TIME TO BUILD AND THE BUSINESS CYCLE*

– Job Market Paper –

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Abstract

Investment is central for business cycles and a defining characteristic of investment is time to build. While existing business cycle models assume constant time to build, I document that time to build is volatile and largest during recessions. Motivated by this finding, I develop a heterogeneous firms general equilibrium model in which time to build fluctuates exogenously. In the model, investment is partially irreversible. The longer time to build, the less frequently firms invest, and the less firm investment reflects firm productivity. As a result, an increase in time to build worsens the allocation of capital across firms and decreases aggregate productivity. In the calibrated model, a shock increasing time to build by one month lowers investment by 2% and output by 0.5%. Structural vector autoregressions corroborate the quantitative importance of time to build shocks.

JEL CLASSIFICATION: C32, C68, D92, E01, E22, E32.

KEYWORDS: Time to Build, Business Cycles, Investment, Misallocation.

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1. INTRODUCTION

Capital goods are complex and manufactured to the specific needs of an investing firm. For example, an assembly line consists of many elements that need to fit together; think of conveyor belts, robotic arms working along these belts, and the concrete foundation that supports the machines. Further, an assembly line needs to fit the specific good it produces. The complexity and specificity of capital cause a time gap between the order of capital goods and their delivery. This time gap is commonly referred to as time to build and is assumed constant in modern business cycle theory.[1] My paper first documents substantial variation in time to build, with peak values in recessions. Second, I ask whether exogenous fluctuations in time to build are of first-order importance for business cycles.

To address this question, I develop a dynamic stochastic general equilibrium model. Firms in my model face persistent shocks to their own productivity. Their investment is partially irreversible. The market for capital goods is characterized by search frictions, which imply time to build. Variations in time to build immediately result from changes in this friction. Calibrating the model to US manufacturing data, I find that time to build fluctuations are quantitatively important. A one month increase in time to build lowers investment by 2% and output by 0.5%.

A lengthening in time to build is contractionary. This is due to two channels. First, later delivery of outstanding investment orders, as follows from longer time to build, mechanically reduces contemporaneous investment and thus production. Second, and this channel is both novel and quantitatively central, longer time to build worsens the allocation of capital across firms. While the efficient allocation dictates that more productive firms use more capital, longer time to build weakens the alignment between capital and productivity. At the core of the mechanism, later delivery of an investment order affects the ex-ante productivity forecasts for the periods the investment good is used as well as the associated forecast uncertainties. In turn, firms invest less frequently and, if they invest, their investment reacts less to their contemporaneous productivity. A lengthening of time to build therefore means capital is less well aligned with firms' productivity, meaning aggregate productivity is lower and so are output, investment, and consumption.

To measure time to build, I use the US Census M3 survey of manufacturing firms. The Census provides publicly available time series for order backlog and shipment

[1] While Kydland and Prescott (1982) assume four quarters time to build, the standard assumption in real business cycle models quickly shifted to one quarter, see Prescott (1986) for example.

in the non-defense equipment goods sector since 1968. These time series allow me to estimate time to build as the time span new capital good orders remain unfilled in capital producers' order books. I document that time to build exhibits substantial variation. It fluctuates between three and nine months. Time to build tends to be largest at the end of recession periods.[2]

The model I develop is a real business cycle model. Households consume and supply labor. The model distinguishes between firms that supply capital and firms that demand capital. On the capital demand side, there are firms that produce consumption goods combining labor with specific capital. To invest in specific capital, they need to hire an engineering firm that devises a blueprint for the investment project. Using the blueprint, the engineering firm searches for a capital supplier to produce the required capital good. Production takes place when engineering firm and capital supplier are matched and goods are delivered at the end of the period. Shocks to the matching technology cause fluctuations in time to build. These shocks may be seen as shortcut for changes in the capital supply network, which make it more difficult to supply the required components. The model further features lumpy firm-level investment in line with the micro-level evidence on capital adjustment. The lumpiness arises because investment is partially irreversible.

To evaluate the quantitative importance of shocks to the matching technology, I calibrate the model to US data. The strategy is to jointly target moments of the investment rate distribution and aggregate fluctuations in time to build. In the calibrated model, shocks to the matching technology that raise time to build by one month cause a sharp 2% drop in investment and a more gradual drop in output that peaks after six quarters at 0.5%. I show that the direct effects of later delivery explain the short-term responses while increased capital misallocation explain nearly all of the persistent responses. Misallocation endogenously lowers measured aggregate productivity. Using the calibrated model, I back out a time series of shocks to the matching technology that explain the measured time to build fluctuations. The model predicts that these shocks account for a third of the decline in output and investment during the early 1990s recession and the Great Recession.

To solve the model, I build on the algorithms in Campbell (1998), Reiter (2009) and Winberry (2016b). The conceptual idea is to combine global projection with

[2]This paper is not the first to document the countercyclicality of the backlog ratio, see, e.g. Nalewaik and Pinto (2015). To the best of my knowledge, however, my paper is the first to relate fluctuations in the backlog ratio to time to build in the context of modern business cycle models.

local perturbation solution methods. Compared to Winberry (2016b), the model in this paper is computationally more involved because the idiosyncratic state additionally consists of outstanding capital good orders. Hence, I show that the algorithms can be applied to solve more involved firm heterogeneity models.

To reassess the results of my business cycle model, I use time series techniques to investigate the importance of time to build shocks. In particular, I fit an eight-variable vector autoregression (VAR) including macroeconomic aggregates, prices, and time to build. To be conservative, I restrict the identified shocks to matching technology to contemporaneously only affect time to build. The restriction also implies that all other shocks may affect time to build contemporaneously.[3]

The results of the structural VAR corroborate the quantitative findings of my business cycle model. I find that adverse shocks to matching technology significantly and persistently lower GDP, investment, and consumption. The identified shock explains more than 20% of the forecast error variance of GDP and consumption. The impulse response functions (IRFs) of output and investment are of similar magnitude as the IRFs in the business cycle model. Moreover, the forecast error variance of time to build explained by the identified shock is almost 50% and the identified shocks are uncorrelated with conventional direct measures of business cycle shocks (e.g., productivity, monetary policy, and tax shocks). This lends support to my business cycle model's assumption of exogenous shocks directly affecting time to build. I further show that my results are robust to relaxing the equality restrictions of the structural VAR by flexible elasticity bounds, using the methods suggested in Gafarov, Meier, and Montiel Olea (2016).

Related Literature

This paper contributes to several literatures. First, this paper contributes to the literature studying the macroeconomic implications of lumpy investment. There is ample evidence for investment lumpiness, see Doms and Dunne (1998), and structural explanations are investment irreversibilities or fixed costs of capital adjustment, see Cooper and Haltiwanger (2006). Recent work has investigated the macroeconomic implications of capital adjustment costs for the response to aggregate productivity shocks, see Khan and Thomas (2008) and Bachmann et al. (2013), and, for the response to uncertainty shocks, see Bloom (2009), Bachmann and Bayer (2013),

[3]The results are robust to the alternative restriction that only shocks to matching technology affect all variables in the VAR contemporaneously.

and Bloom et al. (2014). In my model, the interaction between time to build and investment irreversibilities is key for the transmission of shocks to the matching technology. The transmission mechanism shares the real options effect prominent in the uncertainty literature, albeit without inducing the volatility effect that higher uncertainty eventually realizes and leads to reversals and overshooting, see Bloom (2009). To the extent that longer time to build increases the effective forecast uncertainty, this paper also contributes to the endogenous uncertainty literature, see Bachmann and Moscarini (2011) and Fajgelbaum et al. (2014).

Second, my paper relates to recent work studying the interaction between time to build fluctuations and investment irreversibility. Studying time to build for residential housing, Oh and Yoon (2016) document a time series pattern fairly similar to the one for equipment capital goods documented in this paper. In their model, higher uncertainty increases time to build because residential construction occurs in stages and each stage involves irreversible investment. Kalouptsidi (2014) studies the bulk shipping industry and shows that procyclical fluctuations in time to build dampen the volatility of investment into ships.

Third, in modeling a frictional market for capital goods, I build on the search literature. Since Mortensen and Pissarides (1994) search frictions are popular in labor market models. For capital markets, Kurmann and Petrosky-Nadeau (2007) and Ottonello (2015) show that search frictions amplify business cycle shocks. Tightness on the capital goods market governs the intensive margin of investments, while in my setup search frictions also affect the extensive margin of investment. Shocks to the matching technology in my model build on the labor market search and matching literature, see Krause et al. (2008), Sedláček (2014), and Sedláček (2016) for example.

The remainder of this paper is organized as follows: Section 2 documents time to build. Section 3 presents the central model mechanism and Section 4 develops the quantitative business cycle model. I discuss the calibration in Section 5 and results in Section 6. Section 7 provides the SVAR evidence. Finally, Section 8 concludes.

2. DOES TIME TO BUILD VARY OVER THE CYCLE?

My goal is to estimate time to build using survey data on the order books of capital good producers. I show that time to build exhibits substantial variation between three and nine months with peak values during recessions.

I use US Census data collected in the Manufacturers' Shipments, Inventories, and Orders Survey (M3). The M3 covers two third of manufacturers with annual sales above 500 million USD and some smaller companies to improve industry coverage. M3 participants are selected from the Economic Census and the Annual Survey of Manufacturers and the M3 is benchmarked against these datasets. US quarterly investments are computed by the Bureau of Economic Analysis using the M3.[4]

The Census provides publicly available data for shipments and order backlog at the sectoral level. Under the premise of excluding defense goods, I use the sector category *non-defense equipment goods*, which is available at monthly frequency from 1968 through 2015.[5] M3 data satisfies a stock-flow equation for equipment good orders, where O denotes new orders net of cancellations, S shipments, and B the beginning-of-period stock of order backlog[6]

$$B_{t+1} = B_t + O_t - S_t. \tag{2.1}$$

My baseline measure of time to build, also called backlog ratio, is

$$TTB_t \equiv \frac{B_t}{S_t}. \tag{2.2}$$

It measures the intensity of flows (shipments) out of the stock of backlogged orders, expressed in months. Figure 1 shows the evolution of time to build, which exhibits substantial variation. Time to build tends to start increasing before recessions and peaks at the end of recession periods. In Appendix A, I plot the component series of (2.1) over time. The correlation of annualized real GDP growth with log time to build is -0.3. Detrending the slow-moving trend from time to build using the HP filter with a smoothing parameter of 810,000, the correlation increases to -0.4. The finding of a countercyclical backlog ratio coincides with previous studies, see Nalewaik and Pinto (2015) for example.

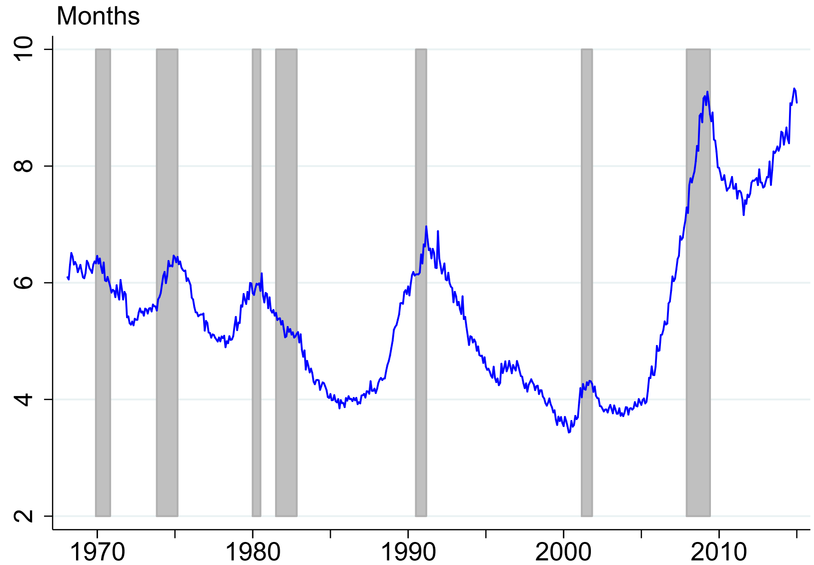
Under two conditions this time to build measure equals the expected waiting time of a new equipment good order: First, the shipment protocol is first-in first-

[4]See *Concepts and Methods of the U.S. National Income and Product Accounts* (2014, ch. 3).

[5]Notice that for finer disaggregation of the equipment goods sector into two-digit sectors, the distinction of defense and non-defense is not always available.

[6]A new order is defined as a legally binding intention to buy for immediate or future delivery, and the survey does not ask separately for order cancellations. Shipments measure the value of goods delivered in a given period, while order backlog measures the value of orders that have not yet fully passed through the sales account.

Figure 1: Time to Build



Notes: Time to build is measured as the ratio of order backlog to monthly shipments, for non-defense equipment goods. Shaded, gray areas indicate NBER recession dates.

out, i.e. new orders are shipped only after backlogged orders are shipped. Second, shipments are expected to be unchanged in the future. While the first condition is plausibly satisfied, the second one is roughly satisfied given that shipments are highly persistent. In Appendix A, I show that an alternative measure of time to build, based on ex-post shipment realizations, closely resembles my baseline measure. Additionally, I provide the evolution of the individual component series defining the order stock-flow equation.

A caveat of estimating time to build using the M3 is that it excludes structure capital and imported equipment capital, which together account for no more than 35% of total non-residential private fixed investments in the US.[7]

Given the aggregate nature of the data I use, my measure is necessarily one of macroeconomic time to build. If there are cross-sectional differences in time to build, this will be different from the average micro-level time to build. Notice that within the model I develop in Section 4, I will recompute the measure of time to build in the exact same way and use that as calibration target.

[7]Out of total private non-residential fixed investment, structure capital constitutes on average 25% over the last 40 years, declining over time with 10% in 2015. Imported equipment capital is on average 10% of total investments, increasing over time with 20% in 2015.

3. FIRM-LEVEL INVESTMENT AND TIME TO BUILD

This section discusses a novel, and quantitatively central, mechanism of my paper. In short, fluctuations in time to build affect how frequently firms invest, and, if they invest, by how much. These changes in the investment policy hamper an efficient reallocation of capital across firms and thereby depress real economic activity.

In general, two key determinants of a firm's investment decision are expected future productivity and uncertainty about future productivity. Higher expected future productivity makes larger investments appear profitable. Higher uncertainty about future productivity may induce the firm to postpone investments if investment is partially irreversible.[8] To understand the specific effects of time to build on a firm's investment decision, it is of central importance that longer time to build shifts the expected usage period of the investment good into the future. Hence, longer time to build affects the expected productivity, and the associated uncertainty, during the usage period.

To illustrate the point, suppose firm productivity follows an AR(1) process

$$x_t = \rho x_{t-1} + \sigma \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0,1).$$

Conditional on the firm's period zero productivity x_0, the forecast of productivity in period $\tau > 0$ and the associated forecast uncertainty are

$$\hat{x}_\tau = \rho^\tau x_0 \qquad \text{and} \qquad \hat{s}_\tau^2 = \sigma^2 \sum_{t=1}^{\tau} \rho^{2(t-1)},$$

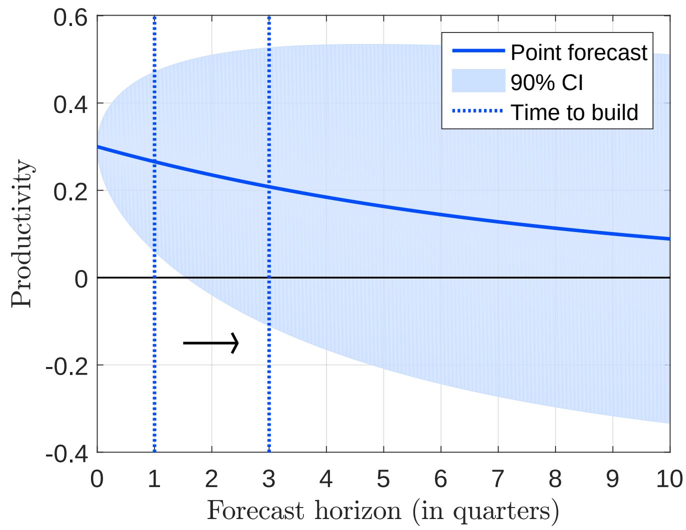
respectively. Consider τ the expected period of investment delivery and $0 < \rho < 1$. Longer time to build, that is larger τ, moves the forecast for productivity after delivery closer to the (zero) long-run mean of productivity and the associated forecast uncertainty increases. Figure 2 illustrates the first and second moment effect of an increase in time to build from one to three quarters.[9]

What are the implications of longer time to build for the firm's investment policy? First, longer time to build reduces the sensitivity of investment to contemporaneous deviations of productivity from its long-run mean. This follows directly from mean-reversion, and I refer to this intensive-margin change in the investment policy as

[8] Abel and Eberly (1996) show analytically that the inaction range, in which not adjusting capital is optimal, expands in uncertainty when capital is partially irreversible.

[9] Longer time to build increases the *relevant* forecast uncertainty by shifting the *relevant* forecast horizon, which is not captured by empirical estimates of forecast uncertainty as Jurado et al. (2015).

Figure 2: Productivity forecasts and time to build



Notes: Starting from an initial productivity level one unconditional standard deviation above zero, the figure plots the productivity forecast, $\hat{x}_\tau$, and its 90% confidence interval (CI), $[x_0 - 1.96\hat{s}_\tau, x_0 + 1.96\hat{s}_\tau]$, per forecast horizon, τ. The arrow illustrates a shift in time to build from one to three quarters, roughly resembling the increase in time to build observed from 2006 to 2009. The figure is based on the parameters for the firm-level productivity process calibrated in Section 5.

regression-to-the-mean effect. Second, higher time to build increases the uncertainty about productivity after delivery. Assuming partial investment irreversibility, the real option value of waiting increases. That is, the firm finds it optimal to tolerate larger deviations of the current capital stock from its optimal size. In turn, the adjustment frequency falls. I refer to this extensive-margin change in the investment policy as *wait-and-see* effect.[10,11]

Increases in time to build have aggregate consequences because the altered investment policy hampers the efficient allocation of capital across firms with different levels of productivity. Intuitively, more of the high productivity firms with low capital stocks do not invest or invest less. Increased capital misallocation endogenously lowers measured aggregate productivity, output, investment, and consumption.

[10] The wait-and-see effect is also prominent to explain contractionary aggregate effects of exogenous uncertainty shocks, see, e.g., Bloom (2009) and Bachmann et al. (2013). In my setup, however, uncertainty is driven by changes in the expected delivery period. The volatility effect, leading to fast reversals as discussed in Bloom (2009), is not present in my setup.

[11] If productivity shocks are permanent, $\rho = 1$, the regression-to-the-mean effect is turned off, while the wait-and-see effect will be strengthened through larger effects on forecast uncertainty.

4. MODELING CYCLICAL FLUCTUATIONS IN TIME TO BUILD

This section develops a model which extends the basic real business cycle model in two ways. First, producers of consumption goods vary in their productivity and use producer-specific capital. Second, investment in specific capital is partially irreversible and supplied through a frictional capital market giving rise to time to build. Shocks to the matching technology cause fluctuations in time to build.

4.1. *Households*

Households value consumption and leisure. I assume the existence of a representative household with separable preferences

$$U(C_t, L_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \psi L_t, \tag{4.1}$$

where C_t is consumption and L_t labor supply in period t. σ denotes the intertemporal substitution elasticity, and ψ parametrizes the disutility of working. I suppose the period utility function is the result of indivisible labor, see Hansen (1985) and Rogerson (1988).[12] The household owns all firms and receives aggregate profits denoted Π_t. The problem of the household is

$$\max_{C_t, L_t} U(C_t, L_t) \quad \text{s.t.} \quad C_t \leq w_t L_t + \Pi_t, \tag{4.2}$$

where w_t is the wage. Due to household ownership, firms discount future profits by

$$Q_{t,t+1} = \beta \frac{p_{t+1}}{p_t}, \tag{4.3}$$

with $p_t = C_t^{-\sigma}$ the marginal utility of consumption. The household's optimal labor supply requires $w_t = \psi / p_t$.

4.2. *Engineering firms and capital suppliers*

To invest in specific capital, producers of consumption goods need to hire an engineering firm that acts as an intermediary on the capital market. Engineering firms search for a capital good producer to supply the required goods. When search succeeds, the capital supplier produces all goods within one period.

[12] These preferences are common in related general equilibrium models with non-convex capital adjustment frictions, see Khan and Thomas (2008) and Bloom et al. (2014) for example.

Let me motivate the setup by the assembly line example in the introduction. Since assembly lines are complex, the investing firm needs to hire an assembly line producer (engineering firm). This producer, in turn requires a network of suppliers that provide the various inputs that compose an assembly line. On top, assembly lines are specific, and thus requires different supplier networks across orders. While many individual business relationships are firmly established, the producer may need to search for some new suppliers given a new assembly line orders. In my model, the capital supplier is a shortcut for a supply network.

In detail, I assume a continuum of capital submarkets differentiated by cost parameter ξ, distributed by G. Consumption good producers randomly access a submarket ξ. The remainder of this subsection focuses on an arbitrary submarket ξ. There is a large mass of engineering firms (short: engineers) and capital suppliers. The mass of active engineers be E_t, and the mass of active capital suppliers S_t. Formally, the matching technology between engineers and capital suppliers is

$$M_t = m_t E_t^{\eta} S_t^{1-\eta}, \tag{4.4}$$

where m_t is stochastic matching efficiency that follows an AR(1) process in logs

$$\log(m_t) = (1-\rho^m)\log(\mu^m) + \rho^m \log(m_{t-1}) + \sigma^m \epsilon_t^m, \quad \epsilon_t^m \overset{iid}{\sim} \mathcal{N}(0,1). \tag{4.5}$$

I define market tightness as $\theta_t = E_t/S_t$. The order filling probability for an engineer is $q_t = m_t \theta_t^{\eta-1}$, and the matching probability for a supplier is $\theta_t q_t$. Once matched, the probability of match separation is χ.

Suppliers and engineers need to hire ξ workers to participate in the market and workers are mobile across sectors so the wage is equal across sectors. When matched for any given investment order i_t, the capital supplier produces within the period and delivers the order to the engineer for unit price p_t^S. Capital suppliers have unit marginal costs to transform consumption goods into capital. Given the stochastic discount factor in (4.3), the value of an unmatched and matched capital supplier is

$$V_t^S = -\xi w_t + \theta_t q_t J_t^S + (1-\theta_t q_t)\mathbb{E}_t[Q_{t,t+1} V_{t+1}^S], \tag{4.6}$$

$$J_t^S = p_t^S i_t - i_t + (1-\chi)\mathbb{E}_t[Q_{t,t+1} J_{t+1}^S], \tag{4.7}$$

respectively. Engineers are hired on a spot market for investment orders, they can perfectly commit and are perfectly competitive. A consumption good producer can only hire one engineering firm. Thus, the number of engineers equals the number of orders. Conditional on delivery, engineers receive unit price p_t^E. To deliver, the engineer needs to find a matching capital supplier. The value of an unmatched and matched engineer is, respectively,

$$V_t^E = -\xi w_t + q_t J_t^E + (1-q_t)\mathbb{E}_t[Q_{t,t+1}V_{t+1}^E], \tag{4.8}$$

$$J_t^E = p_t^E i_t - p_t^S i_t + (1-\chi)\mathbb{E}_t[Q_{t,t+1}J_{t+1}^E]. \tag{4.9}$$

In equilibrium, engineers make zero profits on the spot market for investment orders, and I assume that capital suppliers satisfy a free entry condition.

$$V_t^E = V_t^S = 0. \tag{4.10}$$

When matched, engineer and capital supplier split the match surplus by Nash bargaining over the unit price p_t^S, where ϕ is the engineer's bargaining weight

$$\max_{p_t^S}(J_t^E - V_t^E)^{\phi}(J_t^S - V_t^S)^{1-\phi}. \tag{4.11}$$

The two equations in (4.10) together with the solution to (4.11) jointly define the equilibrium values of θ_t, p_t^S, p_t^E.

Assumption: Matches are formed for a single period, $\chi = 1$.

The assumption considerably simplifies the problem and appears less strong when reconsidering the capital supplier as shortcut for a supplier network. Under $\chi = 1$, the solution to (4.11) is $p_t^S = \phi + (1-\phi)p_t^E$ and the unit price engineers receive becomes $p_t^E = 1 + \frac{\xi w_t}{\phi q_t}\frac{1}{i_t}$. Thus, investment expenditure $p_t^E i_t = i_t + f_t$ consists of a size-dependent component with unit price of one, and a fixed cost component

$$f_t = \frac{\xi w_t}{\phi q_t}. \tag{4.12}$$

It further follows that equilibrium tightness is constant

$$\theta_t = \frac{\phi}{1-\phi}. \tag{4.13}$$

Hence, lower matching efficiency m_t unambiguously lowers delivery probability q_t.

4.3. *Consumption good producers*

The economy consists of a fixed unit mass of perfectly competitive consumption good firms, indexed by j, that produce a homogeneous consumption good

$$y_{jt} = z_t x_{jt} k_{jt}^{\alpha} \ell_{jt}^{\nu}, \tag{4.14}$$

using firm-specific capital, k_{jt}, labor, ℓ_{jt}, and subject to aggregate productivity, z_t, and idiosyncratic productivity, x_{jt}. The production function has decreasing returns to scale in the control variables, $0 < \alpha + \nu < 1$. Aggregate productivity has a deterministic trend but throughout the paper, the model is formulated along the balanced growth path. Both idiosyncratic and aggregate productivity follow an AR(1) process

$$\log(z_t) = \rho^z \log(z_{t-1}) + \sigma^z \epsilon_t^z, \quad \epsilon_t^z \overset{iid}{\sim} \mathcal{N}(0,1), \tag{4.15}$$

$$\log(x_{jt}) = \rho^x \log(x_{jt-1}) + \sigma^x \epsilon_{jt}^x, \quad \epsilon_{jt}^x \overset{iid}{\sim} \mathcal{N}(0,1), \tag{4.16}$$

respectively. Labor adjustment is frictionless and I define gross cash flow as

$$cf_{jt} \equiv \max_{\ell_{jt} \in \mathbb{R}^+} \Big\{ z_t x_{jt} k_{jt}^{\alpha} \ell_{jt}^{\nu} - w_t l_{jt} \Big\}. \tag{4.17}$$

Capital adjustment is not frictionless. Firm-specific capital evolves over time according to $\gamma k_{jt+1} = (1-\delta) k_{jt} + i_{jt}$, where δ denotes the depreciation rate, i_{jt} is investment, and γ denotes constant, aggregate growth of labor productivity.

Let me detail the capital adjustment frictions. First, to invest, consumption good producers need to hire an engineering firm that searches for capital suppliers to supply the required capital goods. As a result of frictional capital markets, investment orders are not delivered instantaneously, but with probability q_t implying average time to build of $1/q_t$. Second, investment entails a fixed cost, f_t, see (4.12). The fixed cost depends on capital submarket ξ. Which submarket the consumption good producer can invest on is random and iid across firms and investment orders. Third, re-adjusting an outstanding order before delivery is prohibitively costly. Fourth, I assume resale losses of capital.[13]

In the dynamic firm problem, I distinguish between consumer good producers with and without outstanding orders. For firms without outstanding orders, the

[13] I assume reselling is also subject to time to build: Disinvesting producers need to hire an engineer that searches for a capital supplier that transforms the capital into consumption goods.

idiosyncratic state is described by $(k_{jt}, x_{jt}, \xi_{jt})$ with probability distribution μ^V defined for space $S^V = \mathbb{R}^+ \times \mathbb{R}^+ \times \mathbb{R}^+$. For firms with outstanding order, the idiosyncratic state consists of $(k_{jt}, i^o_{jt}, x_{jt}, \xi_{jt})$, where i^o_{jt} denotes the outstanding investment order. The probability distribution is μ^W defined for space is $S^W = \mathbb{R}^+ \times \mathbb{R} \times \mathbb{R}^+ \times \mathbb{R}^+$. The cross-sectional distribution of all consumption good firms over their idiosyncratic states is $\mu_t = (\mu^V_t, \mu^W_t)$ defined for $S = S^V \times S^W$. The economy's aggregate state is denoted by $\mathbf{s}_t = (\mu_t, z_t, m_t)$. In the following, I drop time and firm indices and use $'$ notation to indicate subsequent periods. The value of a firm without outstanding order is given by

$$V(k, x, \xi, \mathbf{s}) = \max \Big\{ V^A(k, x, \xi, \mathbf{s}), V^{NA}(k, x, \mathbf{s}) \Big\}. \tag{4.18}$$

Conditional on not ordering investment (not adjusting), the firm value is

$$V^{NA}(k, x, \mathbf{s}) = cf(k, x, \mathbf{s}) + \mathbb{E}\big[Q(\mathbf{s}, \mathbf{s}')V((1-\delta)k/\gamma, x', \xi', \mathbf{s}') | x, k, \mathbf{s}\big]. \tag{4.19}$$

Conditional on ordering investment (adjusting), the firm value is

$$V^A(k, x, \xi, \mathbf{s}) = \max_{i^o \in \mathbb{R}} \Big\{ W(k, i^o, x, \xi, \mathbf{s}) \Big\}, \tag{4.20}$$

The resale loss of divestment is expressed by the investment price function $p^i(i^o)$, which equals $0 \leq \bar{p}^i \leq 1$ if investment $i^o < 0$, and which equals one if investment is positive. Total investment expenditure is

$$ac(i^o, \xi, \mathbf{s}) = (1 - p^i(i^o))i^o + f(\xi, \mathbf{s}) \tag{4.21}$$

The value of the consumption good firm with outstanding orders is

$$\begin{aligned} W(k, i^o, x, \xi, \mathbf{s}) &= cf(k, x, \mathbf{s}) \\ &+ q(\mathbf{s})\Big[- ac(i^o, \xi, \mathbf{s}) + \mathbb{E}\big[Q(\mathbf{s}, \mathbf{s}')V\left(((1-\delta)k + i^o)/\gamma, x', \xi', \mathbf{s}'\right) | x, \mathbf{s}\big]\Big] \\ &+ (1 - q(\mathbf{s}))\Big[\mathbb{E}\big[Q(\mathbf{s}, \mathbf{s}')W\left((1-\delta)k/\gamma, i^o/\gamma, x', \xi, \mathbf{s}'\right) | k, i^o, x, \mathbf{s}\big]\Big]. \end{aligned} \tag{4.22}$$

The extensive margin of the capital adjustment decision is described by the threshold value $\hat{\xi}(k, x, \mathbf{s})$ that satisfies

$$V^A(k, x, \hat{\xi}(k, x, \mathbf{s}), \mathbf{s}) = V^{NA}(k, x, \mathbf{s}). \tag{4.23}$$

Adjustment is optimal whenever fixed adjustment costs $\xi < \hat{\xi}(k, x, \mathbf{s})$. Note that this formulation of the firm problem nests the conventional firm problem with one period time to build whenever $q(\mathbf{s}) = 1\ \forall \mathbf{s}$.

4.4. *Recursive Competitive Equilibrium (RCE)*

Before I define the equilibrium conditions, I define important macroeconomic aggregates. The aggregate production of the consumption good is

$$Y(\mathbf{s}) = \int_S zxk^\alpha \ell(k, x, \mathbf{s})^\nu \mu(d[k \times i^o \times x \times \xi]), \tag{4.24}$$

where $\ell(k, x, \mathbf{s})$ is the solution to (4.17). Aggregate investment expenditure is

$$\begin{aligned} I(\mathbf{s}) = &\int_{S^V} \mathbf{1}_{\{\xi < \hat{\xi}(k,x,\mathbf{s})\}} q(\mathbf{s}) ac(i^o(k, x, \mathbf{s}), \xi, \mathbf{s}) \mu^V(d[k \times x \times \xi]) \\ &+ \int_{S^W} q(\mathbf{s}) ac(i^o, \xi, \mathbf{s}) \mu^W(d[k \times i^o \times x \times \xi]). \end{aligned} \tag{4.25}$$

$\mathbf{1}_{\{\cdot\}}$ is an indicator function, that equals one if the argument is true and zero otherwise. I define aggregate order backlog as the total volume of investment orders at the beginning of the period, after new orders have been made

$$\begin{aligned} B(\mathbf{s}) = &\int_{S^V} \mathbf{1}_{\{\xi < \hat{\xi}(k,x,\mathbf{s})\}} ac(i^o(k, x, \mathbf{s}), \xi, \mathbf{s}) \mu^V(d[k \times x \times \xi]) \\ &+ \int_{S^W} ac(i^o, \xi, \mathbf{s}) \mu^W(d[k \times i^o \times x \times \xi]), \end{aligned} \tag{4.26}$$

A *RCE* is a list of functions $(w, f, q, \ell, i^o, \hat{\xi}, C, L, \Pi, Q, V, W, \mu')$ that satisfies:

(i) *Consumption good producers*: Labor demand ℓ, intensive and extensive margin investment demand $(i^o, \hat{\xi})$, and value function (V, W) solve (4.17)–(4.23).

(ii) *Engineering firms and capital good producers*: Capital prices f and delivery probability q satisfy (4.12) and (4.13).

(iii) *Household*: Consumption demand C and labor supply L solve (4.2).

(iv) *Consistency*:

(a) Π is consistent with profit maximization of consumption good firms.

(b) Q is given by (4.3).

(c) μ', the law of motion of μ, is consistent with functions $(q, i^o, \hat{\xi})$ describing capital adjustment.

(v) *Labor market clearing*: Labor supply L equals labor demand for consumption good production ℓ and labor demand for fixed costs of engineers and suppliers, described by $\hat{\xi}$ and G, the distribution of ξ.

(vi) *Goods market clearing*: $C = Y - I$, with Y and I given by (4.24) and (4.25).

4.5. ***Solution***

The recursive competitive equilibrium is not computable, because the solution depends on the infinite-dimensional distribution μ. Instead, I solve for an approximate equilibrium building on the algorithms in Campbell (1998) and Reiter (2009). The general idea is to use global approximation methods with respect to the individual states, but local approximation methods with respect to the aggregate states. I solve the steady state of my model using projection methods and perturb the model locally around the steady state to solve for the model dynamics in response to aggregate shocks.

Compared to the Krusell-Smith algorithm, see Krusell et al. (1998), the perturbation approach does not require simulating the model with respect to aggregate shocks (in order to update the parameters of the forecasting rules). Further it can easily handle a large number of aggregate shocks. Terry (2015) compares the Krusell-Smith algorithm with the Campbell-Reiter algorithm for a Khan and Thomas (2008) economy. He finds that the Campbell-Reiter algorithm is more than 100 times faster. Ahn et al. (2016) combine the Campbell-Reiter algorithm to compute aggregate dynamics for a general class of heterogeneous agent economies in continuous time. More closely related to this paper, Winberry (2016a) uses (and extends) the Campbell-Reiter algorithm to solve a variation of the Khan and Thomas (2008) economy.

My adaption of the Reiter method uses cubic B-splines with collocation to approximate the value functions. For the baseline calibration of the model, it takes one minute to solve the steady state, aggregate dynamics, and compute the impulse response functions. Appendix B contains the details of my solution method.

5. CALIBRATION

This section discusses the model calibration, which broadly follows the literature on non-convex capital adjustment frictions in general equilibrium model, see Khan and Thomas (2008) for example. I calibrate the model at quarterly frequency. I set the discount factor $\beta = 0.99$ to match an annual risk-free rate of 4%. I assume log-utility in consumption, $\sigma = 1$. The parameter governing the household's disutility from work, ψ, is calibrated to match one third of time spent working.

TABLE I

QUARTERLY MODEL CALIBRATION

Description	Parameter	Value
Households		
Discount factor	β	0.990
Intertemporal elasticity	σ	1.000
Preference for leisure	ψ	2.400
Engineers and capital suppliers		
Bargaining power	ϕ	0.500
Mean matching efficiency	μ^m	0.542
Persistence of matching efficiency	ρ^m	0.959
Dispersion of matching efficiency	σ^m	0.041
Consumption good producers		
Capital share	α	0.250
Labor share	ν	0.580
Depreciation rate	δ	0.025
Aggregate growth	γ	1.004
Idiosyncratic persistence	ρ^x	0.970
Idiosyncratic dispersion	σ^x	0.065
Aggregate persistence	ρ^z	0.950
Aggregate dispersion	σ^z	0.007
Fixed adjustment cost	$\bar{\xi}$	0.010
Resale loss	$\bar{p}^i$	0.830

The parameters that describe the technology of consumption good producers are set to $\alpha = 0.25$ and $\nu = 0.58$. These values are well within the range of estimates in Cooper and Haltiwanger (2006) and Kehrig (2015), and similar to the values assumed in Khan and Thomas (2008) and Bachmann et al. (2013).[14] I assume $\delta = 0.025$ consistent with an annual depreciation rate of 10%. Following Khan and Thomas (2008), I calibrate γ to an annualized aggregate labor productivity growth of 1.6%.

On capital markets, I assume symmetric Nash bargaining between engineers and suppliers, $\phi = 0.5$. This implies delivery probability $q_t = m_t$, which is independent

[14] Interpreting the production function as revenue production function derived in a model of monopolistic competition, the value for output elasticities would imply a markup of roughly 20%.

of η. To calibrate mean, persistence, and variance of matching efficiency, I target the corresponding first and second moments of the empirical baseline measure of time to build, the backlog ratio. To this end, I use (4.26) to compute aggregate order backlog. Since the delivery probability is state-independent and since shipments equal investment in the model, the backlog ratio in the model is $B_t/S_t = q_t$. I set the mean matching efficiency to satisfy an average time to build of 5.5 months corresponding to the mean of the backlog ratio. Given that the backlog ratio has a weak, non-linear time trend, I detrend the monthly time series using a HP filter with $\lambda = 8,100,000$ and fit persistence and standard deviations to the residual. This yields ρ^m and σ^m for the quarterly matching efficiency process.

I assume that G, the distribution of ξ, is uniform with lower bound zero and upper bound $\bar{\xi}$. To calibrate $\bar{\xi}$ and resale loss $\bar{p}^i$, I target the share of spike investment rates in micro data. Since the idiosyncratic productivity process, described by ρ^x and σ^x, also determines the investment rate distribution, it is key to calibrate these four parameters jointly using the same dataset. I use manufacturing establishment-level data from the Longitudinal Research Database. In particular, I use the estimates in Cooper and Haltiwanger (2006) based on revenue function $\tilde{x}k^\theta$, which I take as the production technology after maximizing out labor with $\theta = \alpha/(1-\nu)$. Given ν, I translate their estimates of the profitability process at annual frequency into the parameters describing the quarterly process of x, where $x = \tilde{x}^{1-\nu}$. This yields $\rho^x = 0.97$ and $\sigma^x = 0.065$. To calibrate adjustment cost parameters $\bar{\xi}$ and $\bar{p}^s$ I target the share of positive and negative spike adjusters, documented in Cooper and Haltiwanger (2006). The two model parameters can exactly match the 18.6% share of positive spikes and the 1.5% share of negative spikes. The fixed cost is important to generate fat tails, while the resale loss is particularly important in generating the large difference between positive and negative spikes. Appendix C provides more details and robustness on the calibration.

6. MACROECONOMIC EFFECTS OF SHOCKS TO MATCHING TECHNOLOGY

This section discusses the quantitative effects of shocks to the matching technology. In short, shock to the matching technology that raises time to build by one month lowers investment by 2 percent and output by 0.5 percent. These shocks explain up to one third of the decline in output and investment during the early 1990s recession and the 2007-09 Great Recession.

In more detail, Figure 3 shows the responses to an adverse shock to the matching technology. The shock increases time to build by exactly one month, which is roughly an increase by one standard deviation of the filtered time to build series. The shock causes substantial fluctuations in output, investment, and consumption. Investment is most directly affected by the match efficiency shocks. It falls by 2 percent on impact and remains strongly depressed during the first two years after the shock. Output falls by 0.3 percent on impact and reaches its trough of 0.55 percent five quarters later. Measured aggregate total factor productivity declines gradually and reaches its trough at 0.3 percent 5 quarters after the shock.

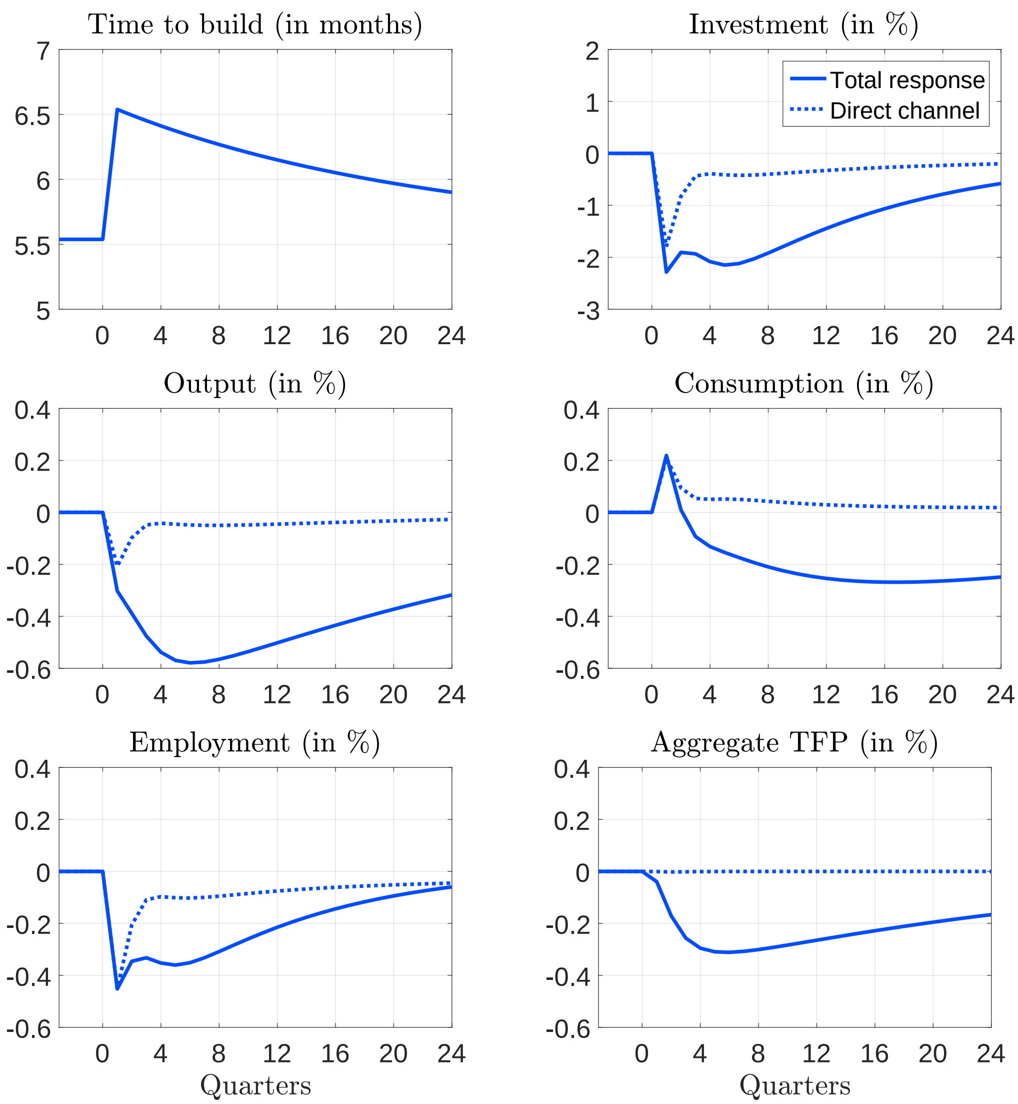
The aggregate effects of adverse shock to the matching technology are explained by a *direct* and an *indirect* channel. The direct channel captures that longer time to build delays delivery of outstanding investment orders and thus reduces investment and output. The indirect channel captures that longer time to build affects firm-level investment policies: firms invest less frequently and, if they invest, their investment reflect less their contemporaneous productivity. In turn, the alignment between firm-level capital and productivity weakens. Thus, longer time to build lowers measured aggregate total factor productivity. A more detailed discussion of the indirect channel is provided in Section 3.

To understand the relative quantitative importance of the two transmission channels, I suggest a simple exercise. While the indirect channel affects measured aggregate total factor productivity, the direct channel has no impact on measured productivity. To isolate the direct channel, I compute a series of exogenous shocks to aggregate productivity that exactly offset the effects on measured aggregate productivity. Measured aggregate productivity then remains at its steady state level. The effects of the direct channel are the macroeconomic responses to the joint occurrence of the initial match efficiency shock and the series of productivity shocks.

The resulting 'direct channel' responses are shown as dotted lines in Figure 3. Note that this exercise is an approximation, which gives an upward bias to the effects of the direct channel. The reason is that the series of productivity shock only offsets the effects on realized aggregate productivity, but not the effects on expected future aggregate productivity. Even so, the direct channel is only central to understand the immediate responses, while the medium-term effect is by and large explained by the indirect channel operating through capital misallocation. The prominent role of aggregate productivity in my model corresponds to the finding in Chari et al. (2007) on the efficiency wedge.

The direct channel is most important on impact of the shock because in subsequent

Figure 3: Responses to an adverse shock to the matching technology



Notes: The impulse response functions are for a shock to the matching technology that decreases time to build by one (unconditional) standard deviations starting from steady state and using the baseline calibration. 'Direct channel' denotes the impulse responses when aggregate TFP changes are eliminated through opposing aggregate productivity (z) shocks. Aggregate TFP is computed as $TFP = \log(Y_t) - \alpha \log(K_t) - \nu \log(L_t)$.

periods firms adjust their investment policies. Firms prepone investment orders as delivery takes longer, see Figure 7 in Appendix D. Abstracting from the on-impact effect, capital adjustment frequency falls consistent with wait-and-see behavior.

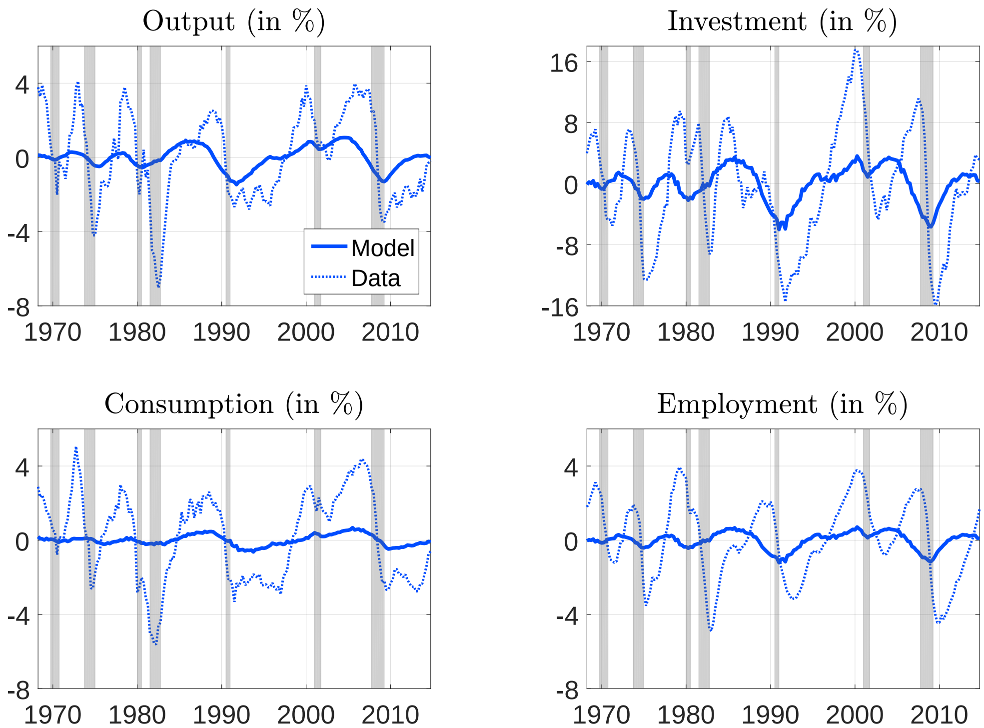
Note that I evaluate the quantitative impact of shocks to the matching technology in general equilibrium. Accounting for general equilibrium effects is important, because household consumption smoothing motives may substantially dampen the investment and output responses that would arise in partial equilibrium, see Khan and Thomas (2008). The initial increase in consumption reflects a general equilibrium mechanism. Since prices are flexible in my model, the intratemporal household optimality condition dictates that consumption has to increase initially in response to the initial decrease in investment, because the capital input in production is predetermined and labor demand falls.

As robustness to the results, I consider a model driven by an exogenous process for the delivery probability q_t. By contrast, shocks to the matching technology in my baseline model also have an effect on fixed capital adjustment costs f_t. Figure 9 in the Appendix shows that the responses are somewhat weaker, but overall, the effects of time to build fluctuations remain quantitatively important.

The responses in Figure 3 show quantitatively important and persistent effects of match efficiency shocks. Next, I assess the importance of time to build to understand past business cycles. To this end, I compute a matching technology shock series that fits the empirical time to build series. This confines my analysis to the period from 1968 through 2015. Using the model, I compute the time series for output, investment, consumption, and employment. To be clear, fluctuations in these series are only driven by shocks to the matching technology. To make the quarterly series comparable to the data, I HP filter both the simulated series and their empirical counterparts using the same low-frequency filter ($\lambda = 100{,}000$) I use in the calibration. More details on the empirical time series are provided in Section 7.

Figure 4 plots the model-implied time series against their empirical counterparts. Two observations stand out. First, the official recession periods (grey-shaded areas) are relatively well matched by periods where shocks to the matching technology induce below-trend output growth. However, the figure also reveals some phase shifts for the timing of expansions and contractions in aggregate production. This may not be surprising given that this paper does not claim shocks to the matching technology are the sole driver of business cycles and other business cycle shocks may follow distinct time patterns. Second, shocks to the matching technology explain an important share of the observed business cycle variations. These shocks alone explains a drop in investments of more than 5% during the Great Recession and the early 1990s recession, compared to a drop of 16% in the data. For output, the model also explains more than a third of the empirically observed drop during these two recessions and for consumption it is almost a quarter.

Figure 4: Role of time to build in understanding past business cycles



Notes: These time series are computed matching the empirically observed (filtered) movements in time to build through shocks to the matching technology and otherwise using the baseline model calibration. Grey-shaded areas indicate NBER recession dates.

Finally, Table V in Appendix D reports business cycle moments for both the empirical data and based on simulations of the model. The model generates autocorrelation in the detrended series for output, consumption, investment, and employment close to the empirical estimates. Further, the volatility of investment relative to output in the model is very similar to the data. The magnitudes of fluctuations generated by the model are between five and ten times lower than in the data. This reflects the observation that time to build exhibits large fluctuations only in two of the seven recessions for which data is available. Conversely, while shocks to the matching technology account for an important share of the early 1990 recession and the Great Recession, these shocks are less important for other recessions.

7. TIME SERIES EVIDENCE ON SHOCKS TO TIME TO BUILD

In this section, I assess the importance of structural time to build shocks using vector autoregressions. The identification requires few assumptions and I compare the identified shocks to the shocks to matching technology in my general equilibrium model. The qualitative effects of time to build shocks are similar to the effects of matching technology shocks, and the quantitative effects are even larger. In addition, the identified shocks are largely uncorrelated with various external measures of business cycles, which supports the notion that time to build is driven by an independent source of variation.

7.1. *Baseline model*

I estimate a medium-scale, eight-variable vector autoregression (VAR) that allows for rich dynamic interactions between the baseline time to build measure, see Section 2, and several macroeconomic series, prominent in both structural and empirical business cycle models. The vector of endogenous variables is:

$$Y = \begin{bmatrix} \text{Time to Build} \\ \text{Real GDP} \\ \text{Real Consumption} \\ \text{Real Investment} \\ \text{Consumer Prices} \\ \text{Real Wage} \\ \text{Federal Funds Rate} \\ \text{Labor Productivity} \end{bmatrix}$$

I use data at quarterly frequency that covers 1968Q1 through 2014Q4. All macroeconomic series except time to build are sourced from FRED.[15] All variables but the federal funds rate are transformed by the natural logarithm. Notice that I use nondurable consumption goods, because durable consumption goods include equipment goods that time to build shocks may directly affect. Similarly, my preferred investment time series is nonresidential investments because only firms invest in my model.

[15]The FRED series names are GDPC96 (*Real GDP*), DNDGRA3Q086SBEA (*Real Personal Consumption Expenditures: Nondurable goods*), B008RA3Q086SBEA (*Real Private Fixed Investment: Nonresidential*), CPI, AHETPI/CPI (*Average Hourly Earnings of Production and Nonsupervisory Employees: Total Private; deflated by CPI*), FEDFUNDS (*Effective Federal Funds Rate*), PAYEMS (*All Employees: Total Nonfarm Payrolls*). Labor productivity is real GDP over employment.

The results are robust against using total consumption and total investment instead.

The baseline structural VAR model is in levels with linear time trend (D)

$$Y_t = A_0 + Dt + \sum_{j=1}^{4} A_j Y_{t-j} + Bu_t, \quad \mathrm{Cov}(u_t) = \mathbb{I}_8, \quad \Sigma = \mathrm{Cov}(Bu_t) = BB', \tag{7.1}$$

where Bu_t denotes reduced-form shocks and u_t structural shocks. The covariance matrix of u_t is the identity matrix of dimension eight, $\mathbb{I}_8$. I assume the match efficiency shock is the last element in vector u_t. The structural impulse responses of Y_t to the match efficiency shock are identified by the last vector in B, denoted B_8.

7.2. *A conservative identification scheme*

The baseline identification assumption is that time to build increases in response to a structural time to build shock while all other macroeconomic time series do not respond contemporaneously, i.e. $B_8 = [0, \dots, 0, b_{88}]'$. Combining this identification restriction with $BB' = \Sigma$, it follows that $b_{88} = \sqrt{e_8' \Sigma^{-1} e_8}$, where e_i is the i-th column of $\mathbb{I}_8$. B_8 is point-identified by the identification restriction.

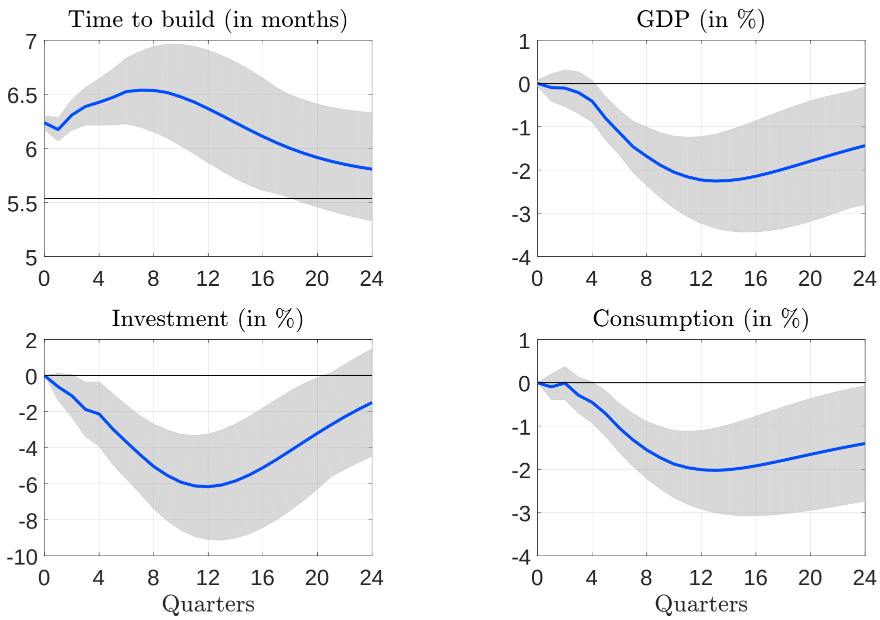
This identification scheme is conservative in the following sense. Except for the time to build shock, all structural shocks may affect time to build contemporaneously, while the time to build shock may affect all variables except time to build only through a one-quarter lag.[16] The identification is also conservative relative to the general equilibrium model where all variables are contemporaneously affected by shocks to the matching technology. I also reassess the importance of time to build shocks using a model-consistent identification scheme.

Figure 5 shows the impulse responses to a positive time to build shock that raises time to build by one month at peak. I have chosen the size of the shock to mimic the exercise in the general equilibrium model. The magnitude of the shock corresponds to five standard deviations of the time to build shock. The shock has a persistent, significant effect on time to build. More interestingly, GDP and its two main components, investment and consumption, significantly fall in response to the match efficiency shock. Not only are the responses statistically significant, but their magnitudes are also economically relevant: GDP and consumption fall by up to 2%, and investment by up to 6% within the first three years.

To get a sense of the role of time to build shocks to explain variation in macroe-

[16] The identification strategy resembles Christiano et al. (2005) for monetary policy shocks.

Figure 5: Impulse responses to a *one month* time to build shock



Notes: Solid, blue lines show (selected) responses to a match efficiency shock, under the baseline identification scheme. Shaded, gray areas illustrate the associated 90% confidence intervals.

TABLE II

Forecast error variance decomposition

	1 year	*2 years*	*3 years*	*4 years*	*5 years*	∞
GDP	0.2	7.6	18.1	22.6	23.4	18.2
Investment	0.3	0.9	2.8	4.9	6.6	6.7
Consumption	0.8	9.8	22.2	26.9	28.2	24.6
Time to build	73.4	57.0	48.8	44.8	42.2	31.1

Note: The shares of forecast error variance explained by time to build shocks are expressed as percentages for different forecast horizons ranging from 1 year to infinity.

conomic, Table II shows the shares of forecast error variance explained by time to build shocks. Albeit conservatively identified, the time to build shock explains an important fraction of macroeconomic fluctuations: more than 20% of GDP and consumption, and 7% of investment. This provides further evidence in support of this paper's suggestion that time to build fluctuations are important for a better understanding of business cycle fluctuations. Importantly, at business cycle frequency the time to build shock explains almost 50% of the forecast error variance of time to build itself. That is, other structural shocks explain only 50%. This supports the modeling choice of the general equilibrium model, in which time to build is directly driven by a shock to the matching technology, and not by a conventional business cycle shock, such as a shock in aggregate productivity.

The importance of time to build shocks could potentially reflect other structural shocks that are not well identified in my model. To address this concern, I correlate my identified time to build shock series with various business cycles shocks, constructed in a number of papers outside my empirical framework. These business cycles shocks include direct estimates of productivity shocks and numerous policy shocks. Table III provides the correlation of the time to build shock series with leads and lags of the external business cycle shock series. By and large, I find time to build shocks uncorrelated with external shocks. This finding further supports to the importance of exogenous shocks to time to build.

TABLE III

CORRELOGRAM OF TIME TO BUILD SHOCKS WITH EXTERNAL BUSINESS CYCLE SHOCKS

	quarterly lags/leads								
	-4	*-3*	*-2*	*-1*	*0*	*+1*	*+2*	*+3*	*+4*
TFP	-0.07	-0.05	0.00	0.00	-0.03	-0.04	-0.07	-0.08	0.00
UA-TFP	-0.09	-0.13*	0.04	0.07	-0.05	-0.06	-0.04	0.00	0.07
UA-TFP-I	-0.03	-0.15**	0.02	0.11	0.03	-0.03	0.03	0.00	0.05
UA-TFP-C	-0.10	-0.08	0.04	0.03	-0.09	-0.07	-0.07	0.00	0.05
MP	0.02	0.08	0.04	0.02	-0.01	0.06	0.04	0.09	0.11
Oil	-0.01	0.00	-0.02	-0.02	0.01	0.00	0.01	0.09	-0.04
Defense	-0.12	-0.15**	-0.02	-0.03	-0.16**	-0.04	-0.08	-0.01	-0.10
Tax	0.02	-0.06	0.02	0.04	0.01	-0.13	0.09	0.04	-0.02

Note: The table shows the correlation of time to build shocks with various shock series at lags/forwards between -4 and +4 quarters. */**/*** denote 10%/5%/1% significance levels, respectively. Productivity shock series are from Fernald (2014): TFP, Utilization-Adjusted (UA) TFP, UA-TFP in equipment and durables, and UA-TFP in non-durables. Monetary policy shocks (MP) are based on Romer and Romer (2004) and Coibion (2012). Oil price shocks are based on Ramey and Vine (2010). Surprise defense expenditures as fiscal shocks are from Ramey and Shapiro (1998), and tax shocks from Mertens and Ravn (2011).

The identified time to build shock series appears not to reflect investment-specific productivity shocks along the lines of Justiniano et al. (2010) and Justiniano et al. (2010). Beyond the evidence in Table III, this conclusion is supported by the finding that extending my VAR model by the relative price of investment goods only marginally affects the results presented here. By the same argument, identified time to build shocks appear not to reflect uncertainty shocks. The identified shocks further do not appear to reflect changes in aggregate financial conditions. This conclusion is based on the following result. The VAR exercise in Gilchrist et al. (2014) finds that uncertainty shocks are crucially transmitted through credit spreads. When replacing uncertainty by time to build, I do not find evidence for the transmission of time to build shocks through credit spreads.

7.3. ***Robustness***

Appendix E provides robustness for the empirical results. First, I evaluate the importance of the linear time trend assumption by estimating the VAR under the same identification restrictions but expressing all variables in first differences. The results are broadly robust. Within the first three years, GDP, investment, and consumption respond significantly to a time to build shock, and the magnitudes are similar to the baseline model in levels. Second, I compare the role of my identification scheme by suggesting an alternative identification scheme, in which time to build shocks can affect all variables contemporaneously, but no other structural shock can affect time to build contemporaneously. Importantly, this restriction is consistent with the restrictions imposed in the general equilibrium model. The responses to a time to build shock tend to be stronger under the alternative restriction, albeit the differences are small. Third, I suggest a new robustness for frequentist, point-identified structural VARs. Based on the findings in Gafarov, Meier, and Montiel Olea (2016), I replace zero restrictions by elasticity bounds. To provide robustness for time to build shocks, I replace the contemporaneous zero restrictions of the baseline restriction by constraining the elasticity of the contemporaneous response of variables other than time to build to be bounded by $\pm 1\%$. I do find my baseline results to be robust against such relaxation of identification restrictions.

Beyond the robustness in Appendix E, the results are also robust against estimating a monthly VAR, in which I replace GDP by IP and investment by new orders for non-defense capital goods. Further, the results are not solely driven by the Great Recession period. The VAR results are robust against cutting the sample from 2008.

8. CONCLUSION

This paper contributes to our understanding of business cycles by addressing a novel question: What are the business cycle implications of fluctuations in time to build? To address this question, I develop a dynamic stochastic general equilibrium model, in which capital good markets are characterized by search frictions. Fluctuations in time to build are driven by shocks to the matching technology. Calibrating the model to US data, I show that the empirically observed fluctuations in time to build are quantitatively of first-order importance for business cycles. Of particular quantitative importance is the interaction of time to build and firm investment policies leading to capital misallocation. To corroborate the model-implied results, I provide time series evidence on the importance of structural time to build shocks. I find that the effects of time to build shocks are even stronger than in the structural model.

An important follow-up question is to better understand the micro-foundations behind fluctuations in time to build. In particular, it may be useful to study capital good supply networks. Small changes at critical points in such networks, for example the exit of an important supplier, could have non-trivial aggregate implications for time to build. A complementary explanation may revolve around trade credit. While the empirical evidence rejects an important role for aggregate financial conditions, trade credit in capital good production networks might be important to understand the observed time to build fluctuations. For example, suppose capital suppliers produce subject to cash-in-advance constraints. During recessions short-run liquidity in the form of trade credit may become scarce. As a result, suppliers may need to slow down production despite long order books.

The long-run time series pattern of time to build shows that it has become more volatile since the mid-1980s. In fact, this coincides with the Great Moderation period from the mid-1980s until 2007. The Great Moderation is characterized by less volatile business cycles. One popular explanation for the decline in volatility is 'just-in-time' inventory practices, which mitigates inventory volatility. Possibly, the flip side of lower inventory volatility is larger volatility in order backlog, and thus time to build.

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Appendix

APPENDIX A: TIME TO BUILD FLUCTUATIONS

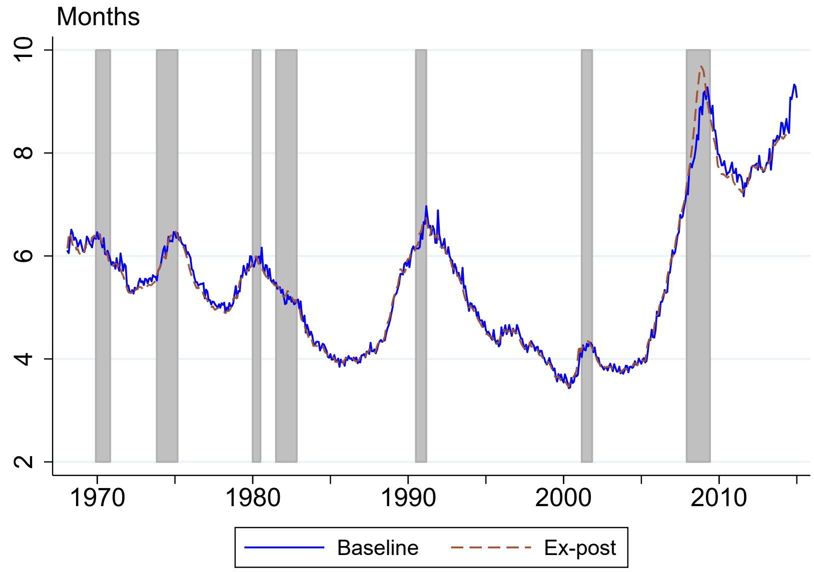
My *ex-post* measure of time to build captures the time which new orders remain in the capital good producers' order books using ex-post realizations of shipments (instead of current shipments). To be precise, I compute the lowest number of future periods required to deplete the given order backlog

$$\widetilde{TTB}_t \equiv \min_{\tau} \left(\sum_{j=1}^{\lfloor \tau \rfloor} S_{t+j} + (\tau - \lfloor \tau \rfloor)(S_{t+\lfloor \tau \rfloor + 1} - S_{t+\lfloor \tau \rfloor}) - B_t \right)^2,$$

where $\lfloor \cdot \rfloor$ denotes the floor function. The second term in above formula captures a linear interpolation of shipments between two periods, by which the ex-post time to build measure becomes continuous.

Figure 6 compares my baseline measure with the ex-post measure of time to build. Differences between the two series are barely visible, which mainly reflects the high auto-correlation of monthly shipments.

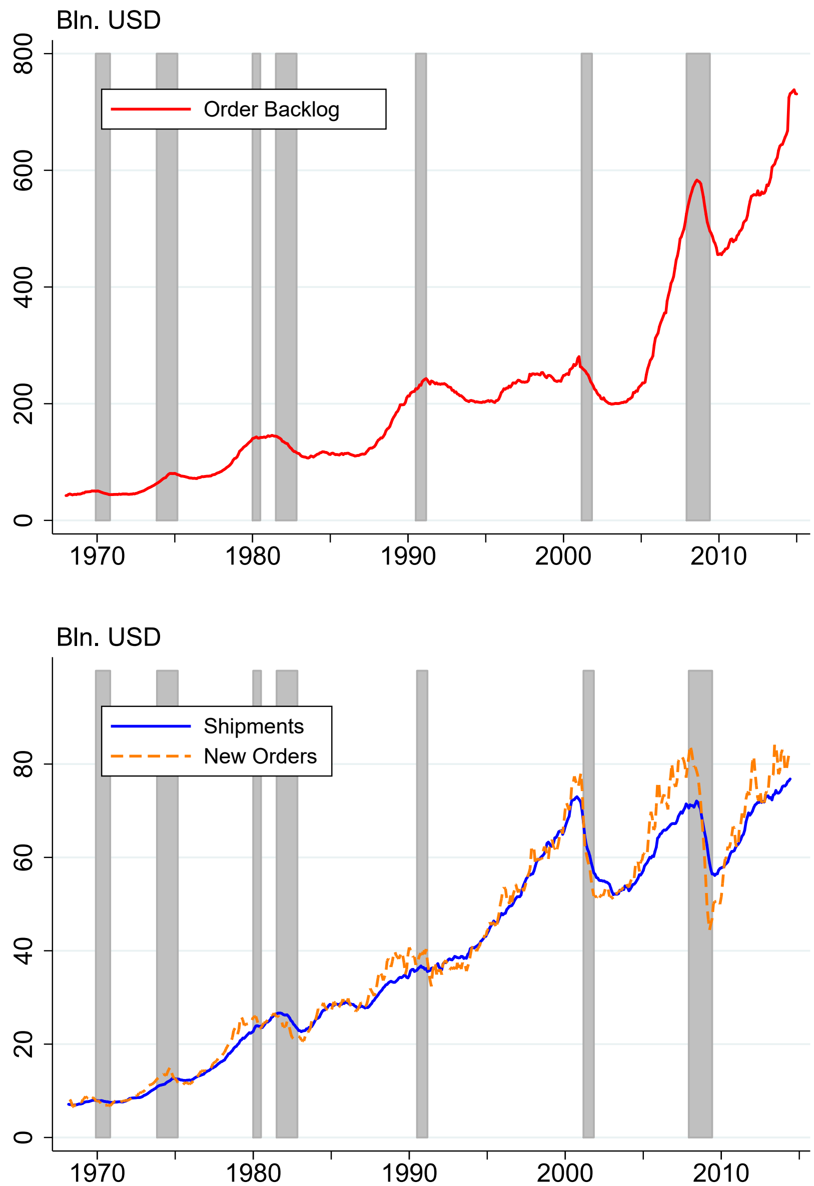
Figure 6: Time to Build



Notes: Time to build is measured as the ratio of order backlog to monthly shipments, for non-defense equipment goods. Shaded, gray areas indicate NBER recession dates.

The two panels of Figure 7 show the individual series defining the order stock-flow equation. The series are plotted in nominal values because the stock-flow equation is defined over nominal values.

Figure 7: Responses of investment orders to an adverse match efficiency shock



Notes: The time series for order backlog, shipments, and new orders refer to the non-defense equipment goods sector and are expressed in nominal values. Shaded, gray areas indicate NBER recession dates.

APPENDIX B: SOLUTION ALGORITHM

B.1. *Simplified consumption good firm problem*

To solve the model most efficiently, I rewrite the firm problem. First, I transform the firm problem. Instead of i^o, the investment order, I let firms choose k^o, the new capital stock upon delivery. Computationally, this transformation has the advantage that I can use the same grid for k^o as for k, and this grid can be tighter than the one for i^o. To leave the firm problem unchanged, k^o needs to evolve over time to guarantee the implicitly defined investment order satisfies $i^{o\prime} = \frac{i^o}{\gamma}$. Using the identity, $i^o = \gamma k^o + (1-\delta)k$, the evolution of k^o over time (conditional on no delivery) according to $k^{o\prime} = \frac{k^o}{\gamma} - \frac{\delta(1-\delta)k}{\gamma^2}$. Second, in slight abuse of notation, I drop the aggregate state $\mathbf{s}$ and instead use time subscripts for functions that depend on the aggregate state. I express the firm value functions in utils, see Khan and Thomas (2008), and redefine the value function such that the expectation with respect to idiosyncratic productivity does not have to be computed within the maximization problem. This raises computational efficiency and it tends to smooth the value functions. More precisely, I define $\tilde{V}_t(k,x,\xi) = p_t \mathbb{E}_x \mathbb{E}_\xi V(k,x',\xi')$, $\tilde{V}_t^A(k,x,\xi) = p_t V_t^A(k,x,\xi)$, $\tilde{V}_t^{NA}(k,x) = p_t V^{NA}(k,x)$, $\tilde{W}_t(k,x,\xi) = \mathbb{E}_x \bar{W}_t(k,x',\xi)$, $\bar{W}_t(k,x,\xi) = p_t W_t(k,x,\xi)$, where $\mathbb{E}_x$ ($\mathbb{E}_\xi$) denotes the expectation with respect to x' (ξ') conditional on x (ξ) and $p_t = C_t^{-\sigma}$ as before. Then equations (4.19), (4.18), (4.20), and (4.22) can be rewritten as:

$$
\begin{aligned}
\tilde{V}_t(k,x,\xi) &= \mathbb{E}_x \mathbb{E}_\xi \max\left\{\tilde{V}_t^A(k,x',\xi'), \tilde{V}_t^{NA}(k,x')\right\} \\
\tilde{V}_t^{NA}(k,x) &= p_t c f_t(k,x) + \beta \mathbb{E}_t\left[\tilde{V}_{t+1}((1-\delta)k/\gamma, x, \xi)\right] \\
\tilde{V}_t^A(k,x,\xi) &= \max_{k_t^o \in \mathbb{R}^+}\left\{\bar{W}_t(k,k_t^o,x,\xi)\right\} \\
\bar{W}_t(k,k^o,x,\xi) &= p_t c f_t(k,x) \\
&\quad + q_t\Big[-p_t[(1-p^i(k,k^o))(\gamma k^o - (1-\delta)k) + f_t^E(\xi)] + \beta \mathbb{E}_t\left[\tilde{V}_{t+1}(k^o,x,\xi)\right]\Big] \\
&\quad + (1-q_t)\Big[\beta \mathbb{E}_t\left[\tilde{W}_{t+1}\left((1-\delta)k/\gamma, k^o/\gamma - \delta(1-\delta)k/\gamma^2, x, \xi\right)\right]\Big] \\
\tilde{W}_t(k,k^o,x,\xi) &= \mathbb{E}_x \bar{W}_t(k,x',\xi)
\end{aligned}
$$

where $\mathbb{E}_t$ denotes the expectation with respect to aggregate state $\mathbf{s}_{t+1}$ conditional on $\mathbf{s}_t$. The net present value of the fixed adjustment cost can be expressed by $fac_t\xi$, where fac_t is defined recursively

$$
fac_t = q_t p_t \frac{w_t}{\phi q_t} + (1-q_t)\beta \mathbb{E}_t fac_{t+1}.
$$

In turn, this allows me to simplify the firm problem as

$$
\begin{aligned}
\tilde{V}_t(k,x) &= \mathbb{E}_x \mathbb{E}_\xi \max \left\{ \tilde{V}_t^A(k,x') - fac_t \xi', \tilde{V}_t^{NA}(k,x') \right\} \\
fac_t &= q_t p_t \frac{w_t}{\phi q_t} + (1-q_t)\beta \mathbb{E}_t fac_{t+1} \\
\tilde{V}_t^{NA}(k,x) &= p_t cf_t(k,x) + \beta \mathbb{E}_t \left[\tilde{V}_{t+1}((1-\delta)k/\gamma, x) \right] \\
\tilde{V}_t^A(k,x) &= \max_{k_t^o \in \mathbb{R}^+} \left\{ \bar{W}_t(k, k_t^o, x) \right\} \\
\bar{W}_t(k,k^o,x) &= p_t cf_t(k,x) \\
&\quad + q_t \Big[-p_t(1-p^i(k,k^o))(\gamma k^o - (1-\delta)k) + \beta \mathbb{E}_t \left[\tilde{V}_{t+1}\left(k^o, x\right) \right] \Big] \\
&\quad + (1-q_t) \Big[\beta \mathbb{E}_t \left[\tilde{W}_{t+1}\left((1-\delta)k/\gamma, k^o/\gamma - \delta(1-\delta)k/\gamma^2, x \right) \right] \Big] \\
\tilde{W}_t(k,k^o,x) &= \mathbb{E}_x \bar{W}_t(k,x')
\end{aligned}
$$

Importantly, this allows me to compute the extensive margin adjustment policy in closed form,

$$
\hat{\xi}_t = \frac{\tilde{V}_t^A(k,x') - \tilde{V}_t^{NA}(k,x')}{fac_t}.
$$

Next, I approximate firm values using collocation where Φ denotes basis functions in matrix representation and c denotes vectors of coefficients

$$
\begin{aligned}
\tilde{V}_t(k,x) &\simeq \Phi^V(k,x) c_t^V \\
\tilde{W}_t(k,k^o,x) &\simeq \Phi^W(k,k^o,x) c_t^W
\end{aligned}
$$

The approximations are exact at the n_k collocation nodes $k_1, ..., k_{n_k}$ and $k_1^o, ..., k_{n_k}^o$. In practice, I choose the same collocation nodes for k and k^o.

As baseline we use cubic B-splines to approximate the firm value functions. This does not only have the advantage of being computationally fast, but also conditional on the coefficients we know the Jacobian in closed form. In particular, I can write down the optimality condition for intensive margin capital adjustment (k_t^o) as

$$
q_t p_t p^s(k, k_t^o)\gamma = q_t \beta \mathbb{E}_t \Phi_k^V(k_t^o, x) c_{t+1}^V + (1-q_t)\beta \mathbb{E}_t \Phi_{k^o}^W((1-\delta)k_t/\gamma, k_t^o, x) c_{t+1}^W,
$$

where $\Phi_k^V = (\partial \Phi^V)/(\partial k)$ and $\Phi_{k^o}^W = (\partial \Phi^W)/(\partial k^o)$.

I approximate the AR(1) process of idiosyncratic productivity using Tauchen's algorithm. I denote the discrete grid points of x by $x_1, ..., x_{n_x}$ consisting of n_x grid points and the transition probability from state x_j to state $x_{j'}$ one period later by $\pi_x(x_{j'}|x_j)$.

To render the infinite-dimensional distribution μ_t tractable, I approximate it with a discrete histogram. That is, μ_t measures the share of firms for each discrete combination of capital stock k_{i_1}, outstanding order $k_{i_2}^o$ (both correspond to the collocation nodes), and productivity x_j. A further distinction is useful: Let μ_t^V denote the cross-sectional distribution of firms without outstanding orders over idiosyncratic states (k_i, x_j) and μ_t^W the distribution of firms with outstanding orders over $(k_{i_1}, k_{i_2}^o, x_j)$. It holds that $\mu_t = (\mu_t^V, \mu_t^W)$.

B.2. *Campbell-Reiter algorithm*

Using the preceding approximation and simplification steps, the model equilibrium is described by the following non-linear equations:

$$\Phi^V(k,x)c_t^V = \mathbb{E}_x\mathbb{E}_\xi \max\left\{\tilde{V}_t^A(k,x') - fac_t\xi', \tilde{V}_t^{NA}(k,x')\right\} \tag{B.1}$$

$$\hat{\xi}_t(k,x) = (\tilde{V}_t^A(k,x) - \tilde{V}_t^{NA}(k,x))/fac_t$$

$$\tilde{V}_t^{NA}(k,x) = p_t cf_t(k,x) + \beta\mathbb{E}_t\Phi^V((1-\delta)k/\gamma, x)c_{t+1}^V$$

$$\tilde{V}_t^A(k,x) = \bar{W}_t(k,k_t^o,x)$$

$$\begin{aligned}\bar{W}_t(k,k^o,x) = p_t cf_t(k,x) &\\ &+ q_t\Big[-p_t(1-p^i(k,k^o))(\gamma k^o - (1-\delta)k) + \beta\mathbb{E}_t\Phi^V(k^o,x)c_{t+1}^V\Big] \\ &+ (1-q_t)\Big[\beta\mathbb{E}_t\Phi^W((1-\delta)k/\gamma, k^o/\gamma - \delta(1-\delta)k/\gamma^2, x)c_{t+1}^W\Big]\end{aligned}$$

$$cf_t(k_t,x_t) = (1-\nu)\,(\nu/w_t)^{\nu/(1-\nu)}\,(z_t x_t)^{1/(1-\nu)} k_t^{\alpha/(1-\nu)}$$

$$w_t = \psi/p_t$$

$$q_t = m_t(\phi/(1-\phi))^{\eta-1}$$

$$\Phi^W(k,k^o,x)c_t^W = \mathbb{E}_x\bar{W}_t(k,x') \tag{B.2}$$

$$fac_t = q_t p_t \frac{w_t}{\phi q_t} + (1-q_t)\beta\mathbb{E}_t fac_{t+1} \tag{B.3}$$

$$q_t p_t p^s(k,k_t^o)\gamma = q_t\beta\mathbb{E}_t\Phi_k^V(k_t^o,x)c_{t+1}^V + (1-q_t)\beta\mathbb{E}_t\Phi_{k^o}^W((1-\delta)k_t/\gamma, k_t^o, x)c_{t+1}^W \tag{B.4}$$

$$\frac{1}{p_t} = Y_t - I_t \tag{B.5}$$

$$Y_t = \sum_{i_1,i_2,j}\mu_t(k_{i_1},k_{i_2},x_j)\,(\nu/w_t)^{\nu/(1-\nu)}\,(z_t x_j)^{1/(1-\nu)} k_{i_1}^{\alpha/(1-\nu)}$$

$$\begin{aligned}I_t = &\sum_{i,j}\mu_t^V(k_i,x_j)G(\hat{\xi}_t(k_i,x_j))q_t p^s(k_i,k_t^o(x_j))\,[\gamma k_t^o(x_j) - (1-\delta)k_i] \\ &+ \sum_{i_1,i_2,j}\mu_t^W(k_{i_1},k_{i_2}^o,x_j)q_t p^s(k_{i_1},k_{i_2}^o)\,[\gamma k_{i_2}^o - (1-\delta)k_{i_1}]\end{aligned}$$

$$\begin{aligned}\mu_{t+1}^V(k_{i'},x_{j'}) = &\sum_{i,j}\pi_x(x_{j'}|x_j)\mu_t^V(k_i,x_j)[\omega_t^{V,V,A}(i,i',j) + \omega_t^{V,V,NA}(i,i',j)] \\ &+ \sum_{i_1,i_2,j}\pi_x(x_{j'}|x_j)q_t\mu_t^W(k_{i_1},k_{i_2}^o,x_j)\omega_t^{W,V}(i_1,i_2,i',j)\end{aligned} \tag{B.6}$$

$$\begin{aligned}\mu_{t+1}^W(k_{i_1'},k_{i_2'},x_{j'}) = &\sum_{i,j}\pi_x(x_{j'}|x_j)\mu_t^V(k_i,x_j)\omega_t^{V,W}(i,i_1',i_2',j) \\ &+ \sum_{i_1,i_2,j}\pi_x(x_{j'}|x_j)\mu_t^W(k_{i_1},k_{i_2},x_j)\omega_t^{W,W}(i_1,i_2,i_1',i_2',j)\end{aligned} \tag{B.7}$$

$$\log(m_{t+1}) = (1-\rho^m)\log(\mu^m) + \rho^m\log(m_t) \tag{B.8}$$

$$\log(z_{t+1}) = \rho^z\log(z_t) \tag{B.9}$$

With the following auxiliary equations for the law of motion of the distribution:

$$\omega_t^{V,V,A}(i,i',j)=\begin{cases} G(\hat{\xi}_t(k_i,x_j))q_t\frac{k_{i'}-k_t^o(x_j)}{k_{i'}-k_{i'-1}} & \text{if} \quad k_t^o(x_j)\in[k_{i'-1},k_{i'}] \\ G(\hat{\xi}_t(k_i,x_j))q_t\frac{k_t^o(x_j)-k_{i'}}{k_{i'+1}-k_{i'}} & \text{if} \quad k_t^o(x_j)\in[k_{i'},k_{i'+1}] \\ 0 & \text{else} \end{cases}$$

$$\omega_t^{V,V,NA}(i,i',j)=\begin{cases} [1-G(\hat{\xi}_t(k_i,x_j))]\frac{k_{i'}-(1-\delta)k_i/\gamma}{k_{i'}-k_{i'-1}} & \text{if} \quad (1-\delta)k_i/\gamma\in[k_{i'-1},k_{i'}] \\ [1-G(\hat{\xi}_t(k_i,x_j))]\frac{(1-\delta)k_i/\gamma-k_{i'}}{k_{i'+1}-k_{i'}} & \text{if} \quad (1-\delta)k_i/\gamma\in[k_{i'},k_{i'+1}] \\ 0 & \text{else} \end{cases}$$

$$\omega_t^{V,W}(i,i_1',i_2',j)=\begin{cases} G(\hat{\xi}_t(k_i,x_j))(1-q_t)\frac{k_{i_1'}-(1-\delta)k_i/\gamma}{k_{i_1'}-k_{i_1'-1}}\frac{k_{i_2'}-k_t^o(x_j)}{k_{i_2'}-k_{i_2'-1}} \\ \qquad \text{if} \quad k_t^o(x_j)\in[k_{i_2'-1},k_{i_2'}] \quad \text{and} \quad (1-\delta)k_i/\gamma\in[k_{i'-1},k_{i'}] \\ G(\hat{\xi}_t(k_i,x_j))(1-q_t)\frac{(1-\delta)k_i/\gamma-k_{i_1'}}{k_{i_1'+1}-k_{i_1'}}\frac{k_{i_2'}-k_t^o(x_j)}{k_{i_2'}-k_{i_2'-1}} \\ \qquad \text{if} \quad k_t^o(x_j)\in[k_{i_2'-1},k_{i_2'}] \quad \text{and} \quad (1-\delta)k_i/\gamma\in[k_{i'},k_{i'+1}] \\ G(\hat{\xi}_t(k_i,x_j))(1-q_t)\frac{k_{i_1'}-(1-\delta)k_i/\gamma}{k_{i_1'}-k_{i_1'-1}}\frac{k_t^o(x_j)-k_{i_2'}}{k_{i_2'+1}-k_{i_2'}} \\ \qquad \text{if} \quad k_t^o(x_j)\in[k_{i_2'},k_{i_2'+1}] \quad \text{and} \quad (1-\delta)k_i/\gamma\in[k_{i'-1},k_{i'}] \\ G(\hat{\xi}_t(k_i,x_j))(1-q_t)\frac{(1-\delta)k_i/\gamma-k_{i_1'}}{k_{i_1'+1}-k_{i_1'}}\frac{k_t^o(x_j)-k_{i_2'}}{k_{i_2'+1}-k_{i_2'}} \\ \qquad \text{if} \quad k_t^o(x_j)\in[k_{i_2'},k_{i_2'+1}] \quad \text{and} \quad (1-\delta)k_i/\gamma\in[k_{i'},k_{i'+1}] \\ 0 \quad \text{else} \end{cases}$$

$$\omega_t^{W,V}(i_1,i_2,i',j)=\begin{cases} q_t\frac{k_{i'}-k_{i_2}}{k_{i'}-k_{i'-1}} & \text{if} \quad k_{i_2}\in[k_{i'-1},k_{i'}] \\ q_t\frac{k_{i_2}-k_{i'}}{k_{i'+1}-k_{i'}} & \text{if} \quad k_{i_2}\in[k_{i'},k_{i'+1}] \\ 0 & \text{else} \end{cases}$$

$$\omega_t^{W,W}(i_1,i_2,i_1',i_2',j)=\begin{cases} (1-q_t)\frac{k_{i_1'}-(1-\delta)k_{i_1}/\gamma}{k_{i'}-k_{i_1'-1}} & \text{if} \quad (1-\delta)k_{i_1}/\gamma\in[k_{i_1'-1},k_{i_1'}] \quad \text{and} \quad i_{2'}=i_2 \\ (1-q_t)\frac{(1-\delta)k_{i_1}/\gamma-k_{i_1'}}{k_{i_1'+1}-k_{i_1'}} & \text{if} \quad (1-\delta)k_{i_1}/\gamma\in[k_{i_1'},k_{i_1'+1}] \quad \text{and} \quad i_{2'}=i_2 \\ 0 & \text{else} \end{cases}$$

Labeled equations (B.1)–(B.9) are the main equations, and all other unlabeled equations are auxiliary in defining the model equilibrium. Given n_k collocation nodes and n_x discrete grid points of x, equations (B.1)–(B.9) are $n_f = 2n_k^2 n_x + 3n_k n_x + 4$. I organize these equations in

$$\text{(B.10)} \qquad \mathbb{E}_t[f(\mathbf{x}_t,\mathbf{x}_{t+1},\mathbf{y}_t,\mathbf{y}_{t+1})]=0,$$

where $\epsilon_t = (\epsilon_t^m, \epsilon_t^z) \in \mathbb{R}^2$ denotes the vector of aggregate shocks. $\mathbf{x}_t$ denotes predetermined state variables and $\mathbf{y}_t$ denotes non-predetermined state variables

$$\text{(B.11)} \qquad \mathbf{x}_t=[\mu_t;\log(m_t);\log(z_t)]\in\mathbb{R}^{n_x\equiv n_k^2 n_x+n_k n_x+2},$$

$$\text{(B.12)} \qquad \mathbf{y}_t=[c_t^V;c_t^W;\log(ac_t);\log(k_t^o);\log(p_t)]\in\mathbb{R}^{n_y\equiv n_k^2 n_x+2n_k n_x+2}.$$

The non-stochastic steady state is defined as $f(\bar{\mathbf{x}},\bar{\mathbf{x}},\bar{\mathbf{y}},\bar{\mathbf{y}}) = 0$. In the general case, the model solution is given by

$$\mathbf{y}_t = g(\mathbf{x}_t, \zeta), \tag{B.13}$$

$$\mathbf{x}_{t+1} = h(\mathbf{x}_t, \zeta) + \zeta\tilde{\sigma}\epsilon_{t+1}, \tag{B.14}$$

where ζ is the perturbation parameter and $g : \mathbb{R}^{n_x} \times \mathbb{R}^+ \to \mathbb{R}^{n_y}$ and $f : \mathbb{R}^{n_x} \times \mathbb{R}^+ \to \mathbb{R}^{n_x}$. The exogenous shocks are collected in $\epsilon_{t+1} \in \mathbb{R}^{n_\epsilon}$, and $\tilde{\sigma} \in \mathbb{R}^{n_x \times n_\epsilon}$ attributes shocks to the right equations while also scaling them (by σ^m, σ^z). To solve the two policy functions, I use a first-order approximation. I follow the perturbation algorithm in Schmitt-Grohe and Uribe (2004). This requires to compute the Jacobians of function f (locally) at steady state. Importantly, the algorithm in Schmitt-Grohe and Uribe (2004) checks for existence and uniqueness of a model solution.

B.3. *Krusell-Smith algorithm*

This subsection suggests how the model can be solved using the Krusell-Smith algorithm. Following Krusell et al. (1998), and the adaption for heterogeneous firms by Khan and Thomas (2008), I assume agents in my model only observe a finite set of moments, informative about the entire distribution, instead of observing μ directly. The agents approximate equilibrium prices and the evolution of the observed moments by a log-linear rule.

I approximate the distribution μ by the aggregate capital stock,

$$K_t = \int_S k d\mu, \tag{B.15}$$

and the stock of investments outstanding from the preceding period

$$I^o_t = \int_{S^W} (\gamma k^o - (1-\delta)k) d\mu^W. \tag{B.16}$$

If time-to-build dropped to zero $q = 1$, I^o_t would constitute the investments activated in addition to new orders. I suggest the following log-linear forecast rules

$$\log K_{t+1} = \beta^0_k(z_t, m_t) + \beta^1_k(z_t, m_t)\log K_t + \beta^2_k(z_t, m_t)\log I^o_t, \tag{B.17}$$

$$\log I^o_{t+1} = \beta^0_i(z_t, m_t) + \beta^1_i(z_t, m_t)\log K_t + \beta^2_i(z_t, m_t)\log I^o_t, \tag{B.18}$$

and the log-linear pricing rule

$$\log p_t = \beta^0_p(z_t, m_t) + \beta^1_p(z_t, m_t)\log K_t + \beta^2_p(z_t, m_t)\log I^o_t. \tag{B.19}$$

The forecasting and pricing rules are described by coefficients that depend on the exogenous aggregate shock. For discretized processes of z and m, the equilibrium under bounded rationality with the above rules becomes computable. I use these rules to solve for the optimal policy functions and then simulate the economy and compute equilibrium prices p_t in every period t. The simulated economy allows price series are then used to update the coefficients of the log-linear rules. I stop the procedure when the coefficients have converged.

APPENDIX C: ADDITIONAL INFORMATION ON THE MODEL CALIBRATION

Cooper and Haltiwanger (2006) targets the spike investment shares, but also persistence of investment rates and the correlation of investment rates with idiosyncratic productivity, when estimating a richer specification of capital adjustment costs including convex adjustment costs. I exclude the latter two moments because they may depend sensitively on the specific time to build setup. Nonetheless, the model matches these moments reasonably well with a persistence of 1.6% (empirically 5.8%), and a productivity correlation of 24% (empirically 14%).

An alternative strategy to calibrate adjustment costs is to target cross-sectional skewness and kurtosis of investment rates, see Bachmann and Bayer (2013). In fact, our calibrated model closely matches these moments in the data: skewness/kurtosis in the model are 5.1/48.3, while in a balanced panel of Census data these are 6.5/67.4 for total investment and 5.5/47.9 for equipment investment, see Kehrig and Vincent (2016). Since skewness and kurtosis monotonically increase in the adjustment cost parameters, this indicates the calibrated adjustment costs may be too low.

TABLE IV

CALIBRATION TARGETS

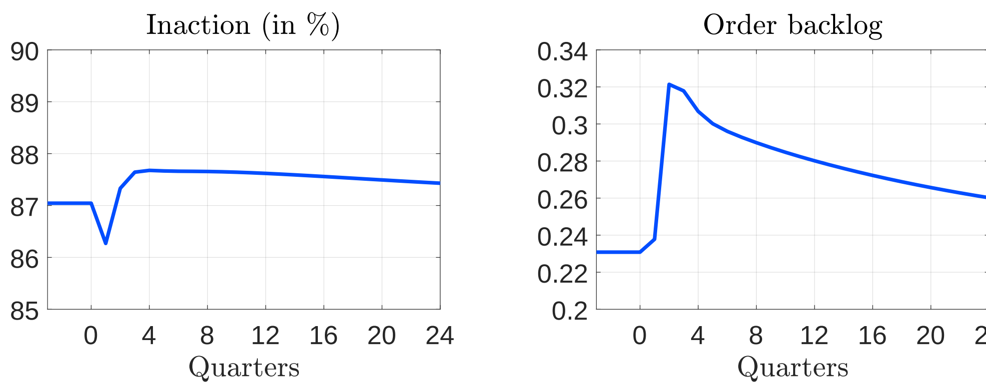
	Model	Data
Targeted (LRD)		
Positive spikes	18.6%	18.6%
Negative spikes	1.5%	1.5%
Non-targeted (LRD)		
Persistence	0.016	0.058
Productivity correlation	0.14	0.24
Non-targeted (Census)		
Skewness	5.1	6.5
Kurtosis	48.3	67.4

Notes: All moments relate to annual investment rates computed as I/K. Positive and negative spikes denote the share of investment rates larger than 20% and smaller than -20%, resp. LRD moments are from Cooper, Haltiwanger (2006), Census moments are from Kehrig, Vincent (2016).

Alternative data sources used to calibrate and estimate similar models are the IRS tax data, see, e.g., Winberry (2016b), and Compustat data, see, e.g., Bloom (2009). Both datasets are at the firm-level. The IRS does includes only positive investments, and Compustat is biased to large private firms. The main disadvantage of the LRD dataset is that it covers manufacturing only.

APPENDIX D: ADDITIONAL RESULTS FROM THE MODEL SIMULATION

Figure 8: Responses of investment orders to an adverse match efficiency shock



Notes: The impulse response functions are based on a *decrease* in match efficiency by one (unconditional) standard deviations starting from steady state and using the baseline calibration. Inaction measures the share of firms without outstanding orders that do not make a new order in a given period. The order backlog is the total of investments outstanding for delivery.

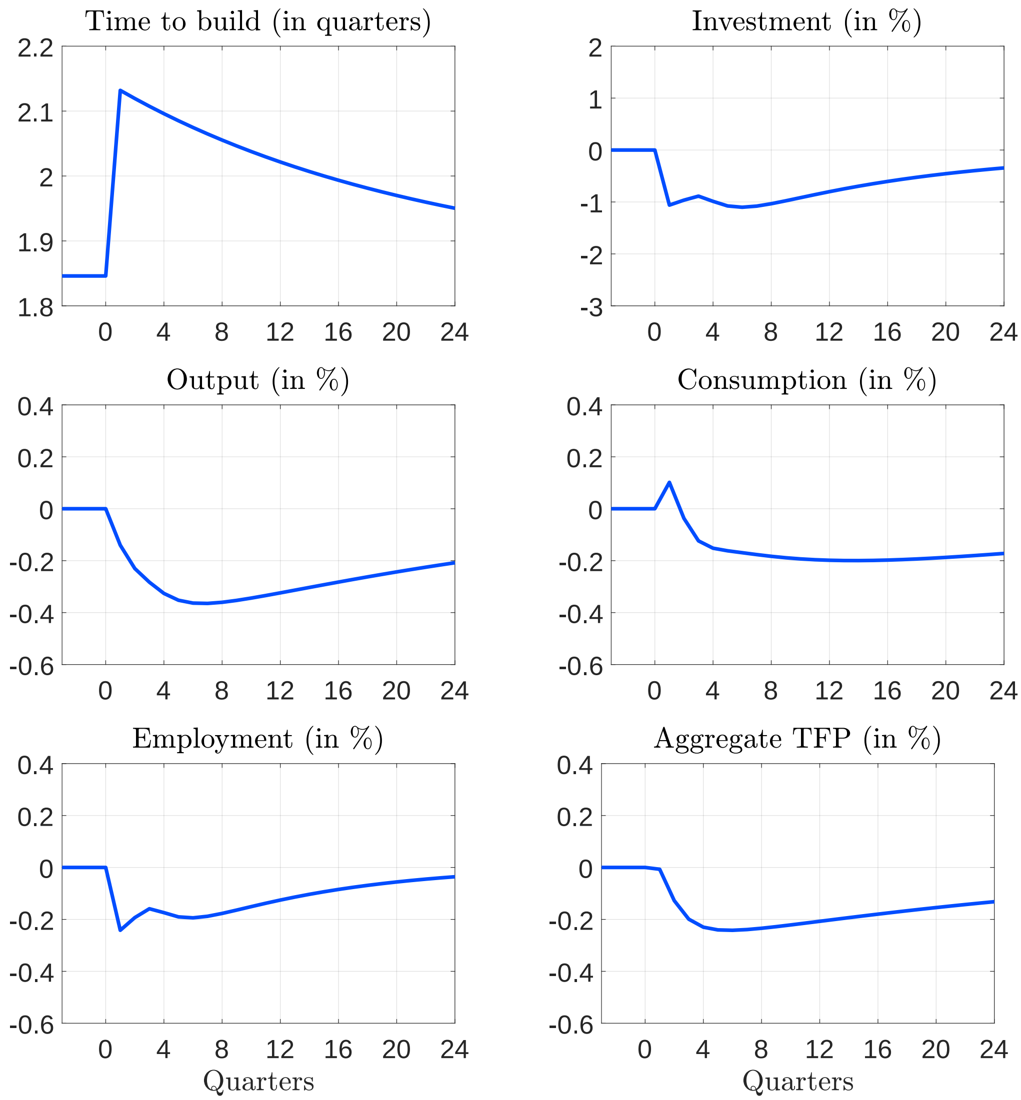
TABLE V

Business cycle statistics

	Data	Model
Volatility of output (%)	2.37	0.31
Volatility of consumption (%)	2.08	0.16
Volatility of investment (%)	7.27	1.23
Volatility of employment (%)	2.11	0.24
Autocorrelation of output	0.94	0.96
Autocorrelation of consumption	0.94	0.87
Autocorrelation of investment	0.96	0.89
Autocorrelation of employment	0.97	0.89
Correlation of consumption with output	0.86	0.61
Correlation of investment with output	0.72	0.92
Correlation of employment with output	0.71	0.89

Note: All series, from data and model simulations, are expressed in logs and HP-filtered with a quarterly smoothing parameter of 100,000.

Figure 9: Responses under alternative fixed adjustment costs:
$f(\xi, \mathbf{s}) = \frac{\xi w(\mathbf{s}}{\phi \bar{q}}$ with $q(\mathbf{s}) = \bar{q}$ in steady state



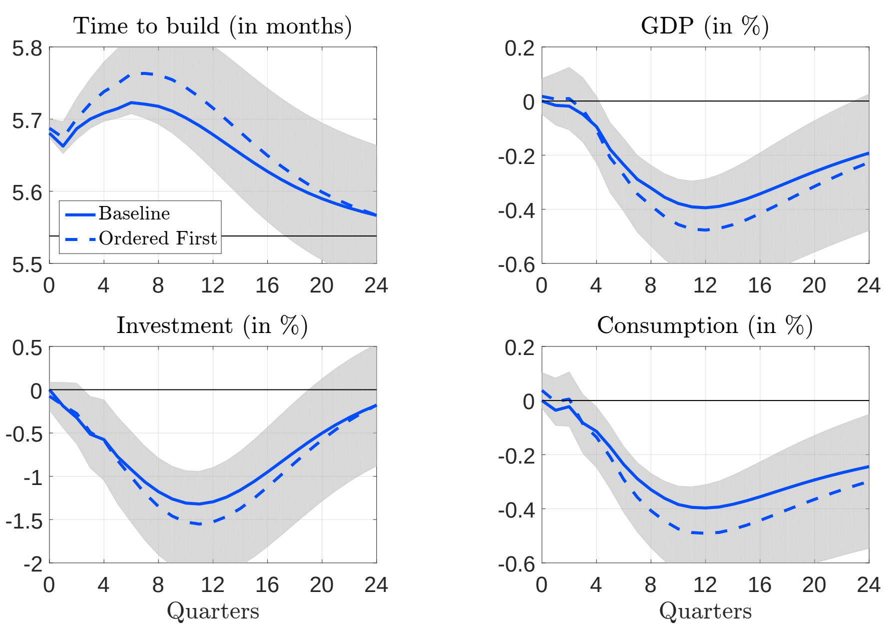
Notes: The impulse response functions are based on a *decrease* in match efficiency by one (unconditional) standard deviations starting from steady state and using the baseline calibration. 'Direct effect' are the impulse responses when aggregate TFP changes are eliminated through opposing aggregate productivity (z) shocks. Aggregate TFP is computed as $TFP = \log(Y_t) - \alpha \log(K_t) - \nu \log(L_t)$.

APPENDIX E: ROBUSTNESS OF THE STRUCTURAL VAR RESULTS

E.1. *Alternative identification scheme and first differences*

First, I investigate the results under an alternative identification assumption. While the baseline identification scheme tends to be conservative, its restrictions are stronger than the restrictions of the general equilibrium model. As alternative identification, I suggest to have the time to build shock *'ordered first'*. This term refers to the ordering of variables in the VAR. It means that time to build shocks can contemporanously affect all other variables in the VAR, but no shock other than time to build shocks can affect time to build contemporaneously. Figure 10 shows that the baseline identification implies smaller macroeconomic respones to time to build shocks compared to the alternative identification, albeit the differences are not large. Impulse responses under the alternative identification remain significant.

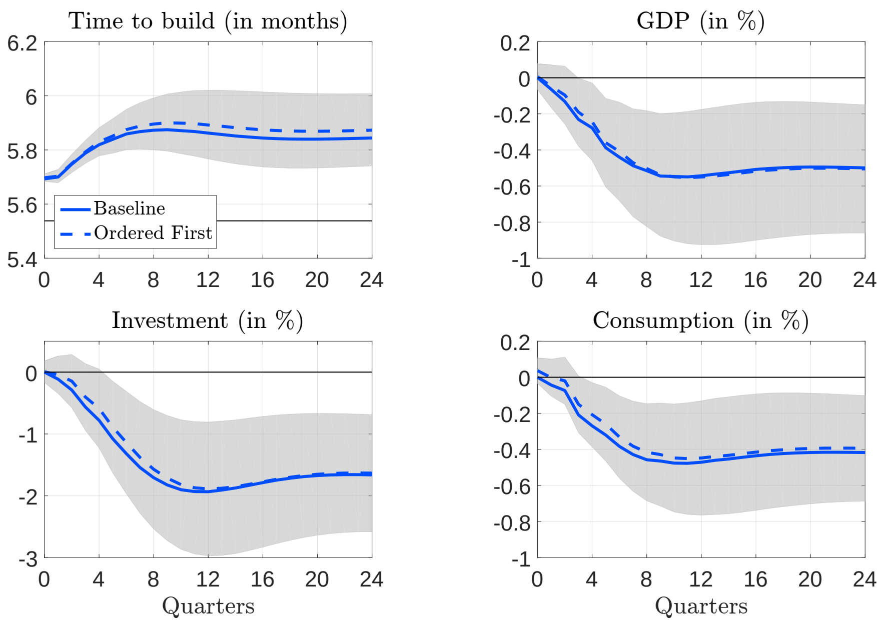
Figure 10: Impulse responses to a *one standard deviation* time to build shock (model in levels with linear time trend, alternative identification schemes)



Notes: Solid lines show (selected) impulse responses to a time to build shock under the baseline identification scheme. Dashed lines show the impulse responses under the alternative identification scheme, in which time to build is *'ordered first'*. Shaded, gray areas illustrate the 90% confidence intervals associated with the alternative identification scheme.

Figure 11 shows the cumulative impulse responses when estimating a VAR, in which all variables enter in first differences and the linear time trend is dropped. At the same time, the figure compares the two identification schemes. The differences of the impulse responses across identification schemes appears negligible. The important finding is that the impulse responses are similar to the ones in Figure 10. While I assumed a linear time trend for the latter, the findings on time to build shocks appear robust to non-linear time trends.

Figure 11: Cumulative impulse responses to a *one standard deviation* time to build shock (model in first differences, two alternative identification schemes)



Notes: Solid lines show (selected) cumulative impulse responses to a time to build shock under the baseline identification scheme. Dashed lines show the impulse responses under the alternative identification scheme, in which time to build is *'ordered first'*. Shaded, gray areas illustrate the 90% confidence intervals associated with the alternative identification scheme.

E.2. *Elasticity bounds*

In this subsection, I propose a new approach to provide robustness for point-identified structural VAR models in a frequentist setup. Structural VAR models, such as Gali (1999), Christiano et al. (2005), and Bloom (2009), impose various zero restrictions on contemporaneous and long-run responses to obtain point identification. As robustness, I propose to replace some or all of the zero restrictions by bounds on the elasticity with respect to the shock of interest.[17] For example, instead of assuming an uncertainty shock does not contemporaneously affect GDP, as robustness I would restrict the elasticity of GDP with respect to a change in uncertainty due to an uncertainty shock to be bounded between $\pm c\%$. This nests the point-identified model in the limit case when all bounds are zero ($c = 0$). The structural VAR model is no longer point-identified when replacing a zero restriction with strictly positive bounds on the elasticities ($c > 0$).

[17] Elasiticity bounds have recently gained popularity in the Bayesian structural VAR literature, see, e.g., Kilian and Murphy (2012) and Baumeister and Hamilton (2015).

I implement this robustness exercise using the results in Gafarov, Meier, and Montiel Olea (2016), which provide inference for set-identified structural VAR models. Formally, to apply their results, I need to assume that for a given IRF either the lower and upper elasticity bound may not hold jointly. Notice that confidence sets are estimated based on Delta method inference. In fact, bootstrap inference is not necessarily valid here because the endpoints of the identified sets are not fully differentiable.

The suggested robustness is similar to Conley et al. (2012) which proposes as robustness to relax the exclusion restriction when using IV methods. I suggest the following robustness for the conservative baseline identification. Instead of zero restrictions on contemporaneous responses, I constrain the elasticity of all variables (except for the backlog ratio) with respect to the match efficiency shock to be between -1% and +1%, see Table VI. For an increase in the backlog ratio of 2.5%, the contemporaneous responses are bound to be between -0.025% and +0.025%.

TABLE VI

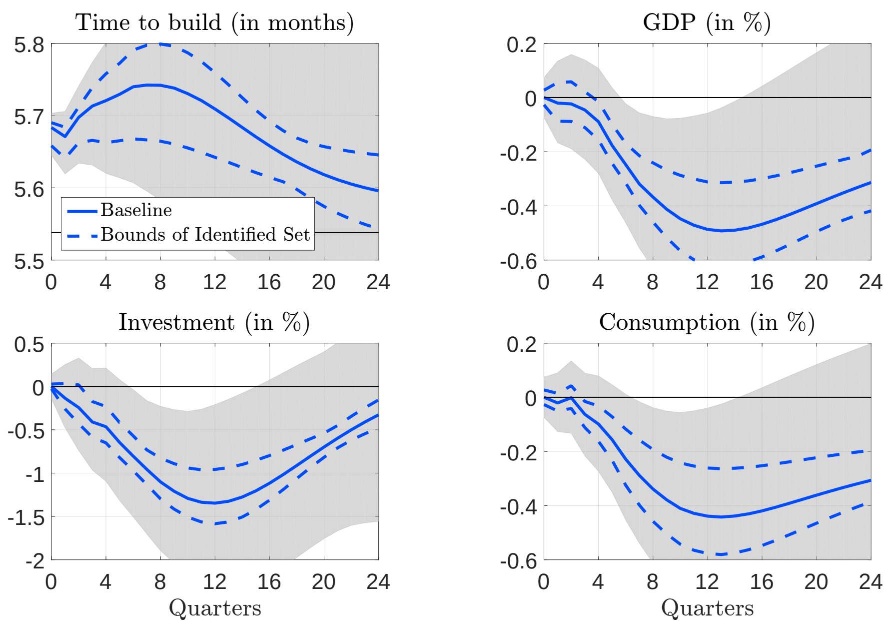
IDENTIFICATION SCHEMES: CONSTRAINTS ON CONTEMPORANEOUS ELASTICITIES

	TTB	GDP	Inv	Con	CPI	Wag	FFR	LaP
Baseline	+	0	0	0	0	0	0	0
Robustness	+	±1%	±1%	±1%	±1%	±1%	±1%	±1%

Notes: +/0/±1% indicate that the elasticity is constrained to be positive/exactly zero/between -1% and +1%, respectively. The contemporaneous elasticity of variable i and time to build in response to time to build shocks is given by $(e_i' B_1)/(e_1' B_1)$, where e_i is the i-th column of the identity matrix $\mathbb{I}_8$. *TTB*: Time to build, *GDP*: Real GDP, *Con*: Real Consumption, *Inv*: Real Investment, *CPI*: Consumer Prices, *Wag*: Real Wage, *FFR*: Federal Funds Rate, *LaP*: Labor Productivity.

Figure 12 shows the resulting impulse responses under the *robustness* identification scheme. Instead of a single impulse response, there is an interval with admissible impulse responses (dotted lines). The confidence set is adjusted accordingly based on Gafarov, Meier, and Montiel Olea (2016). Notice that the main findings of the baseline model in Figure 12 are 'robust' in the sense that the declines in GDP, investment, and consumption remain significant.

Figure 12: Impulse responses to a *one standard deviation* time to build shock (model in levels with linear time trend, two alternative identification schemes)



Notes: Solid lines show (selected) responses to a time to build shock under the baseline identification scheme. Dashed lines show the bounds of the identified set under elasticity constraints, see Table VI. Shaded, gray areas illustrate the 90% confidence intervals for the identified sets.