124 MidTerm One REVIEW OUTLINE ©

Contents

1 Precalculus Material

1.1 Formulas for Circles

(a) Algebraic: $(x-a)^2+(y-b)^2=r^2$.

(b) Parametric equations; $(x(t),y(t)) = (A\cos(\omega t-\theta)+B, A\sin(\omega t-\theta)+C)$

1.2 Tangent Lines to Circles

Find the slopes of the two tangent lines to $x^2+y^2=2$ through the point $(4,2)$. $m_1=0$ and $m_2=\frac{4}{3}$.

Let C_r be a circle centered at $(0,0)$ with radius r and L_r be the line $y=\sqrt{r}\cdot x$. Call the point of intersection in the first quadrant P and the tangent line to C_r at P we will call T_r. If $Q=(x_r,0)$ is the x-intercept of T_r find $r>0$ so that $x_r=3r$. $r=8$ What is $\lim_{r\to 0} r^{-3/2}x_r$? $=1$.

2 Limits

2.1 Bag of Tricks

2.1.1 Rationalize

$$\lim_{x\to 1}\frac{\sqrt{x+3}-2}{x-1}=\frac{1}{4}.$$

$$\lim_{x\to 2}\frac{\sqrt{6-x}-2}{\sqrt{3-x}-1}=\frac{1}{2}$$

2.1.2 Just Plug In

$$\lim_{\sigma\to e^2}\frac{\ln\sigma}{e^{\sqrt{\sigma}}}=\frac{\ln(e^2)}{e^{\sqrt{e^2}}}=\frac{2}{e^e}$$

2.1.3 Left and Right Limits

$$\lim_{x\to 2}\frac{x}{x-2}=DNE.$$

$$\lim_{x\to 0}\frac{(x^2-2)^8}{(x^3-2x)^7\sin x}=-\infty$$

2.1.4 Divide Out

$$\lim_{x\to\infty}\frac{x^2+2}{4x^2+3x+1}=\frac{1}{4}.$$

$$\lim_{x\to\infty}\frac{\sqrt{4x^2-1}+x\ln\left(\frac{x^{2/3}+2}{\sqrt[3]{e^3x^2-x+\pi}}\right)}{\sqrt{9x^2-x+3}}=\frac{1}{9}$$

2.1.5 Factor and Cancel

$$\lim_{x\to 1} \frac{x^2-2x+1}{x-1} = 0.$$

$$\lim_{\theta\to 0} \frac{\sin^2\theta}{1-\cos\theta} = 2$$

$$\lim_{x\to 2} \frac{(x-2)\sin(\pi x)}{\sqrt{x^2-4}} = 0$$

2.1.6 Take a limit inside a continuous function

$$\lim_{t\to\infty} e^{\sin(\frac{\pi}{t})} = 1$$

$$\lim_{t\to\infty} \cos(e^{-t}) = 1$$

2.1.7 $\frac{\sin x}{x}$ limits

$$\lim_{\theta\to 0} \frac{\sin(2x)}{4x} = \frac{1}{2}$$

$$\lim_{\theta\to 0} \frac{\sin(\pi\theta^2)\sin(\theta)}{e\theta^3} = \frac{\pi^2}{e}$$

$$\lim_{x\to 0} \frac{\sin^3(4x)}{3x^3} = \frac{64}{3}.$$

2.1.8 Limits that Look Like Derivatives

$$\lim_{h\to 0} \frac{(4+h)^2-16}{h} = 8.$$

$$\lim_{h\to 0} \frac{\frac{1}{\sqrt{2(3+h)^2-4}} - \frac{1}{\sqrt{14}}}{h} = -\frac{1}{2}(14)^{-3/2}$$

2.1.9 Horizontal Asymptotes

Find the horizontal asymptotes of the following functions: (Note: f(x) is difficult)

$$g(x) = \frac{x^3-3}{4x^3+3} = \frac{1}{4} \text{ for both } x\to\infty \text{ and } x\to-\infty$$

$$f(x) = \frac{e^x(\sin x - 1)}{e^{x^2}} = 0 \text{ you need to use squeeze theorem}$$

2.1.10 Vertical Asymptotes

Find the vertical asymptotes of

$$\frac{x+2}{(x-1)(x+4)(x-3)}, \qquad x = 1, -4, 3.$$

Find the vertical asymptotes of

$$h(t) = \frac{t(t-2)(t^3-t+1)}{\sin(\frac{\pi}{2}t)(t^2-3t-2)}; \quad 0 \le t \le 6, \;\; t = 2, 4, 6$$

2.2 Continuity

2.2.1 Definition

f is continuous if for all points, a, in the domain $\lim_{x\to a^-} f(x) = f(a) = \lim_{x\to a^+} f(x)$.

$$h(x) = \begin{cases} \frac{x^2 \sin\frac{1}{x}}{\sin x}, & x > 0 \\ a, & x = 0 \end{cases}.$$

Find a so that h is continuous at $x = 0$? a= 0

$$g(y) = \begin{cases} e^y - \sin(\pi y), & y < 1 \\ e^y - \ln y, & y \ge 1 \end{cases}$$

Is g continuous? = yes

2.2.2 Known Continuous Functions

Polynomials. $\sin x$ and $\cos x$. Rational functions when not dividing by zero. e^x. $\ln(x)$ when $x > 0$. $\sqrt{x}$ when $x \ge 0$.

3 Derivatives

3.1 Limit Definition of Derivative

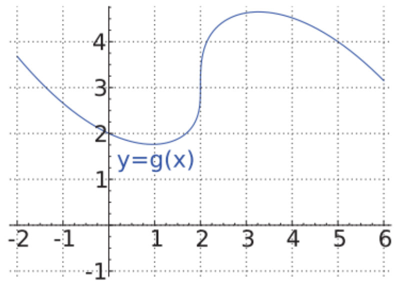
Using the limit definition of the derivative calculate the following derivates

$$f(x) = \frac{1}{\sqrt{1-x^2}} = \frac{-x}{\sqrt{1-x^2}}$$

$$g(t) = t^2 - t + 2 = 2t - 1$$

3.2 Drawing f'

Below is a graph of a function $g(x)$. Draw $g'(x)$.



3.3 Finding Tangent Lines

- Find the equation of the tangent line at $x = \pi$ to $g(x) = \sin x + \cos x$. $y + 1 = -(x - \pi)$
- Find the equation of the tangent line at $x = 0$ to $h(x) = \dfrac{e^x + 5x}{x^2 + 4}$ $y - \frac{1}{4} = \frac{3(5+e)}{25}(x - 0)$
- Find the equation of the tangent lines to $f(x) = x^2 + 2x + 3$ which pass through $(1, 1)$. $y - 3 = 2(x - 0)$; $y - 11 = 6(x - 2)$

3.4 Computing Derivatives with Rules

3.4.1 Power Rule, Product Rule, Quotient Rule, Trig Derivatives

Compute the derivatives of the following functions:

- $f(x) = x^3 e^x \sin x \cos x$
  $f'(x) = (2x^2 e^x + x^3 \cdot e^x)(\sin x \cos x) + (\cos x \cos x - \sin x \sin x)(x^3 \ln x)$
- $h(z) = \frac{\sin z + z^4}{\sqrt{z} - 3z^{4/5}}$ $h'(z) = \frac{(\cos z + 4z^3)(\sqrt{z} - 3z^{4/5}) - (\frac{1}{2\sqrt{z}} - \frac{12}{5} z^{-1/5})(\sin z + z^4)}{(\sqrt{z} - 3z^{4/5})^2}$.
- $g(t) = \frac{\sin^2(t)}{\cos t}$ $g'(t) = \frac{2\sin(t)\cos(t)\cos(t) + \sin(t)\sin^2(t)}{\cos^2(t)}$.