shows the interaction between two scale scores, one computed as the sum of four ordinal items and the other computed as the sum of six items.

```
ORDINAL: x1:x4 m1:m6;
MODEL:
y ~ x1+:x4 m1+:m6 (( x1+:x4 ) * ( m1+:m6 ));
```

Alternatively, the same analysis can be expressed by first defining the sum function as a new random variable as follows.

```
ORDINAL: x1:x4;
MODEL:
xsum = x1+:x4;
msum = m1+:m6;
y ~ xsum msum xsum*msum;
```

Blimp also allows embedded functions on the left side of the tilde, but the range of allowable functions is limited (e.g., to basic mathematical operations and transformations). Moreover, the embedded function can only reference a single (dependent) variable. A common application of this functionality involves transformations to a skewed outcome variable. As an example, the following code block applies a natural log transformation to a positively-valued dependent variable.

MODEL :

```
ln(y) ~ x1 x2 x3;
```

As a second example, the code below applies the Yeo-Johnson (Yeo & Johnson, 2000) transformation to a skewed outcome variable.

MODEL :

```
yjt(y) ~ x1 x2 x3;
```

The Yeo-Johnson procedure estimates the shape of the data as the MCMC algorithm iterates and produces imputations from a skewed distribution. The analysis examples in Chapter 3 illustrate the procedure in more detail.

The description of the TRANSFORM command in Section 2.2 outlined Boolean operators (true/false functions) that can be used to recode an existing variable into a new variable. Boolean operators can also be used to define predictors in a regression equation. To illustrate, consider a binary dummy variable that equals 0 if X is less than 50 and 1 if X is greater than or equal to 50. In lieu of using the TRANSFORM command to compute a new variable, listing a Boolean operator as a predictor similarly creates a dummy variable predictor.

MODEL:

```
y ~ (x >= 50);
```

As explained previously, the operator evaluates to 1 or 0 if the condition in parentheses is true or false, respectively. Note that variables created with TRANSFORM are saved to the imputed data sets, whereas variables created via inline functions are not. When X has missing values, the continuous version of the variable links to other predictors, whereas the discrete version predicts Y . The imputation algorithm uses a Metropolis sampling step that allows X to simultaneously function as a continuous and binary variable.

SIMPLE Command

The SIMPLE command is used to request conditional effects (e.g., simple intercepts and simple slopes) from a regression model with an interaction effect. At each MCMC iteration, Blimp computes conditional effects by applying an appropriate contrast vector to the analysis model's regression coefficients. These additional auxiliary parameters thus have their own distribution, credible intervals, et cetera. The PARAMETERS command described next can also be used to compute contrasts.

The code block below shows the basic specification where the SIMPLE command requests the conditional effects of X (the focal predictor) at different values of M (the moderator).

```
CENTER: x m;
```

```
MODEL: y ~ x m x*m;
```

```
SIMPLE: x | m;
```

Multiple sets of conditional effects can be separated by commas

```

CENTER: x m;
MODEL: y ~ x m x*m;
SIMPLE: x | m, m | x;

```

or listed on separate lines, as follows.

```

CENTER: x m;
MODEL: y ~ x m x*m;
SIMPLE:
x | m;
m | x;

```

When a continuous moderator is listed to the right of the vertical pipe, Blimp automatically reports conditional effects at 0, plus or minus 1, and plus or minus 2 standard deviations from 0. We highly recommend centering the focal predictor and moderator such that zero represents the mean. In a multilevel model, the standard deviation is determined by the type of centering. A continuous moderator centered at its group means has only within-cluster variation, so the pooled within-cluster standard deviation is used. A continuous moderator centered at its grand mean has both within-cluster and between-cluster variation, so the total standard deviation is used. The number of standard deviation units can also be specified. For example, the code block below requests the simple slopes of X at one half of a standard deviation above and below the mean of M .

```

CENTER: x m;
MODEL: y ~ x m x*m;
SIMPLE:
x | m@.5SD;
x | m@-.5SD;

```

When a nominal moderator variable is listed to the right of the vertical pipe, Blimp automatically computes and reports conditional effects for every group. When an ordinal variable is listed to the right of the vertical pipe, the pick-a-point score values must be specified. To illustrate, the following code block specifies conditional effects at $M = 0$ and $M = 1$ (e.g., conditional effects at each level of a binary dummy code). The same method identifies specific points for continuous moderators.

```
CENTER: x m;
MODEL: y ~ x m x*m;
SIMPLE:
x | m@0;
x | m@1;
```

The main restriction with the SIMPLE command occurs in models with multiple equations. In this case, a dependent variable in one equation can serve as the moderator in another equation, but the user must specify the values being conditioned on. The code block below shows a specification where M is the dependent variable in one equation and a moderator in the other. The variable M appears to the right of the vertical pipe along with the fixed values to condition in (i.e., default standard deviation units are not an option).

```
CENTER: x m;
MODEL:
m ~ x z;
y ~ x m x*m;
SIMPLE:
x | m@0;
x | m@1;
```

No such specification is necessary if the dependent variable in one equation is the focal predictor in the other (i.e., appears to the left of the vertical pipe). The code block below shows a specification where M is the dependent variable in one equation and the focal predictor in the other. The variable X appears to the right of the vertical pipe, and the output would return the conditional effect of M at standard deviation units above and below X 's mean.

```
CENTER: x m;
MODEL:
m ~ x z;
y ~ x m x*m;
SIMPLE:
m | x;
```

PARAMETERS Command

The PARAMETERS command is used to (a) define auxiliary parameters that are functions of a model's estimated parameters, and (b) specify custom prior distributions. The command uses the same mathematical operators and accesses the same functions as the TRANSFORM command described earlier. Auxiliary parameters are computed by attaching alphanumeric labels to model parameters, then using those labels in an equation that defines a new parameter. Auxiliary parameters are computed at every MCMC iteration, and thus they have their own distributions and summary tables in the output.

As a first example, recall from the MODEL command section that Blimp links dependent variables from separate equations with correlations or residual correlations (instead of covariances). One use of the PARAMETERS command is to compute covariances. The

code block below labels the variances and correlation and uses the labels to compute the covariance, which is the product of the correlation and the standard deviations.

MODEL:

```
y ~~ y@yvar;
x ~~ x@xvar;
y ~~ x@yxcorr;
```

PARAMETERS:

```
yxcov = yxcorr * sqrt(yvar * xvar);
```

As a second example, the following code block labels the three slope coefficients in a moderated regression model and uses the PARAMETERS command to compute the conditional effect of X (i.e., simple slope) at values of $M = 0$ and $M = 1$.

MODEL:

```
y ~ x@beta1 m@beta2 x*m@beta3;
```

```
CENTER: x m;
```

PARAMETERS:

```
m0 = 0;
m1 = 1;
slope.at.m0 = beta1 + m0 * beta3;
slope.at.m1 = beta1 + m1 * beta3;
```

As a final example, the code block below labels pathways from a single-mediator model and uses the PARAMETERS command to compute the product of coefficients estimator (i.e., the product of the X to M and M to Y paths; Mackinnon, 2008).

MODEL:

```
m ~ x@apath;
y ~ x@cpath m@bpath;
```

PARAMETERS:

```
indirect = apath * bpath;
```

The second major use for the `PARAMETERS` command is to introduce custom prior distributions. This functionality is currently restricted to the following list of univariate prior distributions.

- ❖ `normal(mu, var)` or `N(mu, var)` = normal distribution with `mu` as the mean and `var` as the variance
- ❖ `invgamma(a, b)` = inverse gamma distribution with `a` = scale (i.e., α ; prior degrees of freedom $\div 2$) and `b` = shape (i.e., β ; prior sums of squares $\div 2$)
- ❖ `gamma(k, theta)` = gamma distribution with `k` = shape and `theta` = scale
- ❖ `uniform(a, b)` or `unif(a, b)` = uniform distribution with lower bound `a` and upper bound `b`
- ❖ `beta(a, b)` = beta distribution with `a` = α and `b` = β
- ❖ `laplace(mu, b)` = laplace distribution with `mu` = location and `b` = scale
- ❖ `cauchy(a, g)` = cauchy distribution with `a` = location and `g` = scale
- ❖ `truncate(a, b)` or `trunc(a, b)` = truncate function to generate truncated distributions with `a` = lower bound and `b` = upper bound. To obtain one sided truncation, you can set either parameter to `-Inf` or `Inf` for positive and negative infinity.

To illustrate, the following code block shows a simple regression model with informative normal priors on the regression coefficients and an inverse gamma prior for the variance with `a` = 1 and `b` = .5 (i.e., 2 additional degrees of freedom and unit sum of squares). This prior specification for the variance is identical to listing `prior1` on the `OPTIONS` line.

MODEL:

```
y ~ 1@beta0prior x@beta1prior;
y ~~ y@resvarprior;
```

PARAMETERS:

```
beta0prior ~ normal(2,20);
```

```
beta1prior ~ normal(5,10);
resvarprior ~ invgamma(1,.5);
```

In addition to the mathematical operators and functions described in the TRANSFORM command section, the PARAMETERS command can also access the following model-predicted variance expressions. These expressions could be used, for example, to create custom R^2 statistics beyond those included in the default output.

- ❖ `varname.totalvar` = model-predicted total variance of an outcome variable (a variable to the left of a tilde) named `varname`
- ❖ `varname.coefvar` = explained variance in an outcome variable named `varname` by the fixed effects coefficients
- ❖ `varname.slopevar` = explained variance in an outcome variable named `varname` by the fixed effects coefficients via random slopes in a multilevel model
- ❖ `varname.iceptvar` = explained variance in an outcome variable (a variable to the left of a tilde) named `varname` by the random intercepts
- ❖ `varname.residvar` = residual variance in an outcome variable named `varname`

TEST Command

The TEST command is used to perform the Bayesian Wald test described by Asparouhov and Muthén (2021). This command is available in models with a single outcome and multivariate models with several outcomes (e.g., path models). The TEST command can be implemented in a variety of ways. One approach is to label parameters in the MODEL statement and use the TEST command to specify the null hypothesis or condition to be evaluated. The example below illustrates a test of whether two slopes simultaneously equal 0.

MODEL :

```
y ~ x1@b1 x2@b2 x3@b3;  
TEST:  
b1 = 0;  
b2 = 0;
```

Tests of multiple parameters can also be specified using the following shortcut.

```
MODEL:  
y ~ x1@b1 x2@b2 x3@b3;  
TEST:  
b1:b3 = 0;
```

More than one test can be performed by specifying multiple TEST commands. For example, the following code block yields two tests, the first of which involves a single parameter, the second of which involves two slopes.

```
MODEL:  
y ~ x1@b1 x2@b2 x3@b3;  
TEST:  
b1 = 0;  
TEST:  
b2:b3 = 0;
```

The TEST command can also evaluate equality and other types of constraints. For example, the following code block tests an equality constraint on two regression slopes.

```
MODEL:  
y ~ x1@b1 x2@b2 x3@b3;  
TEST:  
b1 = b2;
```


Complex hypotheses are specified by listing multiple conditions in a single TEST command. For example, the following code block evaluates whether one slope differs from 0 and whether two slopes differ from one another.

MODEL :

```
y ~ x1@b1 x2@b2 x3@b3;
```

TEST:

```
b1 = b2;
```

```
b3 = 0;
```

The second way to implement the TEST command is similar to the MODEL statement—it specifies a regression model. However, the model listed on the TEST line must be nested within the model listed on the MODEL statement. The first way to specify the nested model is to exclude parameters from the nested model. The code block below illustrates a comparison involving a full model with three predictors and a restricted model with only an intercept.

MODEL :

```
y ~ x1 x2 x3;
```

TEST:

```
y ~ 1;
```

Alternatively, a nested model can be specified by fixing parameters to desired test values by appending an @ and a numeric label to a variable or effect. For example, the following code block illustrates an equivalent specification that fixes three slope coefficients to 0.

MODEL :

```
y ~ x1 x2 x3;
```

TEST:

```
y ~ x1@0 x2@0 x3@0;
```

The TEST command produces the output table below. The Wald test statistic is a chi-square variable, and the test's degrees of freedom equals the number of parameters by which the two models differ. The probability value is *not* a frequentist *p*-value because it makes no reference to test statistics from other random samples. Rather, the probability is an index of support for the proposed constraints, where support is defined as the area above the test statistic in a chi-square distribution.

MODEL FIT:

Asparouhov & Muthén Wald Tests

Test #1

Wald Statistic (Chi-Square)	133.705
Number of Parameters Tested (df)	3
Probability	0.000

The TEST command can also compare nested models with different variances. To illustrate, the code block below shows a two-level model with random coefficients, where the TEST command is used to specify a random intercept model with two fewer parameters.

```
CLUSTERID: level2id;
```

```
MODEL:
```

```
y ~ x1 x2 x3 | x1;
```

```
TEST:
```

```
y ~ x1 x2 x3 | x1@0;
```

FCS Command

The FCS command invokes a fully conditional specification multiple imputation (FCS–MI) approach similar to that described by Stef van Buuren and colleagues (van Buuren, 2007; van Buuren, Brand, Groothuis-Oudshoorn, & Rubin, 2006). This command cannot be used in conjunction with the MODEL command. Rather, FCS deploys an MCMC algorithm that cycles through incomplete variables one at a time, imputing each variable from an additive equation that features the incomplete variable regressed on all other variables listed on the FCS line. This algorithm makes no distinction between outcomes and regressors in the subsequent analysis model; all entities listed on the FCS line are simply variables to be imputed or complete variables that contribute to imputation. The SAVE command outputs the filled-in data sets for reanalysis using frequentist methods (Rubin, 1987). FCS–MI is known to introduce bias when applied to analysis models with nonlinear terms such as interactions, polynomial effects, or random coefficients. The model-based imputation routines illustrated in Chapter 3 are far superior.

To illustrate FCS–MI, consider a simple scenario with one continuous variable *X*, one binary dummy variable *D*, and one 7-category ordinal variable *O*. The code block below shows a basic script (which could also include nominal variables).

```
DATA: data.dat;  
VARIABLES: id a1:a5 x d o z;  
ORDINAL: d o;  
MISSING: 999;  
FCS: x d o;  
NIMPS: 100;  
CHAINS: 100;
```

```

BURN: 1000;
ITERATIONS: 10000;
OPTIONS: savelatent;
SAVE: stacked = imps.dat;

```

At a minimum, the FCS command should include all variables and effects of interest in the analysis model(s), but the list may also include additional auxiliary variables. The commands following the FCS line in the script are described later in this section.

Blimp's FCS-MI routine primarily differs from the classic MICE (Multiple Imputation by Chained Equations; van Buuren & Groothuis-Oudshoorn, 2011) approach in two ways. First, Blimp's algorithm is a true Gibbs sampler; this is a small technical nuance that makes no difference in practice. Second, Blimp adopts a fully latent specification for all categorical variables. As noted previously, Blimp uses a probit regression framework that views discrete scores as arising from one or more normally distributed latent response variables (or latent response difference scores in the case of multicategorical nominal variables). Applied to the previous example, the binary dummy variable D and the 7-category ordinal variable O have corresponding latent response variables D^* and O^* , respectively. Blimp's FCS-MI routine uses the latent variables both as predictors and as outcomes. The round robin imputation models for this example are as follows.

$$X = \mu_X + \gamma_{11} (D^* - \mu_{D^*}) + \gamma_{21} (O^* - \mu_{O^*}) + r_1$$

$$D^* = \mu_{D^*} + \gamma_{12} (O^* - \mu_{O^*}) + \gamma_{22} (X - \mu_X) + r_2$$

$$O^* = \mu_{O^*} + \gamma_{13} (X - \mu_X) + \gamma_{23} (D^* - \mu_{D^*}) + r_3$$

The latent response models also incorporate threshold parameters that divide the latent distributions into discrete segments, and the residual variances of r_2 and r_3 are fixed at 1 to establish the latent variable metrics.

Listing the `savelatent` keyword on the `OPTIONS` line saves both the discrete and latent response variables to the imputed data files (by default, only the discrete imputes are written to the imputed data files). The imputed latent scores (plausible values) could be used in lieu of the discrete scores in a subsequent analysis. For example, the analysis in Section 5.7 illustrates an item-level factor analysis that uses imputed latent response scores. In a similar vein, Muthén and Asparouhov (2016) describe an application that replaces a binary mediator with a latent response variable. As an aside, the `savelatent` keyword can also be used in conjunction with imputations generated by the `MODEL` command.

If desired, listing the `mice` and `manifest` keywords on the `OPTIONS` line alters Blimp's default behaviors and invokes an algorithm that is equivalent to the one in the MICE package in R (van Buuren & Groothuis-Oudshoorn, 2011). In addition to a slight algorithmic modification, this specification uses discrete variables as predictors on the right side of equations. The round robin imputation models for this example are as follows.

$$X = \gamma_{01} + \gamma_{11}D + \gamma_{21}O + r_1$$

$$D^* = \gamma_{02} + \gamma_{12}O + \gamma_{22}X + r_2$$

$$O^* = \gamma_{03} + \gamma_{13}X + \gamma_{23}D + r_3$$

The MICE package deploys logistic rather than probit models for categorical variables, but this distinction tends to make little difference in practice.

The multilevel version of fully conditional specification (Enders et al., 2018) automatically introduces the latent group means of all lower-level variables in the imputation model (i.e., latent contextual effects); this is true for both continuous and latent response variables. Including the group means in the imputation model allows all between-cluster associations to vary independently of the within-cluster associations. Listing the `noclmean` keyword on the `OPTIONS` line removes the latent group means from the regression models, producing a more restrictive imputation model where the within- and between-cluster regressions are assumed to be equal. For two-level models, a heterogeneous level-1 variance structure is invoked by listing the `hev` keyword on the `OPTIONS` line. This method is described in Kasim and Raudenbush (1998).

BURN Command

The BURN command specifies the number of burn-in iterations. Bayesian analysis results summarize estimates taken from iterations following the burn-in period, and multiple imputations (via FCS or MODEL) are saved after the burn-in period. To illustrate, the following code block illustrates a 5,000-iteration burn-in period.

```
BURN: 5000;
```

The number of burn-in iterations should always be determined by examining the potential scale reduction factor diagnostic (Gelman & Rubin, 1992) from the Blimp output. Material at the beginning of Chapter 3 describes how to use these diagnostics.

ITERATIONS Command

The ITERATIONS command specifies the number of iterations after the burn-in period. The tabular summaries reflect Bayesian analysis results taken from the post burn-in period. To illustrate, the following code block specifies 10,000 MCMC iterations following an initial burn-in period of 5,000 iterations.

```
BURN: 5000;  
ITERATIONS: 10000;
```

Note that the total number of iterations is distributed equally across the number of MCMC chains, the default value of which is two (see the CHAINS command). In our experience, 10,000 iterations is usually more than sufficient, but material at the beginning of Chapter 3 describes how to verify that this is the case.

CHAINS Command

The CHAINS command is used to specify the number of MCMC processes (and optionally, the number of processors used for computation). The default number of chains is two, and the total number of computational cycles specified on the ITERATIONS line is always divided equally across chains. By default, Blimp attempts to distribute MCMC chains across physical cores, resulting in faster computation (e.g., on a 10-core machine, specifying 10 chains would automatically assign one MCMC process per core). Because Blimp automatically uses the maximum available cores, this specification would primarily be used to specify fewer resources. For example, the code block below specifies 10,000 iterations spread across 10 unique MCMC chains. The MCMC processes are completed sequentially using two physical cores.

```
ITERATIONS: 10000;  
CHAINS: 10 processors 2;
```

By default, each chain will have a different seeding value and different random starting values. Random starting values can be disabled by specifying the `norandomstarts` keyword on the `OPTIONS` line.

NIMPS Command

The NIMPS command is used to specify the desired number of multiple imputation data sets to save during MCMC estimation (saving imputed data sets is optional). Graham, Olchowski, and Gilreath (2007) suggest using at least 20 imputed data sets to maximize power, and other studies have shown that 100 or more imputations may be necessary to reduce the impact of Monte Carlo simulation error on standard errors and get precise estimates of confidence interval half-widths and probability values (Bodner, 2008; Harel, 2007; von Hippel, 2018). Imputations can be saved at regular intervals during a single MCMC chain, at the final iteration of multiple MCMC processes, or some combination of the two. The code block below saves 100 imputed data sets from the final iteration of 100 MCMC chains, each with 5,000 burn-in iterations and 100 iterations thereafter (i.e., 10,000 total iterations spread across 100 MCMC processes).

```
BURN: 5000;  
ITERATIONS: 10000;  
NIMPS: 100;  
CHAINS: 100;
```


THIN Command

The THIN command is used to specify the between-imputation interval when saving multiple imputations from the same MCMC chain. For example, the following code block deploys two MCMC chains (the default) that create 100 filled-in data sets by saving imputations every 1,000 iterations after the 5,000-iteration burn-in period.

```
NIMPS: 100;  
BURN: 5000;  
THIN: 1000;
```

Saving multiple imputations is optional, and this command is not necessary; however, either THIN or ITERATIONS must be specified when saving filled-in data sets. The THIN command has no impact on printed parameter summaries, which are always based on the post burn-in iterations.

OPTIONS Command

The following keywords are used in conjunction with either the FCS or MODEL commands. Bolded keywords are default and do not require explicit specification.

- ❖ `prior1/prior2/prior` = Three common prior distributions for the residual variances and covariances of dependent variables; `prior1` is more informative because it adds to the degrees of freedom and sums of squares, `prior2` is less informative because subtracts from the degrees of freedom, and `prior3` has zero degrees of freedom and adds zero to the sums of squares
- ❖ `xprior1/xprior2/xprior3` = Three common prior distributions for the residual variances of predictor variables with unspecified associations
- ❖ `psr/nopsr` = Compute the potential scale reduction factor diagnostic
- ❖ `hov/hev` = homogenous versus heterogeneous within-cluster variances

- ❖ **randomstarts/norandomstarts** = Enable/disable random starting values for different MCMC chains
- ❖ **listwise** = Enable listwise deletion (off by default).
- ❖ **saveVariableNames** or **saveVarNames** = write variable names as column headers when saving imputed data sets.

The following keywords are used in conjunction with the FCS command to alter the behavior of the fully conditional specification imputation algorithm.

- ❖ **mice** = classic mice algorithm instead of a Gibbs sampler
- ❖ **manifest** = manifest categorical rather than latent response variables as predictors in the imputation model
- ❖ **noclmean** = exclude latent cluster means from level-2 (and level-3) imputation models

The following keywords are used in conjunction with the SAVE command to alter the composition of imputed data files.

- ❖ **saveLatent** = save factor or latent variable scores (measurement models), random effects (two-level models), latent response variables (categorical variables), and normalized values from the Yeo-Johnson transformation
- ❖ **savepredicted** = save the predicted values of continuous outcomes, predicted probabilities for binary and nominal outcomes, and predicted latent response variable scores for ordinal outcomes
- ❖ **saveresidual** = save residuals (or within cluster residuals)
- ❖ **csv** = save data sets as comma separated .csv files (instead of space delimited .dat files) and write variable names as column headers

OUTPUT Command

The OUTPUT command is used to customize the printed parameter summaries. By default, Blimp prints the posterior median, posterior standard deviation, 95% credible

interval limits, split chain potential scale reduction factor, and effective number of MCMC samples (estimated number of independent MCMC iterations using split chain approach) for each parameter. Listing any of the following keywords on the OUTPUT command overrides Blimp's default output tables with new tables containing the requested quantities.

- ❖ `default` = posterior median, standard deviation, 95% credible interval, split chain potential scale reduction factor, effective number of MCMC samples
- ❖ `default_mean` = posterior mean, standard deviation, 95% credible interval, split chain potential scale reduction factor, effective number of MCMC samples
- ❖ `default_median` = posterior median, posterior median absolute deviation (scaled to be same metric as std. dev.), 95% credible interval, split chain potential scale reduction factor, effective number of MCMC samples
- ❖ `mean` = posterior mean
- ❖ `median` = posterior median
- ❖ `stddev` = posterior standard deviation
- ❖ `mad_sd` = posterior median absolute deviation (scaled to be same metric as the standard deviation)
- ❖ `quant` = 2.5%, 25%, 50%, 75%, 97.5% quantiles
- ❖ `quant50` = 25% and 50% quantiles
- ❖ `quant95` = 2.5% and 97.5% quantiles
- ❖ `psr` = potential scale reduction factor computed after the burn-in period
- ❖ `n_eff` = print effective number of MCMC samples
- ❖ `mcmc_se` = print MCMC simulation standard error

To illustrate, the code block below creates a custom table displaying only the median, a set of quantiles (2.5%, 25%, 50%, 75%, and 97.5%), and potential scale reduction factors computed following the burn-in period.

```
OUTPUT: median quant psr;
```

The code block below specifies Blimp's default output with the additional quantities.

```
OUTPUT: default median quant psr;
```

SAVE Command

The SAVE command is used to save byproducts of MCMC estimation. The principal use for this command is to save multiply imputed data sets, but the command also saves parameter estimates from the burn-in and post burn-in iterations, posterior summaries, and potential scale reduction factors. Unless a full file path is specified, Blimp saves the specified files to the directory that contains the input script.

Multiple imputations can be saved in three different formats: (a) as separate data files (ideal for analysis in *Mplus* or HLM), (b) in a single stacked file with an additional identifier variable that indexes imputations (ideal for analysis in R, SPSS, and SAS), and (c) a single stacked file that includes the original data indexed with a zero value (ideal for analysis in Stata). The following code block illustrates all three specifications.

```
SAVE:  
separate = imp*.dat;  
stacked = imps.dat;  
stacked0 = imps0.dat;
```

When saving imputations to separate files, the asterisk in the file path is replaced with an integer in the file name (e.g., specifying `imp*.dat` produces imputed data sets named `imp1.dat`, `imp2.dat`, `imp3.dat`, et cetera). The separate-file specification

also generates a text file that contains the names of the individual data files (this file functions as the input data when analyzing imputations in *Mplus*).

The imputed data sets include all variables from the input data (regardless of whether they were used in an analysis or imputation routine) along with the values of any latent variables, predicted scores, and residuals specified on the `OPTIONS` line (the `savelatent`, `savepredicted`, and `saveresidual` keywords). The stacked format adds a variable to the first column of the data that indexes the data sets. The order of the variables in the imputed data sets is listed at the bottom of the Blimp output. The output excerpt below provides an illustration.

VARIABLE ORDER IN IMPUTED DATA:

```
stacked = 'imps.dat'
```

```
imp# id n1 d1 o1 y x1 d2 x2 x3
```

In addition to creating imputed data sets, the `SAVE` command can produce files containing the estimated parameters for burn-in iterations (`burn = filename;`), estimated parameters for the post burn-in iterations (`iterations = filename;`), posterior summaries of the parameter estimates as they appear on the Blimp output (`estimates = filename;`), starting values (`starts = filename;`), the potential scale reduction factor values for all parameters (`psr = filename;`), the Bayesian Wald test statistic (`waldtest = filename;`), and the average imputation across the post burn-in iterations (`avgimp = averageimps.dat;`). The code block below illustrates these options.

SAVE:

```
burn = burnin.dat;  
iterations = iterations.dat;  
estimates = estimates.dat;  
starts = starts.dat;  
psr = psr.dat;  
waldtest = wald.dat;  
avgimp = averageimps.dat;
```

When using multiple MCMC chains, chain-specific quantities can be saved by specifying an asterisk in the filename. Blimp replaces this symbol in the filename with a numeric value that indexes the chains. The following code block illustrates this specification.

SAVE:

```
burn = burnin*.dat;  
iterations = iterations*.dat;  
estimates = estimates.dat;  
starts = starts.dat;  
psr = psr.dat;  
avgimp = averageimps.dat;
```

Parameter summaries and starting values are saved in a single file regardless of the number of MCMC chains used for computations.

3 Diagnosing Convergence and Specifying the Number of Iterations

Diagnosing the MCMC algorithm's convergence and determining the total number of computational cycles is an important part of any analysis. The initial burn-in (trial) period should be long enough for the algorithm to achieve independence from its random starting values and achieve a steady state (i.e., converge in distribution); the total number of iterations after the burn-in period should be large enough to provide adequate precision. This section describes this process of determining these two quantities. These steps are applicable to any analysis, including all the ensuing examples. Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex3a.imp](#) [Ex3b.imp](#) [data8.dat](#)

The first step in an analysis is to perform a preliminary diagnostic run to determine the length of the burn-in period. This initial period should be long enough for the MCMC algorithm to converge. As a starting point, we find it useful to specify 10,000 burn-in cycles for the preliminary analysis. The code block below estimates a two-level random coefficient model (see Example 6.3) with this setting on the BURN line. The default number of chains is two, and the number of iterations after the burn-in period (the ITERATIONS line) is not important at this point.

```
DATA: data8.dat;  
VARIABLES: level1id level2id x1.i x2.i y.i x3.i x4.i d1.j  
              n1.j x5.j x6.j x7.j x8.j x9.j;  
CLUSTERID: level2id;  
ORDINAL: d1.j;
```

```

MISSING: 999;
FIXED: d1.j;
CENTER:
groupmean = x1.i;
grandmean = x2.i x7.j d1.j;
MODEL: y.i ~ x1.i x2.i x7.j d1.j | x1.i;
SEED: 90291;
BURN: 10000;
ITERATIONS: 10000;
OPTIONS: labels;

```

Blimp divides the burn-in period into 20 equal segments and computes the split-chain potential scale reduction factor (Gelman et al., 2014) at the end of each interval. The table below shows the highest (worst) potential scale reduction factor across all model parameters.

BURN-IN POTENTIAL SCALE REDUCTION (PSR) **OUTPUT:**

NOTE: Split chain PSR is being used. This splits each chain's iterations to create twice as many chains.

Comparing iterations across 2 chains	Highest PSR	Parameter #
251 to 500	1.461	12
501 to 1000	1.303	13
751 to 1500	1.157	13
1001 to 2000	1.313	5
1251 to 2500	1.085	13
1501 to 3000	1.055	5
1751 to 3500	1.096	13
2001 to 4000	1.090	13
2251 to 4500	1.051	13
2501 to 5000	1.024	13
2751 to 5500	1.020	13
3001 to 6000	1.015	13
3251 to 6500	1.041	5
3501 to 7000	1.011	5
3751 to 7500	1.015	8

4001 to 8000	1.009	3
4251 to 8500	1.030	8
4501 to 9000	1.028	14
4751 to 9500	1.032	8
5001 to 10000	1.024	8

The table shows that the index drops to acceptable levels (e.g., less than 1.05, where 1 is the theoretical minimum) by iteration 5,000. A good rule of thumb is to set the burn-in period for the final run to a value at least as large as 5,000. If the value in the bottom row of the table (the final checkpoint) exceeds 1.05, increase the number of burn-in iterations (e.g., to 20,000) and rerun the model.

The potential scale reduction factor table indicates that the highest (worst) values prior to convergence are primarily associated with parameter numbers 13 and 5. Listing the optional `label`s keyword on the `OPTIONS` line prints a table of potential scale reduction factors for all model parameters along with their numeric indices. In some cases (e.g., latent variable models), very high potential scale reduction factors will be associated with standardized regression weights (e.g., due to scaling constraints). In general, these can be ignored, and the focus should be on the unstandardized parameters. The table for the focal regression model is shown below (unspecified predictor models also have similar tables). The table indicates that parameter numbers 13 and 5 correspond to the standardized coefficient for a level-2 predictor and the intercept, respectively. The columns of the table give the potential scale reduction factors for the final five checkpoints during the burn-in period.

PARAMETER LABELS:

Printing out PSR for last 5 comparisons:

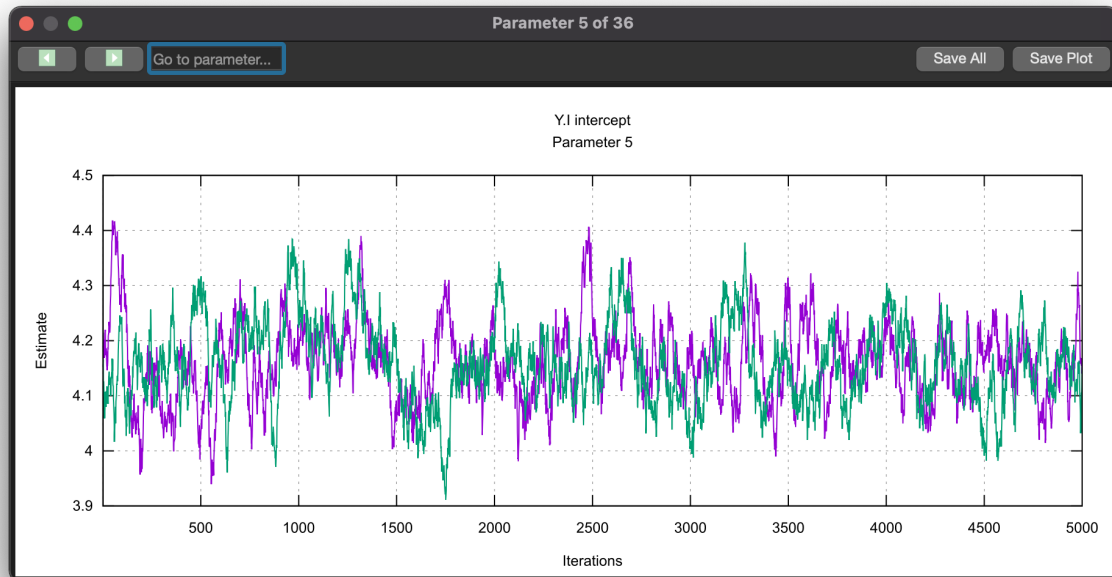
NOTE: Split chain PSR is being used. This splits each chain's iterations to create twice as many chains.

Comparing iterations across 2 chains

[1]	4001 to 8000
[2]	4251 to 8500
[3]	4501 to 9000
[4]	4751 to 9500
[5]	5001 to 10000

	[1]	[2]	[3]	[4]	[5]
Outcome Variable: y.i					
Variances:					
1 L2 : Var(Intercept)	1.00	1.00	1.00	1.00	1.00
2 L2 : Cov(x1.i,Intercept)	1.00	1.00	1.00	1.00	1.00
3 L2 : Var(x1.i)	1.01	1.01	1.02	1.01	1.00
4 Residual Var.	1.00	1.00	1.00	1.00	1.00
Coefficients:					
5 Intercept	1.01	1.00	1.01	1.01	1.01
6 x1.i	1.00	1.00	1.00	1.00	1.00
7 x2.i	1.00	1.00	1.00	1.00	1.00
8 x7.j	1.01	1.03	1.03	1.03	1.02
9 d1.j	1.00	1.00	1.00	1.00	1.00
Standardized Coefficients:					
10 x1.i	1.00	1.00	1.00	1.00	1.00
11 x2.i	1.00	1.00	1.00	1.00	1.00
12 x7.j	1.01	1.03	1.03	1.03	1.02
13 d1.j	1.00	1.00	1.00	1.00	1.00
Proportion Variance Explained					
14 by Coefficients	1.01	1.02	1.03	1.02	1.01
15 by Level-2 Random Intercepts	1.00	1.00	1.00	1.00	1.00
16 by Level-2 Random Slopes	1.01	1.01	1.02	1.01	1.00
17 by Level-1 Residual Variation	1.00	1.00	1.00	1.00	1.00

A trace plot of the intercept estimates from the first 5,000 computational cycles is shown below. Plot features such as the number of chains or iterations printed can be set in the Blimp Studio > Preferences pull-down menu. Plotting can also be turned off completely in these settings (this can reduce post-processing time considerably).



The next step is to set the burn-in period and total number of iterations for the final analysis. We find it useful to specify 10,000 iterations following the initial burn-in period, which for this example we set at 5,000 based on the preliminary diagnostic run. The code block below reflects these settings on the BURN and ITERATIONS line. The labels keyword and OPTIONS line are no longer needed.

```
DATA: data8.dat;
VARIABLES: level1id level2id x1.i x2.i y.i x3.i x4.i d1.j
              n1.j x5.j x6.j x7.j x8.j x9.j;
CLUSTERID: level2id;
```

```

ORDINAL: d1.j;
MISSING: 999;
FIXED: d1.j;
CENTER:
groupmean = x1.i;
grandmean = x2.i x7.j d1.j;
MODEL: y.i ~ x1.i x2.i x7.j d1.j | x1.i;
SEED: 90291;
BURN: 5000;
ITERATIONS: 10000;

```

The Blimp output tables include point estimates and measures of uncertainty (posterior median and standard deviation), 95% credible interval limits, potential scale reduction factors for the iterations following the burn-in period, and the effective number of MCMC samples. The output tables generally include a section for variances, coefficients, standardized estimates, and variance explained effect sizes (Rights & Sterba, 2019).

OUTCOME MODEL ESTIMATES:

Summaries based on 10000 iterations using 2 chains.

Outcome Variable: y.i

Grand Mean Centered: d1.j x2.i x7.j

Group Mean Centered: x1.i

Parameters	Median	StdDev	2.5%	97.5%	PSR	N_Eff

Variances:						
L2 : Var(Intercept)	0.619	0.083	0.484	0.811	1.001	4793.680
L2 : Cov(x1.i,Intercept)	0.013	0.016	-0.018	0.045	1.000	1569.242
L2 : Var(x1.i)	0.020	0.006	0.011	0.034	1.002	715.523
Residual Var.	0.358	0.011	0.336	0.382	1.000	4620.046
Coefficients:						
Intercept	4.168	0.070	4.030	4.305	1.015	169.900
x1.i	-0.094	0.019	-0.132	-0.056	1.000	1841.459
x2.i	0.086	0.008	0.071	0.102	1.000	2622.345

x7.j	0.056	0.066	-0.072	0.194	1.025	149.321
d1.j	-0.104	0.150	-0.398	0.193	1.003	143.175
Standardized Coefficients:						
x1.i	-0.094	0.020	-0.133	-0.056	1.001	1870.786
x2.i	0.184	0.019	0.148	0.222	1.001	2510.830
x7.j	0.054	0.064	-0.069	0.183	1.025	151.449
d1.j	-0.050	0.071	-0.188	0.093	1.003	143.392
Proportion Variance Explained						
by Coefficients	0.061	0.017	0.038	0.105	1.011	186.000
by Level-2 Random Intercepts	0.580	0.034	0.515	0.646	1.001	2791.706
by Level-2 Random Slopes	0.020	0.006	0.011	0.034	1.003	655.997
by Level-1 Residual Variation	0.335	0.027	0.281	0.389	1.002	1828.375

The rightmost column of the table—the effective number of MCMC samples—is essentially the number of independent estimates on which the parameter summaries are based after removing autocorrelations from the MCMC process. Gelman et al. (2014, p. 287) recommend values greater than 100. All values in the example table exceed this recommended minimum. Increasing the total number of iterations would provide more precise summaries.

4 Analysis Examples: Regression Models

The analysis examples in this chapter primarily illustrate different types of univariate regression models. Univariate regressions are the basic building blocks of more complicated multivariate and latent variable models, which are just collections of univariate equations. In general, it is possible to mix and match features from any examples to easily create complex analysis models that honor features of the data. The examples use a generic notation system where variable names usually consist of an alphanumeric prefix and a numeric suffix (e.g., Y_1 , X_1 , N_1 , D_1 , D_2 , V_1 , V_2 , V_3). The letter Y designates a dependent variable, a D prefix denotes a binary dummy variable, an O prefix indicates an ordinal variable, and an N prefix indicates a multicategorical nominal variable. Other letters generally represent continuous variables. Finally, the model equations use a “cgm” superscript to indicate grand mean centering. The following list outlines the examples in this section. The following list outlines the examples in this section.

- ❖ 4.1: Correlations and Descriptive Statistics
- ❖ 4.2: Polychoric Correlations With Latent Response Variables
- ❖ 4.3: Linear Regression
- ❖ 4.4: Model-Based Multiple Imputation
- ❖ 4.5: Linear Regression With Nominal Predictors
- ❖ 4.6: Fully Conditional Specification Multiple Imputation
- ❖ 4.7: Regression With Auxiliary Variables
- ❖ 4.8: Linear Regression With an Interaction
- ❖ 4.9: Multiple Imputation Within Subgroups
- ❖ 4.10: Curvilinear Regression

- ❖ 4.11: Probit Regression With a Binary Outcome
- ❖ 4.12: Probit Regression With an Ordinal Outcome
- ❖ 4.13: Logistic Regression With a Binary Outcome
- ❖ 4.14: Logistic Regression With a Multicategorical Outcome
- ❖ 4.15: Negative Binomial Regression With a Count Outcome
- ❖ 4.16: Linear Regression With Scale Scores
- ❖ 4.17: Linear Regression With Scale Score Interaction
- ❖ 4.18: Skewed Predictor and Yeo-Johnson Transform
- ❖ 4.19: Skewed Outcome and Yeo-Johnson Transform
- ❖ 4.20: Bayesian Wald Test
- ❖ 4.21: Propensity Score Estimation With Missing Data
- ❖ 4.22: Linear Regression With Sampling Weights

4.1: Correlations and Descriptive Statistics

This example illustrates correlations and descriptive statistics. Blimp has two ways of estimating associations among a set of variables. Whenever possible, the software assigns a multivariate distribution with a mean vector and covariance matrix as the parameters. The covariance matrix approach invokes a Wishart prior distribution. The second approach uses phantom latent factors to correlate dependent variables from different regression equations. Variances and correlations are the parameters of this specification. In a single-level model, the procedure is the “srs” specification described in Merkle and Rosseel (2018), and Blimp extends their approach to two- and three-level models. A path diagram of the underlying model is shown in the MODEL section of Chapter 2. The phantom variable approach uses a so-called separation strategy (Barnard, McCulloch, & Meng, 2000; Liu, Zhang, & Grimm, 2016) that assigns distinct priors to the diagonal and off-diagonal elements of the covariance matrix.

Computer simulation studies suggest that the separation strategy gives more accurate estimates of the variance components, although the correlation estimate may be attenuated when the number of level-2 units is small (Keller & Enders, 2021).

Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex4.1a.imp](#) [Ex4.1b.imp](#) [Ex4.1c.imp](#) [data1.dat](#)

The following code block uses the default approach to estimate means, covariances, and correlations.

```
DATA: data1.dat;  
VARIABLES: id n1 d1 y1 y2 x1 d2 x2 x3;  
MISSING: 999;  
MODEL:  
x1 y1 y2 <-> x1 y1 y2;  
SEED: 90291;  
BURN: 10000;  
ITERATIONS: 10000;
```

Adding the `use_phantom` keyword to the `OPTIONS` line invokes the phantom variable approach with separation priors.

```
DATA: data1.dat;  
VARIABLES: id n1 d1 y1 y2 x1 d2 x2 x3;  
MISSING: 999;  
MODEL:  
x1 y1 y2 <-> x1 y1 y2;  
SEED: 90291;  
BURN: 10000;
```



```
ITERATIONS: 10000;  
OPTIONS: use_phantom;
```

Finally, the code block below labels variance parameters and uses the **PARAMETERS** command to compute standard deviations. Omitting the `use_phantom` argument defaults to a standard multivariate model.

```
DATA: data1.dat;  
VARIABLES: id n1 d1 y1 y2 x1 d2 x2 x3;  
MISSING: 999;  
MODEL:  
x1 y1 y2 <-> x1 y1 y2;  
x1 <-> x1@varx1;  
y1 <-> y1@vary1;  
y2 <-> y2@vary2;  
PARAMETERS:  
sd.x1 = sqrt(varx1);  
sd.y1 = sqrt(vary1);  
sd.y2 = sqrt(vary2);  
SEED: 90291;  
BURN: 10000;  
ITERATIONS: 10000;
```

4.2: Polychoric Correlations With Latent Response Variables

This example illustrates polychoric correlations among continuous variables and latent response scores from binary and ordinal variables. Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex4.2.imp](#) [data1.dat](#)

The syntax highlights are as follows.

- ❖ ORDINAL command identifies binary and ordinal variables
- ❖ Longer burn-in period for estimating threshold parameters

```

DATA: data1.dat;
VARIABLES: id n1 d1 o1 y1 x1 d2 x2 x3;
ORDINAL: d1 o1;
MISSING: 999;
MODEL:
d1 o1 y1 x1 <-> d1 o1 y1 x1;
SEED: 90291;
BURN: 25000;
ITERATIONS: 10000;

```

4.3: Linear Regression

This example illustrates a linear regression analysis. Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex4.3.imp](#) [data1.dat](#)

The model features a pair of continuous predictors and a binary dummy code, as follows. The *cgm* superscript denotes variables centered at their grand means.

$$Y = \beta_0 + \beta_1 X_1^{cgm} + \beta_2 X_2^{cgm} + \beta_3 D_2 + \varepsilon$$

The syntax highlights are as follows.

- ❖ ORDINAL command identifies a binary predictor
- ❖ FIXED command identifies a complete predictor
- ❖ CENTER command applies grand mean centering to predictors
- ❖ Unspecified associations for predictor variables

```
DATA: data1.dat;  
VARIABLES: id n1 d1 o1 y x1 d2 x2 x3;  
ORDINAL: d2;  
MISSING: 999;  
FIXED: d2;  
CENTER: x1 x2;  
MODEL: y ~ x1 d2 x2;  
SEED: 90291;  
BURN: 1000;  
ITERATIONS: 10000;
```

4.4: Model-Based Multiple Imputation

Blimp can save multiple imputations from any model it estimates. This example illustrates a model-based multiple imputation procedure tailored around the linear regression model from Example 4.3. Clicking the links below downloads the scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex4.4.imp](#) [Ex4.4.R](#) [data1.dat](#)

The syntax highlights are as follows.

- ❖ **ORDINAL** command identifies a binary predictor
- ❖ **FIXED** command identifies a complete predictor
- ❖ **CENTER** command grand mean centers predictors in the Bayesian output, saved imputations are on the original metric
- ❖ Unspecified associations for predictor variables
- ❖ **NIMPS** command specifies 20 imputed data sets

- ❖ Setting CHAINS equal to NIMPS saves one data set from the final iteration of each MCMC chain (avoids autocorrelated imputations)
- ❖ Imputations are stacked in a single file with an index variable added in the first column

```

DATA: data1.dat;
VARIABLES: id n1 d1 o1 y x1 d2 x2 x3;
ORDINAL: d2;
MISSING: 999;
FIXED: d2;
CENTER: x1 x2;
MODEL: y ~ x1 d2 x2;
SEED: 90291;
BURN: 1000;
ITERATIONS: 10000;
CHAINS: 20;
NIMPS: 20;
SAVE: stacked = imps.dat;

```

Blimp lists the order of the variables in the imputed data sets at the bottom of the output file, and all variables in the input file appear in the output file regardless of whether they were imputed.

VARIABLE ORDER IN IMPUTED DATA:

```
stacked = 'imps.dat'
```

```
imp# id n1 d1 o1 y x1 d2 x2 x3
```

The imputed data sets can be analyzed in other software packages. For example, the script below uses the R package `mi tml` (Grund, Robitzsch, & Lüdke, 2021) to fit the

linear regression model to the filled-in data sets. The resulting estimates are numerically equivalent to the Bayesian results from Example 4.3.

```
# set working directory
fdir::set()

# read data from working directory
imps <- read.table("imps.dat")
names(imps) <- c("imputation","id","n1","d1","o1","y","x1",
  "d2","x2","x3")

# center predictors
imps$x1.cgm <- imps$x1 - mean(imps$x1)
imps$x2.cgm <- imps$x2 - mean(imps$x2)

# analysis and pooling with mitml
implist <- mitml::as.mitml.list(split(imps, imps$imputation))
results <- with(implist, lm(y ~ x1.cgm + d2 + x2.cgm))
mitml::testEstimates(results, extra.pars = T, df.com = 626)
```

4.5: Linear Regression With Nominal Predictors

This example illustrates a linear regression model with a multicategorical nominal predictor. Clicking the links below downloads the scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex4.4.imp](#) [data2.dat](#)

The regression model is

$$Y = \beta_0 + \beta_1 X_1^{cgm} + \beta_2 D_1 + \beta_3 N_{1.2} + \beta_3 N_{1.3} + \beta_3 N_{1.4} + \varepsilon_i$$

where Y is a continuous outcome, X_1 is a continuous predictor, D_1 is a dummy code, and $N_{1,2}$, $N_{1,3}$, and $N_{1,4}$ are dummy codes that represent a four-category nominal predictor ($N_1 = 1, 2, 3, 4$). The *cgm* superscript denotes variables centered at their grand means. The syntax highlights are listed below, and adding the NIMPS and SAVE commands generates model-based multiple imputations for a frequentist analysis (see Example 4.4).

- ❖ ORDINAL command identifies a binary predictor
- ❖ NOMINAL command identifies a 4-category discrete predictor that Blimp automatically converts to dummy codes with the lowest numeric value as the reference group
- ❖ FIXED command identifies a complete predictor
- ❖ CENTER command grand mean centering to predictors
- ❖ Unspecified associations for predictor variables

```
DATA: data2.dat;
VARIABLES: id y1 y2 x1 d1 d2 n1 x2 n2;
ORDINAL: d1;
NOMINAL: n1;
MISSING: 999;
FIXED: x1;
CENTER: x1;
MODEL: y1 ~ x1 d1 n1;
SEED: 90291;
BURN: 2000;
ITERATIONS: 10000;
```

4.6: Fully Conditional Specification Multiple Imputation

The model-based multiple imputation procedure illustrated in Example 4.4 creates filled-in data sets tailored to the analysis specified on the MODEL line. The resulting imputations are appropriate for fitting the identical model (or one that is nested within the target model) in the frequentist framework. Fully conditional specification multiple imputation instead uses a round robin sequence of regression models, each of which features an incomplete variable regressed on all other variables (complete or previously imputed). Blimp's implementation of fully conditional specification is described in Chapter 2 (see the FCS command).

This example illustrates a fully conditional specification imputation routine that would yield appropriate imputations for the linear regression model from Example 4.5 (or any additive model that includes the variables listed on the FCS line). Note that fully conditional specification should not be applied to analysis models with interactive or nonlinear effects, as it is prone to bias in such cases (Bartlett et al., 2015; Seaman, Bartlett, & White, 2012). The model-based multiple imputation procedure illustrated in Example 4.8 is a better option. Clicking the links below downloads the scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex4.6.imp](#) [data2.dat](#)

The syntax highlights are as follows.

- ❖ ORDINAL command identifies binary variables
- ❖ NOMINAL command identifies a 4-category nominal variable
- ❖ FIXED command identifies complete variables

- ❖ FCS command includes all analysis variables plus two auxiliary variables
- ❖ NIMPS command specifies 20 imputed data sets
- ❖ Setting CHAINS equal to NIMPS saves one data set from the final iteration of each MCMC chain (avoids autocorrelated imputations)
- ❖ Imputations are stacked in a single file with an index variable added in the first column

```

DATA: data2.dat;
VARIABLES: id y1 y2 x1 d1 d2 n1 x2 n2;
ORDINAL: d1 d2;
NOMINAL: n1;
FIXED: x1 d2;
MISSING: 999;
FCS: y1 x1 d1 d2 n1 x2;
SEED: 90291;
BURN: 1000;
ITERATIONS: 1000;
CHAINS: 20;
NIMPS: 20;
SAVE: stacked = imps.dat;

```

Blimp lists the order of the variables in the imputed data sets at the bottom of the output file, and all variables in the input file appear in the output file regardless of whether they were imputed.

VARIABLE ORDER IN IMPUTED DATA:

```
stacked = 'imps.dat'
```

```
imp# id y1 y2 x1 d1 d2 n1 x2 n2
```


The imputed data sets can be analyzed in other software packages. Example 4.4 illustrated an analysis using the R package `mitml` (Grund et al., 2021).

4.7: Regression With Auxiliary Variables

This example illustrates how to add auxiliary variables to a regression model. Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex4.7a.imp](#) [Ex4.7b.imp](#) [data3.dat](#)

The model analysis model features a continuous variable and dummy code as predictors. The *cgm* superscript denotes variables centered at their grand means.

$$Y = \beta_0 + \beta_1 X_1^{cgm} + \beta_2 D_1 + \varepsilon_i$$

In Blimp, auxiliary variables are introduced via a factored regression (sequential) specification where analysis variables predict the auxiliary variables and auxiliary variables predict each other in a cascading pattern (i.e., the first auxiliary predicts the second, the first and second predict the third, and so on).

$$A_1 = \gamma_{01} + \gamma_{11} Y + \gamma_{21} X_1^{cgm} + \gamma_{31} D_2 + r_1$$

$$A_2 = \gamma_{02} + \gamma_{12} A_1 + \gamma_{22} Y + \gamma_{32} X_1^{cgm} + \gamma_{42} D_2 + r_2$$

$$A_3 = \gamma_{03} + \gamma_{13} A_2 + \gamma_{23} A_1 + \gamma_{33} Y + \gamma_{43} X_1^{cgm} + \gamma_{53} D_2 + r_3$$

The syntax highlights are as follows.

- ❖ ORDINAL command identifies binary variables
- ❖ FIXED command identifies a complete predictor
- ❖ CENTER command applies grand mean centering to a predictor

- ❖ MODEL command uses labels ending in a colon to group models and order their summary tables on the output
- ❖ MODEL command features a factored regression (sequential specification) for auxiliary variables
- ❖ Unspecified associations for predictor variables

```
DATA: data3.dat;  
VARIABLES: id x1 a1 a2 y d1 a3 v1:v4;  
MISSING: 999;  
ORDINAL: d1 a3;  
FIXED: d1;  
CENTER: x1;  
MODEL:  
  focal.model:  
  y ~ x1 d1;  
  auxiliary.model:  
  a1 ~ y x1 d1;  
  a2 ~ a1 y x1 d1;  
  a3 ~ a1 a2 y x1 d1;  
SEED: 90291;  
BURN: 1000;  
ITERATIONS: 10000;
```

The script below illustrates a syntax shortcut that specifies the sequential specification by listing all auxiliary variables to the left of the tilde sign.

```
DATA: data3.dat;  
VARIABLES: id x1 a1 a2 y d1 a3 v1:v4;  
MISSING: 999;  
ORDINAL: d1 a3;  
FIXED: d1;  
CENTER: x1;  
MODEL:  
  focal.model:
```

```

y ~ x1 d1;
auxiliary.model:
a3 a2 a1 ~ y x1 d1;
SEED: 90291;
BURN: 1000;
ITERATIONS: 10000;

```

Adding the NIMPS, CHAINS, and SAVE commands to the script creates model-based multiple imputations that can be analyzed in the frequentist framework (see Example 4.4).

4.8: Linear Regression With an Interaction

This example illustrates a moderated regression with an interaction between a continuous predictor and binary moderator and an incomplete binary covariate. Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex4.8a.imp](#) [Ex4.8b.imp](#) [Ex4.8b.R](#) [data4.dat](#)

The model is as follows, and the *cgm* superscript denotes variables centered at their grand means.

$$Y = \beta_0 + \beta_1 X_1^{cgm} + \beta_2 D_2 + \beta_3 X_1 D_2 + \beta_4 D_1^{cgm} + \varepsilon$$

The syntax highlights are as follows.

- ❖ ORDINAL command identifies a binary predictor
- ❖ NOMINAL command identifies a binary predictor
- ❖ FIXED command identifies a complete variable

- ❖ CENTER command applies grand mean centering to predictors
- ❖ MODEL command features a product term
- ❖ SIMPLE command produces conditional effects (simple slopes) at each level of the nominal moderator
- ❖ Unspecified associations for predictor variables

```

DATA: data4.dat;
VARIABLES: id a1:a3 y x1 x2 n1 d1 d2 o1:o19;
ORDINAL: d1;
NOMINAL: d2;
MISSING: 999;
FIXED: d2;
CENTER: x1 d1;
MODEL: y ~ x1 d2 x1*d2 d1;
SIMPLE: x1 | d2;
SEED: 90291;
BURN: 1000;
ITERATIONS: 10000;

```

Blimp can save multiple imputations from any model it estimates. The script below illustrates model-based multiple imputation (imputation tailored around one specific analysis) for the linear moderated regression model. The new syntax features are as follows.

- ❖ CENTER command grand mean centers predictors in the Bayesian output, but saved imputations are on the original metric
- ❖ NIMPS command specifies 20 imputed data sets
- ❖ Setting CHAINS equal to NIMPS saves one data set from the final iteration of each MCMC chain (avoids autocorrelated imputations)
- ❖ Imputations are stacked in a single file with an index variable added in the first column

```

DATA: data4.dat;
VARIABLES: id a1:a3 y x1 x2 n1 d1 d2 o1:o19;
ORDINAL: d1;
NOMINAL: d2;
MISSING: 999;
FIXED: d2;
CENTER: x1 d1;
MODEL: y ~ x1 d2 x1*d2 d1;
SIMPLE: x1 | d2;
SEED: 90291;
BURN: 1000;
ITERATIONS: 10000;
CHAINS: 20;
NIMPS: 20;
SAVE: stacked = imps.dat;

```

Blimp lists the order of the variables in the imputed data sets at the bottom of the output file, and all variables in the input file appear in the output file regardless of whether they were imputed.

VARIABLE ORDER IN IMPUTED DATA:

```
stacked = 'imps.dat'
```

```

imp# id a1 a2 a3 y x1 x2 n1 d1 d2 o1 o2 o3 o4 o5 o6 o7 o8
      o9 o10 o11 o12 o13 o14 o15 o16 o17 o18 o19

```

The imputed data sets can be analyzed in other software packages. For example, the script below uses the R package `mi` `tm1` (Grund et al., 2021) to fit the moderated regression model to the filled-in data sets. The resulting estimates are numerically equivalent to the Bayesian results.

```
# set working directory
fdir::set()

# read data from working directory
imps <- read.table("imps.dat")
names(imps) <- c("imputation","id","a1","a2","a3","y","x1",
  "x2","n1","d1","d2",paste0("o", 1:19))

# center predictors
imps$x1.cgm <- imps$x1 - mean(imps$x1)
imps$d1.cgm <- imps$d1 - mean(imps$d1)

# analysis and pooling with mitml
implist <- mitml::as.mitml.list(split(imps, imps$imputation))
results <- with(implist, lm(y ~ x1.cgm + d2 + x1.cgm*d2 + d1.cgm))
mitml::testEstimates(results, extra.pars = T, df.com = 295)
```

4.9: Multiple Imputation Within Subgroups

Fully conditional specification multiple imputation is generally inappropriate for interactive effects because it is prone to bias. The moderated regression in Example 4.8 is an exception that could be handled by imputing the data separately within each group of the complete moderator variable (Enders & Gottschall, 2011; Graham, 2009). This example illustrates a multiple-group multiple imputation strategy that stratifies the data by subgroup and imputes within each strata. Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex4.9.imp](#) [Ex4.9.R](#) [data4.dat](#)

The syntax highlights are as follows.

- ❖ ORDINAL command identifies a binary variable

- ❖ **FIXED** command identifies a complete variable
- ❖ **BYGROUP** identifies complete, nominal strata variable not listed on the **ORDINAL** (or **NOMINAL**) command
- ❖ **FCS** command includes all analysis variables (other than the one listed on the **BYGROUP** line) plus two auxiliary variables
- ❖ **NIMPS** command specifies 20 imputed data sets
- ❖ Setting **CHAINS** equal to **NIMPS** saves one data set from the final iteration of each MCMC chain (avoids autocorrelated imputations)
- ❖ Imputations are stacked in a single file with an index variable added in the first column

```
DATA: data4.dat;  
VARIABLES: id a1:a3 y x1 x2 n1 d1 d2 o1:o19;  
ORDINAL: d1;  
MISSING: 999;  
FIXED: d2;  
BYGROUP: d2;  
FCS: a1:a3 y x1 d1;  
SEED: 90291;  
BURN: 1000;  
ITERATIONS: 10000;  
CHAINS: 20;  
NIMPS: 20;  
SAVE: stacked = imps.dat;
```

Blimp lists the order of the variables in the imputed data sets at the bottom of the output file, and all variables in the input file appear in the output file regardless of whether they were imputed.

```
VARIABLE ORDER IN IMPUTED DATA:
```

```
stacked = 'imps.dat'
```

```
imp# id a1 a2 a3 y x1 x2 n1 d1 d2 o1 o2 o3 o4 o5 o6 o7 o8
      o9 o10 o11 o12 o13 o14 o15 o16 o17 o18 o19
```

The imputed data sets can be analyzed in other software packages. The R script from Example 4.8 fits a moderated regression model to the filled-in data sets from this run.

4.10: Curvilinear Regression

This example illustrates a curvilinear regression with a quadratic term and continuous and binary covariates. Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex4.10.imp](#) [data5.dat](#)

The regression model is as follows, and the *cgm* superscript denotes variables centered at their grand means.

$$Y = \beta_0 + \beta_1 X_1^{cgm} + \beta_2 (X_1^{cgm})^2 + \beta_3 X_2^{cgm} + \beta_4 D_1 + \beta_5 D_2 + \varepsilon$$

The syntax highlights are listed below, and adding the NIMPS and SAVE commands generates model-based multiple imputations for a frequentist analysis (see Example 4.8).

- ❖ ORDINAL command identifies binary predictors
- ❖ FIXED command identifies complete predictors
- ❖ CENTER command applies grand mean centering to predictors
- ❖ MODEL command features an embedded function that squares a predictor
- ❖ Unspecified associations for predictor variables


```

DATA: data5.dat;
VARIABLES: id d1 d2 v1:v3 x1 x2 y;
MISSING: 999;
ORDINAL: d1 d2;
FIXED: d1 x2;
CENTER: x1 x2;
MODEL: y2 ~ x1 (x1^2) x2 d1 d2;
SEED: 12345;
BURN: 1000;
ITERATIONS: 10000;

```

4.11: Probit Regression With a Binary Outcome

This example illustrates probit regression for a binary outcome. Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex4.11a.imp](#) [Ex4.11b.imp](#) [data1.dat](#)

The model features a latent response variable regressed on continuous predictors and a binary dummy code, and the *cgm* superscript denotes variables centered at their grand means.

$$Y^* = \beta_0 + \beta_1 X_1^{cgm} + \beta_2 X_2^{cgm} + \beta_3 D_1 + \varepsilon$$

A single threshold value fixed at 0 is automatically included and does not require specification. The syntax highlights are listed below, and adding the NIMPS and SAVE commands generates model-based multiple imputations for a frequentist analysis (see Example 4.8).

- ❖ **ORDINAL** command identifies a binary outcome and predictor

- ❖ FIXED command identifies a complete predictor
- ❖ CENTER command applies grand mean centering to predictors
- ❖ Unspecified associations for predictor variables

```

DATA: data1.dat;
VARIABLES: id n1 y o1 x1 x2 d1 x3 x4;
ORDINAL: y d1;
MISSING: 999;
FIXED: d1;
CENTER: x1 x2;
MODEL:
y ~ x1 x2 d1;
SEED: 90291;
BURN: 1000;
ITERATIONS: 10000;

```

Blimp can also create auxiliary parameters that are functions of the estimated model parameters. To illustrate, the following script uses parameter labels, built-in functions, and the PARAMETERS command to compute the predicted probability of a “success” or “case” at each level of the D_1 dummy code (and at the means of the continuous predictors). The additional syntax highlights are as follows.

- ❖ MODEL command labels the intercept and the binary predictor’s slope
- ❖ PARAMETERS command defines news parameters that give the predicted probability of a “success” (outcome = 1) at each level of the dummy code and the group difference on the probability metric

```

DATA: data1.dat;
VARIABLES: id n1 y o1 x1 x2 d1 x3 x4;
ORDINAL: y d1;

```

```

MISSING: 999;
FIXED: d1;
CENTER: x1 x2;
MODEL:
y ~ 1@b0 x1 x2 d1@b3;
PARAMETERS:
pp.d0 = phi(b0);
pp.d1 = phi(b0 + b3);
pp.diff = pp.d1 - pp.d0;
SEED: 90291;
BURN: 1000;
ITERATIONS: 10000;

```

4.12: Probit Regression With an Ordinal Outcome

This example illustrates a probit regression for an ordered categorical outcome with seven response options (e.g., a Likert scale). Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex4.12.imp](#) [data1.dat](#)

The model features a latent response variable regressed on continuous predictors and a binary dummy code, and the *cgm* superscript denotes variables centered at their grand means.

$$Y^* = \beta_0 + \beta_1 X_1^{cgm} + \beta_2 X_2^{cgm} + \beta_3 D_1 + \varepsilon$$

Six threshold parameters that divide the latent response distribution into seven bins are automatically included and do not require specification (the lowest is fixed at 0 for identification). The syntax highlights are listed below, and adding the NIMPS and SAVE

commands generates model-based multiple imputations for a frequentist analysis (see Example 4.8).

- ❖ **ORDINAL** command identifies an ordinal outcome and a binary predictor
- ❖ Automatic threshold specification for binary and ordinal variables
- ❖ **FIXED** command identifies a complete predictor
- ❖ **CENTER** command applies grand mean centering to predictors
- ❖ Unspecified associations for predictor variables
- ❖ Longer burn-in period required for estimating threshold parameters

```
DATA: data1.dat;  
VARIABLES: id n1 n2 y x1 x2 d1 x3 x4;  
ORDINAL: y d1;  
MISSING: 999;  
FIXED: d1;  
CENTER: x1 x2;  
MODEL:  
y ~ x1 x2 d1;  
SEED: 90291;  
BURN: 20000;  
ITERATIONS: 10000;
```

4.13: Logistic Regression With a Binary Outcome

This example illustrates logistic regression for a binary outcome. Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex4.13a.imp](#) [Ex4.13b.imp](#) [data1.dat](#)

The model features a binary outcome regressed on continuous predictors and a binary dummy code, and the *cgm* superscript denotes variables centered at their grand means.

$$\ln \left(\frac{\Pr(Y = 1)}{1 - \Pr(Y = 1)} \right) = \beta_0 + \beta_1 X_1^{cgm} + \beta_2 X_2^{cgm} + \beta_3 D_1$$

The syntax highlights are listed below, and adding the NIMPS and SAVE commands generates model-based multiple imputations for a frequentist analysis (see Example 4.8). When saving imputations, adding the `savepredicted` keyword to the `OPTIONS` command saves predicted probabilities (see Example 4.20).

- ❖ `ORDINAL` command identifies a binary outcome and predictor
- ❖ `FIXED` command identifies a complete predictor
- ❖ `CENTER` command applies grand mean centering to predictors
- ❖ Applying the `logit` function to the dependent variable on the `MODEL` line requests a logit rather than probit link
- ❖ Unspecified associations for predictor variables

```
DATA: data1.dat;
VARIABLES: id n1 y o1 x1 x2 d1 x3 x4;
ORDINAL: y d1;
MISSING: 999;
FIXED: d1;
CENTER: x1 x2;
MODEL:
logit(y) ~ x1 x2 d1;
SEED: 90291;
BURN: 2000;
ITERATIONS: 10000;
```

Note that the outcome variable must be coded as 0 and 1 when using the logit command in conjunction with ORDINAL. If the outcome has different codes (e.g., 1 and 2), either use TRANSFORM to recode the variable or use the NOMINAL command to identify the outcome as categorical.

Blimp can also create auxiliary parameters that are functions of the estimated model parameters. To illustrate, the following script uses parameter labels, built-in functions, and the PARAMETERS command to compute the predicted probability of a “success” or “case” at each level of the D_1 dummy code (and at the means of the continuous predictors). The additional syntax highlights are as follows.

- ❖ MODEL command labels the intercept and the binary predictor’s slope
- ❖ PARAMETERS command defines news parameters that give the predicted probability of a “success” (outcome = 1) at each level of the dummy code and the group difference on the probability metric

```
DATA: data1.dat;
VARIABLES: id n1 y o1 x1 x2 d1 x3 x4;
ORDINAL: y d1;
MISSING: 999;
FIXED: d1;
CENTER: x1 x2;
MODEL:
logit(y) ~ 1@b0 x1 x2 d1@b3;
PARAMETERS:
pp.d0 = exp(b0) / (1 + exp(b0));
pp.d1 = exp(b0 + b3) / (1 + exp(b0 + b3));
```

4.14: Logistic Regression With a Multicategorical Outcome

This example illustrates logistic regression for a multicategorical outcome with three levels. Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex4.14.imp](#) [data4.dat](#)

The model features a 3-category outcome ($Y = 1, 2, 3$) regressed on three continuous predictors, with the lowest numeric code (e.g., $Y = 1$) as the reference group. The *cgm* superscript denotes variables centered at their grand means.

$$\ln \left(\frac{Pr(Y = 2)}{Pr(Y = 1)} \right) = \beta_{02} + \beta_{12}X_1^{cgm} + \beta_{22}X_2^{cgm} + \beta_{32}X_3^{cgm}$$

$$\ln \left(\frac{Pr(Y = 3)}{Pr(Y = 1)} \right) = \beta_{03} + \beta_{13}X_1^{cgm} + \beta_{23}X_2^{cgm} + \beta_{33}X_3^{cgm}$$

The syntax highlights are listed below, and adding the NIMPS and SAVE commands generates model-based multiple imputations for a frequentist analysis (see Example 4.8).

- ❖ NOMINAL command identifies a multicategorical outcome, which automatically invokes a logit link when the categorical variable is an outcome (applying the `logit` function to the dependent variable is optional)
- ❖ FIXED command identifies a complete predictor
- ❖ CENTER command applies grand mean centering to predictors
- ❖ Unspecified associations for predictor variables

```
DATA: data4.dat;
```

```

VARIABLES: id x1:x6 y d1 d2 o1:o19;
ORDINAL: d1;
NOMINAL: y;
MISSING: 999;
FIXED: x2 x3;
CENTER: x1 x2 x3;
MODEL: logit(y) ~ x1 x2 x3;
SEED: 90291;
BURN: 2000;
ITERATIONS: 10000;

```

4.15: Regression With a Count Outcome

This example illustrates a regression analysis with a count outcome. Blimp uses the negative binomial regression model described by Asparouhov and Muthén (2021). The negative binomial (NB) model is similar to the Poisson model, but it incorporates an additional overdispersion term that accounts for heterogeneity among individuals with the same predicted score. The interpretation of the coefficients is the same as a Poisson regression (Coxe, West, & Aiken, 2009). Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface. Note that the COUNT command currently requires Blimp's beta computational engine. If it is not already installed, you can install it by (a) launching the `blimp-updater` file from within the Applications directory, (b) clicking the `Add or remove components` radio button, then (c) clicking the `Blimp Base (Beta)` radio button.

[Ex4.15.imp](#) [data22.dat](#)

The model features a pair of continuous predictors and a binary dummy code, as follows. The *cgm* superscript denotes variables centered at their grand means.

$$\ln(\hat{\mu}) = \beta_0 + \beta_1 D_1 + \beta_2 D_2 + \beta_3 X_1^{cgm} + \beta_4 X_2^{cgm}$$

The natural log on the left side of the equation is the predicted count based on the constellation of predictors on the right side of the equation. As noted earlier, the model incorporates an overdispersion parameter that accommodates heterogeneity among individuals with the same predicted value.

The syntax highlights are as follows.

- ❖ ORDINAL command identifies binary predictors
- ❖ COUNT command identifies a count outcome
- ❖ FIXED command identifies complete predictors
- ❖ CENTER command applies grand mean centering to predictors
- ❖ Unspecified associations for predictor variables

```
#!beta
DATA: data22.dat;
VARIABLES: id d1 x1 n1 d2 x2 x3 y1 y2 y3;
ORDINAL: d1 d2;
COUNT: y1;
MISSING: 999;
FIXED: d1 x1;
CENTER: x1 x2;
MODEL: y1 ~ d1 d2 x1 x3;
SEED: 90291;
BURN: 5000;
ITERATIONS: 10000;
```

4.16: Linear Regression With Scale Scores

This example illustrates a regression analysis that features a 6-item sum (scale) score as the outcome, a 7-item sum score as a predictor, and two binary covariates. The ordered categorical (e.g., questionnaire) items that determine the sum are incomplete. Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex4.16a.imp](#) [Ex4.16b.imp](#) [data4.dat](#)

The analysis model is

$$\begin{aligned} Y &= \beta_0 + \beta_1 X + \beta_2 D_1 + \beta_3 D_2 + \varepsilon \\ &= \beta_0 + \beta_1 (X_1 + \dots + X_7) + \beta_2 D_1 + \beta_3 D_2 + \varepsilon \end{aligned}$$

where X is the scale (sum) score, and X_1 to X_7 are its ordinal components. It is important to treat missing data at the item level when analyzing incomplete composite scores, as doing so maximizes power and precision. This example illustrates the approach from Alacam, Du, Enders, and Keller (2023) and Enders (2022). It is important to reiterate that the scale score (represented as an embedded function) is essentially a random variable that depends on its constituent parts rather than a deterministic computation or passive imputation (see the Functions Embedded in Equations section from Chapter 2). The syntax highlights are listed below, and adding the NIMPS and SAVE commands generates model-based multiple imputations for a frequentist analysis (see Example 4.8).

- ❖ ORDINAL command identifies binary and ordinal variables
- ❖ Automatic threshold specification for binary and ordinal variables

- ❖ MODEL command uses labels ending in a colon to group models and order their summary tables on the output
- ❖ MODEL command features a syntax shortcut that creates a factored regression (sequential) specification for all predictors
- ❖ MODEL command features an embedded function that defines the sum of ordinal items as a predictor
- ❖ MODEL command defines the sum of ordinal items as a random variable
- ❖ Longer burn-in period for estimating threshold parameters

```

DATA: data4.dat;
VARIABLES: id a1 a2 a3 yscale v zscale n1 d1 d2
             y1:y6 x1:x7 z1:z6;
ORDINAL: x1:x7 d1 d2;
MISSING: 999;
MODEL:
  focal.model:
  # embedded sum variable as a predictor
  xscale = x1+:x7;
  yscale ~ xscale d1 d2;
  predictor.model:
  x1:x7 d1 d2 ~ 1;
SEED: 90291;
BURN: 5000;
ITERATIONS: 10000;

```

The previous script used a composite score as the dependent variable but did not incorporate the dependent variable's component items into the model. Doing so would improve precision because the items are strong correlates of the sum score. The code block below leverages item-level correlations by introducing five of the six outcome items as auxiliary variables (Eekhout et al., 2015). The component items are added

using the same auxiliary variable approach from Example 4.7. The additional syntax highlights are as follows.

- ❖ MODEL command features a factored regression (sequential specification) for the dependent variable's scale score and its items
- ❖ All but one of the dependent variable's scale items are used as auxiliary variables (using all items induces linear dependencies)

```
DATA: data4.dat;
VARIABLES: id a1 a2 a3 yscale v zscale n1 d1 d2
             y1:y6 x1:x7 z1:z6;
ORDINAL: x1:x7 d1 d2;
MISSING: 999;
MODEL:
  focal.model:
  xscale = x1+:x7;
  yscale ~ xscale d1 d2;
  predictor.model:
  x1:x7 d1 d2 ~ 1;
  auxiliary.models:
  # sequential specification for y scale items
  y1:y5 ~ yscale;
SEED: 90291;
BURN: 20000;
ITERATIONS: 10000;
```

4.17: Linear Regression With Scale Score Interaction

This example illustrates a moderated regression with an interaction between a 7-item sum score predictor and binary moderator. Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex4.17.imp](#) [data4.dat](#)

The analysis model is

$$Y = \beta_0 + \beta_1 D_1 + \beta_2 X + \beta_3 D_2 + \beta_4 D_2 X + \varepsilon$$

$$= \beta_0 + \beta_1 D_1 + \beta_2 (X_1 + \dots + X_7) + \beta_3 D_2 + \beta_4 D_2 (X_1 + \dots + X_7) + \varepsilon$$

where X is the scale (sum) score, and X_1 to X_7 are its ordinal components. It is important to treat missing data at the item level when analyzing incomplete composite scores, as doing so maximizes power and precision. This example extends the approach from Alacam, Du, Enders, and Keller (2023) to include interaction effects involving an incomplete sum score. Enders (2022) summarizes the approach. It is important to reiterate that the scale score (represented as an embedded function) is essentially a random variable that depends on its constituent parts rather than a deterministic computation or passive imputation (see the Functions Embedded in Equations section from Chapter 2). The syntax highlights are listed below, and adding the NIMPS and SAVE commands generates model-based multiple imputations for a frequentist analysis (see Example 4.8).

- ❖ ORDINAL command identifies binary and ordinal variables
- ❖ Automatic threshold specification for binary and ordinal variables
- ❖ MODEL command uses labels ending in a colon to group models and order their summary tables on the output
- ❖ MODEL command features a syntax shortcut that creates a factored regression (sequential) specification for all predictors
- ❖ MODEL command features an embedded function that defines the sum of ordinal items as a predictor
- ❖ MODEL command defines the sum of ordinal items as a random variable

- ❖ MODEL command features a factored regression (sequential specification) for the dependent variable's scale score and its items
- ❖ All but one of the dependent variable's scale items are used as auxiliary variables (using all items induces linear dependencies)
- ❖ Longer burn-in period for estimating threshold parameters

```

DATA: data4.dat;
VARIABLES: id a1 a2 a3 yscale xscale zscale n1 d1 d2
               y1:y6 x1:x7 z1:z6;
ORDINAL: y1:y5 x1:x7 d1 d2;
MISSING: 999;
MODEL:
  focal.model:
  # scale score predictor and interaction with embedded function
  xscale = x1+:x7;
  yscale ~ xscale d2 d2*xscale d1;
  predictor.model:
  # sequential specification for x scale items and dummy codes
  x1:x7 d1 d2 ~ 1;
  auxiliary.model:
  # sequential specification for y scale items
  y1:y5 ~ yscale;
SEED: 90291;
BURN: 20000;
ITERATIONS: 10000;

```

4.18: Skewed Predictor and Yeo-Johnson Transform

This example illustrates a Yeo-Johnson (Yeo & Johnson, 2000) transformation that samples imputations from a skewed distribution. Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex4.18.imp](#) [data6.dat](#)

The analysis model is a logistic regression with two continuous variables and two binary dummy codes as predictors.

$$\ln \left(\frac{\Pr(Y = 1)}{1 - \Pr(Y = 1)} \right) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 D_1 + \beta_4 D_2$$

X_2 's distribution is markedly peaked and positively skewed, and drawing imputations from a normal distribution would likely distort the variable's distribution.

The Yeo-Johnson procedure estimates the variable's shape and draws imputations from a nonnormal distribution. Applying the Yeo-Johnson transformation normalizes the predictor variable, such that the resulting linear regression reflects associations between the normalized variable and other predictors. The sequential specification in the code block below invokes the following regression equation for the normalized predictor.

$$\tilde{X}_2 = \gamma_0 + \gamma_1 X_1 + \gamma_2 D_1 + r$$

However, skewed imputations on the raw score metric always appear on the right side of any regression equation (e.g., the focal regression model). Normalized imputations can be saved by adding the `save latent` keyword to the `OPTIONS` line.

The Yeo-Johnson transformation can be very slow (or fail) to converge if the skewed variable's mean is far from zero. To facilitate interpretation, the code block below centers the predictor scores at the median value of 16. Additional details about the procedure are available in the literature (Enders, 2022; Lüdtke et al., 2020b). The syntax highlights are listed below, and adding the `NIMPS` and `SAVE` commands

generates model-based multiple imputations for a frequentist analysis (see Example 4.8).

- ❖ **ORDINAL** command identifies a binary outcome and predictors
- ❖ **FIXED** command identifies complete predictors
- ❖ **MODEL** command uses labels ending in a colon to group models and order their summary tables on the output
- ❖ **MODEL** command features a factored regression (sequential specification) for incomplete predictor variables
- ❖ Unspecified associations for complete predictor variables
- ❖ Applying the `yj t` function to the skewed predictor on the **MODEL** line requests a Yeo-Johnson transformation
- ❖ Applying a subtraction function to center the skewed predictor at its median facilitates convergence
- ❖ Applying the `logit` function to the dependent variable on the **MODEL** line requests a logit rather than probit link

```
DATA: data6.dat;
VARIABLES: id d1 x1 n1 d2 a1 x2 x3 x4 y;
ORDINAL: y d1 d2;
MISSING: 999;
FIXED: d1 x1;
MODEL:
  focal.model:
    logit(y) ~ x1 x2 d1 d2;
  predictor.model:
    # sequential predictor models with yeo-johnson transform for x2
    yjt(x2 - 16) ~ x1 d1;
    d2 ~ x2 x1 d1;
SEED: 90291;
BURN: 1000;
```


ITERATIONS: 10000;

4.19: Skewed Outcome and Yeo-Johnson Transform

This example applies the Yeo–Johnson transformation to a nonnormal dependent variable. Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex4.19a.imp](#) [Ex4.19b.imp](#) [Ex4.19.R](#) [data2.dat](#)

The untransformed analysis model features two continuous variables and one binary dummy code as predictors, where the *cgm* superscript denotes variables centered at their grand means.

$$Y = \beta_0 + \beta_1 X_1^{cgm} + \beta_2 X_2^{cgm} + \beta_3 D_1 + \varepsilon$$

The outcome variable's distribution is markedly peaked and positively skewed. Applying the Yeo-Johnson transformation normalizes the dependent variable, such that the resulting linear regression reflects associations between the normalized outcome and the predictors.

$$\tilde{Y} = \gamma_0 + \gamma_1 X_1^{cgm} + \gamma_2 X_2^{cgm} + \gamma_3 D_1 + r$$

Normalized imputations can be saved by adding the `saveLatent` keyword to the `OPTIONS` line. The Yeo–Johnson transformation can be very slow (or fail) to converge if the skewed variable's mean is far from zero. To facilitate interpretation, the code block below centers the outcome at the median value of 9. Additional details about the procedure are available in the literature (Enders, in press; Lüdtke et al., 2020b).

The syntax highlights are listed below.

- ❖ **ORDINAL** command identifies a binary predictor
- ❖ **FIXED** command identifies complete predictors
- ❖ Applying **yj t** function to the skewed outcome on the **MODEL** line requests a Yeo-Johnson transformation
- ❖ Applying a subtraction function to center the skewed outcome at its median facilitates convergence
- ❖ Unspecified associations for predictor variables

```
DATA: data2.dat;
VARIABLES: id y n1 x1 d1 d2 n2 x2 n3;
ORDINAL: d1;
MISSING: 999;
FIXED: x1;
CENTER: x1 x2;
MODEL:
yj t(y - 9) ~ x1 x2 d1;
SEED: 90291;
BURN: 1000;
ITERATIONS: 10000;
```

Blimp can save multiple imputations from any model it estimates. Adding the **NIMPS** and **SAVE** commands generates model-based multiple imputations for a frequentist analysis, and listing the **save latent** keyword on the **OPTIONS** command saves the normalized imputes from the Yeo-Johnson transformation alongside the skewed imputes on the raw score metric (this keyword also saves the latent response scores for the binary predictor).

```
DATA: data2.dat;
```

```

VARIABLES: id y n1 x1 d1 d2 n2 x2 n3;
ORDINAL: d1;
MISSING: 999;
FIXED: x1;
CENTER: x1 x2;
MODEL:
yjt(y - 9) ~ x1 x2 d1;
SEED: 90291;
BURN: 3000;
ITERATIONS: 10000;
# save model-based multiple imputations;
CHAINS: 20;
NIMPS: 20;
OPTIONS: savelatent;
SAVE: stacked = imps.dat;

```

Blimp lists the order of the variables in the imputed data sets at the bottom of the output file, and all variables in the input file appear in the output file regardless of whether they were imputed.

VARIABLE ORDER IN IMPUTED DATA:

```

stacked = 'imps.dat'
imp# id y n1 x1 d1 d2 n2 x2 n3 yjt(yjt(y-9)) d1.latent

```

The variable `y` contains skewed imputations on the raw score metric, and the variable `yjt(yjt(y-9))` contains the normalized imputes. The imputed data sets can be analyzed in other software packages. To illustrate, the script below uses the R package `mitml` (Grund et al., 2021) to fit the regression model to the filled-in data sets. The positively skewed raw score imputations are on the original metric, whereas the transformed imputations are approximately normal.

```
# set working directory
fdir::set()

# read data from working directory
imps <- read.table("imps.dat")
names(imps) <- c("imputation","id","y","n1","x1","d1","d2","n2",
               "x2","n3","ytransform","d1.latent")

# plot raw and transformed scores
hist(imps$y)
hist(imps$ytransform)

# center predictors
imps$x1.cgm <- imps$x1 - mean(imps$x1)
imps$x2.cgm <- imps$x2 - mean(imps$x2)

# analysis and pooling with mitml
implist <- mitml::as.mitml.list(split(imps, imps$imputation))

# analyze skewed outcome
results <- with(implist, lm(y ~ x1.cgm + x2.cgm + d1))
mitml::testEstimates(results, extra.pars = T, df.com = 1996)

# analyze transformed outcome
results <- with(implist, lm(ytransform ~ x1.cgm + x2.cgm + d1))
mitml::testEstimates(results, extra.pars = T, df.com = 1996)
```

4.20: Bayesian Wald Test

This example illustrates the linear regression analysis from Example 4.3 with the Bayesian Wald test described by Asparouhov and Muthén (2021). Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex4.20.imp](#) [data1.dat](#)

The model features a pair of continuous predictors and a binary dummy code, where the *cgm* superscript denotes variables centered at their grand means.

$$Y = \beta_0 + \beta_1 X_1^{cgm} + \beta_2 X_2^{cgm} + \beta_3 D_2 + \varepsilon$$

The syntax highlights are as follows.

- ❖ ORDINAL command identifies a binary predictor
- ❖ FIXED command identifies a complete predictor
- ❖ CENTER command applies grand mean centering to predictors
- ❖ TEST command specifies a nested model with all slopes fixed at 0
- ❖ Unspecified associations for predictor variables

```
DATA: data1.dat;
VARIABLES: id n1 d1 o1 y x1 d2 x2 x3;
ORDINAL: d2;
MISSING: 999;
FIXED: d2;
CENTER: x1 x2;
MODEL: y ~ x1@b1 x2@b2 d2@b3;
TEST:
b1:b3 = 0;
SEED: 90291;
BURN: 1000;
ITERATIONS: 10000;
```

The TEST command produces the output table below. The Wald test statistic is a chi-square variable, and the test's degrees of freedom equals the number of parameters by which the two models differ. The probability value is *not* a frequentist *p*-value because it makes no reference to test statistics from other random samples.

Rather, the probability is an index of support for the proposed constraints, where support is defined as the area above the test statistic value in a chi-square distribution.

MODEL FIT:

Asparouhov & Muthén Wald Tests

Test #1

Wald Statistic (Chi-Square)	158.363
Number of Parameters Tested (df)	3
Probability	0.000

NOTE: Wald tests are printed in the order specified on the WALDTEST command.

4.21: Propensity Score Estimation With Missing Data

This example illustrates propensity score estimation with missing data. Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex4.21.imp](#) [data4.dat](#)

The focal model features a binary dummy code (the “treatment” indicator) predicting a continuous outcome.

$$Y = \beta_0 + \beta_1 D_1 + \varepsilon$$

The propensity score model features the treatment indicator regressed on potential confounder variables and their higher-order interaction terms.

$$\ln \left(\frac{Pr(D_1 = 1)}{1 - Pr(D_1 = 1)} \right) = \gamma_0 + \gamma_1 X_1 + \gamma_2 X_2 + \gamma_3 X_3 + \gamma_4 X_4 + \\ \gamma_5 X_1 X_2 + \gamma_6 X_1 X_3 + \gamma_7 X_1 X_4 + \gamma_8 X_2 X_3 + \gamma_9 X_2 X_4 + \gamma_{10} X_3 X_4$$

Because the treatment indicator D_1 consists of naturally occurring groups, this variable could be incomplete, which it is here. In this case, it is important for propensity score estimation to account for both models.

- ❖ ORDINAL command identifies a binary outcome and predictor
- ❖ FIXED command identifies a complete predictor
- ❖ Applying the `logit` function to the dependent variable on the `MODEL` line requests a logit rather than probit link
- ❖ FIXED command identifies complete predictors
- ❖ CENTER command applies grand mean centering to predictors
- ❖ MODEL command uses labels ending in a colon to group models and order their summary tables on the output
- ❖ MODEL command features a product term
- ❖ The `savepredicted` keyword on the `OPTIONS` line saves the predicted probabilities of treatment group membership, which are the propensity scores
- ❖ Unspecified associations for predictor variables
- ❖ NIMPS command specifies 20 imputed data sets
- ❖ Setting `CHAINS` equal to `NIMPS` saves one data set from the final iteration of each MCMC chain (avoids autocorrelated imputations)
- ❖ Imputations are stacked in a single file with an index variable added in the first column

```
DATA: data4.dat;
VARIABLES: id x1:x4 y x5 n1 d1 d2 o1:o19;
ORDINAL: d1;
```

```

MISSING: 999;
FIXED: x2 x3;
MODEL:
  focal.model:
  y ~ d1;
  propensity.model:
  logit(d1) ~ x1 x2 x3 x4 x1*x2 x1*x3 x1*x4 x2*x3 x2*x4 x3*x4;
SEED: 90291;
BURN: 2000;
ITERATIONS: 10000;
OPTIONS: savepredicted;
CHAINS: 20;
NIMPS: 20;
SAVE: stacked = imps.dat;

```

Blimp lists the order of the variables in the imputed data sets at the bottom of the output file, and all variables in the input file appear in the output file regardless of whether they were imputed.

VARIABLE ORDER IN IMPUTED DATA:

```

stacked = 'imps.dat'

  imp# id x1 x2 x3 x4 y x5 n1 d1 d2 o1 o2 o3 o4 o5 o6 o7 o8
  o9 o10 o11 o12 o13 o14 o15 o16 o17 o18 o19
  d1.probability y.predicted

```

The variable `d1.probability` contains the propensity scores. Following earlier examples, imputed data sets from Blimp can be analyzed in other software packages.

4.22: Linear Regression With Sampling Weights

This example illustrates a linear regression analysis with sampling (i.e., inverse probability) weights. Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex4.22.imp](#) [data21.dat](#)

The analysis model features three continuous predictors.

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \varepsilon$$

Blimp's MCMC estimation routine incorporates sampling weights using the procedure described in Goldstein (2011, Section 3.4.2). The syntax highlights are as follows.

- ❖ **WEIGHT** command identifies the inverse probability sampling weights
- ❖ Unspecified associations for predictor variables

```
DATA: data21.dat;  
VARIABLES: v1 v2 v3 wght y x1 x2 x3 x4 x5;  
WEIGHTS: wght;  
MISSING: 999;  
MODEL: y ~ x1 x2 x3;  
SEED: 90291;  
BURN: 1000;  
ITERATIONS: 10000;
```

5 Analysis Examples: Path Analysis and Latent Variable Models

This section illustrates path analyses and latent variable models in Blimp. These multivariate analyses are specified as collections of univariate equations. In general, it is possible to mix and match features from any examples to easily create complex analysis models that honor features of the data. Additional details about fitting path and latent variable models in Blimp can be found in Keller (2022), which is available for download [here](#).

Following the previous chapter, the examples in this section use a generic notation system where variable names usually consist of an alphanumeric prefix and a numeric suffix (e.g., Y_1 , X_1 , N_1 , D_1 , D_2 , V_1 , V_2 , V_3). The letter Y designates a dependent variable, a D prefix denotes a binary dummy variable, an O prefix indicates an ordinal variable, and an N prefix indicates a multicategorical nominal variable. Other letters generally represent continuous variables. Finally, the model equations use a “cgm” superscript to indicate grand mean centering. The following list outlines the examples in this section.

- ❖ 5.1: Mediation Analysis
- ❖ 5.2: Mediation Analysis With Moderated Paths
- ❖ 5.3: Mediation Analysis With a Binary Outcome
- ❖ 5.4: Mediation Analysis With a Categorical Mediator
- ❖ 5.5: Factor Analysis With Continuous Indicators
- ❖ 5.6: Factor Analysis With Binary Indicators (IRT Model)
- ❖ 5.7: Factor Analysis With Ordinal Indicators
- ❖ 5.8: Imputing Latent Response Scores for Item-Level Factor Analysis
- ❖ 5.9: Factor Analysis With Skewed Indicators and Yeo-Johnson Transform
- ❖ 5.10: Latent Variable Regression Model

- ❖ 5.11: Latent-by-Manifest Variable Interaction
- ❖ 5.12: Latent-by-Latent Variable Interaction
- ❖ 5.13: Moderated Factor Analysis for Measurement Invariance Testing
- ❖ 5.14: Latent Growth Curve Model

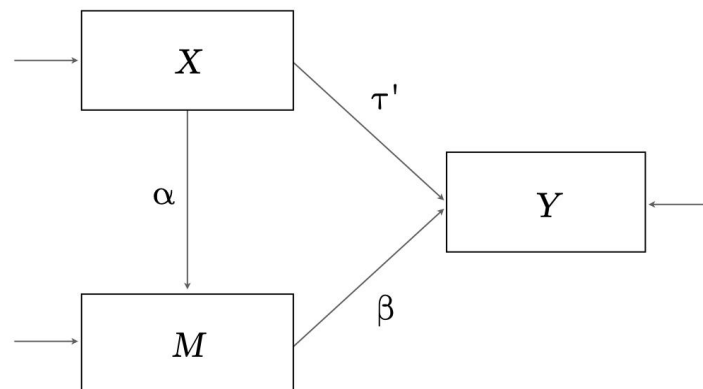
5.1: Mediation Analysis

This example illustrates a single-mediator path model. The regression models are shown below

$$M = \beta_{01} + \beta_{11}X + \varepsilon_1 = I_M + \alpha X + \varepsilon_M$$

$$Y = \beta_{02} + \beta_{12}M + \beta_{22}X + \varepsilon_2 = I_Y + \beta M + \tau' X + \varepsilon_Y$$

where α and β are slope coefficients that define the indirect effect or product of the coefficients estimator, and τ' is the direct effect of X on Y . A path diagram of the analysis is shown below. The model also incorporates three auxiliary variables following the procedure from Example 4.7.



Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex5.1.imp](#) [data4.dat](#)

The syntax highlights are as follows.

- ❖ MODEL command uses labels ending in a colon to group models and order their summary tables on the output
- ❖ MODEL command labels the indirect effect's component pathways
- ❖ MODEL command features a syntax shortcut that creates a factored regression (sequential) specification for auxiliary variables
- ❖ PARAMETERS command uses labeled quantities to compute the product of coefficients estimator

```
DATA: data4.dat;
VARIABLES: id a1:a3 zscale y m n1 x d1 o1:o19;
MISSING: 999;
MODEL:
mediation.model:
m ~ x@alpha;
y ~ m@beta x;
auxiliary.model:
# sequential specification for auxiliary variables
a1:a3 ~ y m x;
PARAMETERS:
indirect = alpha * beta;
SEED: 90291;
BURN: 1000;
ITERATIONS: 10000;
```

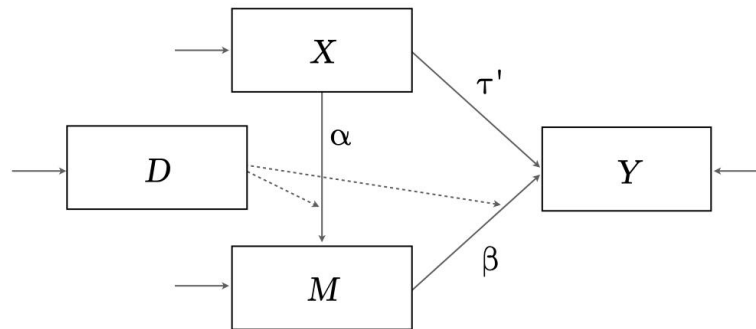
5.2: Mediation Analysis With Moderated Paths

This example adds moderated pathways to the single-mediator model from the previous example. The regression models are shown below

$$M = \beta_{01} + \beta_{11}X + \beta_{21}D + \beta_{31}(X \times D) + \varepsilon_1$$

$$Y = \beta_{02} + \beta_{12}M + \beta_{22}X + \beta_{32}D + \beta_{42}(M \times D) + \varepsilon_2$$

and the corresponding path diagram is as follows.



The dashed lines pointing from D to the directed arrows convey that D moderates the mediation model paths.

Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex5.2.imp](#) [data4.dat](#)

The syntax highlights are as follows.

- ❖ ORDINAL command identifies a binary predictor
- ❖ FIXED command identifies a complete predictor
- ❖ MODEL command uses labels ending in a colon to group models and order their summary tables on the output
- ❖ MODEL command labels the indirect effect's component pathways
- ❖ MODEL command features a product term

- ❖ MODEL command features a syntax shortcut that creates a factored regression (sequential) specification for auxiliary variables
- ❖ PARAMETERS command uses labeled quantities to compute the product of coefficients estimator at each level of the binary moderator

```

DATA: data4.dat;
VARIABLES: id a1 a2 a3 zscale y m n1 x d o1:o19;
ORDINAL: d;
MISSING: 999;
FIXED: d;
MODEL:
  mediation.model:
  m ~ x@alpha d x*d@alphamod;
  y ~ m@beta x d m*d@betamod;
  auxiliary.model:
  # sequential specification for auxiliary variables
  a1:a3 ~ y m x d;
PARAMETERS:
  indirect.d0 = alpha * beta;
  indirect.d1 = ( alpha + alphamod ) * ( beta + betamod );
SEED: 90291;
BURN: 1000;
ITERATIONS: 10000;

```

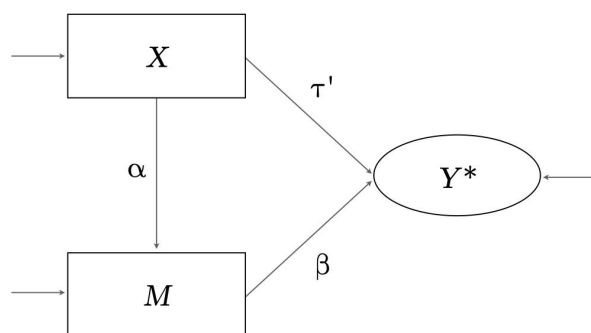
5.3: Mediation Analysis With a Binary Outcome

This example illustrates a single-mediator model with a binary outcome. The regression models are shown below

$$M = \beta_{01} + \beta_{11}X + \varepsilon_1 = I_M + \alpha X + \varepsilon_M$$

$$Y^* = \beta_{02} + \beta_{12}M + \beta_{22}X + \varepsilon_2 = I_Y + \beta M + \tau'X + \varepsilon_Y$$

where Y^* denotes the underlying latent response variable for a binary outcome Y , and all other features of the model are the same as Example 5.1. A path diagram of the mediation model is shown below, with the ellipse denoting the latent response variable, the residual variance of which is fixed at 1 for identification.



Muthén, Muthén, and Asparouhov (2016) show that the indirect effect alone does not capture the fact that changes in Y are non-constant on the probability metric. They give the following expression for computing the probability that $Y = 1$ given a particular value of X (Equations 8.4 to 8.6). The script below uses these equations to compute the probability at different values of X .

$$E(Y^*|X) = I_Y + I_M\beta + \alpha\beta X + \tau'X =$$

$$V(Y^*|X) = \beta^2\sigma_M^2 + 1$$

$$P(Y = 1|X) = \phi[E(Y^*|X)/\sqrt{V(Y^*|X)}]$$

Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex5.3a.inp](#) [Ex5.3b.inp](#) [data20.dat](#)

The syntax highlights are as follows.

- ❖ **ORDINAL** command identifies a binary outcome and predictor
- ❖ **FIXED** command identifies a complete predictor
- ❖ **CENTER** command applies grand mean centering to predictors
- ❖ Unspecified associations for predictor variables
- ❖ **MODEL** command uses labels ending in a colon to group models and order their summary tables on the output
- ❖ **MODEL** command labels intercepts, slopes, and the residual variance of the mediator
- ❖ **MODEL** command features a syntax shortcut that creates a factored regression (sequential) specification for auxiliary variables
- ❖ **PARAMETERS** command uses labeled quantities to compute the product of coefficients estimator
- ❖ **PARAMETERS** command uses labeled quantities to compute the the probability that Y equals 1 at three values of X (the mean and plus/minus one standard deviation from the mean)

```
DATA: data20.dat;
VARIABLES: id a1:a3 z y m n1 x d1 o1:o19;
MISSING: 999;
ORDINAL: y;
MODEL:
mediation.model:
# single-mediator model with parameter labels
m ~ 1@m.icept x@alpha;
m ~~ m@m.resvar;
y ~ 1@y.icept m@beta1 x@beta2;
auxiliary.model:
# sequential specification for auxiliary variables
a1:a3 ~ y m x;
```


PARAMETERS:

```

indirect = alpha * beta1;
xvalue1 = -.50;
xvalue2 = 0;
xvalue3 = .50;
pyeq1.xvalue1 = phi((y.icept + beta1*m.icept +
  beta1*alpha*xvalue1 + beta2*xvalue1)/
  sqrt(beta1^2*m.resvar + 1));
pyeq1.xvalue2 = phi((y.icept + beta1*m.icept +
  beta1*alpha*xvalue2 + beta2*xvalue2)/
  sqrt(beta1^2*m.resvar + 1));
pyeq1.xvalue3 = phi((y.icept + beta1*m.icept +
  beta1*alpha*xvalue3 + beta2*xvalue3)/
  sqrt(beta1^2*m.resvar + 1));

```

SEED: 90291;

BURN: 1000;

ITERATIONS: 10000;

The script above defines the binary outcome as a latent response variable (i.e., probit regression). Applying the `logit` function to the dependent variable on the `MODEL` line requests a logit rather than probit link. The conditional probabilities described above do not have a simple expression when using a logit link for the outcome.

DATA: data20.dat;

VARIABLES: id a1:a3 zscale y mscale n1 x d1 o1:o19;

MISSING: 999;

ORDINAL: y;

MODEL:

```

# single-mediator model with parameter labels
mscale ~ x@alpha;
logit(y) ~ mscale@beta x;
# sequential specification for auxiliary variables
a1:a3 ~ y mscale x;

```

PARAMETERS:

indirect = alpha * beta;

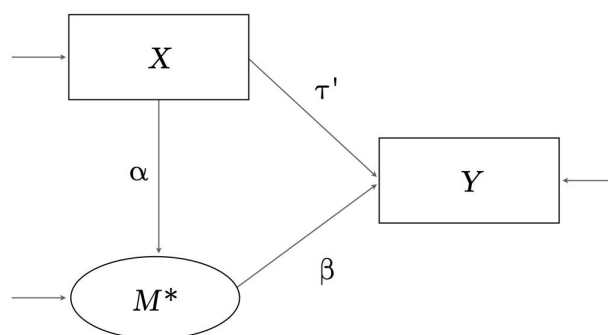
SEED: 90291;**BURN:** 1000;**ITERATIONS:** 10000;**5.4: Mediation Analysis With a Categorical Mediator**

This example illustrates a single-mediator path model with an ordered categorical mediator (the mediator could also be binary). The regression models are shown below

$$M^* = \beta_{01} + \beta_{11}X + \varepsilon_1 = I_M + \alpha X + \varepsilon_M$$

$$Y = \beta_{02} + \beta_{12}M^* + \beta_{22}X + \varepsilon_2 = I_Y + \beta M^* + \tau' X + \varepsilon_Y$$

where α and β are slope coefficients that define the indirect effect or product of the coefficients estimator, and τ' is the direct effect of X on Y . A path diagram of the analysis is shown below. The model also incorporates three auxiliary variables following the procedure from Example 4.7.



When M is binary or ordinal, the α path represents the regression of a latent response variable on X . Typically, the discrete M would then serve as a predictor of Y , thus leading to an awkward situation where M essentially has two different metrics within the same model (i.e., M is latent when it is an outcome variable but ordinal when it is a

predictor). Alternatively, Blimp can use the latent response variable in both regressions, effectively converting a complicated categorical variable regression into a straightforward linear regression with latent response variables. This idea was proposed in Muthén, Muthén, and Asparouhov (2016). Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex5.4.imp](#) [data4.dat](#)

The syntax highlights are as follows.

- ❖ MODEL command uses labels ending in a colon to group models and order their summary tables on the output
- ❖ MODEL command labels the indirect effect's component pathways
- ❖ MODEL command features a syntax shortcut that creates a factored regression (sequential) specification for auxiliary variables
- ❖ Appending the `.latent` suffix to the mediator's variable name in the MODEL statement accesses the latent response variable instead of the discrete responses
- ❖ PARAMETERS command uses labeled quantities to compute the product of coefficients estimator

```
DATA: data4.dat;
VARIABLES: id a1:a3 zscale y n1 n2 x d1 o1:o18 m;
MISSING: 999;
ORDINAL: m;
MODEL:
  mediation.model:
  m ~ x@alpha;
  y ~ m.latent@beta x;
  auxiliary.model:
  # sequential specification for auxiliary variables
```

```
a1:a3 ~ y m.latent x;
```

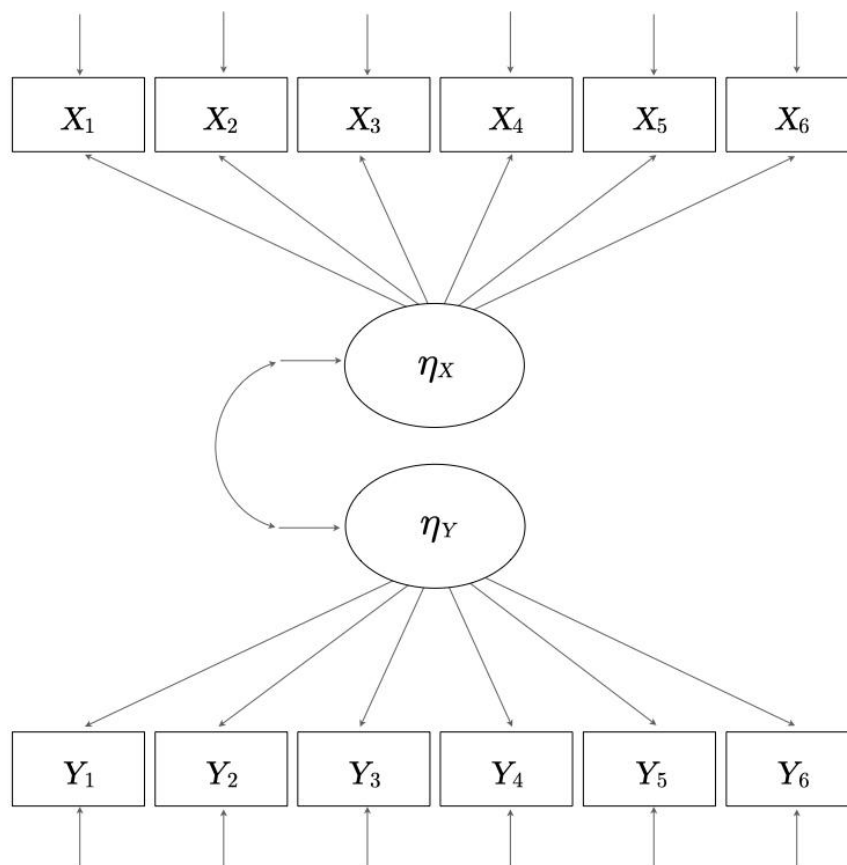
PARAMETERS:

```
indirect = alpha * beta;
```

SEED: 90291;**BURN:** 5000;**ITERATIONS:** 10000;

5.5: Factor Analysis With Continuous Indicators

This example illustrates a two-factor measurement model with correlated latent variables, each measured by six continuous indicators. A path diagram of the analysis model is shown below.



Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex5.5.imp](#) [data4.dat](#)

The syntax highlights are as follows.

- ❖ LATENT command defines two latent variables
- ❖ Default specification fixes the first loading of each factor to 1 and sets the latent means equal to 0
- ❖ Longer burn-in period for estimating latent variables

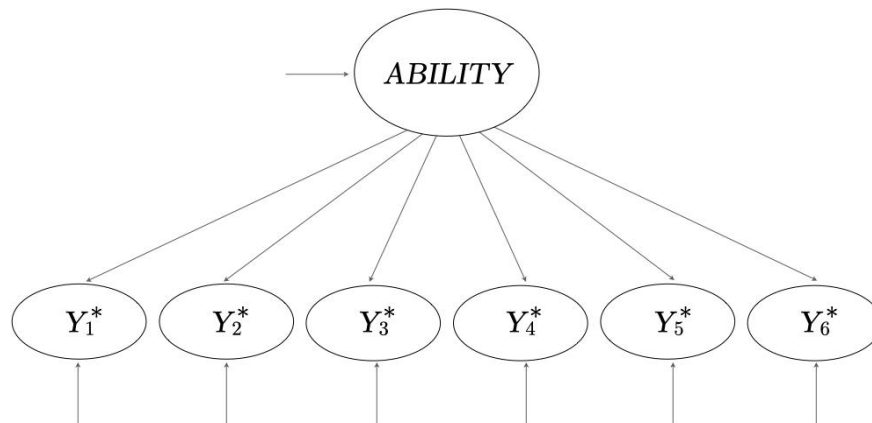
```
DATA: data4.dat;  
VARIABLES: id a1 a2 a3 yscale mscale xscale n1 d1 d2  
          y1:y6 m1:m7 x1:x6;  
MISSING: 999;  
LATENT: latency latentx;  
MODEL:  
latentx -> x1:x6;  
latency -> y1:y6;  
latentx <-> latency;  
SEED: 90291;  
BURN: 10000;  
ITERATIONS: 10000;
```

5.6: Factor Analysis With Binary Indicators (IRT Model)

This example illustrates a unidimensional measurement model with binary indicators and IRT scaling. Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex5.6a.imp](#) [Ex5.6b.imp](#) [data4.dat](#)

A path diagram of the analysis model is shown below, with ellipses denoting latent response variables, the residual variances of which are fixed scaling constants.



Blimp can use either a logit or probit link. The syntax highlights for the logistic link are as follows.

- ❖ `ORDINAL` command identifies binary variables
- ❖ `LATENT` command defines a latent (ability) variable
- ❖ `MODEL` command fixes the mean and variance of the latent variable to 0 and 1, respectively
- ❖ `MODEL` command labels measurement intercepts and factor loadings
- ❖ Applying the `logit` function to the dependent variable on the `MODEL` line requests a logit rather than probit link
- ❖ `PARAMETERS` command uses labeled quantities to compute item discrimination and difficulty indices for 2-parameter IRT scaling

```

DATA: data14.dat;
VARIABLES: id y1:y6;
ORDINAL: y1:y6;
  
```

```

MISSING: 999;
LATENT: ability;
MODEL:
ability ~ 1@0;
ability ~~ ability@1;
logit(y1) ~ 1@icept1 ability@load1;
logit(y2) ~ 1@icept2 ability@load2;
logit(y3) ~ 1@icept3 ability@load3;
logit(y4) ~ 1@icept4 ability@load4;
logit(y5) ~ 1@icept5 ability@load5;
logit(y6) ~ 1@icept6 ability@load6;
PARAMETERS:
discrim1 = load1;
discrim2 = load2;
discrim3 = load3;
discrim4 = load4;
discrim5 = load5;
discrim6 = load6;
difficulty1 = - icept1 / load1;
difficulty2 = - icept2 / load2;
difficulty3 = - icept3 / load3;
difficulty4 = - icept4 / load4;
difficulty5 = - icept5 / load5;
difficulty6 = - icept6 / load6;
SEED: 90291;
BURN: 2000;
ITERATIONS: 10000;

```

The script below is identical but uses a probit rather than logit link (i.e., a normal ogive model specification). The logistic coefficients differ by a factor of approximately 1.7.

```

DATA: data14.dat;
VARIABLES: id y1:y6;
ORDINAL: y1:y6;
MISSING: 999;
LATENT: ability;
MODEL:

```

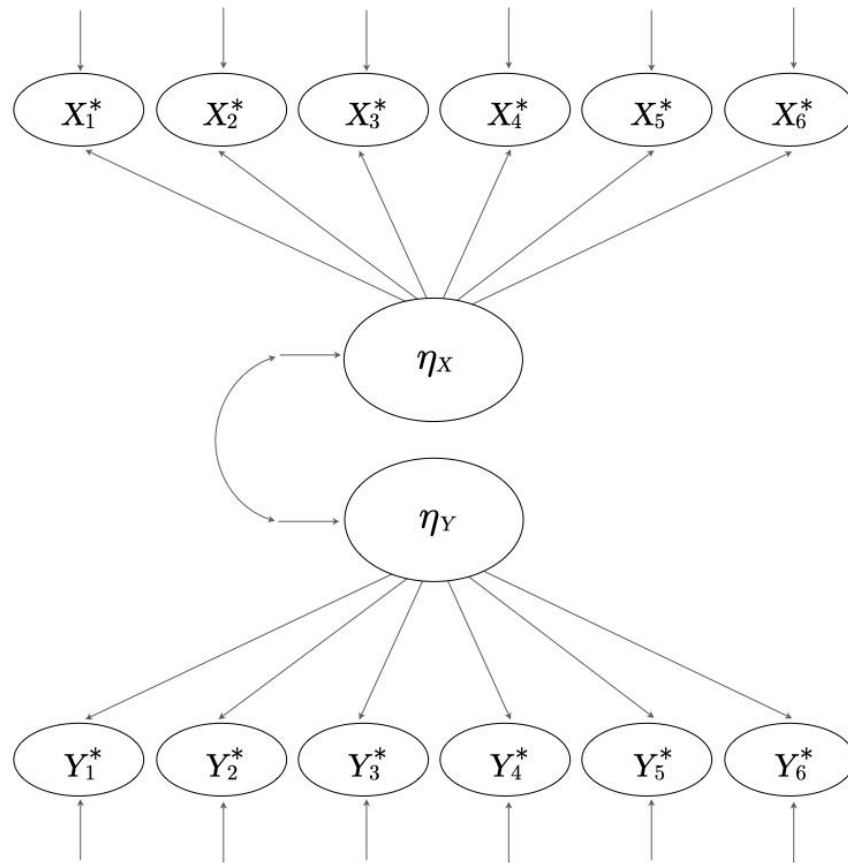
```

ability ~ 1@0;
ability ~~ ability@1;
y1 ~ 1@icept1 ability@load1;
y2 ~ 1@icept2 ability@load2;
y3 ~ 1@icept3 ability@load3;
y4 ~ 1@icept4 ability@load4;
y5 ~ 1@icept5 ability@load5;
y6 ~ 1@icept6 ability@load6;
PARAMETERS:
discrim1 = load1;
discrim2 = load2;
discrim3 = load3;
discrim4 = load4;
discrim5 = load5;
discrim6 = load6;
difficulty1 = - icept1 / load1;
difficulty2 = - icept2 / load2;
difficulty3 = - icept3 / load3;
difficulty4 = - icept4 / load4;
difficulty5 = - icept5 / load5;
difficulty6 = - icept6 / load6;
SEED: 90291;
BURN: 3000;
ITERATIONS: 10000;

```

5.7: Factor Analysis With Ordinal Indicators

This example illustrates a two-factor measurement model with correlated latent variables, each measured by six ordinal indicators. A path diagram of the analysis model is shown below, with ellipses denoting latent response variables, the residual variances of which are fixed at 1.



Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex5.7.imp](#) [data4.dat](#)

The syntax highlights are as follows.

- ❖ ORDINAL command identifies ordinal variables
- ❖ Automatic threshold specification for binary and ordinal variables
- ❖ LATENT command defines two latent variables
- ❖ Default specification fixes the first loading of each factor to 1 and sets the latent means equal to 0

- ❖ Longer burn-in period for estimating latent variables

```
DATA: data4.dat;  
VARIABLES: id a1 a2 a3 yscale mscale xscale n1 d1 d2  
             y1:y6 m1:m7 x1:x6;  
ORDINAL: x1:x6 y1:y6;  
MISSING: 999;  
LATENT: latency latentx;  
MODEL:  
latentx -> x1:x6;  
latency -> y1:y6;  
latentx <-> latency;  
SEED: 90291;  
BURN: 50000;  
ITERATIONS: 10000;
```

5.8: Imputing Latent Response Scores for Item-Level Factor Analysis

Examples 5.5 and 5.6 illustrated item-level factor analyses that imposed a measurement model on latent response variables. This example illustrates a latent variable imputation scheme from Enders (2022) that creates multiple imputation data sets containing categorical items as well as their underlying latent response variables (i.e., plausible values). The goal is to convert a categorical factor analysis problem into a normal-theory multiple imputation analysis that uses the latent response scores as indicators in lieu of discrete items. Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex5.8.imp](#) [Ex5.8.R](#) [data4.dat](#)

The syntax highlights are as follows.

- ❖ **ORDINAL** command identifies ordinal variables
- ❖ Automatic threshold specification for binary and ordinal variables
- ❖ **FCS** command specifies fully conditional specification multiple imputation
- ❖ Longer burn-in period for estimating thresholds
- ❖ `savelatent` keyword on the **OPTIONS** line saves latent response scores
- ❖ **NIMPS** command specifies 20 imputed data sets
- ❖ Setting **CHAINS** equal to **NIMPS** saves one data set from the final iteration of each MCMC chain (avoids autocorrelated imputations)
- ❖ Imputations are stacked in a single file with an index variable added in the first column

```

DATA: data4.dat;
VARIABLES: id a1 a2 a3 yscale mscale xscale d1 d2
               y1:y6 m1:m7 x1:x6;
ORDINAL: x1:x6 y1:y6;
MISSING: 999;
FCS: x1:x6 y1:y6;
SEED: 90291;
BURN: 25000;
ITERATIONS: 10000;
CHAINS: 100;
NIMPS: 100;
OPTIONS: savelatent;
SAVE: stacked = imps.dat;

```

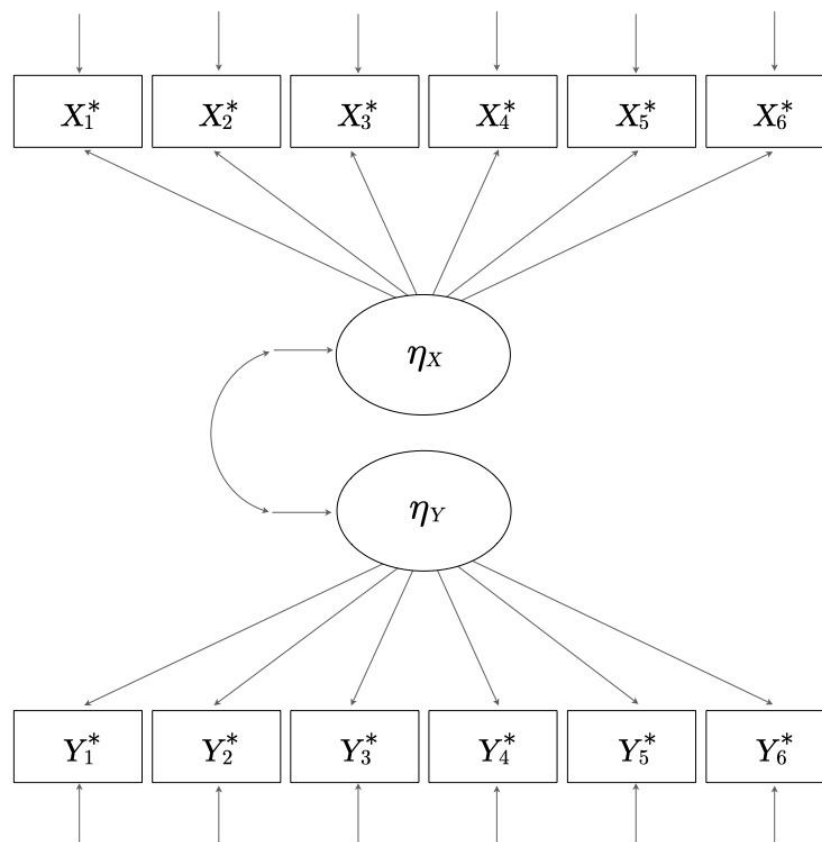
Blimp lists the order of the variables in the imputed data sets at the bottom of the output file, and all variables in the input file appear in the output file regardless of whether they were imputed. The latent response variables have a `.latent` suffix appended to the discrete variable's name.

VARIABLE ORDER IN IMPUTED DATA:

```
stacked = 'imps.dat'
```

```
imp# id a1 a2 a3 yscale mscale xscale d1 d2 y1 y2 y3 y4 y5
y6 m1 m2 m3 m4 m5 m6 m7 x1 x2 x3 x4 x5 x6
y1.latent y2.latent y3.latent
y4.latent y5.latent y6.latent
x1.latent x2.latent x3.latent
x4.latent x5.latent x6.latent
```

In the analysis phase, a normal-theory item-level confirmatory factor analysis is fit to the imputed latent response scores using other software packages. A path diagram is as follows.



The code block below uses the R packages `mitml` (Grund, Robitzsch, & Lüdtke, 2021), `lavaan` (Rosseel, Jorgensen, & Rockwood, 2021), and `semTools` (Jorgensen, Pornprasertmanit, Schoemann, & Rosseel, 2021) to fit a two-factor measurement model to the latent normal imputations. The resulting estimates are numerically equivalent to applying full information maximum likelihood analysis (FIML) with a probit link to the categorical data, but the FIML analysis often doesn't provide fit indices because the saturated model is too complex to estimate.

```
# load packages
library(semTools)
library(lavaan)
library(mitml)

# set working directory
fdir::set()

# read data from working directory
imps <- read.table("imps.dat")

names(imps) <- c("Imputation","id","a1","a2","a3","yscale","mscale",
  "xscale","d1","d2",paste0("y",seq(1:6)), paste0("m",seq(1:7)),
  paste0("x",seq(1:6)),paste0("laty",seq(1:6)),paste0("latx",seq(1:6)))

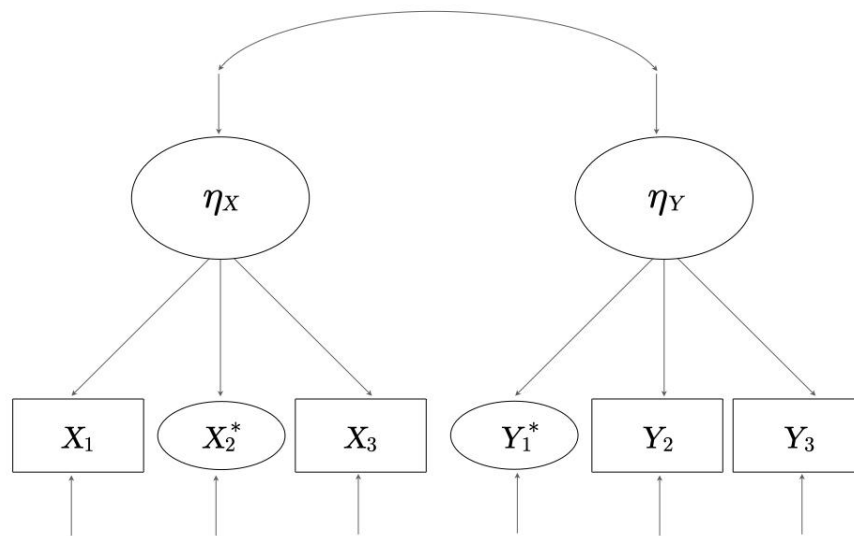
# specify lavaan model
ylatent <- paste("ylatent =~", paste0("laty", 1:6, collapse = " + "))
xlatent <- paste("xlatent =~", paste0("latx", 1:6, collapse = " + "))
model <- c(ylatent,xlatent)

# fit model with semtools
implist <- as.mitml.list(split(imps, imps$Imputation))
analysis <- cfa.mi(model, data = implist, estimator = "ml")
summary(analysis, standardized = T, fit = T)

# imputation-based modification indices
modindices.mi(analysis, op = c("~","="), minimum.value = 3, sort. = T)
```

5.9: Factor Analysis With Skewed Indicators and Yeo-Johnson Transform

This example illustrates a two-factor model with correlated latent variables, each measured by three continuous indicators. One indicator from each latent factor is skewed, and a Yeo-Johnson (Yeo & Johnson, 2000) normalizing transformation is applied to these indicators. A path diagram of the analysis model is shown below.



The ellipses indicate normalized indicators, which are essentially latent normal variables that have a nonlinear mapping to the nonnormal manifest variables. Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex5.9.imp](#) [Ex5.9.R](#) [data12.dat](#)

The syntax highlights are as follows.

- ❖ LATENT command defines two latent variables
- ❖ Individual regression equations specified for each indicator (instead of the \rightarrow convention for latent factors)

- ❖ Applying `yj t` function to skewed indicators on the MODEL line requests a Yeo-Johnson transformation
- ❖ MODEL command uses labels ending in a colon to group models and order their summary tables on the output
- ❖ MODEL command fixes the latent variable means to 0 and fixes one loading from each factor to 1
- ❖ Longer burn-in period for estimating latent variables
- ❖ `save latent` keyword on the OPTIONS line saves transformed (normalized) variables
- ❖ NIMPS command specifies 100 imputed data sets (more imputations recommended when analyzing latent response variables)
- ❖ Setting CHAINS equal to NIMPS saves one data set from the final iteration of each MCMC chain (avoids autocorrelated imputations)
- ❖ Imputations are stacked in a single file with an index variable added in the first column

```

DATA: data12.dat;
VARIABLES: x1:x3 y1:y3;
MISSING: 999;
LATENT: latentx latency;
MODEL:
  structural.model:
  latentx <-> latency;
  latentx ~ 1@0;
  latency ~ 1@0;
  measurement.model:
  x1 ~ latentx@1;
  yjt(x2) ~ latentx;
  x3 ~ latentx;
  yjt(y1) ~ latency;
  y2 ~ latency@1;

```

```

y3 ~ latent;
SEED: 90291;
BURN: 20000;
ITERATIONS: 10000;
OPTIONS: savelatent;
CHAINS: 20;
NIMPS: 20;
SAVE: stacked = imps.dat;

```

Blimp can save multiple imputations from any model it estimates. In addition to producing Bayesian estimates of the factor model parameters, the previous code block saves normalized imputations for a frequentist analysis. Blimp lists the order of the variables in the imputed data sets at the bottom of the output file, and all variables in the input file appear in the output file regardless of whether they were imputed. The variables `yjt(yjt(x2))` and `yjt(yjt(y1))` are the normalized variables.

VARIABLE ORDER IN IMPUTED DATA:

```
stacked = 'imps.dat'
```

```

imp# x1 x2 x3 y1 y2 y3 latentx.latent latency.latent
    yjt(yjt(x2)) yjt(yjt(y1))

```

In the analysis phase, a normal-theory item-level confirmatory factor analysis is fit to the original and normalized variables using other software packages. For example, the script shown in the code block below uses the R packages `mi` `tml` (Grund, Robitzsch, & Lüdtke, 2021), `lavaan` (Rosseel, Jorgensen, & Rockwood, 2021), and `semTools` (Jorgensen, Pornprasertmanit, Schoemann, & Rosseel, 2021) to fit a two-factor measurement model.


```

# load packages
library(semTools)
library(lavaan)
library(mitml)

# set working directory
fdir::set()

# plot original and normalized variables
hist(imps$x2)
hist(imps$x2norm)
hist(imps$y1)
hist(imps$y1norm)

# read data from working directory
imps <- read.table("imps.dat")
names(imps) <- c("Imputation","x1","x2","x3","y1","y2","y3",
  "latentx","latenty","x2norm","y1norm")

# specify lavaan model
ylatent <- paste("ylatent =~ x1 + x2norm + x3")
xlatent <- paste("xlatent =~ y1norm + y2 + y3")
model <- c(ylatent,xlatent)

# fit model with semtools
implist <- as.mitml.list(split(imps, imps$Imputation))
analysis <- cfa.mi(model, data = implist, estimator = "ml")
summary(analysis, standardized = T, fit = T)

```

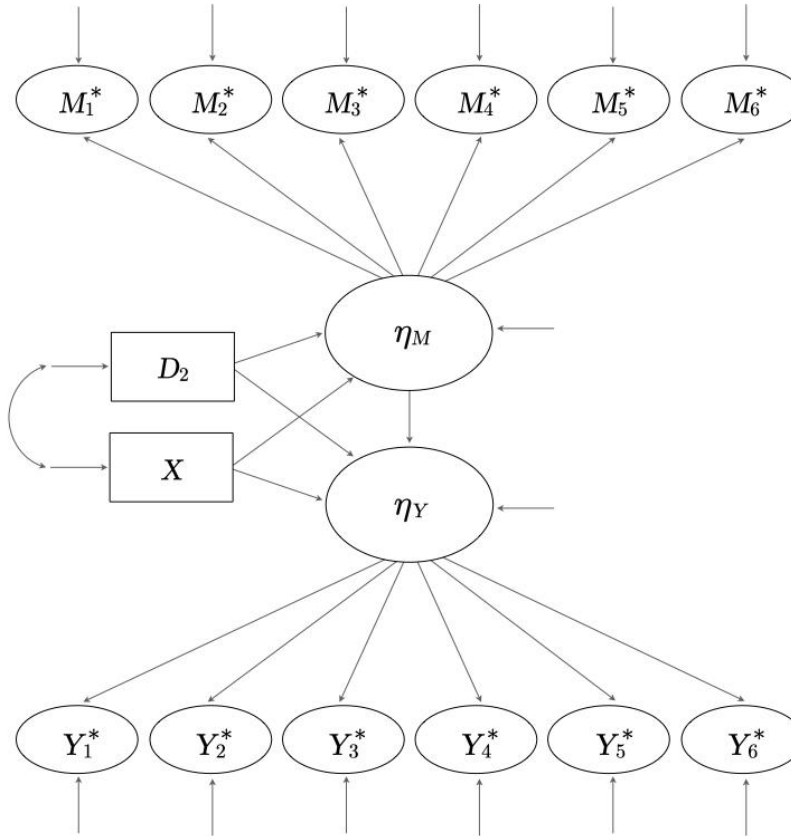
5.10: Latent Variable Regression Model

This example illustrates a latent variable mediation model where both the mediator and outcome are latent variables, each with six ordinal indicators. The structural regression equations are as follows

$$\eta_M = \beta_{01} + \beta_{11}X + \beta_{21}D_2 + \varepsilon_1$$

$$\eta_Y = \beta_{02} + \beta_{12}\eta_M + \beta_{22}X + \beta_{32}D_2 + \varepsilon_2$$

and a path diagram for the full model is shown below.



The residual variances of all latent response variances are fixed at values of 1, and mediated pathways can be computed following Example 5.1. Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex5.10.imp](#) [data4.dat](#)

The syntax highlights are as follows.

- ❖ ORDINAL command identifies binary and ordinal variables
- ❖ FIXED command defines a complete predictor

- ❖ Automatic threshold specification for binary and ordinal variables
- ❖ LATENT command defines two latent variables
- ❖ MODEL command uses labels ending in a colon to group models and order their summary tables on the output
- ❖ Default specification fixes the first loading of each factor to 1 and sets the latent means equal to 0
- ❖ Longer burn-in period for estimating latent variables and threshold parameters

```

DATA: data4.dat;
VARIABLES: id a1 a2 a3 yscale mscale xscale n1 d1 d2
              y1:y6 m1:m7 x1:x6;
ORDINAL: d2 y1:y6 m1:m7;
MISSING: 999;
FIXED: d2;
LATENT: latency latentm;
MODEL:
  structural.model:
    latentm ~ xscale d2;
    latency ~ latentm xscale d2;
  measurement.model:
    latentm -> m1:m6;
    latency -> y1:y6;
SEED: 90291;
BURN: 50000;
ITERATIONS: 10000;

```

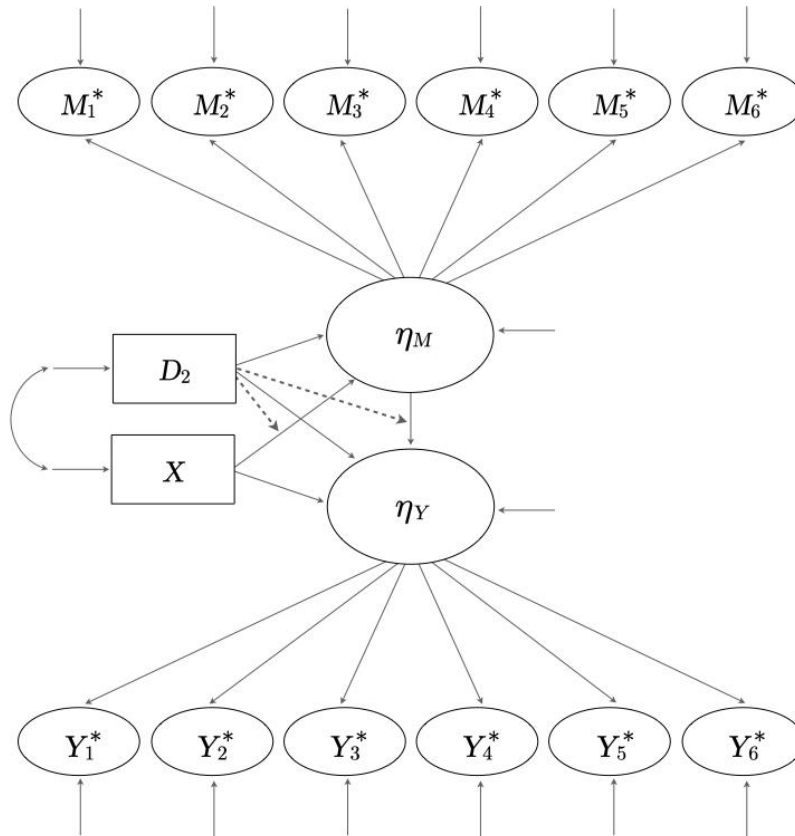
5.11: Latent-by-Manifest Variable Interaction

This example adds moderated paths to the latent variable mediation model from the previous example. The structural regression equations feature an interaction between two manifest variables and an interaction between a manifest and latent variable.

$$\eta_M = \beta_{01} + \beta_{11}X + \beta_{21}D_2 + \beta_{31}(X \times D_2) + \varepsilon_1$$

$$\eta_Y = \beta_0 + \beta_1\eta_X + \beta_2D_2 + \beta_3(\eta_X \times D_2) + \varepsilon_2$$

The path diagram of the full model is shown below.



The dashed lines pointing from D_2 to the directed arrows convey that D_2 moderates the association between X and the latent mediator as well as the association between the latent mediator and the outcome. The residual variances of all latent response variances are fixed at values of 1, and mediated pathways can be computed following Example 5.2. Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex5.11.imp](#) [data4.dat](#)

The syntax highlights are as follows.

- ❖ ORDINAL command identifies a binary predictor
- ❖ FIXED command defines a complete predictor
- ❖ LATENT command defines two latent variables
- ❖ MODEL command uses labels ending in a colon to group models and order their summary tables on the output
- ❖ Default specification fixes the first loading of each factor to 1 and sets the latent means equal to 0
- ❖ MODEL command features product terms
- ❖ Longer burn-in period for estimating latent variables

```

DATA: data4.dat;
VARIABLES: id a1 a2 a3 xscale zscale yscale n1 d1 d2 x1:x6
              z1:z7 y1:y6;
ORDINAL: d2;
MISSING: 999;
LATENT: latentx latency;
MODEL:
  structural.model:
  # label factor variances for simple slopes
  latentx ~~ latentx@xvar;
  latentm ~~ latentm@mvar;
  # latent correlation
  latentx <-> latentm;
  # regression model with interaction
  latency ~ latentx@b1 latentm@b2 latentx*latentm@b3;
  measurement.model:
  latentx -> x1:x3;
  latentm -> m1:m3;

```

```

latency -> y1:y3;
SEED: 90291;
BURN: 25000;
ITERATIONS: 10000;

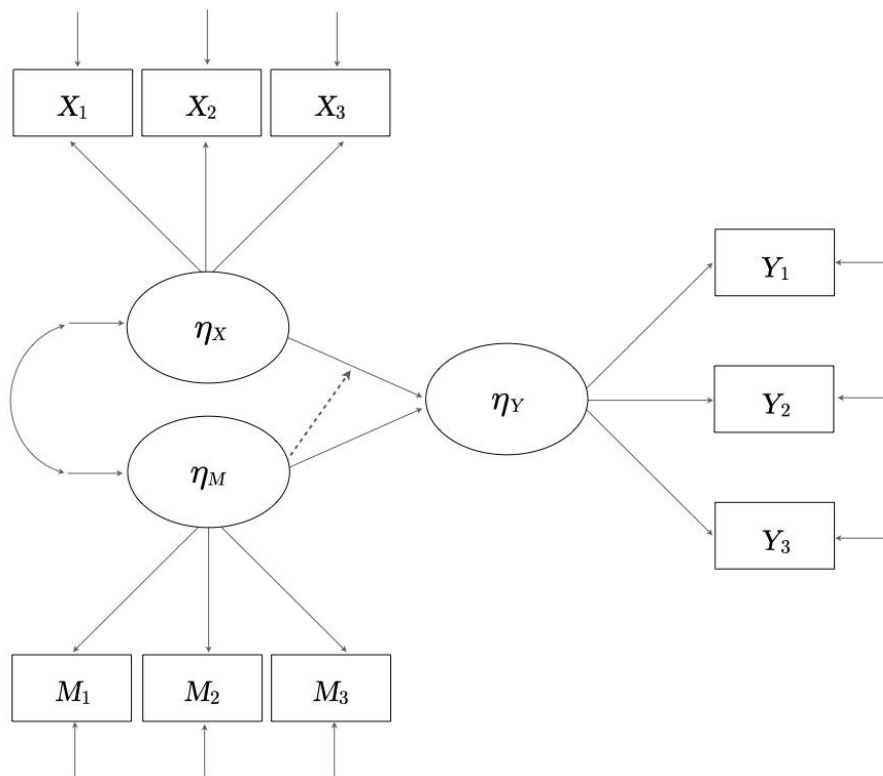
```

5.12: Latent-by-Latent Variable Interaction

This example illustrates a latent variable regression model with two latent predictors and their interaction influencing a latent outcome variable. The structural regression equation is as follows.

$$\eta_Y = \beta_0 + \beta_1\eta_X + \beta_2\eta_M + \beta_3(\eta_X \times \eta_M) + \varepsilon$$

A path diagram of the full model is shown below.



The dashed line pointing from the latent variable to the directed arrow conveys that one latent predictor is moderating the influence of the other. Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex5.12.imp](#) [data13.dat](#)

The syntax highlights are as follows.

- ❖ LATENT command defines three latent variables
- ❖ Default specification fixes the first loading of each factor to 1 and sets the latent means equal to
- ❖ MODEL command uses labels ending in a colon to group models and order their summary tables on the output
- ❖ MODEL command features a product term
- ❖ MODEL command labels the latent variable variances and structural regression slopes
- ❖ PARAMETERS command uses labeled quantities to compute conditional effects (simple slopes) at plus and minus one standard deviation above the latent moderator's mean
- ❖ Longer burn-in period for estimating latent variables

```
DATA: data13.dat;  
VARIABLES: x1:x3 m1:m3 y1:y3;  
MISSING: 999;  
LATENT: latentx latentm latenty;  
MODEL:  
# label factor variances for simple slopes  
latentx ~~ latentx@xvar;  
latentm ~~ latentm@mvar;  
# measurement models
```

```

latentx -> x1:x3;
latentm -> m1:m3;
latenty -> y1:y3;
# latent correlation
latentx <-> latentm;
# regression model with interaction
latenty ~ latentx@b1 latentm@b2 latentx*latentm@b3;
PARAMETERS:
xslp.mlo = b1 - b3 * sqrt(mvar);
xslp.mhi = b1 + b3 * sqrt(mvar);
mslp.xlo = b2 - b3 * sqrt(xvar);
mslp.xhi = b2 + b3 * sqrt(xvar);
SEED: 90291;
BURN: 10000;
ITERATIONS: 10000;

```

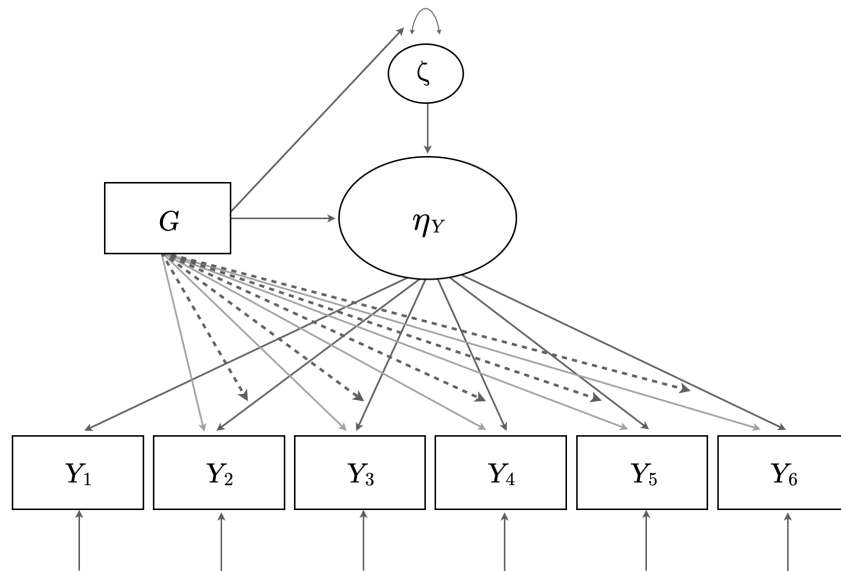
5.13: Moderated Factor Analysis for Measurement Invariance Testing

This example illustrates a moderated factor analysis for measurement invariance testing. The method is conceptually similar to nonlinear moderated factor analysis (Bauer, 2017) but does not require complicated constraints. Rather, the binary grouping variable exerts direct effects on factor model indicators and it interacts with the latent factor to produce group-specific factor loadings (the predictor need not be binary). The latent variable's variance and the indicator variances can also be modeled as a function of the background variable, which does not need to be categorical.

The measurement model equation for a manifest indicator k is as follows.

$$Y_k = \beta_{0k} + \beta_{1k}\eta_Y + \beta_{2k}G + \beta_{3k}(G \times \eta_Y) + \varepsilon_k$$

A path diagram of the model is shown below.



The straight lines from G to the indicators introduce group differences in measurement intercepts, and the dashed lines from G to the directed arrows reflect manifest-by-latent interaction terms (factor loading differences). Unlike a conventional multiple-group model, G could be a continuous dimension, although it is binary in this example. Finally, the model includes a loglinear equation that relates the variance of the latent factor to the background variable.

$$\ln(\sigma_{\zeta}^2) = \gamma_0 + \gamma_1 G$$

For identification, the intercept γ_0 is fixed at zero on the logarithmic metric, which sets the factor variance of the $G = 0$ group to one.

Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex5.13.imp](#) [data4.dat](#)

The syntax highlights are as follows.

- ❖ **ORDINAL** command identifies a binary predictor
- ❖ **FIXED** command identifies a complete predictor
- ❖ **LATENT** command defines a latent variable
- ❖ Individual regression equations specified for each indicator (instead of the \rightarrow convention for latent factors)
- ❖ **MODEL** command uses labels ending in a colon to group models and order their summary tables on the output
- ❖ **MODEL** command labels intercept group differences and loading group differences
- ❖ **MODEL** command features product terms
- ❖ **MODEL** command uses an @ and a label to assign a positive-valued prior distribution to the first factor loading
- ❖ **MODEL** command features a regression equation with the natural logarithm of the factor variance as the outcome
- ❖ **PARAMETER** command defines a positive-valued uniform prior distribution
- ❖ **TEST** commands specify Bayesian Wald tests that evaluate the null hypothesis that intercept and loading differences equal 0
- ❖ **SIMPLE** command produces conditional effects (group-specific intercepts and loadings) at each level of the binary moderator
- ❖ Longer burn-in period for estimating latent variables

```
DATA: data4.dat;  
VARIABLES: id a1 a2 a3 yscale mscale xscale n1 d1 g  
             y1:y6 m1:m7 x1:x6;  
ORDINAL: g;  
MISSING: 999;  
FIXED: g;  
LATENT: latency;
```

MODEL:

```
structural.model:
```

```
latency ~ 1@0 g;
```

```
var(latency) ~ 1@0 g;
```

```
measurement.model:
```

```
y1 ~ 1 latency@load1prior;
```

```
y2 ~ g@diffcept1 latency g*latency@diffload1;
```

```
y3 ~ g@diffcept2 latency g*latency@diffload2;
```

```
y4 ~ g@diffcept3 latency g*latency@diffload3;
```

```
y5 ~ g@diffcept4 latency g*latency@diffload4;
```

```
y6 ~ g@diffcept5 latency g*latency@diffload5;
```

```
TEST: diffload1:diffload5 = 0;
```

```
TEST: diffcept1:diffcept5 = 0;
```

```
PARAMETER;;
```

```
load1prior ~ truncate(0, Inf);
```

SIMPLE:

```
latency | g;
```

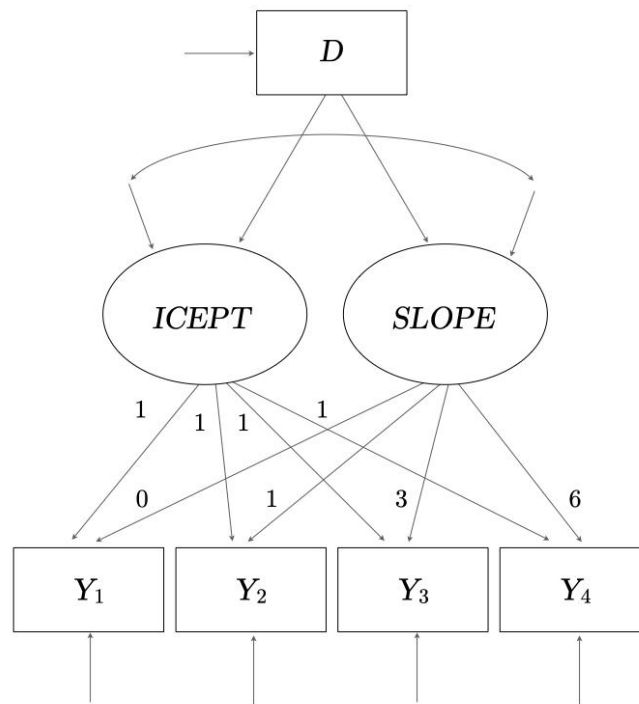
```
SEED: 90291;
```

```
BURN: 20000;
```

```
ITERATIONS: 10000;
```

5.14: Latent Growth Curve Model

This example illustrates a two-factor latent growth curve model with unequally spaced repeated measurements and a binary predictor of the random intercepts and slopes. A path diagram of the model is shown below.



Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex5.14.imp](#) [data3.dat](#)

The syntax highlights are as follows.

- ❖ `ORDINAL` command identifies a binary predictor
- ❖ `FIXED` command identifies complete predictors
- ❖ `LATENT` command defines two latent variables
- ❖ `MODEL` command uses labels ending in a colon to group models and order their summary tables on the output
- ❖ Individual regression equations specified for each indicator (instead of the \rightarrow convention for latent factors)

- ❖ MODEL command estimates the latent variable means, fixes the intercept factor loadings to 1, fixes the growth factor loadings to the time scores (0, 1, 3, and 6), and fixes the measurement intercepts to 0
- ❖ MODEL command uses a label to impose equality constraint on residual variance
- ❖ Longer burn-in period for estimating latent variables

```
DATA: data3.dat;  
VARIABLES: id y0 y1 y3 y6 d1 d2 v1:v4;  
ORDINAL: d1;  
MISSING: 999;  
FIXED: d1;  
LATENT: icept growth;  
MODEL:  
  structural.model:  
  icept ~ 1 d1;  
  slope ~ 1 d1;  
  icept <-> slope;  
  measurement.model:  
  y0 ~ 1@0 icept@1 slope@0;  
  y1 ~ 1@0 icept@1 slope@1;  
  y3 ~ 1@0 icept@1 slope@3;  
  y6 ~ 1@0 icept@1 slope@6;  
  # common residual variance  
  y0 ~~ y0@resvar;  
  y1 ~~ y1@resvar;  
  y3 ~~ y3@resvar;  
  y6 ~~ y6@resvar;  
SEED: 90291;  
BURN: 10000;  
ITERATIONS: 10000;
```

6 Analysis Examples: Multilevel Models

This section illustrates multilevel models in Blimp. In general, it is possible to mix and match features from any examples to create complex analysis models that honor features of the data. Following the previous chapter, the examples in this section use a generic notation system where variable names usually consist of an alphanumeric prefix and a numeric suffix (e.g., Y_1 , X_1 , N_1 , D_1 , D_2 , V_1 , V_2 , V_3). The letter Y designates a dependent variable, a D prefix denotes a binary dummy variable, an O prefix indicates an ordinal variable, and an N prefix indicates a multicategorical nominal variable. Other letters generally represent continuous variables. Additionally, the examples use a “.i” suffix to denote level-1 variables, “.j” for level-2 variables, and “.k” for level-3 variables (e.g., $d1.j$ is a level-2 dummy variable, $x1.i$ is a continuous variable measured at level-1). Blimp determines the levels automatically, so the suffixes are meant as a visual aid for understanding the scripts. Finally, the model equations use “cgm” and “cwc” superscripts to indicate grand and group mean centering, respectively. The following list outlines the examples in this section.

- ❖ 6.1: Two-Level Regression With Random Intercepts
- ❖ 6.2: Two-Level Fully Conditional Specification Multiple Imputation
- ❖ 6.3: Two-Level Regression With Random Coefficients
- ❖ 6.4: Alternate Prior Distributions
- ❖ 6.5 Inspecting Residuals
- ❖ 6.6: Two-Level Regression With Random (Heterogeneous) Within-Cluster Variances
- ❖ 6.7: Two-Level Model With Random Effects Predicting a Level-2 Outcome
- ❖ 6.8: Two-Level Regression With Latent Contextual Effect

- ❖ 6.9: Two-Level Regression With Cross-Level Interaction
- ❖ 6.10: Two-Level 1-1-1 Mediation With Random Slopes
- ❖ 6.11: Two-Level 1-1-1 Mediation With Moderated Paths
- ❖ 6.12: Within- and Between-Level Mediation
- ❖ 6.13: Two-Level Mediation With a Binary Outcome
- ❖ 6.14: Two-Level Linear Growth Model
- ❖ 6.15: Three-Level Growth Model
- ❖ 6.16: Two-Level Measurement Model With Predictors
- ❖ 6.17: Two-Level Regression With Random Intercepts and Sampling Weights
- ❖ 6.18: Partially Nested Designs (Singleton Clusters)
- ❖ 6.19: Discrete-Time Survival Model
- ❖ 6.20: Two-Level Dynamic Structural Equation Model With Random Intercepts
- ❖ 6.21: Two-Level Dynamic Structural Equation Model With Random Coefficients
- ❖ 6.22: Two-Level Dynamic Structural Equation Model With Random Within-Cluster Variances
- ❖ 6.23: Two-Level Dynamic Structural Equation Model With a Cross-Level Interaction Effect

6.1: Two-Level Regression With Random Intercepts

This example illustrates a two-level regression model with random intercepts. The regression model is shown below.

$$Y_{ij} = (\beta_0 + b_{0j}) + \beta_1 X_{1ij}^{cgm} + \beta_2 X_{4ij}^{cgm} + \beta_3 D_{2ij}^{cgm} + \beta_4 X_{5j}^{cgm} + \beta_5 D_{3j} + \varepsilon_{ij}$$

Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex6.1.imp](#) [data7.dat](#)

The syntax highlights are as follows.

- ❖ CLUSTERID command identifies a level-2 identifier, automatically inducing random intercepts for all incomplete level-1 variables
- ❖ ORDINAL command identifies binary predictors
- ❖ FIXED command defines complete predictors
- ❖ CENTER command applies grand mean centering to predictors
- ❖ Unspecified associations for predictor variables

```

DATA: data7.dat;
VARIABLES: level1id level2id n1.i d1.i d2.i n2.i x1.i x2.i
               x3.i x4.i y.i d3.j x5.j x6.j;
CLUSTERID: level2id;
ORDINAL: d2.i d3.j;
MISSING: 999;
FIXED: x4.i d3.j;
CENTER: grandmean = x1.i x4.i d2.i x5.j;
MODEL: y.i ~ x1.i x4.i d2.i x5.j d3.j;
SEED: 90291;
BURN: 2000;
ITERATIONS: 10000;

```

6.2: Two-Level Fully Conditional Specification Multiple Imputation

This example illustrates multilevel fully conditional specification multiple imputation as an approach to getting frequentist inference for the analysis from Example 6.1. The analysis model is shown below.

$$Y_{ij} = (\beta_0 + b_{0j}) + \beta_1 X_{1ij}^{cgm} + \beta_2 X_{4ij}^{cgm} + \beta_3 D_{2ij}^{cgm} + \beta_4 X_{5j}^{cgm} + \beta_5 D_{3j} + \varepsilon_{ij}$$

Fully conditional specification should be reserved for random intercept analyses, as applying the procedure to models with random coefficients or interaction terms is known to induce bias (Enders et al., 2020; Grund, Lüdke, & Robitzsch, 2016).

Model-based multiple imputation is recommended for such analyses (see Example 6.3). Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex6.2.imp](#) [Ex6.2.R](#) [data7.dat](#)

The syntax highlights are as follows.

- ❖ CLUSTERID command identifies a level-2 identifier, automatically inducing random intercepts (latent group means) for all level-1 variables
- ❖ ORDINAL command identifies binary predictors
- ❖ FCS command specifies fully conditional specification multiple imputation with a saturated model at level-1 and level-2 (unstructured within- and between-cluster covariance matrices)
- ❖ FCS command includes all analysis variables
- ❖ NIMPS command specifies 20 imputed data sets
- ❖ Setting CHAINS equal to NIMPS saves one data set from the final iteration of each MCMC chain (avoids autocorrelated imputations)
- ❖ Imputations are stacked in a single file with an index variable added in the first column

```
DATA: data7.dat;  
VARIABLES: level1id level2id n1.i d1.i d2.i n2.i x1.i x2.i  
              x3.i x4.i y.i d3.j x5.j x6.j;  
CLUSTERID: level2id;
```

```

ORDINAL: d2.i d3.j;
MISSING: 999;
FCS: y.i x1.i x4.i d2.i x5.j d3.j;
SEED: 90291;
BURN: 25000;
ITERATIONS: 10000;
CHAINS: 20;
NIMPS: 20;
SAVE: stacked = imps.dat;

```

Blimp lists the order of the variables in the imputed data sets at the bottom of the output file, and all variables in the input file appear in the output file regardless of whether they were imputed.

VARIABLE ORDER IN IMPUTED DATA:

```
stacked = 'imps.dat'
```

```

imp# level1id level2id n1.i d1.i d2.i n2.i x1.i x2.i x3.i
x4.i y.i d3.j x5.j x6.j

```

The imputed data sets can be analyzed in other software packages. The script below uses the R packages `lme4` (Bates et al., 2021) and `mitml` (Grund et al., 2021) to fit the multilevel regression model to the filled-in data sets. The resulting estimates are numerically equivalent to the Bayesian results from the Blimp output.

```

# set working directory
fdir::set()

# read data from working directory
imps <- read.table("imps.dat")
names(imps) <- c("imputation","level1id","level2id","n1.i","d1.i",
  "d2.i","n2.i","x1.i","x2.i","x3.i","x4.i", "y.i","d3.j","x5.j","x6.j")

```

```
# center predictors
imps$x1.i.cgm <- imps$x1.i - mean(imps$x1.i)
imps$x4.i.cgm <- imps$x4.i - mean(imps$x4.i)
imps$d2.i.cgm <- imps$d2.i - mean(imps$d2.i)
imps$x5.j.cgm <- imps$x5.j - mean(imps$x5.j)

# analysis and pooling
implist <- mitml::as.mitml.list(split(imps, imps$imputation))
mod <- "y.i ~ x1.i.cgm + x4.i.cgm + d2.i.cgm + x5.j.cgm + d3.j +
(1|level2id)"
ddf <- 23
results <- with(implist, lme4::lmer(mod, REML = T))
mitml::testEstimates(results, extra.pars = T, df.com = ddf)
```

6.3: Two-Level Regression With Random Coefficients

This example illustrates a two-level regression model with random intercepts and random slopes. The analysis model is shown below.

$$Y_{ij} = (\beta_0 + b_{0j}) + (\beta_1 + b_{1j}) X_{1ij}^{cwc} + \beta_2 X_{2ij}^{cgm} + \beta_3 X_{7j}^{cgm} + \beta_4 D_{1j}^{cgm} + \varepsilon_{ij}$$

Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex6.3a.imp](#) [Ex6.3b.imp](#) [Ex6.3.R](#) [data8.dat](#)

The syntax highlights are as follows.

- ❖ CLUSTERID command identifies a level-2 identifier, automatically inducing random intercepts for all level-1 variables
- ❖ ORDINAL command identifies a binary predictor
- ❖ FIXED command identifies a complete predictor

- ❖ CENTER command applies grand mean and latent group mean centering to predictors
- ❖ MODEL command features a random coefficient listed after the vertical pipe
- ❖ Unspecified associations for predictor variables

```
DATA: data8.dat;  
VARIABLES: level1id level2id x1.i x2.i y.i x3.i x4.i d1.j  
              n1.j x5.j x6.j x7.j x8.j x9.j;  
CLUSTERID: level2id;  
ORDINAL: d1.j;  
MISSING: 999;  
FIXED: d1.j;  
CENTER:  
groupmean = x1.i;  
grandmean = x2.i x7.j d1.j;  
MODEL: y.i ~ x1.i x2.i x7.j d1.j | x1.i;  
SEED: 90291;  
BURN: 5000;  
ITERATIONS: 10000;
```

Blimp can save multiple imputations from any model it estimates. Model-based multiple imputations can be saved for a frequentist analysis by adding the SAVE and NIMPS commands. The additional syntax highlights are as follows.

- ❖ CENTER command grand mean centers predictors in the Bayesian output, but saved imputations are on the original metric
- ❖ NIMPS command specifies 20 imputed data sets
- ❖ Setting CHAINS equal to NIMPS saves one data set from the final iteration of each MCMC chain (avoids autocorrelated imputations)

- ❖ `saveLatent` keyword on the `OPTIONS` line saves the latent group means of the level-1 predictors and the analysis model's random intercept and random slope residuals
- ❖ Imputations are stacked in a single file with an index variable added in the first column

```

DATA: data8.dat;
VARIABLES: level1id level2id x1.i x2.i y.i x3.i x4.i d1.j
               n1.j x5.j x6.j x7.j x8.j x9.j;
CLUSTERID: level2id;
ORDINAL: d1.j;
MISSING: 999;
FIXED: d1.j;
CENTER:
groupmean = x1.i;
grandmean = x2.i x7.j d1.j;
MODEL: y.i ~ x1.i x2.i x7.j d1.j | x1.i;
SEED: 90291;
BURN: 5000;
ITERATIONS: 10000;
CHAINS: 20;
NIMPS: 20;
OPTIONS: saveLatent;
SAVE: stacked = imps.dat;

```

Blimp lists the order of the variables in the imputed data sets at the bottom of the output file, and all variables in the input file appear in the output file regardless of whether they were imputed. The `saveLatent` keyword also saves the latent group means of any level-1 predictors, and these can be used to center variables prior to analyzing the imputations. This example uses X_1 's latent group means, which are referred to by the name `x1.i.mean[level2id]`.

VARIABLE ORDER IN IMPUTED DATA:

```
stacked = 'imps.dat'

imp# level1id level2id x1.i x2.i y.i x3.i x4.i d1.j n1.j
x5.j x6.j x7.j x8.j x9.j y.i[level2id] y.i$x1.i[level2id]
x1.i.mean[level2id] x2.i.mean[level2id]
```

The imputed data sets can be analyzed in other software packages. The script below uses the R packages `lme4` (Bates et al., 2021) and `mitml` (Grund et al., 2021) to fit the multilevel regression model to the filled-in data sets. The resulting estimates are numerically equivalent to the Bayesian results from the Blimp output.

```
# set working directory
fdir::set()

# read data from working directory
imps <- read.table("imps.dat")
names(imps) <-
c("imputation", "level1id", "level2id", "x1.i", "x2.i", "y.i", "x3.i", "x4.i",
  "d1.j", "n1.j", "x5.j", "x6.j", "x7.j", "x8.j", "x9.j",
  "y.ranicept", "x1.ranslp", "x1.mean.j", "x2.mean.j")

# grand center predictors
imps$x1.i.cwc <- imps$x1.i - imps$x1.mean.j
imps$x2.i.cgm <- imps$x2.i - mean(imps$x2.i)
imps$x7.j.cgm <- imps$x7.j - mean(imps$x7.j)
imps$d1.j.cgm <- imps$d1.j - mean(imps$d1.j)

# analysis and pooling
implist <- mitml::as.mitml.list(split(imps, imps$imputation))
mod <- "y.i ~ x1.i.cwc + x2.i.cgm + x7.j.cgm + d1.j + (1 +
x1.i.cwc|level2id)"
ddf <- 127
results <- with(implist, lme4::lmer(mod, REML = T))
mitml::testEstimates(results, extra.pars = T, df.com = ddf)
```

6.4: Alternate Prior Distributions for Random Effect Covariance Matrix

This example illustrates how to examine the influence of different prior distributions on the level-2 covariance matrix of the random effects. The analysis model is the following two-level regression with random intercepts and random slopes.

$$Y_{ij} = (\beta_0 + b_{0j}) + (\beta_1 + b_{1j}) X_{1ij}^{cwc} + \beta_2 X_{2ij}^{cgm} + \varepsilon_{ij}$$

The between-cluster covariance matrix of the random effects is a 2 by 2 matrix in this example. Blimp offers three “off-the-shelf” inverse Wishart priors for the covariance matrix, and it is also possible to use a so-called separation strategy that applies distinct priors to variances and the intercept-slope correlation. Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex6.4a.prior2.imp](#) [Ex6.4b.prior1.imp](#) [Ex6.4c.prior3.imp](#)

[Ex6.4d.separation.imp](#) [data8.dat](#)

Considering the inverse Wishart options, the default `prior2` setting is less informative because it subtracts the number of dimensions plus 1 from the degrees of freedom, and it adds nothing to the sum of squares and cross-products; `prior1` is more informative because it adds the number of dimensions plus 1 to the degrees of freedom, and it adds an identity matrix to the sum of squares and cross-products; `prior3` adds zero degrees of freedom and adds zero to the sums of squares. The code block below shows the default specification, the syntax highlights for which are as follows.

- ❖ `CLUSTERID` command identifies a level-2 identifier, automatically inducing random intercepts for all level-1 variables

- ❖ CENTER command applies grand mean and latent group mean centering
- ❖ MODEL command features a random coefficient listed after the vertical pipe
- ❖ Unspecified associations for predictor variables
- ❖ prior2 keyword on the OPTIONS line (optional) specifies the default inverse Wishart prior

```
DATA: data8.dat;  
VARIABLES: level1id level2id x1.i x2.i y.i x3.i x4.i d1.j  
              n1.j x5.j x6.j x7.j x8.j x9.j;  
CLUSTERID: level2id;  
MISSING: 999;  
CENTER:  
groupmean = x1.i;  
grandmean = x2.i;  
MODEL: y.i ~ x1.i x2.i | x1.i;  
SEED: 90291;  
BURN: 10000;  
ITERATIONS: 10000;  
OPTIONS: prior2;
```

Similarly, the code block below shows the specification for the more informative prior1 inverse Wishart option.

```
DATA: data8.dat;  
VARIABLES: level1id level2id x1.i x2.i y.i x3.i x4.i d1.j  
              n1.j x5.j x6.j x7.j x8.j x9.j;  
CLUSTERID: level2id;  
MISSING: 999;  
CENTER:  
groupmean = x1.i;  
grandmean = x2.i;  
MODEL: y.i ~ x1.i x2.i | x1.i;
```



```

SEED: 90291;
BURN: 10000;
ITERATIONS: 10000;
OPTIONS: prior1;

```

Comparing the magnitude of the point estimates provides a gauge about the prior distribution's impact. The output table for the default `prior2` specification is shown immediately below, and the second output table shows the results from the more informative `prior1` specification.

```

# prior2

Parameters                Median      StdDev      2.5%      97.5%
-----
Variances:
  L2 : Var(Intercept)      0.613      0.083      0.478      0.804
  L2 : Cov(x1.i,Intercept) 0.016      0.016     -0.014      0.048
  L2 : Var(x1.i)           0.020      0.006      0.010      0.034
  Residual Var.           0.358      0.011      0.337      0.381

...

Proportion Variance Explained
  by Fixed Effects         0.048      0.009      0.032      0.065
  by Level-2 Random Intercepts 0.587      0.033      0.524      0.653
  by Level-2 Random Slopes    0.021      0.006      0.011      0.036
  by Level-1 Residual Variation 0.343      0.027      0.288      0.395

```

```

# prior1

Parameters                Median      StdDev      2.5%      97.5%
-----
Variances:
  L2 : Var(Intercept)      0.591      0.078      0.464      0.771

```

L2 : Cov(x1.i,Intercept)	0.017	0.018	-0.017	0.053
L2 : Var(x1.i)	0.039	0.007	0.028	0.056
Residual Var.	0.354	0.011	0.333	0.377
...				
Proportion Variance Explained				
by Fixed Effects	0.048	0.009	0.033	0.068
by Level-2 Random Intercepts	0.569	0.033	0.505	0.635
by Level-2 Random Slopes	0.041	0.008	0.028	0.059
by Level-1 Residual Variation	0.341	0.026	0.289	0.392

The default prior 2's random slope variance is roughly half as large as that of the more informative prior (0.020 vs. 0.039), and the two estimates differed by about 2.7 posterior standard deviation units (a very large difference). As a proportion of the total variance, the R^2 effect sizes attributable to the random slopes (Rights & Sterba, 2019) were also quite different (2.1% vs. 4.1%).

The separation strategy (Barnard, McCulloch, & Meng, 2000; Liu, Zhang, & Grimm, 2016) assigns distinct priors to the diagonal and off-diagonal elements of the covariance matrix. An analogous strategy can be implemented in Blimp by specifying the random intercepts and slopes as a pair of level-2 latent variables. The focal model is cast as a multilevel structural equation model with a pair of normally distributed level-2 latent variables representing the random intercepts and slopes. The focal model features these latent variables as predictors, as shown below.

$$\beta_{0j} = \beta_0 + b_{0j}$$

$$\beta_{1j} = \beta_1 + b_{1j}$$

$$Y_{ij} = \beta_{0j} + \beta_{1j}X_{1ij}^{cwc} + \beta_2X_{2ij}^{cgm} + \varepsilon_{ij}$$

Note that the coefficient of the random slope predictor is implicitly fixed to 1 in this specification.

This specification assigns separate inverse gamma priors to the random intercept and slope variances, and it specifies a beta prior distribution to their correlation. Blimp uses a multilevel extension of the procedure described in Merkle and Rosseel (2018).

Computer simulation studies suggest that the separation strategy gives more accurate estimates of the variance components, although the correlation estimate may be attenuated when the number of level-2 units is small (Keller & Enders, 2021). The unique syntax highlights for the code block are as follows.

- ❖ LATENT command defines two between-cluster latent variables representing the random intercepts and slopes
- ❖ MODEL command uses labels ending in a colon to group models and order their summary tables on the output
- ❖ MODEL command estimates the random intercept and slope means
- ❖ MODEL command sets the intercept of the regression equation equal to the level-2 latent mean (`1@beta0.j`)
- ❖ MODEL command omits the random coefficient listed after the vertical pipe
- ❖ MODEL command includes the interaction between a level-1 predictor and the level-2 latent slope (`x1.i*beta1.j`)
- ❖ MODEL command fixes the interaction coefficient for the product of the random slope predictor and its level-2 latent variable to 1
- ❖ MODEL command specifies correlation between random intercepts and random slopes (level-2 latent variables)
- ❖ OPTIONS command lists the `use_phantom` keyword to invoke a phantom variable specification that assigns distinct priors to latent variable variances and their correlation

```

DATA: data8.dat;
VARIABLES: level1id level2id x1.i x2.i y.i x3.i x4.i d1.j
               n1.j x5.j x6.j x7.j x8.j x9.j;
CLUSTERID: level2id;
MISSING: 999;
LATENT: level2id = beta0.j beta1.j;
CENTER:
groupmean = x1.i;
grandmean = x2.i;
MODEL:
latent.variables:
beta0.j ~ 1;
beta1.j ~ 1;
beta0.j ~~ beta1.j;
focal.model:
y.i ~ 1@beta0.j x1.i*beta1.j@1 x2.i;
SEED: 90291;
BURN: 10000;
ITERATIONS: 10000;
OPTIONS: use_phantom;

```

The random effect parameter estimates no longer appear on the same table when employing the separation strategy because the random intercepts and slopes are latent variables with their own equations and summary tables. The analysis model table shows the random intercept variance, and the level-2 latent variable's (random slope) variance and correlation appear in separate tables.

```
# separation strategy
```

```
Latent Variable:  beta0.j
```

Parameters	Median	StdDev	2.5%	97.5%

Variances:				
Residual Var.	0.605	0.081	0.472	0.792

Coefficients:				
Intercept	4.170	0.071	4.031	4.309
Proportion Variance Explained				
by Coefficients	0.000	0.000	0.000	0.000
by Residual Variation	1.000	0.000	1.000	1.000

Latent Variable: beta1.j				
Parameters	Median	StdDev	2.5%	97.5%

Variances:				
Residual Var.	0.019	0.005	0.011	0.031
Coefficients:				
Intercept	-0.094	0.020	-0.132	-0.055
Proportion Variance Explained				
by Coefficients	0.000	0.000	0.000	0.000
by Residual Variation	1.000	0.000	1.000	1.000

Phantom Variable Correlations:				
Parameters	Median	StdDev	2.5%	97.5%

beta0.j <-> beta1.j	0.112	0.123	-0.127	0.348

...				
Outcome Variable: y.i				
Grand Mean Centered: x2.i				
Group Mean Centered: x1.i				
Parameters	Median	StdDev	2.5%	97.5%

Variances:				
Residual Var.	0.358	0.011	0.337	0.381
Coefficients:				

beta0.j	@1.000	---	---	---
x2.i	0.087	0.008	0.071	0.102
x1.i*beta1.j	@1.000	---	---	---
Standardized Coefficients:				
x2.i	0.291	0.025	0.242	0.339
x1.i*beta1.j	0.284	0.025	0.235	0.334
Proportion Variance Explained				
by Coefficients	0.177	0.020	0.140	0.218
by Residual Variation	0.823	0.020	0.782	0.860

6.5: Inspecting Residuals

This example illustrates how to inspect the level-1 and level-2 residuals (random effects) from a two-level regression model with random intercepts and random slopes. The analysis model, shown below, is the same as the one from Example 6.4.

$$Y_{ij} = (\beta_0 + b_{0j}) + (\beta_1 + b_{1j}) X_{1ij}^{cwc} + \beta_2 X_{2ij}^{cgm} + \varepsilon_{ij}$$

Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex6.5.imp](#) [Ex6.5.R](#) [data8.dat](#)

The syntax highlights are as follows.

- ❖ CLUSTERID command identifies a level-2 identifier, automatically inducing random intercepts for all level-1 variables
- ❖ CENTER command applies grand mean and latent group mean centering to predictors in the Bayesian output, but saved imputations are on the original metric
- ❖ MODEL command features a random coefficient listed after the vertical pipe
- ❖ Unspecified associations for predictor variables

- ❖ `savelatent` keyword on the `OPTIONS` line saves the latent group means of the level-1 predictors and the analysis model's random intercept and random slope residuals
- ❖ `saveresidual` keyword on the `OPTIONS` line saves level-1 residuals
- ❖ `NIMPS` command specifies 20 imputed data sets
- ❖ Setting `CHAINS` equal to `NIMPS` saves one data set from the final iteration of each MCMC chain (avoids autocorrelated imputations)
- ❖ Imputations are stacked in a single file with an index variable added in the first column

```

DATA: data8.dat;
VARIABLES: level1id level2id x1.i x2.i y.i x3.i x4.i d1.j
               n1.j x5.j x6.j x7.j x8.j x9.j;
CLUSTERID: level2id;
MISSING: 999;
CENTER:
groupmean = x1.i;
grandmean = x2.i;
MODEL: y.i ~ x1.i x2.i | x1.i;
SEED: 90291;
BURN: 5000;
ITERATIONS: 10000;
CHAINS: 20;
NIMPS: 20;
OPTIONS: savelatent saveresidual;
SAVE: stacked = imps.dat;

```

Blimp lists the order of the variables in the imputed data sets at the bottom of the output file, and all variables in the input file appear in the output file regardless of whether they were imputed. The latent group means, random effects, and level-1 residuals are appended to the end of the file. Latent group means are designated by

appending the level-2 identifier in square brackets to the end of a predictor variable's name (e.g., `x1.i.mean[level2id]` and `x2.i.mean[level2id]`). The analysis model's random intercepts are denoted by appending the level-2 identifier in square brackets to the end of an outcome variable's name (e.g., `y.i[level2id]`). Random slope residuals are indicated by joining the outcome and random predictor variables with a `$` sign (e.g., `y.i$x1.i[level2id]`). Finally, level-1 residuals are indicated by appending `.residual` to the end of the outcome variable's name (e.g., `y.i.residual`).

VARIABLE ORDER IN IMPUTED DATA:

```
stacked = 'imps.dat'

imp# level1id level2id x1.i x2.i y.i x3.i x4.i d1.j n1.j
x5.j x6.j x7.j x8.j x9.j y.i[level2id]
y.i$x1.i[level2id] x1.i.mean[level2id]
x2.i.mean[level2id] y.i.residual
```

The imputed data sets can be analyzed in other software packages. The script below uses the R package `rockchalk` (Johnson, 2019) to compute excess skewness and kurtosis, and it uses base R functions to plot the residuals.

```
# set working directory
fdir::set()

# read data from working directory
imps <- read.table("imps.dat")
names(imps) <- c("imputation","level1id","level2id","x1.i","x2.i",
  "y.i","x3.i","x4.i","d1.j","n1.j","x5.j","x6.j","x7.j",
  "x8.j","x9.j","y.ranicept","x1.ranslope","x1.mean.j",
  "x2.mean.j","y.l1residual")
```



```
# skewness and kurtosis of residuals
rockchalk::skewness(imps$y.ranicept)
rockchalk::kurtosis(imps$y.ranicept)
rockchalk::skewness(imps$x1.ranslope)
rockchalk::kurtosis(imps$x1.ranslope)
rockchalk::skewness(imps$y.l1residual)
rockchalk::kurtosis(imps$y.l1residual)

# plot distribution of level-1 residuals
hist(imps$y.l1residual)
plot(density(imps$y.l1residual))

# plot distribution of level-2 random effects
hist(imps$y.ranicept)
hist(imps$x1.ranslope)
plot(density(imps$y.ranicept))
plot(density(imps$x1.ranslope))

# qq plots
qqnorm(imps$y.ranicept); qqline(imps$y.ranicept)
qqnorm(imps$x1.ranslope); qqline(imps$x1.ranslope)
```

6.6: Two-Level Regression With Random (Heterogeneous) Within-Cluster Variation

This example illustrates a two-level regression model with random intercepts and slopes and heterogeneous within-cluster variances. The analysis model below is the same one as Example 6.3, but the variance of the within-cluster residuals differs across clusters.

$$Y_{ij} = (\beta_0 + b_{0j}) + (\beta_1 + b_{1j}) X_{1ij}^{cwc} + \beta_2 X_{2ij}^{cgm} + \beta_3 X_{7j}^{cgm} + \beta_4 D_{1j}^{cgm} + \varepsilon_{ij}$$

Blimp provides two methods for modeling heterogeneous within-cluster variation. The first is the so-called location and scale model where observation-level variation is a function of level-1 and/or level-2 predictors and a cluster-level random effect

(Hedeker, Mermelstein, & Demirtas, 2008). A log-linear model expresses the natural log of the variance as a level-2 latent variable with a mean and between-cluster variance that represents cluster-level differences in the within-cluster variances.

$$\gamma_{0j} = \gamma_0 + g_{0j}$$

$$\ln(\sigma_{\varepsilon j}^2) = \gamma_{0j} + \gamma_1 X_{2ij}^{cgm} + \gamma_2 D_{1j}^{cgm}$$

This approach allows within-cluster variation to function as both an outcome and a predictor of distal level-2 outcomes.

A second approach described by Kasim and Raudenbush (1998) views cluster-specific variation as a level-2 variable that follows a distribution. The Kasim and Raudenbush method does not allow links between within-cluster variation and other variables.

Clicking the links below downloads the Blimp scripts and data for these examples, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex6.6a.imp](#) [Ex6.6b.imp](#) [data8.dat](#)

The code block below shows the basic setup for a location-scale model where observation-level variation is a function of a level-1 and level-2 predictor and a level-2 random effect. The syntax highlights are listed below, and adding the NIMPS and SAVE commands generates model-based multiple imputations for a frequentist analysis (see Example 6.3).

- ❖ CLUSTERID command identifies a level-2 identifier, automatically inducing random intercepts for all level-1 variables
- ❖ ORDINAL command identifies a binary predictor

- ❖ LATENT command identifies level-2 latent variable named logvar that represents within-cluster variation on the log metric
- ❖ FIXED command identifies a complete predictor
- ❖ CENTER command applies grand mean and latent group mean centering to predictors
- ❖ MODEL command uses labels ending in a colon to group models and order their summary tables on the output
- ❖ MODEL command features a random coefficient listed after the vertical pipe
- ❖ MODEL command features a second regression equation with the natural logarithm of the level-1 variation as the outcome. The 1@logvar constraint sets the intercept equal to the level-2 latent variable logvar
- ❖ Unspecified associations for predictor variables

```

DATA: data8.dat;
VARIABLES: level1id level2id x1.i x2.i y.i x3.i x4.i d1.j n1.j
               x5.j x6.j x7.j x8.j y2.j;
CLUSTERID: level2id;
ORDINAL: d1.j;
LATENT: level2id = logvar;
MISSING: 999;
FIXED: d1.j;
CENTER:
groupmean = x1.i;
grandmean = x2.i x7.j d1.j;
MODEL:
focal.model:
y.i ~ x1.i x2.i x7.j d1.j | x1.i;
variance.model:
var(y.i ) ~ 1@logvar x2.i d1.j;
SEED: 90291;
BURN: 10000;
ITERATIONS: 10000;

```

As noted previously, within-cluster variation can also predict distal outcomes at level-2. The model command below uses cluster-specific variation to predict a level-2 variable named `y2.j`.

MODEL :

```
y.i ~ x1.i x2.i x7.j d1.j | x1.i;
var(y.i ) ~ 1@logvar x2.i d1.j;
y2.j ~ logvar;
```

MCMC estimation yields a model-predicted variance for each observation (on the natural log scale). To convey the magnitude of the variational differences, Blimp converts the mean and quartiles of the log-variance distribution to the variance metric and includes these summaries on the output. The output excerpt below shows part of the main summary table from the example.

OUTCOME MODEL ESTIMATES:

Summaries based on 10000 iterations using 2 chains.

Outcome Variable: `y.i`

Grand Mean Centered: `d1.j x2.i x7.j`

Group Mean Centered: `x1.i`

Parameters	Median	StdDev	2.5%	97.5%

Variances:				
L2 : Var(Intercept)	0.626	0.085	0.488	0.815
L2 : Cov(x1.i,Intercept)	0.026	0.014	0.000	0.056
L2 : Var(x1.i)	0.015	0.005	0.006	0.028
Mean Residual Var.	0.299	0.010	0.281	0.320
Q25% Residual Var.	0.190	0.010	0.170	0.211
Q50% Residual Var.	0.305	0.015	0.278	0.335
Q75% Residual Var.	0.482	0.025	0.436	0.536
...				

The code block below shows the setup for the Kasim and Raudenbush (1998) approach to modeling heterogeneous variation. The syntax highlights are listed below, and adding the NIMPS and SAVE commands generates model-based multiple imputations for a frequentist analysis (see Example 6.3).

- ❖ CLUSTERID command identifies a level-2 identifier, automatically inducing random intercepts for all level-1 variables
- ❖ ORDINAL command identifies a binary predictor
- ❖ FIXED command identifies a complete predictor
- ❖ CENTER command applies grand mean and latent group mean centering to predictors
- ❖ MODEL command features a random coefficient listed after the vertical pipe
- ❖ Unspecified associations for predictor variables
- ❖ hev keyword on OPTIONS line specifies heterogeneous within-cluster variances (Kasim & Raudenbush, 1998)

```
DATA: data8.dat;  
VARIABLES: level1id level2id x1.i x2.i y.i x3.i x4.i d1.j n1.j  
             x5.j x6.j x7.j x8.j x9.j;  
CLUSTERID: level2id;  
ORDINAL: d1.j;  
MISSING: 999;  
FIXED: d1.j;  
CENTER:  
groupmean = x1.i;  
grandmean = x2.i x7.j d1.j;  
MODEL: y.i ~ x1.i x2.i x7.j d1.j | x1.i;  
SEED: 90291;  
BURN: 5000;  
ITERATIONS: 10000;
```

OPTIONS: hev;

MCMC estimation yields an estimate of the variation within each cluster. To convey the magnitude of the variational differences, Blimp computes the mean and quartiles of the variance distribution and includes these summaries on the output. The output excerpt below shows part of the main summary table from the example.

OUTCOME MODEL ESTIMATES:

Summaries based on 10000 iterations using 2 chains.

Outcome Variable: y.i

Grand Mean Centered: d1.j x2.i x7.j

Group Mean Centered: x1.i

Parameters	Median	StdDev	2.5%	97.5%

Variances:				
L2 : Var(Intercept)	0.648	0.088	0.507	0.851
L2 : Cov(x1.i,Intercept)	0.026	0.014	-0.000	0.056
L2 : Var(x1.i)	0.014	0.006	0.006	0.028
Mean Residual Var.	0.373	0.016	0.345	0.406
Q25% Residual Var.	0.188	0.011	0.168	0.211
Q50% Residual Var.	0.296	0.016	0.267	0.328
Q75% Residual Var.	0.476	0.028	0.426	0.534
...				

6.7: Two-Level Model With Random Effects Predicting a Level-2 Outcome

This example illustrates a two-level path model where the random intercepts and random slopes from one model predict a level-2 outcome in another model. The regression equations are shown below.

$$Y_{ij} = (\beta_0 + b_{0j}) + (\beta_1 + b_{1j}) X_{1ij}^{cwc} + \beta_2 X_{2ij}^{cgm} + \beta_3 X_{7j}^{cgm} + \beta_4 D_{1j}^{cgm} + \varepsilon_{ij}$$

$$Y_{2j} = \gamma_0 + \gamma_1 b_{0j} + \gamma_1 b_{1j} + \gamma_3 X_{7j}^{cgm} + r_j$$

Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex6.7a.imp](#) [Ex6.7b.imp](#) [data8.dat](#)

The syntax highlights are listed below, and adding the NIMPS and SAVE commands generates model-based multiple imputations for a frequentist analysis (see Example 6.3).

- ❖ CLUSTERID command identifies a level-2 identifier, automatically inducing random intercepts for all level-1 variables
- ❖ ORDINAL command identifies a binary predictor
- ❖ FIXED command identifies a complete predictor
- ❖ RANDEFFECT command defines random intercept and slope residuals as level-2 latent variables
- ❖ CENTER command applies grand mean and latent group mean centering to predictors
- ❖ MODEL command features a random coefficient listed after the vertical pipe
- ❖ Unspecified associations for predictor variables

```
DATA: data8.dat;
VARIABLES: level1id level2id x1.i x2.i y1.i x3.i x4.i d1.j n1.j
              x5.j x6.j x7.j x8.j y2.j;
CLUSTERID: level2id;
ORDINAL: d1.j;
MISSING: 999;
FIXED: d1.j;
```

RANDOMEFFECT:

```
raniceps = y1.i | 1 [level2id];
ranslopes = y1.i | x1.i [level2id];
```

CENTER:

```
groupmean = x1.i;
grandmean = x2.i x7.j d1.j;
```

MODEL:

```
y1.i ~ x1.i x2.i x7.j d1.j | x1.i;
y2.j ~ raniceps ranslopes x7.j;
```

SEED: 90291;

BURN: 25000;

ITERATIONS: 25000;

A slightly different way to set up the model is to define the random intercepts and slopes as level-2 latent variables that predict the outcomes from both models. The random effect parameter estimates no longer appear in the same table as the other model parameters because they are distinct latent variables with their own mean (fixed effect) and variance (random effect). This parameterization converges more quickly in this example. The new syntax highlights are as follows.

- ❖ LATENT command defines two between-cluster latent variables
- ❖ MODEL command removes (fixes to 0 using @0) both the fixed intercept in the regression equation and the random intercept listed after the vertical pipe
- ❖ MODEL command estimates the mean and variance of each between-cluster latent variable (the latter specification happens by default)
- ❖ MODEL command specifies correlation between random intercepts and random slopes (level-2 latent variables)
- ❖ MODEL command fixes the coefficient for the random intercept latent variable to 1
- ❖ MODEL command fixes the interaction coefficient for the product of the random slope predictor and the level-2 latent slope variable to 1


```

DATA: data8.dat;
VARIABLES: level1id level2id x1.i x2.i y1.i x3.i x4.i d1.j n1.j
  x5.j x6.j x7.j x8.j y2.j;
CLUSTERID: level2id;
ORDINAL: d1.j;
MISSING: 999;
LATENT:
level2id = ranicept;
level2id = ranslope;
FIXED: d1.j;
CENTER:
groupmean = x1.i;
grandmean = x2.i x7.j d1.j;
MODEL:
ranicept ~ 1;
ranslope ~ 1;
ranicept ~~ ranslope;
y1.i ~ 1@0 ranicept@1 x1.i*ranslope@1 x2.i x7.j d1.j | 1@0;
y2.j ~ ranicept ranslope x7.j;
SEED: 90291;
BURN: 25000;
ITERATIONS: 25000;

```

6.7: Two-Level Model With Random Effects Predicting a Level-2 Outcome

This example illustrates a two-level regression model with random intercepts and random slopes. The focal analysis model is shown below.

$$Y_{1ij} = (\beta_0 + b_{0j}) + (\beta_1 + b_{1j}) X_{1ij}^{cwc} + \beta_2 X_{2ij}^{cgm} + \beta_3 X_{7j}^{cgm} + \beta_4 D_{1j}^{cgm} + \varepsilon_{ij}$$

The random intercepts and random slopes in turn predict a distal outcome, as follows.

$$Y_{2j} = \gamma_0 + \gamma_1 b_{0j} + \gamma_2 b_{1j} + r_i$$

Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex6.7a.imp](#) [Ex6.7b.imp](#) [data8.dat](#)

The syntax highlights are as follows.

- ❖ CLUSTERID command identifies a level-2 identifier, automatically inducing random intercepts for all level-1 variables
- ❖ ORDINAL command identifies a binary predictor
- ❖ RANOMEFFECT command defines random intercepts and slopes as level-2 latent variables
- ❖ FIXED command identifies a complete predictor
- ❖ CENTER command applies grand mean and latent group mean centering to predictors
- ❖ MODEL command uses labels ending in a colon to group models and order their summary tables on the output
- ❖ MODEL command features a random coefficient listed after the vertical pipe
- ❖ Unspecified associations for predictor variables

```
DATA: data8.dat;  
VARIABLES: level1id level2id x1.i x2.i y.i x3.i x4.i d1.j  
              n1.j x5.j x6.j x7.j x8.j x9.j;  
CLUSTERID: level2id;  
ORDINAL: d1.j;  
RANOMEFFECT:  
raniceps = y1.i | 1 [level2id];  
ranslopes = y1.i | x1.i [level2id];  
MISSING: 999;  
FIXED: d1.j;
```

CENTER:

```
groupmean = x1.i;
grandmean = x2.i x7.j d1.j;
```

MODEL:

```
focal.model:
y1.i ~ x1.i x2.i x7.j d1.j | x1.i;
structural.model:
y2.j ~ raniceps ranslopes x7.j;
```

SEED: 90291;

BURN: 25000;

ITERATIONS: 30000;

An alternate approach defines a pair of level-2 latent variables that represent the random intercepts and slopes. The setup of this model mimics the separate specification approach outlined in Section 6.4. The unique syntax highlights for the code block are as follows.

- ❖ LATENT command defines a between-cluster latent variable
- ❖ MODEL command uses labels ending in a colon to group models and order their summary tables on the output
- ❖ MODEL command removes the random coefficient listed after the vertical pipe
- ❖ MODEL command estimates the latent slope mean
- ❖ MODEL command specifies correlation between random intercepts and random slopes (level-2 latent variable)
- ❖ MODEL command fixes the interaction coefficient for the product of the random slope predictor and its level-2 latent variable to 1

DATA: data8.dat;

VARIABLES: level1id level2id x1.i x2.i y1.i x3.i x4.i d1.j n1.j
x5.j x6.j x7.j x8.j y2.j;

CLUSTERID: level2id;

ORDINAL: d1.j;

```

MISSING: 999;
LATENT:
level2id = ranicept ranslope;
FIXED: d1.j;
CENTER:
groupmean = x1.i;
grandmean = x2.i x7.j d1.j;
MODEL:
focal.model:
y1.i ~ 1@ranicept x1.i*ranslope@1 x2.i x7.j d1.j;
structural.model:
y2.j ~ ranicept ranslope x7.j;
random.effect.model:
ranicept ~ 1;
ranslope ~ 1;
ranicept ~~ ranslope;
SEED: 90291;
BURN: 20000;
ITERATIONS: 20000;

```

6.8: Two-Level Regression With Latent Contextual Effect

This example illustrates a two-level regression model that includes within- and between-cluster slopes for a level-1 predictor and a latent contextual effect (Lüdtke et al., 2008).

$$Y_{ij} = (\beta_0 + b_{0j}) + \beta_1 X_{2ij}^{cwc} + \beta_2 \mu_{2j}^{cgm} + \beta_3 X_{7j}^{cgm} + \beta_4 D_{1j}^{cgm} + \varepsilon_{ij}$$

Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex6.8.imp](#) [data8.dat](#)

The syntax highlights are listed below, and adding the NIMPS and SAVE commands generates model-based multiple imputations for a frequentist analysis (see Example 6.3).

- ❖ CLUSTERID command identifies a level-2 identifier, automatically inducing random intercepts for all level-1 variables
- ❖ ORDINAL command identifies a binary predictor
- ❖ FIXED command identifies a complete predictor
- ❖ CENTER command applies grand mean and latent group mean centering to predictors
- ❖ MODEL command features a random coefficient listed after the vertical pipe
- ❖ MODEL command specifies latent group means as a level-2 predictor with the .mean suffix on a level-1 predictor
- ❖ MODEL command labels within- and between-cluster slopes
- ❖ Unspecified associations for predictor variables
- ❖ PARAMETERS command uses labeled quantities to compute latent contextual effect (between- vs. within-cluster slope difference)

```

DATA: data8.dat;
VARIABLES: level1id level2id x1.i x2.i y x3.i x4.i
               d1.j n1.j x5.j x6.j x7.j x8.j x9.j;
CLUSTERID: level2id;
ORDINAL: d1.j;
MISSING: 999;
FIXED: d1.j;
CENTER:
groupmean = x2.i;
grandmean = x2.i.mean x7.j d1.j;
MODEL:
y ~ x2.i@betaw x2.i.mean@betab x7.j d1.j | x2.i;

```

PARAMETERS:

```
x2.contextual = betab - betaw;
```

```
SEED: 90291;
```

```
BURN: 10000;
```

```
ITERATIONS: 10000;
```

6.9: Two-Level Regression With Cross-Level Interaction

This example illustrates a two-level regression model that includes a cross-level interaction involving a continuous level-1 predictor and a continuous level-2 moderator. The regression model is as follows.

$$Y_{ij} = (\beta_0 + b_{0j}) + (\beta_1 + b_{1j}) X_{1ij}^{cwc} + \beta_2 X_{2j}^{cgm} + \beta_3 (X_{1ij}^{cwc} \times X_{2j}^{cgm}) + \varepsilon_{ij}$$

Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex6.9.imp](#) [data1.dat](#)

The syntax highlights are listed below, and adding the NIMPS and SAVE commands generates model-based multiple imputations for a frequentist analysis (see Example 6.3).

- ❖ CLUSTERID command identifies a level-2 identifier, automatically inducing random intercepts for all level-1 variables
- ❖ CENTER command applies grand mean and latent group mean centering to predictors
- ❖ MODEL command features a random coefficient listed after the vertical pipe
- ❖ MODEL command features a product term

- ❖ SIMPLE command produces conditional effects (simple slopes) at different standard deviation units of the continuous moderator
- ❖ Unspecified associations for predictor variables

```

DATA: data1.dat;
VARIABLES: level1id level2id d1.i o1.i y.i x1.i d2.i x2.j x3.j;
CLUSTERID: level2id;
MISSING: 999;
CENTER:
groupmean = x1.i;
grandmean = x2.j;
MODEL:
y.i ~ x1.i x2.j x1.i*x2.j | x1.i;
SIMPLE:
x1.i | x2.j;
SEED: 90291;
BURN: 10000;
ITERATIONS: 10000;

```

6.10: Two-Level 1-1-1 Mediation With Random Slopes

This example illustrates a two-level path model that features an indirect effect of two level-1 predictors, both of which are within-cluster centered at their latent group means. The regression models are as follows.

$$M_{ij} = (\alpha_0 + a_{0j}) + (\alpha_1 + a_{1j}) X_{ij}^{cwc} + r_{ij}$$

$$Y_{ij} = (\beta_0 + b_{0j}) + (\beta_1 + b_{1j}) M_{ij}^{cwc} + (\beta_2 + b_{2j}) X_{ij}^{cwc} + \varepsilon_{ij}$$

Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex6.10.imp](#) [data1.dat](#)

The syntax highlights are listed below, and adding the NIMPS and SAVE commands generates model-based multiple imputations for a frequentist analysis (see Example 6.3).

- ❖ CLUSTERID command identifies a level-2 identifier, automatically inducing random intercepts for all level-1 variables
- ❖ CENTER command applies latent group mean centering to predictors
- ❖ MODEL command features random coefficients listed after the vertical pipe
- ❖ MODEL command labels the indirect effect's component pathways
- ❖ PARAMETERS command uses labeled quantities to compute the product of coefficients estimator
- ❖ Unspecified associations for predictor variables

```
DATA: data1.dat;  
VARIABLES: level1id level2id d1.i y.i m.i x.i d2.i v1.j v2.j;  
CLUSTERID: level2id;  
MISSING: 999;  
CENTER:  
groupmean = x.i m.i;  
MODEL:  
m.i ~ x.i@alpha | x.i;  
y.i ~ m.i@beta1 x.i | m.i x.i;  
PARAMETERS:  
indirect = alpha * beta1;  
SEED: 90291;  
BURN: 5000;  
ITERATIONS: 10000;
```


6.11: Two-Level 1-1-1 Mediation With Moderated Paths

This example illustrates a two-level path model that features an indirect effect of two level-1 predictors, both of which are within-cluster centered at their latent group means, and a level-2 predictor moderating the direct pathways. The regression models are as follows.

$$M_{ij} = (\alpha_0 + a_{0j}) + (\alpha_1 + a_{1j}) X_{1ij}^{cwc} + \alpha_2 X_{2j}^{cgm} + \alpha_3 (X_{1ij}^{cwc} \times X_{2j}^{cgm}) + r_{ij}$$

$$Y_{ij} = (\beta_0 + b_{0j}) + (\beta_1 + b_{1j}) M_{ij}^{cwc} + \beta_2 X_{1ij}^{cwc} + \beta_3 X_{2j}^{cgm} + \varepsilon_{ij}$$

Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex6.11.imp](#) [data1.dat](#)

The syntax highlights are listed below, and adding the NIMPS and SAVE commands generates model-based multiple imputations for a frequentist analysis (see Example 6.3).

- ❖ CLUSTERID command identifies a level-2 identifier, automatically inducing random intercepts for all level-1 variables
- ❖ CENTER command applies grand mean and latent group mean centering to predictors
- ❖ MODEL command features random coefficients listed after the vertical pipe
- ❖ MODEL command features a product term
- ❖ MODEL command labels the indirect effect's component pathways
- ❖ PARAMETERS command uses labeled quantities to compute the product of coefficients estimator

❖ Unspecified associations for predictor variables

```

DATA: data1.dat;
VARIABLES: level1id level2id d1.i y.i m.i x1.i d2.i x2.j x3.j;
CLUSTERID: level2id;
MISSING: 999;
CENTER:
groupmean = x1.i m.i;
grandmean = x2.j;
MODEL:
m.i ~ x1.i@alpha1 x2.j x1.i*x2.j@alpha3 | x1.i;
y.i ~ m.i@beta1 x1.i x2.j | m.i;
PARAMETERS:
x2.sd = 4;
indirect.low = ((alpha1 - (alpha3 * x2.sd)) * beta1);
indirect.med = alpha1 * beta1;
indirect.high = ((alpha1 + (alpha3 * x2.sd)) * beta1);
SEED: 90291;
BURN: 10000;
ITERATIONS: 10000;

```

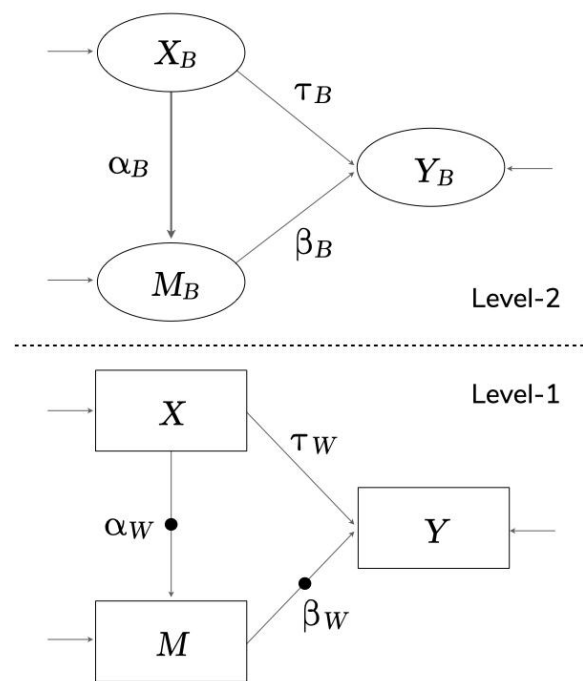
6.12: Within- and Between-Level Mediation

This example illustrates a two-level path model that features a within-cluster indirect effect involving centered level-1 variables and a between-cluster indirect effect involving a pair of latent group means. The regression models are as follows.

$$M_{ij} = (\alpha_0 + a_{0j}) + (\alpha_1 + a_{1j}) X_{1ij}^{cwc} + \alpha_2 \mu_{X1}^{cgm} + r_{ij}$$

$$Y_{ij} = (\beta_0 + b_{0j}) + (\beta_1 + b_{1j}) (M_{ij}^{cwc}) + \beta_2 \mu_{Mj}^{cgm} + \beta_3 X_{1ij}^{cwc} + \beta_4 \mu_{X1}^{cgm} + \varepsilon_{ij}$$

The model features distinct within-cluster and between-cluster mediation processes, as depicted in the path diagram below.



The ellipses in the between-cluster model represent latent group means (i.e., random intercepts). Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex6.12.imp](#) [data1.dat](#)

The syntax highlights are listed below, and adding the NIMPS and SAVE commands generates model-based multiple imputations for a frequentist analysis (see Example 6.3).

- ❖ CLUSTERID command identifies a level-2 identifier, automatically inducing random intercepts for all level-1 variables
- ❖ CENTER command applies grand mean and latent group mean centering to predictors
- ❖ MODEL command features random coefficients listed after the vertical pipe

- ❖ MODEL command specifies latent group means as a level-2 predictor with the .mean suffix on a level-1 predictor
- ❖ MODEL command labels within- and between-cluster slopes
- ❖ PARAMETERS command uses labeled quantities to compute the product of coefficients estimator
- ❖ Unspecified associations for predictor variables

```

DATA: data1.dat;
VARIABLES: level1id level2id d1.i y.i m.i x1.i d2.i x2.j x3.j;
CLUSTERID: level2id;
MISSING: 999;
CENTER:
grandmean = x1.i.mean m.i.mean;
groupmean = x1.i m.i;
MODEL:
m.i ~ x1.i@alpha1 x1.i.mean@alpha2 | x1.i;
y.i ~ m.i@beta1 m.i.mean@beta2 x1.i x1.i.mean | m.i;
PARAMETERS:
indirect.w = alpha1 * beta1;
indirect.b = alpha2 * beta2;
SEED: 90291;
BURN: 10000;
ITERATIONS: 10000;

```

6.13: Two-Level Mediation With a Binary Outcome

This example illustrates a two-level path model that features an indirect effect of two level-1 predictors, both of which are within-cluster centered at their latent group means. The regression models are as follows.

$$M_{ij} = (\alpha_0 + a_{0j}) + (\alpha_1 + a_{1j}) X_{ij}^{cwc} + r_{ij}$$

$$Y_{ij}^* = (\beta_0 + b_{0j}) + (\beta_1 + b_{1j}) M_{ij}^{cwc} + (\beta_2 + b_{2j}) X_{ij}^{cwc} + \varepsilon_{ij}$$

where Y^* denotes the underlying latent response variable for a binary outcome Y . The residual variance of the latent response variable is fixed at 1 for identification. All other features of the model are the same as Example 6.10.

Muthén, Muthén, and Asparouhov (2016) show that the indirect effect alone does not capture the fact that changes in Y are non-constant on the probability metric. They give the following expression for computing the probability that $Y = 1$ given a particular value of X (Equations 8.4 to 8.6). The script below uses these equations to compute the probability at different values of X , conditional on random effect values equal to 0.

$$E(Y^*|X) = I_Y + I_M\beta + \alpha\beta X + \tau'X =$$

$$V(Y^*|X) = \beta^2\sigma_M^2 + 1$$

$$P(Y = 1|X) = \phi[E(Y^*|X)/\sqrt{V(Y^*|X)}]$$

Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex6.13.imp](#) [data1.dat](#)

The syntax highlights are listed below, and adding the NIMPS and SAVE commands generates model-based multiple imputations for a frequentist analysis (see Example 6.3).

- ❖ CLUSTERID command identifies a level-2 identifier, automatically inducing random intercepts for all level-1 variables
- ❖ TRANSFORM command recodes the continuous Y variable into a binary dummy code by splitting at the mean
- ❖ CENTER command applies latent group mean centering to predictors

- ❖ MODEL command features random coefficients listed after the vertical pipe
- ❖ MODEL command uses labels ending in a colon to group models and order their summary tables on the output
- ❖ MODEL command labels intercepts, slopes, and the residual variance of the mediator
- ❖ PARAMETERS command uses labeled quantities to compute the product of coefficients estimator
- ❖ PARAMETERS command uses labeled quantities to compute the the probability that Y equals 1 at three values of X (the mean and plus/minus one standard deviation from the mean)
- ❖ Unspecified associations for predictor variables

```

DATA: data1.dat;
VARIABLES: level1id level2id d1.i y.i m.i x.i d2.i v1.j v2.j;
CLUSTERID: level2id;
MISSING: 999;
TRANSFORM:
ybinary.i = (y.i >= 4);
CENTER:
groupmean = x.i m.i;
MODEL:
m.i ~ 1@m.icept x.i@alpha | x.i;
ybinary.i ~ 1@y.icept m.i@beta1 x.i@beta2 | m.i x.i;
# label residual variance
m.i ~~ m.i@m.resvar;
PARAMETERS:
indirect = alpha * beta1;
xvalue1 = -.50;
xvalue2 = 0;
xvalue3 = .50;
pyeq1.xvalue1 = phi((y.icept + beta1*m.icept +
  beta1*alpha*xvalue1 + beta2*xvalue1)/

```

```

    sqrt(beta1^2*m.resvar + 1));
pyeq1.xvalue2 = phi((y.icept + beta1*m.icept +
    beta1*alpha*xvalue2 + beta2*xvalue2)/
    sqrt(beta1^2*m.resvar + 1));
pyeq1.xvalue3 = phi((y.icept + beta1*m.icept +
    beta1*alpha*xvalue3 + beta2*xvalue3)/
    sqrt(beta1^2*m.resvar + 1));
SEED: 90291;
BURN: 5000;
ITERATIONS: 10000;

```

6.14: Two-Level Linear Growth Model

This example illustrates a two-level linear growth model that includes a cross-level group-by-time interaction involving the temporal predictor ($TIME = 0, 1, 3, 6$) and a binary moderator. The regression model, which is the two-level version of the latent growth model from Example 5.13, is shown below.

$$Y_{ij} = (\beta_0 + b_{0j}) + (\beta_1 + b_{1j}) TIME_{ij} + \beta_2 D_{1j} + \beta_3 (TIME_{ij} \times D_{1j}) + \varepsilon_{ij}$$

Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex6.14.imp](#) [data9.dat](#)

The syntax highlights are listed below, and adding the NIMPS and SAVE commands generates model-based multiple imputations for a frequentist analysis (see Example 6.3).

- ❖ CLUSTERID command identifies a level-2 identifier, automatically inducing random intercepts for all level-1 variables

- ❖ FIXED command identifies complete predictors
- ❖ NOMINAL command identifies a binary predictor
- ❖ MODEL command features a random coefficient listed after the vertical pipe
- ❖ MODEL command features a product term
- ❖ SIMPLE command produces conditional effects (simple slopes) at each level of the nominal moderator

```

DATA: data9.dat;
VARIABLES: level2id y.i time.i n1.i v1.i v2.i d1.j d2.j;
CLUSTERID: level2id;
NOMINAL: d1.j;
MISSING: 999;
FIXED: time.i d1.j;
MODEL: y.i ~ time.i d1.j time.i*d1.j | time.i;
SIMPLE: time.i | d1.j;
SEED: 90291;
BURN: 2000;
ITERATIONS: 10000;

```

6.15: Three-Level Growth Model

This example illustrates a three-level linear growth model that includes a cross-level group-by-time interaction involving the temporal predictor ($TIME = 0, 1, \dots, 5, 6$) and a level-2 binary moderator. The regression model is as follows.

$$\begin{aligned}
 Y_{ijk} = & (\beta_0 + b_{0jk} + b_{0k}) + (\beta_1 + b_{1jk} + b_{1k}) TIME_{ijk} + \beta_2 X_{2jk}^{cgm} \\
 & + \beta_3 D_{3k} + \beta_4 (TIME_{ijk} \times D_{3k}) + \varepsilon_{ij}
 \end{aligned}$$

Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex6.15a.imp](#) [Ex6.15b.imp](#) [data10.dat](#)

The syntax highlights are listed below, and adding the NIMPS and SAVE commands generates model-based multiple imputations for a frequentist analysis (see Example 6.3).

- ❖ CLUSTERID command identifies level-2 and level-3 identifiers (order doesn't matter), automatically inducing random intercepts for all level-1 and level-2 variables
- ❖ FIXED command identifies complete predictors
- ❖ CENTER command applies grand mean centering to predictors
- ❖ NOMINAL command identifies a binary predictor
- ❖ MODEL command features a random coefficient listed after the vertical pipe
- ❖ MODEL command features a product term
- ❖ SIMPLE command produces conditional effects (simple slopes) at each level of the nominal moderator

```

DATA: data10.dat;
VARIABLES: level1id level2id level3id y.i time.i x1.i
               n1.j d1.j d2.j n2.j x2.j d3.k x3.k x4.k;
NOMINAL: d3.k;
CLUSTERID: level2id level3id;
MISSING: 999;
FIXED: time.i d3.k;
CENTER: grandmean = x2.j;
MODEL:
y.i ~ time.i x2.j d3.k time.i*d3.k | time.i;
SIMPLE:
time.i | d3.k;
SEED: 90291;
BURN: 15000;

```

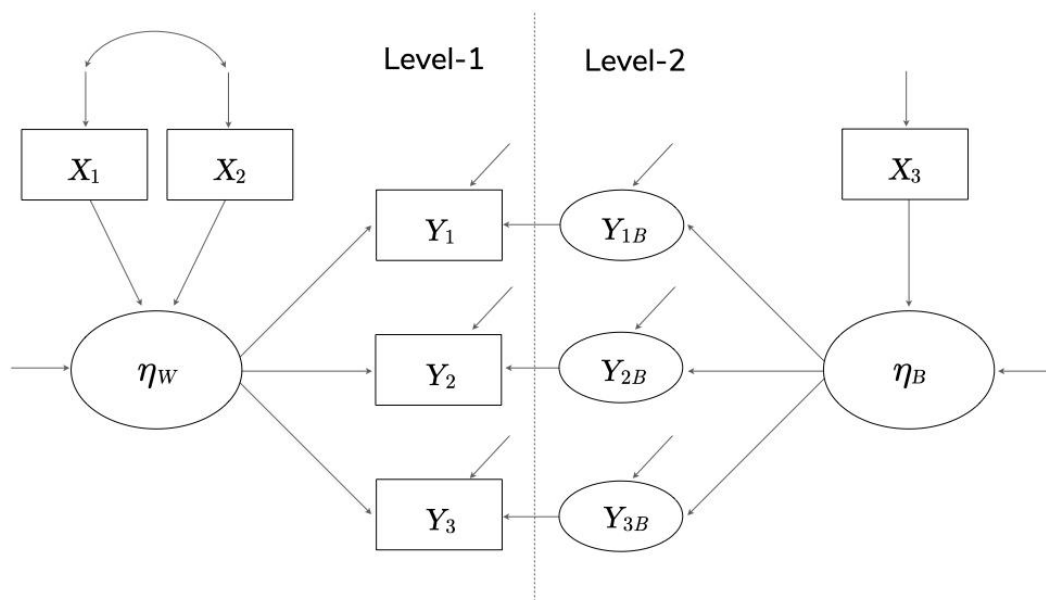
ITERATIONS: 10000;

By default, Blimp estimates random intercepts and random slopes (when specified) at all levels of the data hierarchy. For example, the previous analysis produces a 2 x 2 covariance matrix of random effects at level-2 and level-3. In some situations, it may be desirable or necessary to override Blimp's default behavior and fix certain variance components to zero (or alternatively, select which variances get estimated). This is achieved by listing the desired random effects on the right side of the vertical pipe and appending to the effect's name a cluster-level identifier in square brackets. To illustrate, the following code block illustrates a three-level model with random intercepts at both levels and a random coefficient for the temporal predictor at the second level only.

```
DATA: data10.dat;
VARIABLES: level1id level2id level3id y.i time.i x1.i
              n1.j d1.j d2.j n2.j x2.j d3.k x3.k x4.k;
NOMINAL: d3.k;
CLUSTERID: level2id level3id;
MISSING: 999;
FIXED: time.i d3.k;
CENTER: grandmean = x2.j;
MODEL:
y.i ~ time.i x2.j d3.k time.i*d3.k |
      1[level2id] 1[level3id] time.i[level2id];
SEED: 90291;
BURN: 15000;
ITERATIONS: 10000;
```

6.16: Two-Level MIMIC Measurement Model

This example illustrates a two-level factor analysis model that features a measurement model for the within-cluster scores at level-1 and the between-cluster latent group means at level-2. The model also features predictor variables at each level, as shown in the path diagram below.



The ellipses in the between-cluster model are latent group means (i.e., random intercepts that load on a level-2 latent variable. Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex6.16.imp](#) [data11.dat](#)

The syntax highlights are listed below, and adding the NIMPS and SAVE commands generates model-based multiple imputations for a frequentist analysis (see Example 6.3).

- ❖ CLUSTERID command identifies a level-2 identifier, automatically inducing random intercepts for all level-1 variables
- ❖ LATENT command defines within- and between-cluster latent variables
- ❖ MODEL command uses labels ending in a colon to group models and order their summary tables on the output
- ❖ Individual regression equations specified for each indicator (instead of the \rightarrow convention for latent factors)
- ❖ MODEL command fixes the within- and between-cluster loading of the first indicator to 1
- ❖ Default specification fixes the latent means equal to 0
- ❖ Longer burn-in period for estimating latent variables

```
DATA: data11.dat;  
VARIABLES: level2id y1.i:y4.i x1.i x2.i x3.j;  
MISSING: 999;  
CLUSTERID: level2id;  
LATENT:  
latency.l1;  
level2id = latency.l2;  
CENTER:  
grandmean = x1.i x2.i x3.j;  
MODEL:  
structural.model:  
latency.l1 ~ x1.i x2.i;  
latency.l2 ~ x3.j;  
measurement.model:  
y1.i ~ latency.l1@1 latency.l2@1;  
y2.i ~ latency.l1 latency.l2;  
y3.i ~ latency.l1 latency.l2;  
y4.i ~ latency.l1 latency.l2;  
SEED: 90291;  
BURN: 10000;
```

```
ITERATIONS: 10000;
```

6.17: Two-Level Regression With Random Intercepts and Sampling Weights

This example illustrates a two-level regression model with random intercepts and sampling (inverse probability) weights at each level. The regression model is shown below.

$$Y_{ij} = (\beta_0 + b_{0j}) + \beta_1 X_{1ij} + \beta_2 X_{2ij} + \beta_3 X_{3ij} + \varepsilon_{ij}$$

Goldstein (2011, Section 3.4.2) describes MCMC estimation for multilevel models with sampling weights. Level-1 and level-2 sampling weights are rescaled following Goldstein (2011, Section 3.4.1). At level-1, the rescaled weights within a given cluster sum to the cluster size. This is the same as the so-called “cluster” method from Asparouhov (2006). Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex6.17.imp](#) [data21.dat](#)

The syntax highlights are as follows.

- ❖ CLUSTERID command identifies a level-2 identifier, automatically inducing random intercepts for all incomplete level-1 variables
- ❖ WEIGHTS command identifies level-1 and level-2 weights (order does not matter)
- ❖ Unspecified associations for predictor variables

```
DATA: data21.dat;
```

```
VARIABLES: level2id level1wgt level2wgt v1 y.i x1.i x2.i x3.i  
              x4.i x5.i;
```

```
CLUSTERID: level2id;  
WEIGHTS: level1wgt level2wgt;  
MISSING: 999;  
MODEL: y.i ~ x1.i x2.i x3.i;  
SEED: 90291;  
BURN: 2000;  
ITERATIONS: 10000;
```

Note that some data sets may not include level-2 sampling weights, in which case the weight command simplifies as follows.

```
CLUSTERID: level2id;  
WEIGHT: level1wgt;
```

6.18: Partially Nested Design (Singleton Clusters)

This example illustrates a two-level regression model from a partially nested design. The example below considers a level-2 binary predictor (e.g., a treatment assignment indicator) where participants in group $D = 1$ (e.g., treatment participants) are clustered in level-2 units but observations in group $D = 0$ are not nested (i.e., are singleton clusters). The regression model below features an interaction between the binary indicator and the random intercept, such that the random effect term drops from the equation if $D = 0$.

$$Y_{ij} = \beta_0 + \beta_1 D_j + D_j b_{0j} + \varepsilon_{ij}$$

More generally, the variable D does not need to have a fixed effect. Outside of an intervention context, D could simply be an indicator that differentiates clustered versus singleton observations (e.g., $D = 0$ is a singleton cluster with a single member, $D = 1$ is

an observation that shares cluster membership with other observations). The following random intercept model illustrates this idea.

$$Y_{ij} = \beta_0 + \beta_1 X_{ij} + D_j b_{0j} + \varepsilon_{ij}$$

Blimp estimates this model by defining a level-2 latent variable that interacts with D. Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex6.18.imp](#) [data19.dat](#)

The syntax highlights are as follows.

- ❖ CLUSTERID command identifies a level-2 identifier, automatically inducing random intercepts for all incomplete level-1 variables
- ❖ LATENT command defines a level-2 latent variable (the random intercepts), the mean of which is fixed to 0 in the MODEL section
- ❖ MODEL command eliminates the default random intercept by fixing it to 0 (... | 1@0;), and it adds the interaction between the binary indicator and the level-2 latent variable.

```
DATA: data19.dat;
VARIABLES: level2id y.i d.j;
MISSING: 999;
CLUSTERID: level2id;
LATENT: level2id = ranicepts;
MODEL:
  ranicepts ~ 1@0;
  y.i ~ d.j ranicepts*d.j@1 | 1@0;
SEED: 90291;
BURN: 1000;
ITERATIONS: 10000;
```

6.19: Discrete-Time Survival Model

This example illustrates a discrete-time survival model using Blimp's multilevel modeling features. Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex6.19a.imp](#) [Ex6.19b.imp](#) [data20.dat](#)

The input data set is in stacked (i.e., "person-period") format with each row representing a time interval nested within an individual. The data also include a set of time indicators that dummy code each measurement interval. The example below illustrates a model with six intervals and thus six dummy codes. The outcome variable is an event indicator that equals 0 if the event did not happen in the interval and a 1 if the event did happen in the interval. Figure 11.5 from Singer and Willett (2003) illustrates the data structure.

The basic model is a logistic regression with the binary event indicator regressed on the time dummy codes.

$$\ln \left(\frac{Pr(Event = 1)}{1 - Pr(Event = 1)} \right) = \alpha_1(t_{1j}) + \alpha_2(t_{2j}) + \alpha_3(t_{3j}) + \alpha_4(t_{4j}) \\ + \alpha_5(t_{5j}) + \alpha_6(t_{6j})$$

Note that the model omits the usual regression intercept. The syntax highlights are listed below. Adding the NIMPS and SAVE commands generates imputed data sets, and adding the savepredicted keyword to the OPTIONS command saves predicted probabilities (see Example 4.20).

- ❖ CLUSTERID command identifies a level-2 identifier, automatically inducing random intercepts for all incomplete level-1 variables
- ❖ ORDINAL command identifies a binary outcome and predictors
- ❖ FIXED command identifies a complete predictor
- ❖ CENTER command applies grand mean centering to predictors
- ❖ Applying the `logit` function to the dependent variable on the MODEL line requests a logit rather than probit link
- ❖ MODEL command eliminates the default fixed and random intercepts by fixing both to 0 (`~ 1@0 ... | 1@0;`)
- ❖ MODEL command includes a binary dummy code for each time interval (`t7.i` to `t12.i`)
- ❖ PARAMETERS command computes predicted probability of the event at each time point (i.e., hazard probabilities)

```

DATA: data20.dat;
VARIABLES: level2id time.i t1.i t2.i t3.i t4.i t5.i t6.i
               y.i d.j x.j;
ORDINAL: y.i t1.i t2.i t3.i t4.i t5.i t6.i;
CLUSTERID: level2id;
MISSING: 999;
FIXED: t1.i t2.i t3.i t4.i t5.i t6.i;
MODEL:
logit(y.i) ~ 1@0 t1.i@alpha1 t2.i@alpha2 t3.i@alpha3
            t4.i@alpha4 t5.i@alpha5 t6.i@alpha6 | 1@0;
PARAMETERS:
hazard.1 = exp(alpha1) / (1 + exp(alpha1));
hazard.2 = exp(alpha2) / (1 + exp(alpha2));
hazard.3 = exp(alpha3) / (1 + exp(alpha3));
hazard.4 = exp(alpha4) / (1 + exp(alpha4));
hazard.5 = exp(alpha5) / (1 + exp(alpha5));
hazard.6 = exp(alpha6) / (1 + exp(alpha6));
SEED: 90291;
BURN: 2000;

```

ITERATIONS: 10000;

The next example expands the model by incorporating a person-level dummy code and continuous covariate as predictors of the hazard function.

$$\ln \left(\frac{\Pr(Event = 1)}{1 - \Pr(Event = 1)} \right) = \alpha_1(t_{1j}) + \alpha_2(t_{2j}) + \alpha_3(t_{3j}) + \alpha_4(t_{4j}) \\ + \alpha_5(t_{5j}) + \alpha_6(t_{6j}) + \beta_1(D_{ij}) + \beta_2(X_{ij})$$

As before, the model omits the usual regression intercept and includes a set of six dummy codes that index the intervals. The code block below is identical to the previous example, but it defines the binary predictor as ordinal and grand mean centers the continuous covariate.

```
DATA: data20.dat;
VARIABLES: level2id time.i t1.i t2.i t3.i t4.i t5.i t6.i
  y.i d.j x.j;
ORDINAL: y.i t1.i t2.i t3.i t4.i t5.i t6.i d.j;
CLUSTERID: level2id;
MISSING: 999;
FIXED: t1.i t2.i t3.i t4.i t5.i t6.i d.j x.j;
CENTER: grandmean = x.j;
MODEL:
logit(y.i) ~ 1@0 t1.i@alpha1 t2.i@alpha2 t3.i@alpha3
  t4.i@alpha4 t5.i@alpha5 t6.i@alpha6 d.j x.j | 1@0;
SEED: 90291;
BURN: 2000;
ITERATIONS: 10000;
```

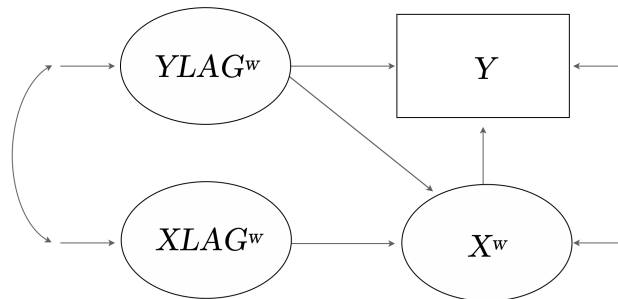
6.20: Two-Level Dynamic Structural Equation Model With Random Intercepts

This example illustrates a two-level dynamic structural equation model with random intercepts. The application of this model involves intensive repeated measures data nested within persons. The first model is a random intercept regression where Y is regressed on the “pure” within-cluster parts of $YLAG$ and X , such that $YLAG$ is a lagged version of Y with each cluster's (e.g., person's) data record shifted down by one row and a missing value in the first row (i.e., the first Y measurement appears in the second row, the second Y measurement in the third row, and so on). Similarly, X is regressed on the within-cluster parts of $YLAG$ and $XLAG$, where $XLAG$ is the lagged version of X . The structural regression equations are

$$Y_{ij} = \beta_{0j} + \beta_1 YLAG_{ij}^w + \beta_2 X_{ij}^w + \varepsilon_{ij}$$

$$X_{ij} = \gamma_{0j} + \gamma_1 YLAG_{ij}^w + \gamma_2 XLAG_{ij}^w + \zeta_{ij}$$

and a path diagram of the focal model is as follows.

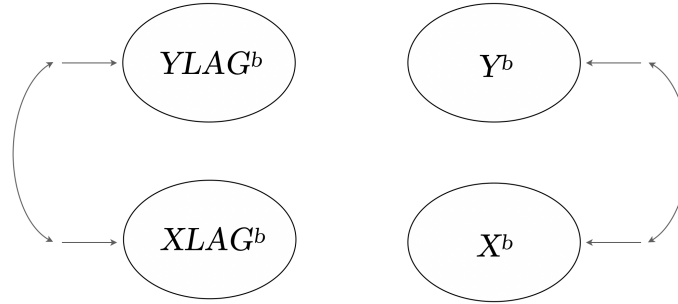


Importantly, the random intercepts— β_{0j} and γ_{0j} —are normally distributed level-2 latent variables (i.e., latent group means)

$$\beta_{0j} = \beta_0 + b_{0j} = Y_j^b$$

$$\gamma_{0j} = \gamma_0 + g_{0j} = X_j^b$$

with means equal to β_0 and γ_0 and an unstructured between-cluster covariance matrix shown in the path diagram below.



The pure within-cluster predictors are created by centering each person's (cluster's) scores at their latent group means. The previous regression equations can thus be written more explicitly as follows.

$$Y_{ij} = \beta_{0j} + \beta_1(YLAG_{ij} - \beta_{0j}) + \beta_2(X_{ij} - \gamma_{0j}) + \varepsilon_{ij}$$

$$X_{ij} = \gamma_{0j} + \gamma_1(YLAG_{ij} - \beta_{0j}) + \gamma_2(XLAG_{ij} - \gamma_{0j}) + \zeta_{ij}$$

To accommodate missing data in the lagged variables, $YLAG$ and $XLAG$ have their own model that consists of a grand mean, a between-cluster deviation (or equivalently, latent group means), and a within-cluster deviation, as follows.

$$YLAG_{ij} = \beta_0 + \dot{b}_{0j} + \dot{\varepsilon}_{ij} = YLAG_j^b + \dot{\varepsilon}_{ij}$$

$$XLAG_{ij} = \gamma_0 + \dot{g}_{0j} + \dot{\zeta}_{ij} = XLAG_j^b + \dot{\zeta}_{ij}$$

The lagged variables have the same means and between-cluster variances as their nonlagged counterparts, and they too have an unstructured between- and within-cluster cluster covariance matrices, as shown in the previous path diagrams.

These equations can be viewed as nuisance models that are necessary for missing data handling.

The data structure for dynamic structural equation models requires a brief discussion. Blimp's `lag1` function essentially assumes that (a) each person has one row of data for every occasion (rows could be filled with missing values), and (b) time scores are discrete and consecutively ordered. For example, the time variable in this example consists of consecutive integers between 0 and 20, and each person has 21 rows of data. If the data do not follow this format, you can use the R function at [Brian Keller's github site](#) to restructure the data into time-balanced format.

Clicking the links below downloads the Blimp scripts and data for these examples, which build on the model above. The full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex6.20.imp](#) [data25.dat](#)

The syntax highlights are listed below, and adding the NIMPS and SAVE commands generates model-based multiple imputations for a frequentist analysis (see Example 6.3).

- ❖ CLUSTERID command identifies a level-2 identifier, automatically inducing random intercepts for all level-1 variables
- ❖ LATENT command introduces four level-2 latent variables (random intercepts) that represent the level-2 means
- ❖ TRANSFORM command uses the `lag1` function to create variables that are lagged (shifted down by one row) within each cluster
- ❖ MODEL command uses labels ending in a colon to group models and order their summary tables on the output

- ❖ MODEL estimates the means of the level-2 latent variables
- ❖ MODEL command uses labels (@y_mean and @x_mean) to impose equality constraints on the means of the original and lagged variables
- ❖ MODEL command sets the intercept of each regression equation equal to a latent group mean (e.g., 1@y_b and 1@x_b)
- ❖ MODEL command centers predictors by subtracting the latent group means
- ❖ MODEL command specifies within- and between-cluster covariances

```

DATA: data25.dat;
VARIABLES: level2id time y x m;
CLUSTERID: level2id;
MISSING: 999;
LATENT: level2id = y_b x_b ylag_b xlag_b;
TRANSFORM:
ylag = lag1(y, time, level2id);
xlag = lag1(x, time, level2id);
MODEL:
y.model:
# between
y_b ~ 1@y_mean;
# within
y ~ 1@y_b (ylag - y_b) (x - x_b);
x.model:
# between
x_b ~ 1@x_mean;
y_b ~~ x_b;
# within
x ~ 1@x_b (xlag - x_b) (ylag - y_b);
ylag.model:
# between
ylag_b ~ 1@y_mean;
ylag ~ 1@ylag_b;

```

```

xlag.model:
# between
xlag_b ~ 1@x_mean;
ylag_b ~~ xlag_b;
# within
xlag ~ 1@xlag_b;
ylag ~~ xlag;
SEED: 90291;
BURN: 2000;
ITERATIONS: 10000;

```

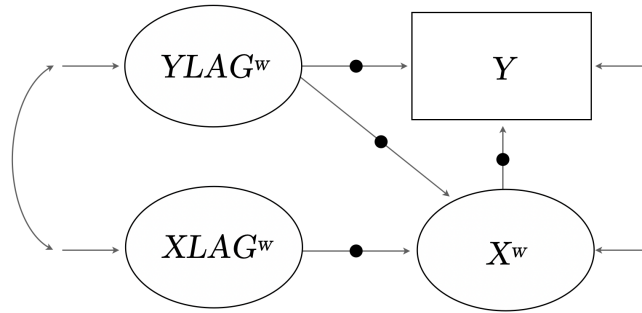
6.21: Two-Level Dynamic Structural Equation Model With Random Coefficients

This example illustrates a two-level dynamic structural equation model with random slopes. The application of this model involves intensive repeated measures data nested within persons. The first model is a random intercept regression where Y is regressed on the “pure” within-cluster parts of $YLAG$ and X , such that $YLAG$ is a lagged version of Y with each cluster’s (e.g., person’s) data record shifted down by one row and a missing value in the first row (i.e., the first Y measurement appears in the second row, the second Y measurement in the third row, and so on). Similarly, X is regressed on the within-cluster parts of $YLAG$ and $XLAG$, where $XLAG$ is the lagged version of X . The structural regression equations are

$$Y_{ij} = \beta_{0j} + \beta_{1j}YLAG_{ij}^w + \beta_{2j}X_{ij}^w + \varepsilon_{ij}$$

$$X_{ij} = \gamma_{0j} + \gamma_{1j}YLAG_{ij}^w + \gamma_{2j}XLAG_{ij}^w + \zeta_{ij}$$

and a path diagram of the focal model is as follows. The solid dots on the arrows represent the random coefficients.



Importantly, the random intercepts— β_{0j} and γ_{0j} —are normally distributed level-2 latent variables (i.e., latent group means)

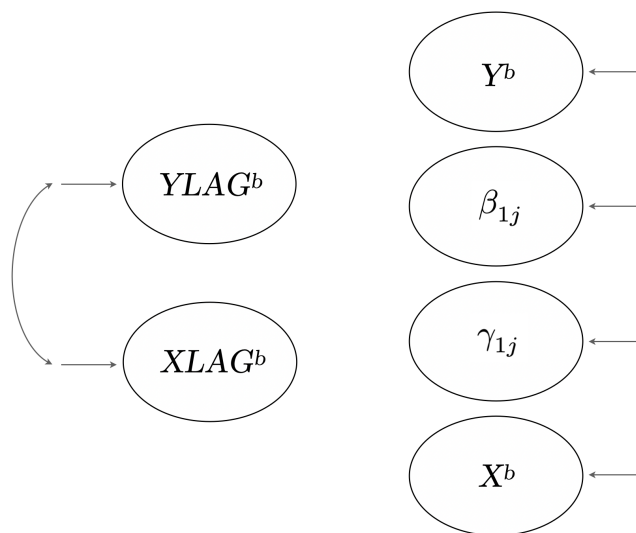
$$\beta_{0j} = \beta_0 + b_{0j} = Y_j^b$$

$$\beta_{1j} = \beta_1 + b_{1j}$$

$$\gamma_{0j} = \gamma_0 + g_{0j} = X_j^b$$

$$\gamma_{1j} = \gamma_1 + g_{1j}$$

with means equal to β_0 and γ_0 and an unstructured between-cluster covariance matrix shown in the path diagram below.



Following Example 6.20, pure within-cluster predictors are created by centering each person's (cluster's) scores at their latent group means. The previous regression equations can thus be written more explicitly as follows.

$$Y_{ij} = \beta_{0j} + \beta_{1j}(YLAG_{ij} - \beta_{0j}) + \beta_{2j}(X_{ij} - \gamma_{0j}) + \varepsilon_{ij}$$

$$X_{ij} = \gamma_{0j} + \gamma_{1j}(YLAG_{ij} - \beta_{0j}) + \gamma_{2j}(XLAG_{ij} - \gamma_{0j}) + \zeta_{ij}$$

To accommodate missing data in the lagged variables, *YLAG* and *XLAG* have their own model that consists of a grand mean, a between-cluster deviation (or equivalently, latent group means), and a within-cluster deviation, as follows.

$$YLAG_{ij} = \beta_0 + \dot{b}_{0j} + \dot{\varepsilon}_{ij} = YLAG_j^b + \dot{\varepsilon}_{ij}$$

$$XLAG_{ij} = \gamma_0 + \dot{g}_{0j} + \dot{\zeta}_{ij} = XLAG_j^b + \dot{\zeta}_{ij}$$

The lagged variables have the same means and between-cluster variances as their nonlagged counterparts, and they too have an unstructured between- and within-cluster cluster covariance matrices, as shown in the previous path diagrams. These equations can be viewed as nuisance models that are necessary for missing data handling.

The data structure for dynamic structural equation models requires a brief discussion. Blimp's `lag1` function essentially assumes that (a) each person has one row of data for every occasion (rows could be filled with missing values), and (b) time scores are discrete and consecutively ordered. For example, the time variable in this example consists of consecutive integers between 0 and 20, and each person has 21 rows of data. If the data do not follow this format, you can use the R function at [Brian Keller's github site](#) to restructure the data into time-balanced format.

Clicking the links below downloads the Blimp scripts and data for these examples, which build on the model above. The full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex6.21.imp](#) [data25.dat](#)

The syntax highlights are listed below, and adding the NIMPS and SAVE commands generates model-based multiple imputations for a frequentist analysis (see Example 6.3).

- ❖ CLUSTERID command identifies a level-2 identifier, automatically inducing random intercepts for all level-1 variables
- ❖ LATENT command introduces four level-2 latent variables (random intercepts) that represent the level-2 means and four level-2 latent variables that represent the random slopes
- ❖ TRANSFORM command uses the `lag1` function to create variables that are lagged (shifted down by one row) within each cluster
- ❖ MODEL command uses labels ending in a colon to group models and order their summary tables on the output
- ❖ MODEL estimates the means of the level-2 latent variables
- ❖ MODEL command uses labels (`@y_mean` and `@x_mean`) to impose equality constraints on the means of the original and lagged variables
- ❖ MODEL command sets the intercept of each regression equation equal to a latent group mean (e.g., `1@y_b` and `1@x_b`)
- ❖ MODEL command centers predictors by subtracting the latent group means
- ❖ MODEL command omits the random coefficient listed after the vertical pipe
- ❖ MODEL command includes the interaction between a level-1 predictor and its level-2 latent slope (e.g., `(y_lag - y_b)*y_on_y_lag`)
- ❖ MODEL command fixes the interaction coefficient for the product of the random slope predictor and its level-2 latent variable to 1

- ❖ MODEL command specifies within- and between-cluster covariances

```

DATA: data25.dat;
VARIABLES: level2id time y x m;
CLUSTERID: level2id;
MISSING: 999;
LATENT: level2id = y_b x_b ylag_b xlag_b
            y_on_ylag y_on_x x_on_xlag x_on_ylag;
TRANSFORM:
ylag = lag1(y, time, level2id);
xlag = lag1(x, time, level2id);
MODEL:
y.model:
# between
y_b ~ 1@y_mean;
y_on_ylag ~ 1;
y_on_x ~ 1;
# within
y ~ 1@y_b (ylag - y_b)*y_on_ylag@1 (x - x_b)*y_on_x@1;
x.model:
# between
x_b ~ 1@x_mean;
x_on_xlag ~ 1;
x_on_ylag ~ 1;
y_b y_on_ylag y_on_x x_b x_on_xlag x_on_ylag ~~
    y_b y_on_ylag y_on_x x_b x_on_xlag x_on_ylag;
# within
x ~ 1@x_b (xlag - x_b)*x_on_xlag@1 (ylag - y_b)*x_on_ylag@1;
ylag.model:
# between
ylag_b ~ 1@y_mean;
# within
ylag ~ 1@ylag_b;
xlag.model:

```

```
# between
xlag_b ~ 1@x_mean;
ylag_b ~~ xlag_b;
# within
xlag ~ 1@xlag_b;
ylag ~~ xlag;
SEED: 90291;
BURN: 20000;
ITERATIONS: 10000;
```

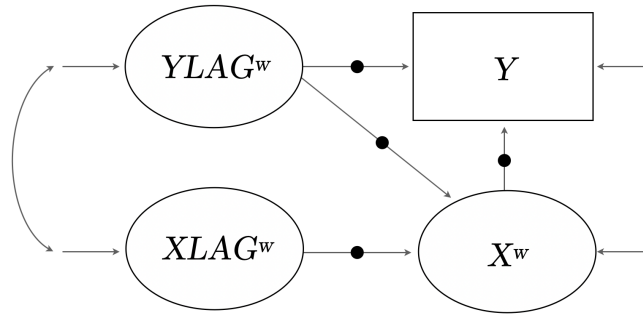
6.22: Two-Level Dynamic Structural Equation Model With Random Within-Cluster Variances

This example illustrates a two-level dynamic structural equation model with random slopes and random within-cluster variances. The application of this model involves intensive repeated measures data nested within persons. The first model is a random intercept regression where Y is regressed on the “pure” within-cluster parts of $YLAG$ and X , such that $YLAG$ is a lagged version of Y with each cluster’s (e.g., person’s) data record shifted down by one row and a missing value in the first row (i.e., the first Y measurement appears in the second row, the second Y measurement in the third row, and so on). Similarly, X is regressed on the within-cluster parts of $YLAG$ and $XLAG$, where $XLAG$ is the lagged version of X . The structural regression equations are

$$Y_{ij} = \beta_{0j} + \beta_{1j}YLAG_{ij}^w + \beta_{2j}X_{ij}^w + \varepsilon_{ij}$$

$$X_{ij} = \gamma_{0j} + \gamma_{1j}YLAG_{ij}^w + \gamma_{2j}XLAG_{ij}^w + \zeta_{ij}$$

and a path diagram of the focal model is as follows. The solid dots on the arrows represent the random coefficients.



Importantly, the random intercepts— β_{0j} and γ_{0j} —are normally distributed level-2 latent variables (i.e., latent group means)

$$\beta_{0j} = \beta_0 + b_{0j} = Y_j^b$$

$$\beta_{1j} = \beta_1 + b_{1j}$$

$$\gamma_{0j} = \gamma_0 + g_{0j} = X_j^b$$

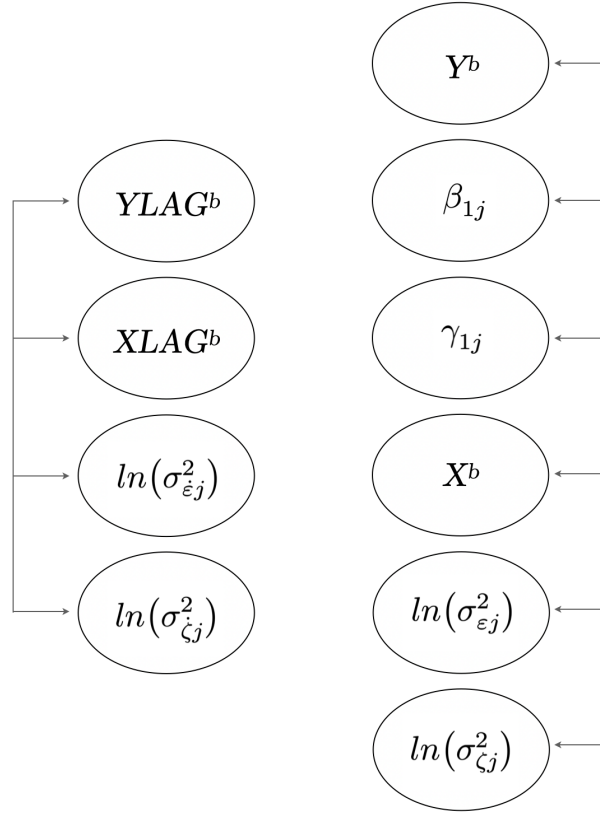
$$\gamma_{1j} = \gamma_1 + g_{1j}$$

with means equal to β_0 and γ_0 . Both X and Y have random within-cluster variances that differ across clusters (persons). A log-linear model expresses the natural log of the variance as a level-2 latent variable with a mean and between-cluster variance that represents cluster-level variance differences.

$$\ln(\sigma_{\varepsilon j}^2) = \alpha_Y + a_{Yj}$$

$$\ln(\sigma_{\zeta j}^2) = \alpha_X + a_{Xj}$$

Level-1 and level-2 predictors of the variance can be incorporated following Example 6.6. The level-2 latent variables (random effects) have an unstructured between-cluster covariance matrix shown in the path diagram below.



Following Example 6.21, pure within-cluster predictors are created by centering each person's (cluster's) scores at their latent group means. The previous regression equations can thus be written more explicitly as follows.

$$Y_{ij} = \beta_{0j} + \beta_{1j}(YLAG_{ij} - \beta_{0j}) + \beta_{2j}(X_{ij} - \gamma_{0j}) + \varepsilon_{ij}$$

$$X_{ij} = \gamma_{0j} + \gamma_{1j}(YLAG_{ij} - \beta_{0j}) + \gamma_{2j}(XLAG_{ij} - \gamma_{0j}) + \zeta_{ij}$$

To accommodate missing data in the lagged variables, $YLAG$ and $XLAG$ have their own model that consists of a grand mean, a between-cluster deviation (or equivalently, latent group means), and a within-cluster deviation, as follows.

$$YLAG_{ij} = \beta_0 + \dot{b}_{0j} + \dot{\varepsilon}_{ij} = YLAG_j^b + \dot{\varepsilon}_{ij}$$

$$XLAG_{ij} = \gamma_0 + \dot{g}_{0j} + \dot{\zeta}_{ij} = XLAG_j^b + \dot{\zeta}_{ij}$$

The lagged variables have the same means and between-cluster variances as their nonlagged counterparts, and they too have random within-cluster variances. The previous path diagrams depict the unstructured within- and between-cluster covariance matrices of the lagged variables.

The data structure for dynamic structural equation models requires a brief discussion. Blimp's `lag1` function essentially assumes that (a) each person has one row of data for every occasion (rows could be filled with missing values), and (b) time scores are discrete and consecutively ordered. For example, the time variable in this example consists of consecutive integers between 0 and 20, and each person has 21 rows of data. If the data do not follow this format, you can use the R function at [Brian Keller's github site](#) to restructure the data into time-balanced format.

Clicking the links below downloads the Blimp scripts and data for these examples, which build on the model above. The full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex6.22.imp](#) [data25.dat](#)

The syntax highlights are listed below, and adding the NIMPS and SAVE commands generates model-based multiple imputations for a frequentist analysis (see Example 6.3).

- ❖ CLUSTERID command identifies a level-2 identifier, automatically inducing random intercepts for all level-1 variables
- ❖ LATENT command introduces four level-2 latent variables (random intercepts) that represent the level-2 means and four level-2 latent variables that represent the random slopes

- ❖ LATENT command identifies level-2 latent variables that represents within-cluster variation on the log metric (e.g., `y_logv` and `x_logv`)
- ❖ TRANSFORM command uses the `lag1` function to create variables that are lagged (shifted down by one row) within each cluster
- ❖ MODEL command uses labels ending in a colon to group models and order their summary tables on the output
- ❖ MODEL estimates the means of the level-2 latent variables
- ❖ MODEL command uses labels (`@y_mean` and `@x_mean`) to impose equality constraints on the means of the original and lagged variables
- ❖ MODEL command sets the intercept of each regression equation equal to a latent group mean (e.g., `1@y_b` and `1@x_b`)
- ❖ MODEL command centers predictors by subtracting the latent group means
- ❖ MODEL command omits the random coefficient listed after the vertical pipe
- ❖ MODEL command includes the interaction between a level-1 predictor and its level-2 latent slope (e.g., `(y_lag - y_b)*y_on_y_lag`)
- ❖ MODEL command fixes the interaction coefficient for the product of the random slope predictor and its level-2 latent variable to 1
- ❖ MODEL command features regression equations with the natural logarithm of the level-1 variation as the outcome. The intercepts of these equations are set equal to the level-2 latent variables (e.g., `1@y_logv` and `1@x_logv`)
- ❖ MODEL command specifies within- and between-cluster covariances

```

DATA: data25.dat;
VARIABLES: level2id time y x m;
CLUSTERID: level2id;
MISSING: 999;
LATENT:
level2id = y_b x_b ylag_b xlag_b y_on_ylag y_on_x x_on_xlag
          x_on_ylag y_logv ylag_logv x_logv xlag_logv;
TRANSFORM:

```



```

ylag = lag1(y, time, level2id);
xlag = lag1(x, time, level2id);
MODEL:
y.model:
# between
y_b ~ 1@y_mean;
y_on_ylag ~ 1;
y_on_x ~ 1;
# within
y ~ 1@y_b (ylag - y_b)*y_on_ylag@1 (x - x_b)*y_on_x@1;
# random within-cluster variances
y_logv ~ 1;
var(y) ~ 1@y_logv;
x.model:
# between
x_b ~ 1@x_mean;
x_on_xlag ~ 1;
x_on_ylag ~ 1;
y_b y_on_ylag y_on_x x_b x_on_xlag x_on_ylag y_logv x_logv ~~
    y_b y_on_ylag y_on_x x_b x_on_xlag x_on_ylag y_logv x_logv;
# within
x ~ 1@x_b (xlag - x_b)*x_on_xlag@1 (ylag - y_b)*x_on_ylag@1;
# random within-cluster variances
x_logv ~ 1;
var(x) ~ 1@x_logv;
ylag.model:
# between
ylag_b ~ 1@y_mean;
# within
ylag ~ 1@ylag_b;
# random within-cluster variances
ylag_logv ~ 1;
var(ylag) ~ 1@ylag_logv;
xlag.model:
# between

```

```

xlag_b ~ 1@x_mean;
ylag_b xlag_b ylag_logv xlag_logv ~~
  ylag_b xlag_b ylag_logv xlag_logv;
# within
xlag ~ 1@xlag_b;
ylag ~~ xlag;
# random within-cluster variances
xlag_logv ~ 1;
var(xlag) ~ 1@xlag_logv;
SEED: 90291;
BURN: 20000;
ITERATIONS: 10000;

```

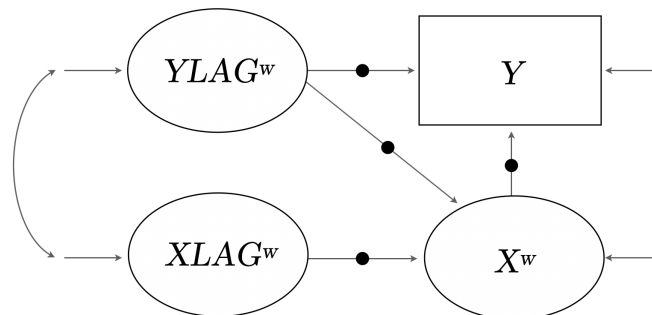
6.23: Two-Level Dynamic Structural Equation Model With a Cross-Level Interaction Effect

This example illustrates a two-level dynamic structural equation model with random slopes, random within-cluster variances, and a cross-level interaction. The example is identical to 6.22 but it incorporates a level-2 moderator M . The application of this model involves intensive repeated measures data nested within persons. The first model is a random intercept regression where Y is regressed on the “pure” within-cluster parts of $YLAG$ and X , such that $YLAG$ is a lagged version of Y with each cluster’s (e.g., person’s) data record shifted down by one row and a missing value in the first row (i.e., the first Y measurement appears in the second row, the second Y measurement in the third row, and so on). Similarly, X is regressed on the within-cluster parts of $YLAG$ and $XLAG$, where $XLAG$ is the lagged version of X . The structural regression equations are

$$Y_{ij} = \beta_{0j} + \beta_{1j}YLAG_{ij}^w + \beta_{2j}X_{ij}^w + \varepsilon_{ij}$$

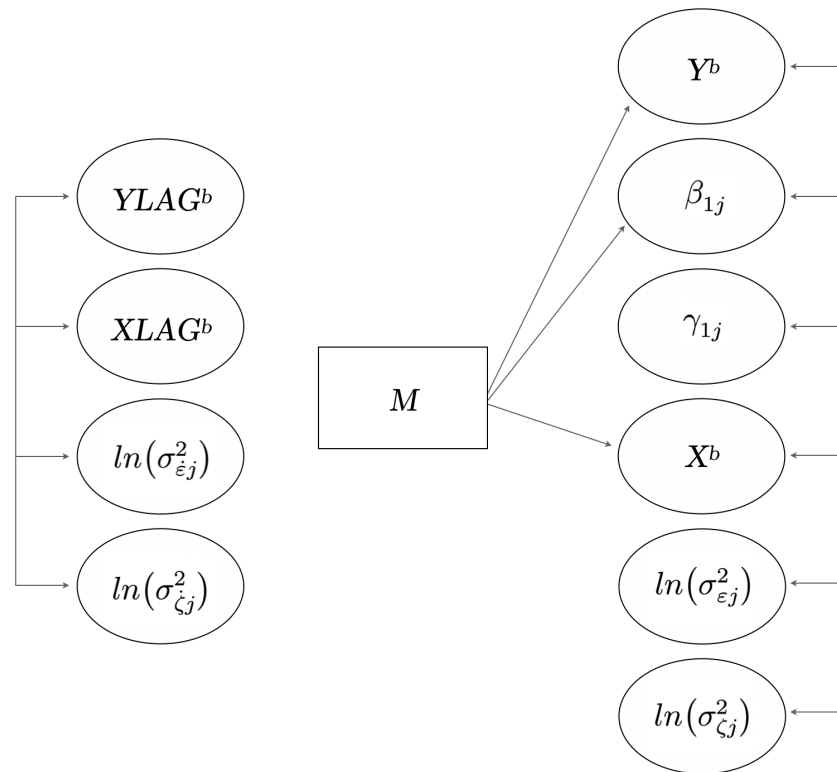
$$X_{ij} = \gamma_{0j} + \gamma_{1j}YLAG_{ij}^w + \gamma_{2j}XLAG_{ij}^w + \zeta_{ij}$$

and a path diagram of the focal model is as follows. The solid dots on the arrows represent the random coefficients.



Following previous examples, the random intercepts— β_{0j} and γ_{0j} —are normally distributed level-2 latent variables (i.e., latent group means).

The level-2 moderator enters the latent variable regression equations as a predictor of the between-cluster parts of X and Y as well as the random coefficients from the regression of Y on X . A path diagram of the between-cluster model is shown below. The cross-level interaction corresponds to the straight arrow connecting M to the random slope latent variable β_{0j} . The model for the random within-cluster variances and lagged variables is identical to Example 6.22, so we omit these details for brevity.



The data structure for dynamic structural equation models requires a brief discussion. Blimp's `lag1` function essentially assumes that (a) each person has one row of data for every occasion (rows could be filled with missing values), and (b) time scores are discrete and consecutively ordered. For example, the time variable in this example consists of consecutive integers between 0 and 20, and each person has 21 rows of data. If the data do not follow this format, you can use the R function at [Brian Keller's github site](#) to restructure the data into time-balanced format.

Clicking the links below downloads the Blimp scripts and data for these examples, which build on the model above. The full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex6.23.imp](#) [data25.dat](#)

The syntax highlights are listed below, and adding the NIMPS and SAVE commands generates model-based multiple imputations for a frequentist analysis (see Example 6.3).

- ❖ CLUSTERID command identifies a level-2 identifier, automatically inducing random intercepts for all level-1 variables
- ❖ LATENT command introduces four level-2 latent variables (random intercepts) that represent the level-2 means and four level-2 latent variables that represent the random slopes
- ❖ LATENT command identifies level-2 latent variables that represents within-cluster variation on the log metric (e.g., y_logv and x_logv)
- ❖ TRANSFORM command uses the `lag1` function to create variables that are lagged (shifted down by one row) within each cluster
- ❖ ORDINAL command identifies a binary predictor
- ❖ FIXED command identifies a complete predictor
- ❖ MODEL command uses labels ending in a colon to group models and order their summary tables on the output
- ❖ MODEL estimates the means of the level-2 latent variables
- ❖ MODEL includes the moderator M in the between-cluster models
- ❖ MODEL command uses labels ($@y_mean$ and $@x_mean$) to impose equality constraints on the means of the original and lagged variables
- ❖ MODEL command sets the intercept of each regression equation equal to a latent group mean (e.g., $1@y_b$ and $1@x_b$)
- ❖ MODEL command centers predictors by subtracting the latent group means
- ❖ MODEL command omits the random coefficient listed after the vertical pipe
- ❖ MODEL command includes the interaction between a level-1 predictor and its level-2 latent slope (e.g., $(ylag - y_b) * y_on_ylag$)
- ❖ MODEL command fixes the interaction coefficient for the product of the random slope predictor and its level-2 latent variable to 1

- ❖ MODEL command features regression equations with the natural logarithm of the level-1 variation as the outcome. The intercepts of these equations are set equal to the level-2 latent variables (e.g., 1@y_logv and 1@x_logv)
- ❖ MODEL command specifies within- and between-cluster covariances

```

DATA: data25.dat;
VARIABLES: level2id time y x m;
CLUSTERID: level2id;
ORDINAL: m;
MISSING: 999;
LATENT:
level2id = y_b x_b ylag_b xlag_b y_on_ylag y_on_x x_on_xlag
          x_on_ylag y_logv ylag_logv x_logv xlag_logv;
TRANSFORM:
ylag = lag1(y, time, level2id);
xlag = lag1(x, time, level2id);
FIXED: m;
CENTER: grandmean = m;
MODEL:
y.model:
# between
y_b ~ 1@y_mean m;
y_on_ylag ~ 1;
y_on_x ~ 1 m;
# within
y ~ 1@y_b (ylag - y_b)*y_on_ylag@1 (x - x_b)*y_on_x@1;
# random within-cluster variances
y_logv ~ 1;
var(y) ~ 1@y_logv;
x.model:
# between
x_b ~ 1@x_mean m;
x_on_xlag ~ 1;
x_on_ylag ~ 1;

```

```
y_b y_on_ylag y_on_x x_b x_on_xlag x_on_ylag y_logv x_logv ~~  
    y_b y_on_ylag y_on_x x_b x_on_xlag x_on_ylag y_logv x_logv;  
# within  
x ~ 1@x_b (xlag - x_b)*x_on_xlag@1 (ylag - y_b)*x_on_ylag@1;  
# random within-cluster variances  
x_logv ~ 1;  
var(x) ~ 1@x_logv;  
ylag.model:  
# between  
ylag_b ~ 1@y_mean;  
# within  
ylag ~ 1@ylag_b;  
# random within-cluster variances  
ylag_logv ~ 1;  
var(ylag) ~ 1@ylag_logv;  
xlag.model:  
# between  
xlag_b ~ 1@x_mean;  
ylag_b xlag_b ylag_logv xlag_logv ~~  
    ylag_b xlag_b ylag_logv xlag_logv;  
# within  
xlag ~ 1@xlag_b;  
ylag ~~ xlag;  
# random within-cluster variances  
xlag_logv ~ 1;  
var(xlag) ~ 1@xlag_logv;  
SEED: 90291;  
BURN: 20000;  
ITERATIONS: 10000;
```

7 Analysis Examples: Missing Not at Random Processes

This section illustrates missing not at random analysis models in Blimp. Following the previous chapters, the examples in this section use a generic notation system where variable names usually consist of an alphanumeric prefix and a numeric suffix (e.g., Y_1 , X_1 , N_1 , D_1 , D_2 , V_1 , V_2 , V_3). The letter Y designates a dependent variable, a D prefix denotes a binary dummy variable, an O prefix indicates an ordinal variable, and an N prefix indicates a multicategorical nominal variable. Other letters generally represent continuous variables. For examples involving multilevel models, the examples use a “.i” suffix to denote level-1 variables and “.j” for level-2 variables (e.g., $d1.j$ is a level-2 dummy variable, $x1.i$ is a continuous variable measured at level-1). Blimp determines the levels automatically, so the suffixes are meant as a visual aid for understanding the scripts. Finally, the model equations use “cgm” and “cwc” superscripts to indicate grand and group mean centering, respectively. The following list outlines the examples in this section.

- ❖ 7.1: Selection Model for Linear Regression
- ❖ 7.2: Pattern Mixture Model for Linear Regression
- ❖ 7.3: Shared Parameter (Wu–Carroll) Latent Curve Model
- ❖ 7.4: Diggle–Kenward Latent Curve Model
- ❖ 7.5: Factor Analysis With a Selection Model
- ❖ 7.6: Mediation Analysis With a Selection Model
- ❖ 7.7: Two-Level Hedeker–Gibbons Pattern Mixture Growth Model
- ❖ 7.8: Selection Model for a Two-Level Regression With Random Coefficients
- ❖ 7.9: Two-Level Shared Parameter Growth Curve Model

7.1: Selection Model for Linear Regression

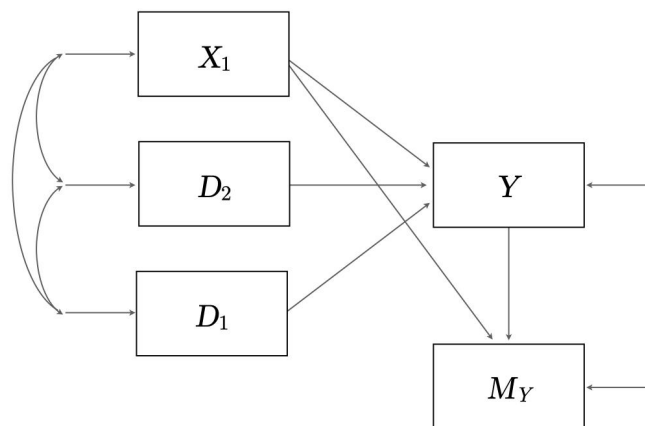
This example illustrates a selection model for a missing not at random process where an incomplete outcome variable predicts its own missingness. The focal analysis model is the linear regression below.

$$Y = \beta_0 + \beta_1 D_1 + \beta_2 D_2 + \beta_3 X_1^{cgm} + \varepsilon$$

The most basic selection model is one where the outcome alone predicts its missingness indicator ($M_Y = 0$ if Y is observed and 1 if it's missing); Gomer and Yuan (2021) refer to this as a focused missing not at random process. The following equation is a probit model where the missingness indicator's latent response variable (denoted by an asterisk superscript) is regressed on the outcome.

$$M_Y^* = \gamma_0 + \gamma_1 Y + r$$

For identification, the residual variance is fixed at 1, and the threshold parameter is fixed at 0. A path diagram of the model is shown below.



The analysis model also incorporates three auxiliary variables using the sequential specification from Example 4.7. Clicking the links below downloads the Blimp scripts

and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex7.1a.imp](#) [Ex7.1b.imp](#) [data3.dat](#)

The syntax highlights are listed below, and adding the NIMPS and SAVE commands generates model-based multiple imputations for a frequentist analysis that no longer requires a missingness model.

- ❖ ORDINAL command identifies binary predictors
- ❖ FIXED command identifies complete predictors
- ❖ CENTER command applies grand mean centering to predictors
- ❖ MODEL command uses labels ending in a colon to group models and order their summary tables on the output
- ❖ MODEL command features a syntax shortcut that creates a factored regression (sequential) specification for auxiliary variables
- ❖ The `.missing` suffix references the dependent variable's missing data indicator, which is automatically defined as ordinal
- ❖ Unspecified associations for predictor variables

```
DATA: data3.dat;  
VARIABLES: id x1 x2 x3 y d1 d2 v1:v4;  
MISSING: 999;  
ORDINAL: d1 d2;  
FIXED: d1 d2;  
CENTER: x1;  
MODEL:  
focal.model:  
y ~ d1 d2 x1;  
missingness.model:  
y.missing ~ y;
```

```

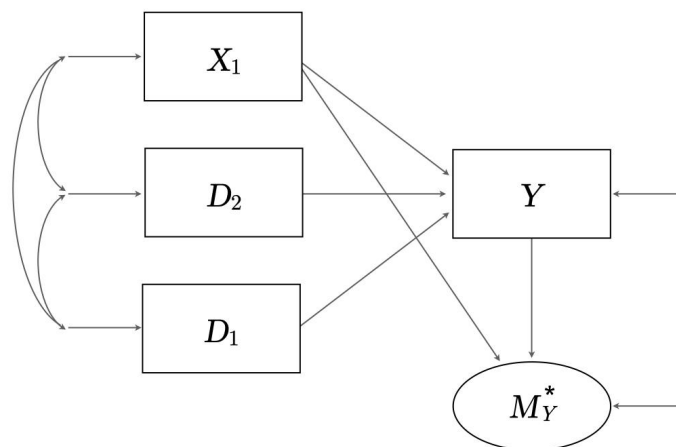
auxiliary.model:
x2 x3 ~ y d1 d2 x1;
SEED: 90291;
BURN: 1000;
ITERATIONS: 10000;

```

A more complex selection model features the outcome predicting its missingness indicator along with other variables, in this case D_1 ; Gomer and Yuan (2021) refer to this as a diffuse missing not at random process. The following equation is a probit model where the missingness indicator's latent response variable is regressed the outcome and D_1 .

$$M_Y^* = \gamma_0 + \gamma_1 Y + \gamma_2 D_1 + r$$

A path diagram of the model is shown below.



Caution is warranted when including too many predictors from the analysis model in the selection equation, as doing so weakens identification. Entering and selecting predictors in a stepwise fashion using fit indices such as the DIC and WAIC is often a good strategy. The code block for the analysis is shown below.

```

DATA: data3.dat;
VARIABLES: id x1 x2 x3 y d1 d2 v1:v4;
MISSING: 999;
ORDINAL: d1 d2;
FIXED: d1 d2;
CENTER: x1;
MODEL:
# focal analysis model
y ~ d1 d2 x1;
# auxiliary variable models
x2 x3 ~ y d1 d2 x1;
# selection model
y.missing ~ y d1;
SEED: 90291;
BURN: 2500;
ITERATIONS: 10000;

```

7.2: Pattern Mixture Model for Linear Regression

This example illustrates a pattern mixture model for a missing not at random process where regression model parameters differ between cases with and without dependent variable scores. Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex7.2a.imp](#) [Ex7.2b.imp](#) [data3.dat](#)

The focal analysis model is the linear regression below

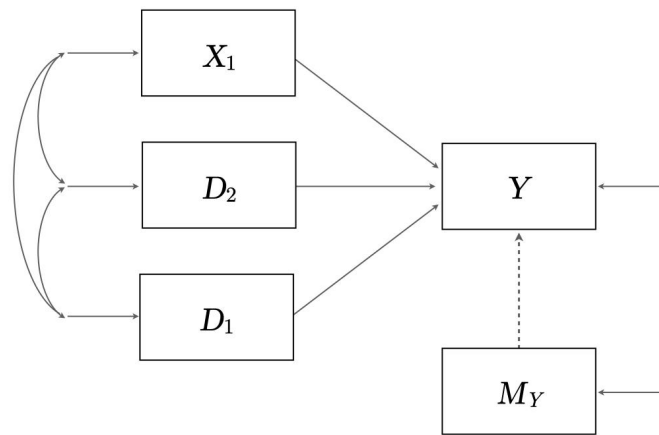
$$Y = \beta_0 + \beta_1 D_1 + \beta_2 D_2^{cgm} + \beta_3 X_1^{cgm} + \varepsilon$$

where D_1 is a dummy code representing a focal group comparison (e.g., a treatment assignment indicator), and D_2 and X_1 are covariates. The most basic pattern mixture

model is one where the intercept (outcome variable mean) differs between people with and without Y values; Gomer and Yuan (2021) characterize this as a focused missing not at random process. The fitted model features a binary missing data indicator ($M_Y = 0$ if Y is observed and $M_Y = 1$ if it's missing) as a predictor, as follows.

$$Y = \beta_{0(obs)} + \beta_{0(diff)}M_Y + \beta_1D_1 + \beta_2D_2^{cgm} + \beta_3X_1^{cgm} + \varepsilon$$

A path diagram of the model is shown below.



The overall population-level intercept estimate is a weighted average of the pattern-specific intercepts, where the weights are the group proportions. The marginal intercept estimate for this example is

$$\beta_0 = p_{(obs)}\beta_{0(obs)} + p_{(mis)}\beta_{0(mis)} = p_{(obs)}\beta_{0(obs)} + p_{(mis)}(\beta_{0(obs)} + \beta_{0(diff)})$$

where $p_{(obs)}$ and $p_{(mis)}$ are the proportions of completers and dropouts, respectively.

Importantly, the intercept difference (the dashed line pointing from M_Y to Y) is inestimable because people in the $M_Y = 1$ group have no data on Y . This parameter must be fixed to a value during estimation, and the magnitude and sign of the coefficient controls the strength and direction of the missing not at random process.

Enders (2022, Section 9.7) illustrates a strategy that uses off-the-shelf effect size benchmarks to determine this parameter. For example, if a researcher felt that the unseen Y scores have a higher mean than the observed data, then the inestimable intercept coefficient could be solved as a function of the standardized mean difference effect size and the dependent variable's standard deviation (or residual standard deviation).

$$\beta_{0(diff)} = d \times \sigma_Y$$

A positive value of d sets the mean of the unseen scores to a higher value than the observed data, and a negative value specifies a lower mean. The code block below sets the effect size equal to +0.20 and uses the residual standard deviation to estimate the spread of Y (Little, 2009, p. 428). This setting corresponds to a sensitivity analysis where persons with incomplete data are hypothesized to have a mean difference roughly equal to Cohen's (1988) small effect size benchmark. The syntax highlights are listed below, and adding the NIMPS and SAVE commands generates model-based multiple imputations for a frequentist analysis that no longer requires the missing data indicator.

- ❖ ORDINAL command identifies binary predictors
- ❖ FIXED command identifies complete predictors
- ❖ CENTER command applies grand mean centering to predictors
- ❖ MODEL command uses labels ending in a colon to group models and order their summary tables on the output
- ❖ MODEL command features a syntax shortcut that creates a factored regression (sequential) specification for all predictor
- ❖ MODEL command features a syntax shortcut that creates a factored regression (sequential) specification for auxiliary variables

- ❖ The TRANSFORM command creates the dependent variable's missing data indicator, `m.y`
- ❖ MODEL command labels the missing data indicator's latent response variable mean and three parameters from the focal analysis model: the residual variance, intercept coefficient, and intercept mean difference
- ❖ PARAMETERS command passes the value of the residual standard deviation into the formula that determines the intercept mean difference
- ❖ PARAMETERS command uses labeled quantities to compute missing data group proportions, pattern-specific intercept coefficients, and a marginal intercept estimate that averages over the missing data patterns

```

DATA: data3.dat;
VARIABLES: id x1 x2 x3 y d1 d2 v1:v4;
MISSING: 999;
ORDINAL: d1 d2 m.y;
CENTER: x1 d2;
TRANSFORM:
m.y = ismissing(y);
MODEL:
focal.model:
y ~ 1@b0obs m.y@b0diff d1 d2 x1 ;
# label residual variance
y ~~ y@resvar;
predictor.model:
# sequential specification for predictors
m.y ~ 1@ymissmean;
x1 d1 d2 ~ m.y;
auxiliary.model:
x2 x3 ~ y d1 d2 x1;
PARAMETERS:
# set b0diff equal to +.20 residual std. dev. units
cohensd = .20;
b0diff = cohensd * sqrt(resvar);

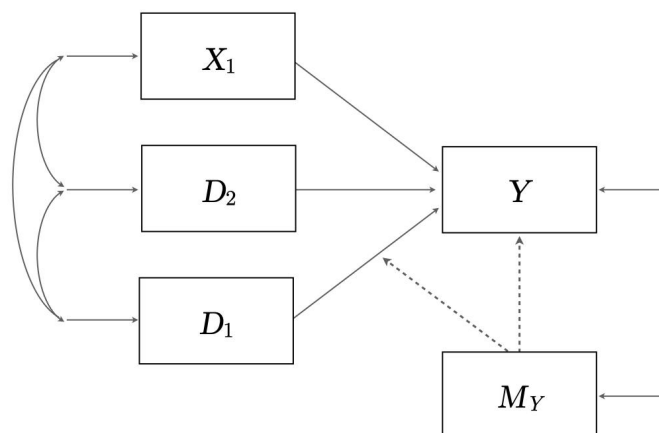
```

```
# missingness group proportions
p.mis = phi(ymissmean);
p.obs = 1 - p.mis;
# compute weighted average intercept
b0.obs = b0obs;
b0.mis = b0obs + b0diff;
b0 = (b0.obs * p.obs) + (b0.mis * p.mis);
SEED: 90291;
BURN: 2000;
ITERATIONS: 10000;
```

A more complex pattern mixture model is one where people with missing outcome scores have different intercepts and slopes than people with data; Gomer and Yuan (2021) characterize this as a diffuse missing not at random process. The fitted model features the missing data indicator and its interaction with the focal predictor, D_1 .

$$Y = \beta_{0(obs)} + \beta_{0(diff)}M_Y + \beta_{1(obs)}D_1 + \beta_{1(diff)}(D_1 \times M_Y) \\ + \beta_2 D_2^{cgm} + \beta_3 X_1^{cgm} + \varepsilon$$

A path diagram of the model is as follows.



The marginal slope that averages over the missing data patterns is a weighted average of the pattern-specific slopes, with weights equal to the group proportions.

$$\beta_1 = p_{(obs)}\beta_{1(obs)} + p_{(mis)}\beta_{1(mis)} = p_{(obs)}\beta_{1(obs)} + p_{(mis)}(\beta_{1(obs)} + \beta_{1(diff)})$$

Importantly, both the intercept and slope difference for the incomplete cases (the dashed lines pointing from M_Y to Y and M_Y to D_1 's slope) are inestimable because people in the $M_Y = 1$ group have no data on Y . As such, these parameters must be fixed to a value during estimation, and their magnitude and sign control the strength and direction of the missing not at random process. The same effect size-based strategy can be applied to the slope difference. In this example, the focal predictor D_1 is binary (e.g, intervention vs. control), in which case $\beta_{1(obs)}$ is the group mean difference for people with data on Y , and $\beta_{1(diff)}$ is the additional group mean difference for persons with missing Y scores. Specifying the inestimable slope as a standardized mean difference effect size gives the following solution.

$$\beta_{1(diff)} = d \times \sigma_Y$$

If the focal predictor is continuous, then the solution is

$$\beta_{1(diff)} = d \times (\sigma_Y \div \sigma_X)$$

in which case d can be viewed as the additional change in the dependent variable (in standard deviation units) for every one standard deviation increase in the predictor.

Setting d to a positive value means that the missing data group's slope is more positive, and a negative value of d means their slope is more negative.

To illustrate, suppose that Y is scaled such that high scores reflect a negative outcome (e.g., greater illness severity, a higher symptom count), and D_1 is a treatment

assignment dummy code ($D_1 = 0$ indicates the control group, and $D_1 = 1$ is the intervention group). Further, consider a missing not at random process where control group participants with the highest Y scores (e.g., most acute symptoms) leave the study to seek treatment elsewhere, whereas intervention group participants with the lowest Y scores (e.g., mildest symptoms) leave the study because they no longer feel treatment is necessary. This scenario requires a positive value of d for the inestimable intercept difference and a negative value of d for the slope difference. The code block below sets both effect sizes equal to 0.20 (they need not be the same) and uses the residual standard deviation to estimate the spread of Y .

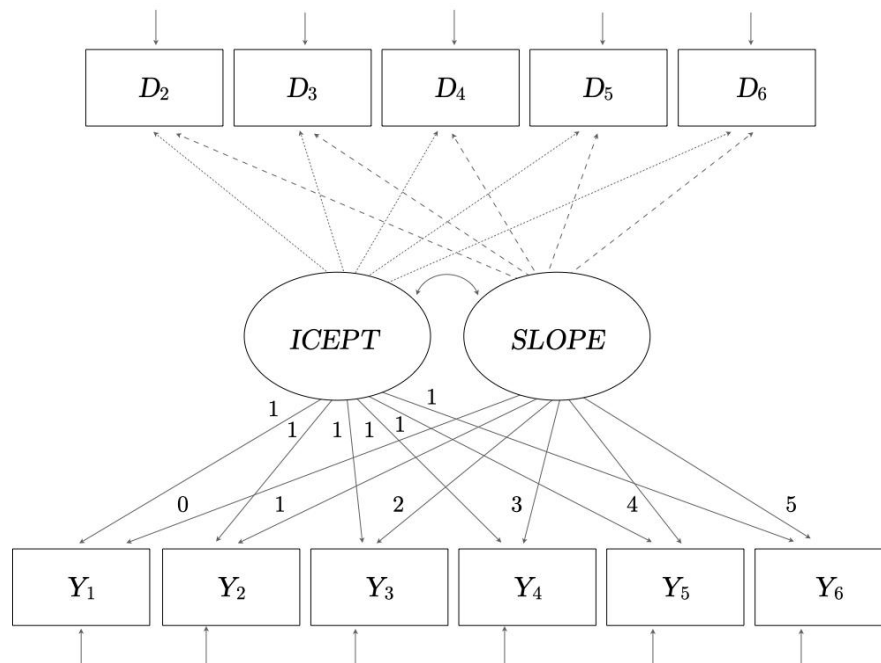
```
DATA: data3.dat;
VARIABLES: id x1 x2 x3 y d1 d2 v1:v4;
MISSING: 999;
ORDINAL: d1 d2 m.y;
TRANSFORM:
m.y = ismissing(y);
CENTER: x1 d2;
MODEL:
focal.model:
y ~ 1@b0diff m.y@b0diff d1@b1diff d1*m.y@b1diff d2 x1;
# label residual variance
y ~~ y@resvar;
predictor.model:
m.y ~ 1@ymissmean;
x1 d1 d2 ~ m.y;
auxiliary.model:
x2 x3 ~ y x1 d1 d2;
PARAMETERS:
# set b0diff equal to +.20 residual std. dev. units
# set b1diff equal to -.20 residual std. dev. units
cohensd = .20;
b0diff = cohensd * sqrt(resvar);
b1diff = - cohensd * sqrt(resvar);
```

```
# missingness group proportions
p.mis = phi(ymissmean);
p.obs = 1 - p.mis;
# compute weighted average intercept and slope
b0.obs = b0obs;
b0.mis = b0obs + b0diff;
b1.obs = b1obs;
b1.mis = b1obs + b1diff;
b0 = (b0.obs * p.obs) + (b0.mis * p.mis);
b1 = (b1.obs * p.obs) + (b1.mis * p.mis);
SEED: 90291;
BURN: 2000;
ITERATIONS: 10000;
```

Linking inestimable parameters to the standardized mean difference provides a practical heuristic for specifying inestimable coefficients, but it is still incumbent on the researcher to choose values that are reasonable for a given application. As mentioned previously, the magnitude of the missing data group's difference parameters dictates the strength of the missing not at random process. It is incorrect to view “small” values of d as unimportant, as standardized differences of this magnitude could be very salient in many situations. For example, consider a randomized intervention where the true effect size is $d = 0.20$ (i.e., a small effect size). Setting the missing data group's coefficient difference to $d = .20$ means that the moderating impact of missing data is just as large as the intervention effect itself. A medium effect size threshold is probably an upper bound for most practical applications, and much smaller values of d could be realistic. A sensitivity analysis strategy would examine the changes in the focal model parameters across a range of d values (see Enders 2022, Section 9.8).

7.3: Shared Parameter (Wu–Carroll) Latent Curve Model

This example illustrates a two-factor latent growth curve model with the Wu–Carroll (1998) model for missing not at random dropout that depends on the growth trajectories. A path diagram of the model is shown below.



The model features binary dropout indicators regressed on the random intercepts and slopes. Importantly, the missing data indicators code attrition or permanent dropout rather than intermittent missingness. Accordingly, the D variables equal 0 prior to dropout, 1 at the occasion participants leave the study, and 999 (missing) at all post-dropout measurements. There is no missing data indicator for Y_1 because this variable is complete. The path coefficients connecting dropout at occasion t to the random intercepts are constrained to equality, as are the coefficients connecting the random slopes to dropout. The dashed lines convey these constraints. Predictor variables can be incorporated into either the outcome or dropout part of the model

(e.g., see Example 5.14). Additional details about this model can be found in Enders (2011) and Muthén, Asparouhov, Hunter, and Leuchter (2011).

Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex7.3.imp](#) [data24.dat](#)

The syntax highlights are as follows.

- ❖ **ORDINAL** command identifies the binary dropout indicators
- ❖ **LATENT** command defines two latent variables
- ❖ **MODEL** command uses labels ending in a colon to group models and order their summary tables on the output
- ❖ Individual regression equations specified for each indicator (instead of the \rightarrow convention for latent factors)
- ❖ **MODEL** command estimates the latent variable means, fixes the intercept factor loadings to 1, fixes the growth factor loadings to the time scores (0, 1, 2, 3, 4, and 5), and fixes the measurement intercepts to 0
- ❖ **MODEL** command uses a label to impose equality constraint on residual variance, a label to constrain associations between the random intercepts and indicators, and a label to constrain associations between the random slopes and indicators
- ❖ Longer burn-in period for estimating latent variables

```
DATA: data24.dat;  
VARIABLES: id y1 y2 y3 y4 y5 y6 d1 d2 d3 d4 d5 d6;  
ORDINAL: d2 d3 d4 d5 d6;  
MISSING: 999;  
LATENT: icept slope;
```

MODEL:

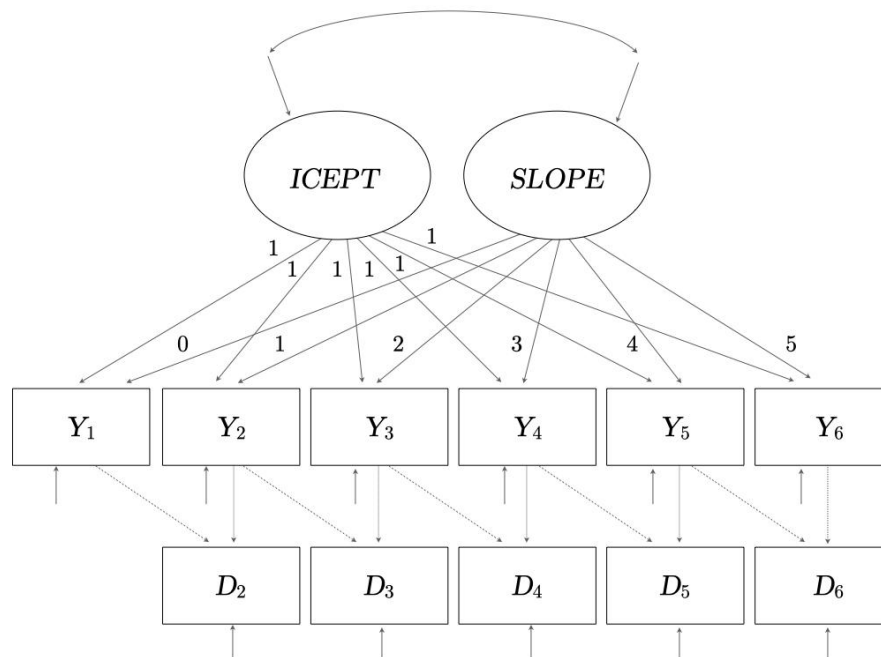
```

structural.model:
icept ~ 1;
slope ~ 1;
icept <-> slope;
measurement.model:
y1 ~ 1@0 icept@1 slope@0;
y2 ~ 1@0 icept@1 slope@1;
y3 ~ 1@0 icept@1 slope@2;
y4 ~ 1@0 icept@1 slope@3;
y5 ~ 1@0 icept@1 slope@4;
y6 ~ 1@0 icept@1 slope@5;
y1 ~~ y1@vconstraint;
y2 ~~ y2@vconstraint;
y3 ~~ y3@vconstraint;
y4 ~~ y4@vconstraint;
y5 ~~ y5@vconstraint;
y6 ~~ y6@vconstraint;
dropout.model:
d2 ~ icept@iconstraint slope@sconstraint;
d3 ~ icept@iconstraint slope@sconstraint;
d4 ~ icept@iconstraint slope@sconstraint;
d5 ~ icept@iconstraint slope@sconstraint;
d6 ~ icept@iconstraint slope@sconstraint;
SEED: 90291;
BURN: 100000;
ITERATIONS: 10000;

```

7.4: Diggle-Kenward Latent Curve Model

This example illustrates a two-factor latent growth curve model with the Diggle–Kenward (1998) model for missing not at random dropout that depends on the unseen outcome score at time t . A path diagram of the model is shown below.



The model features binary dropout indicators regressed on the random intercepts and slopes. Importantly, the missing data indicators code attrition or permanent dropout rather than intermittent missingness. Accordingly, the D variables equal 0 prior to dropout, 1 at the occasion participants leave the study, and 999 (missing) at all post-dropout measurements. There is no missing data indicator for Y_1 because this variable is complete. The path coefficients connecting dropout at occasion t to the observed data from the previous occasion are constrained to equality, as are the paths connecting dropout at occasion t with the concurrent (unseen) scores at occasion t . The dashed lines convey these constraints. Predictor variables can be incorporated into either the outcome or dropout part of the model (e.g., see Example 5.14). Additional details about this model can be found in Enders (2011) and Muthén, Asparouhov, Hunter, and Leuchter (2011).

Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex7.4.imp](#) [data23.dat](#)

The syntax highlights are as follows.

- ❖ **ORDINAL** command identifies the binary dropout indicators
- ❖ **LATENT** command defines two latent variables
- ❖ **MODEL** command uses labels ending in a colon to group models and order their summary tables on the output
- ❖ Individual regression equations specified for each indicator (instead of the \rightarrow convention for latent factors)
- ❖ **MODEL** command estimates the latent variable means, fixes the intercept factor loadings to 1, fixes the growth factor loadings to the time scores (0, 1, 2, 3, 4, and 5), and fixes the measurement intercepts to 0
- ❖ **MODEL** command uses a label to impose equality constraint on residual variance, a label to constrain lagged associations between the outcomes and indicators, and a label to constrain concurrent associations between the outcomes and indicators
- ❖ Longer burn-in period for estimating latent variables

```
DATA: data23.dat;
VARIABLES: id y1 y2 y3 y4 y5 y6 d1 d2 d3 d4 d5 d6;
ORDINAL: d2 d3 d4 d5 d6;
MISSING: 999;
LATENT: icept slope;
MODEL:
  structural.model:
  icept ~ 1;
  slope ~ 1;
```



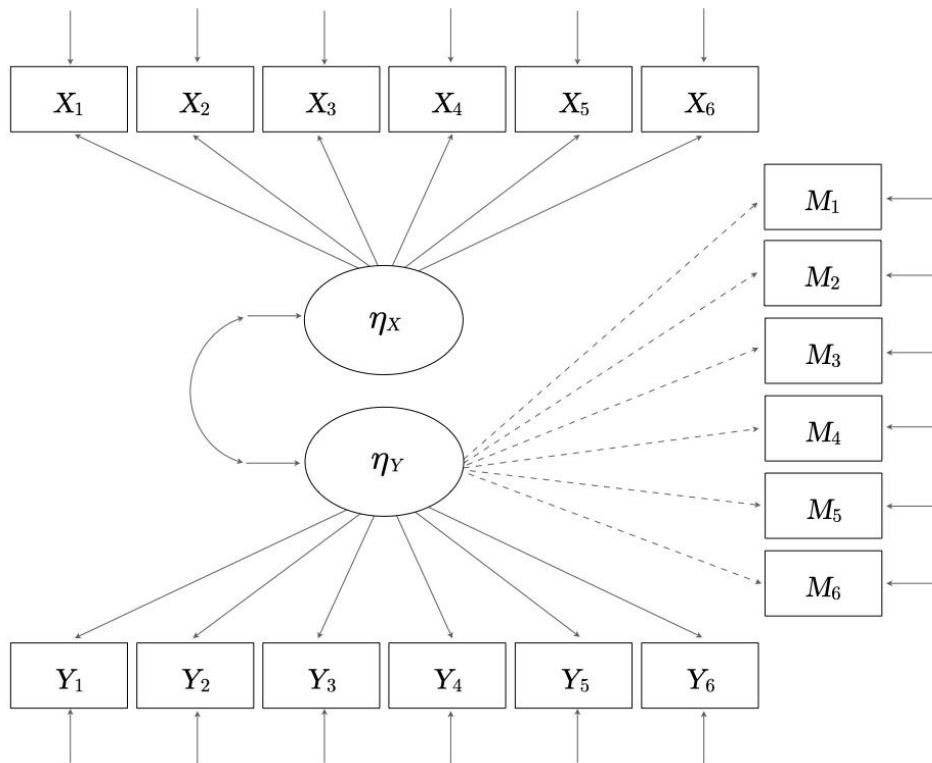
```

icept <-> slope;
measurement.model:
y1 ~ 1@0 icept@1 slope@0;
y2 ~ 1@0 icept@1 slope@1;
y3 ~ 1@0 icept@1 slope@2;
y4 ~ 1@0 icept@1 slope@3;
y5 ~ 1@0 icept@1 slope@4;
y6 ~ 1@0 icept@1 slope@5;
y1 ~~ y1@vconstraint;
y2 ~~ y2@vconstraint;
y3 ~~ y3@vconstraint;
y4 ~~ y4@vconstraint;
y5 ~~ y5@vconstraint;
y6 ~~ y6@vconstraint;
dropout.model:
d2 ~ y1@marconstraint y2@mnarconstraint;
d3 ~ y2@marconstraint y3@mnarconstraint;
d4 ~ y3@marconstraint y4@mnarconstraint;
d5 ~ y4@marconstraint y5@mnarconstraint;
d6 ~ y5@marconstraint y6@mnarconstraint;
SEED: 90291;
BURN: 100000;
ITERATIONS: 10000;

```

7.5: Factor Analysis With a Selection Model

This example illustrates a two-factor measurement model with correlated latent variables, each measured by six continuous indicators. A path diagram of the analysis model is shown in Section 5.5. The main difference is that the analysis incorporates a selection missingness model for the indicators of one latent factor. The selection equations model a missing not at random process where missingness is explained by an individual's standing on the latent trait. A path diagram of the model is shown below.



The M variables are binary missing data indicators coded 0 if a response is complete and 1 if missing. The path coefficients connecting the missingness indicators to the latent factor are constrained to equality (i.e., one's standing on the factor exerts the same influence on the probability of missing not at random missingness). The dashed lines convey these constraints.

Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex7.5.imp](#) [data4.dat](#)

The syntax highlights are as follows.

- ❖ LATENT command defines two latent variables

- ❖ TRANSFORM command uses the `ismissing` function to create missing data indicators for the six Y items
- ❖ Default specification fixes the first loading of each factor to 1 and sets the latent means equal to 0
- ❖ MODEL command uses a label to impose equality constraint on the associations between the latent factor and the missingness indicators
- ❖ Longer burn-in period for estimating latent variables

```

DATA: data4.dat;
VARIABLES: id a1 a2 a3 yscale mscale xscale n1 d1 d2 y1:y6
               m1:m7 x1:x6;
MISSING: 999;
ORDINAL: y1mis y2mis y3mis y4mis y5mis y6mis;
LATENT: latency latentx;
TRANSFORM:
y1mis = ismissing(y1);
y2mis = ismissing(y2);
y3mis = ismissing(y3);
y4mis = ismissing(y4);
y5mis = ismissing(y5);
y6mis = ismissing(y6);
MODEL:
  factor.model:
latentx -> x1:x6;
latency -> y1:y6;
latentx <-> latency;
  missingness.model:
y1mis ~ latency@misconstraint;
y2mis ~ latency@misconstraint;
y3mis ~ latency@misconstraint;
y4mis ~ latency@misconstraint;
y5mis ~ latency@misconstraint;
y6mis ~ latency@misconstraint;

```

SEED: 90291;
BURN: 40000;
ITERATIONS: 10000;

7.6: Mediation Analysis With a Selection Model

This example illustrates a single-mediator path model with selection models for a missing not at random process on the mediator and outcome. The focal regression models are shown below

$$M = \beta_{01} + \beta_{11}X + \varepsilon_1 = I_M + \alpha X + \varepsilon_M$$

$$Y = \beta_{02} + \beta_{12}M + \beta_{22}X + \varepsilon_2 = I_Y + \beta M + \tau' X + \varepsilon_Y$$

where α and β are slope coefficients that define the indirect effect or product of the coefficients estimator, and τ' is the direct effect of X on Y . A path diagram of the analysis is shown in Section 5.1. The analysis additionally incorporates missingness models for both M and Y .

$$M_{mis}^* = \gamma_{01} + \gamma_{11}M + \gamma_{21}X + \varepsilon_1$$

$$Y_{mis}^* = \gamma_{02} + \gamma_{12}Y + \gamma_{22}M + \gamma_{32}X + \varepsilon_2$$

The asterisk superscripts on the binary missing data indicators represent latent response variables from a probit regression. M 's missingness model features M and X as predictors, and Y 's missingness model features Y , M , and X . Finally, the model also incorporates three auxiliary variables following the procedure from Example 4.7.

Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex7.6.imp](#) [data4.dat](#)

The syntax highlights are as follows.

- ❖ TRANSFORM command uses the `ismissing` function to create missing data indicators for *M* and *Y*
- ❖ MODEL command uses labels ending in a colon to group models and order their summary tables on the output
- ❖ MODEL command labels the indirect effect's component pathways
- ❖ MODEL command features a syntax shortcut that creates a factored regression (sequential) specification for auxiliary variables
- ❖ PARAMETERS command uses labeled quantities to compute the product of coefficients estimator

```

DATA: data4.dat;
VARIABLES: id a1:a3 zscale y m n1 x d1 o1:o19;
MISSING: 999;
TRANSFORM:
m.mis = ismissing(m);
y.mis = ismissing(y);
MODEL:
mediation.model:
m ~ x@alpha;
y ~ m@beta x;
missingness.model:
m.mis ~ m x;
y.mis ~ y m x;
auxiliary.model:
# sequential specification for auxiliary variables
a1:a3 ~ y m x;
PARAMETERS:
indirect = alpha * beta;
SEED: 90291;

```

BURN: 2000;
ITERATIONS: 10000;

7.7: Two-Level Hedeker-Gibbons Pattern Mixture Growth Model

This example illustrates the random coefficient pattern mixture model from Hedeker and Gibbons (1997). The model is designed for a missing not at random process where growth model parameters differ between cases who complete the study versus those who dropout. Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex7.7.imp](#) [data18.dat](#)

The complete-data multilevel model features a cross-level (group-by-time) interaction effect involving a level-2 dummy code D_1 (e.g., a treatment assignment indicator) and the level-1 time scores, as follows.

$$Y_{ij} = (\beta_0 + b_{0j}) + (\beta_1 + b_{1j})TIME_{ij} + \beta_2(D_{1j}) + \beta_3(TIME_{ij} \times D_{1j}) + \varepsilon_{ij}$$

The pattern mixture model introduces a dropout indicator that differentiates completers and dropouts, $M = 0$ and 1 , respectively. The fitted model features the dropout indicator and its interaction effects

$$\begin{aligned} Y_{ij} = & \beta_{0(obs)} + \beta_{1(obs)}TIME_{ij} + \beta_{2(obs)}(D_{1j}) + \beta_{3(obs)}(TIME_{ij} \times D_{1j}) \\ & + \beta_{0(diff)}(M_j) + \beta_{1(diff)}(TIME_{ij} \times M_j) + \beta_{2(diff)}(D_{1j} \times M_j) \\ & + \beta_{3(diff)}(TIME_{ij} \times D_{1j})(M_j) + b_{0j} + b_{1j}(TIME_{ij}) + \varepsilon_{ij} \end{aligned}$$

where the “obs” subscript denotes the completer group's ($M = 0$) parameters, and the “diff” subscript denotes coefficient differences for the dropout group ($M = 1$). Following

Example 7.2, the overall population-level estimates (i.e., the marginal estimates that average over the distribution of missingness) are a weighted average of the pattern-specific coefficients, where the weights are the group proportions. The marginal estimates for this example are shown below, where $p_{(obs)}$ and $p_{(mis)}$ are the proportions of completers and dropouts, respectively.

$$\beta_0 = p_{(obs)}\beta_{0(obs)} + p_{(mis)}\beta_{0(mis)} = p_{(obs)}\beta_{0(obs)} + p_{(mis)}(\beta_{0(obs)} + \beta_{0(diff)})$$

$$\beta_1 = p_{(obs)}\beta_{1(obs)} + p_{(mis)}\beta_{1(mis)} = p_{(obs)}\beta_{1(obs)} + p_{(mis)}(\beta_{1(obs)} + \beta_{1(diff)})$$

$$\beta_2 = p_{(obs)}\beta_{2(obs)} + p_{(mis)}\beta_{2(mis)} = p_{(obs)}\beta_{2(obs)} + p_{(mis)}(\beta_{2(obs)} + \beta_{2(diff)})$$

$$\beta_3 = p_{(obs)}\beta_{3(obs)} + p_{(mis)}\beta_{3(mis)} = p_{(obs)}\beta_{3(obs)} + p_{(mis)}(\beta_{3(obs)} + \beta_{3(diff)})$$

The syntax highlights are listed below, and adding the NIMPS and SAVE commands generates model-based multiple imputations for a frequentist analysis (see Example 6.3).

- ❖ CLUSTERID command identifies a level-2 identifier, automatically inducing random intercepts for all level-1 variables
- ❖ ORDINAL command identifies binary predictors
- ❖ CENTER command applies grand mean and latent group mean centering to predictors
- ❖ MODEL command uses labels ending in a colon to group models and order their summary tables on the output
- ❖ MODEL command features a random coefficient listed after the vertical pipe
- ❖ MODEL command labels each fixed effect coefficient
- ❖ MODEL command features a factored regression (sequential) specification for the binary predictors

- ❖ **PARAMETERS** command uses labeled quantities to compute population-average (marginal) coefficients that average over missing data patterns

```

DATA: data18.dat;
VARIABLES: level2id n1.j d1.j y.i time.i n2.j m.j d3.i;
ORDINAL: d1.j m.j;
CLUSTERID: level2id;
MISSING: 999;
FIXED: time.i;
MODEL:
focal.model:
y.i ~ 1@beta0.obs time.i@beta1.obs d1.j@beta2.obs
      (time.i*d1.j)@beta3.obs m.j@beta0.dif (m.j*time.i)@beta1.dif
      (m.j*d1.j)@beta2.dif (m.j*time.i*d1.j)@beta3.dif | time.i;
predictor.model:
m.j ~ 1@ymissmean;
d1.j ~ m.j;
PARAMETERS:
# missingness group proportions
p.mis = phi(ymissmean);
p.obs = 1 - p.mis ;
# population-average estimates
beta0 = p.obs * beta0.obs + p.mis * (beta0.obs + beta0.dif);
beta1 = p.obs * beta1.obs + p.mis * (beta1.obs + beta1.dif);
beta2 = p.obs * beta2.obs + p.mis * (beta2.obs + beta2.dif);
beta3 = p.obs * beta3.obs + p.mis * (beta3.obs + beta3.dif);
SEED: 90291;
BURN: 5000;
ITERATIONS: 10000;

```

7.8: Selection Model for a Two-Level Regression With Random Coefficients

This example illustrates a two-level regression model with random intercepts and random slopes. The analysis model is shown below.

$$Y_{ij} = (\beta_0 + b_{0j}) + (\beta_1 + b_{1j}) X_{1ij}^{cwc} + \beta_2 X_{2ij}^{cgm} + \beta_3 X_{7j}^{cgm} + \beta_4 D_{1j}^{cgm} + \varepsilon_{ij}$$

The analysis additionally incorporates a missingness model for Y .

$$M_{Yij}^* = \gamma_0 + \gamma_1 Y_{ij} + \gamma_2 X_{1ij}^{cwc} + \varepsilon_{1ij}$$

The asterisk superscript on the binary missing data indicator represents a latent response variable from a probit regression. Y 's missingness model features Y and X_1 as predictors. The M variable is a binary missing data indicator coded 0 if Y is observed and 1 if it is missing. This model would be appropriate for a multilevel model that does not involve a time trend and permanent attrition (e.g., intensive longitudinal data).

Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex7.8.imp](#) [data8.dat](#)

The syntax highlights are as follows.

- ❖ CLUSTERID command identifies a level-2 identifier, automatically inducing random intercepts for all level-1 variables
- ❖ ORDINAL command identifies a binary predictor
- ❖ TRANSFORM command uses the `ismissing` function to create a missing data indicator for Y
- ❖ FIXED command identifies a complete predictor
- ❖ CENTER command applies grand mean and latent group mean centering to predictors
- ❖ MODEL command uses labels ending in a colon to group models and order their summary tables on the output
- ❖ MODEL command features a random coefficient listed after the vertical pipe

❖ Unspecified associations for predictor variables

```

DATA: data8.dat;
VARIABLES: level1id level2id x1.i x2.i y.i x3.i x4.i d1.j
               n1.j x5.j x6.j x7.j x8.j x9.j;
CLUSTERID: level2id;
ORDINAL: d1.j;
MISSING: 999;
TRANSFORM:
ymis.i = ismissing(y.i);
FIXED: d1.j;
CENTER:
groupmean = x1.i;
grandmean = x2.i x7.j d1.j;
MODEL:
focal.model:
y.i ~ x1.i x2.i x7.j d1.j | x1.i;
missingness.model:
ymis.i ~ y.i x1.i;
SEED: 90291;
BURN: 5000;
ITERATIONS: 10000;

```

7.9: Two-Level Shared Parameter (Wu–Carroll) Growth Curve Model

This example illustrates how to estimate the Wu–Carroll (1998) model from Section 7.3 as a multilevel regression model. Consistent with that earlier example, the model is for situations where missing not at random dropout depends on the growth trajectories. The model features binary dropout indicators regressed on the random intercepts and slopes. Importantly, the missing data indicators code attrition or permanent dropout rather than intermittent missingness. Accordingly, the D variables equal 0 prior to dropout, 1 at the occasion participants leave the study, and 999

(missing) at all post-dropout measurements. In the multilevel specification, the dropout indicators are stacked in a single column like other time-varying variables.

The growth curve model is cast as a multilevel structural equation model with a pair of normally distributed level-2 latent variables representing the random intercepts and slopes. The focal model features these latent variables as predictors, as shown below.

$$\beta_{0j} = \beta_0 + b_{0j}$$

$$\beta_{1j} = \beta_1 + b_{1j}$$

$$Y_{ij} = \beta_{0j} + \beta_{1j}TIME_{ij} + \varepsilon_{ij}$$

Note that the slope of the time score predictor is implicitly fixed to 1 in this specification.

The latent curve model from the earlier example has features that require special attention when specifying the analysis as a multilevel model. First, there is no dropout indicator for the baseline assessment because this variable is complete. In the multilevel framework, the indicator takes on constant values of zero in the first row of each individual's missingness vector. Second, the random intercepts and slopes cannot influence the omitted (and constant) baseline missingness indicator. Third, each measurement occasion has a unique intercept that determines the occasion-specific missingness rate. The multilevel selection equation must honor these important features.

The multilevel missingness model is shown below.

$$\begin{aligned} M_{Yij}^* = & \gamma_1 T_{1j} + \gamma_2 T_{2j} + \gamma_3 T_{3j} + \gamma_4 T_{4j} + \gamma_5 T_{5j} + \gamma_6 T_{6j} \\ & + \gamma_7 B_{ij} \beta_{0j} + \gamma_8 B_{ij} \beta_{1j} + r_{ij} \end{aligned}$$

To accommodate occasion-specific intercepts, the missingness model includes a dummy code for each measurement occasion. The equation denotes the time codes as T_1 through T_6 . Note that the usual regression intercept is omitted from the equation (alternatively, the intercept could be estimated if T_1 is omitted). Importantly, the coefficient for the first (baseline) dummy code is fixed at $\gamma_1 = -3$ to induce a near-zero probability of missingness at baseline. In the latent curve model, this is equivalent to omitting the first dropout indicator. Next, the variable B_{ij} is a dummy code that equals 0 at the baseline assessment and 1 in all other rows. Including the product of this dummy code and the random effects acts like an on/off switch that sets the influence of the random intercepts and slopes to zero at baseline. Recall that this was an important feature of the latent curve model. The Blimp script below uses Boolean functions in the selection equation to create the dummy codes, but these could also be generated using the TRANSFORM function. See the TRANSFORM section in Chapter 2 for a description of Boolean operators.

Clicking the links below downloads the Blimp scripts and data for this example, and the full set of User Guide examples is available from a pull-down menu in the graphical interface.

[Ex7.9.imp](#) [data17.dat](#)

The syntax highlights are as follows.

- ❖ `ORDINAL` command identifies the binary dropout indicators
- ❖ `FIXED` command identifies a complete time score predictor
- ❖ `LATENT` command defines two between-cluster latent variables representing the random intercepts and slopes

- ❖ MODEL command uses labels ending in a colon to group models and order their summary tables on the output
- ❖ MODEL command estimates the random intercept and slope means
- ❖ MODEL command specifies correlation between random intercepts and random slopes (level-2 latent variables)
- ❖ MODEL command fixes the interaction coefficient for the product of the random slope predictor and its level-2 latent variable to 1
- ❖ MODEL command omits the random coefficient listed after the vertical pipe
- ❖ MODEL command uses Boolean operators in parentheses to create dummy variables
- ❖ MODEL command features product terms between the Boolean operators (dummy codes) and the level-2 latent variables
- ❖ Longer burn-in period for estimating latent variables

```

DATA: data17.dat;
VARIABLES: level2id time.i y.i dropout.i;
ORDINAL: dropout.i;
CLUSTERID: level2id;
MISSING: 999;
LATENT:
level2id = icept.j slope.j;
FIXED: time.i;
MODEL:
latent.variables:
icept.j ~ 1;
slope.j ~ 1;
icept.j ~~ slope.j;
growth.model:
y.i ~ 1@icept.j time.i*slope.j@1;
missingness.model:
dropout.i ~ 1@0 (time.i == 0)@-3 (time.i == 1) (time.i == 2)

```

```
(time.i == 3) (time.i == 4) (time.i == 5)
(time.i > 0)*icept.j (time.i > 0)*slope.j | 1@0;
SEED: 90291;
BURN: 10000;
ITERATIONS: 10000;
```

8 Monte Carlo Studies in Blimp

Blimp also includes a SIMULATE command for creating artificial data sets with Monte Carlo computer simulation. This command creates a data set based on user-specified population parameters, after which it fits the equations from the MODEL command to the simulated data. The simulated data set can be saved by specifying dataset = filename with the SAVE command. Parameter estimates from the fitted model can also be saved following the procedure described in the SAVE section of Chapter 2. Note that each Blimp script generates a single artificial data set. To embed this functionality in a broader Monte Carlo computer simulation (e.g., inside a loop function), see Section 1.7. This chapter is currently under construction, but the Examples pull-down menu in Blimp Studio shows the following examples.

- ❖ 8.1: Simulation With Linear Regression
- ❖ 8.2: Simulation With Bivariate Regression
- ❖ 8.3: Simulation With Bifactor Model
- ❖ 8.4: Simulation With Random Intercept Model
- ❖ 8.5: Simulation With Random Slopes
- ❖ 8.6: Simulation With Linear Growth Model
- ❖ 8.7: Simulation with Two-level Decomposed Effects Model
- ❖ 8.8: Simulation With Missing Data Generation

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